The Effects of Competition and Entry in Multi-sided Markets

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Outline

Introduction

Motivation and Focus

- Multi-sided platforms enable customers from different sides to interact.
- Cross-subsidization, is driven by the existence of crossside externalities, a key feature of the multi-sided markets. So, we focus on the questions below:
  1. Equilibrium Pricing Strategy?
  2. The Effects of Competition on Price and Welfare in the Presence of Cross-subsidization?
  3. Efficient? socially optimal level of welfare?

Main Findings

- Under Such Assumptions and Setup:
  1. customer heterogeneity/platform product differentiation by membership benefits
  2. homogeneous network effects (within/cross-side externalities)
  3. single-homing and full market coverage
- Main Findings:
  1. We establish the existence of a symmetric pricing equilibrium
     the price equals a mark-up(oligopoly market power) minus a subsidy(externalities)
  2. A **perverse** pattern between prices and competition
  3. two U-shaped curves: one for CS and inverted one for Profit
     how fast each term changes with the number of platfoms n matters!
  4. Sufficient conditions under which the overall effect favours excessive entry

Realted Literature

1. Two-sided markets:
   Modelling Externalities and Studying the effects of competition and entry on prices and social welfare
   Platforms' strategies in two-sided markets and corresponding equilibrium concepts
   The relationship between free entry and social efficiency
   The impact of mergers and industry concentration in different industries with two-sidedness features (Empirical Research)
2. Oligopoly theory with discrete choice demand.

Model

Setup

- $n \geq 2$ platforms competing for customers from $s \geq 1$ sides by charging prices/membership fees
- $p^k = \{p_1^k, ..., p_s^k\}$: the prices charged by platform $k \in N = \{1, ..., n\}$
- $x^k = \{x_1^k, ..., x_s^k\}$: the participation profile on platform k, representing the aggregate share of customers who select k
- $\epsilon_i = \{\epsilon_i^1, ..., \epsilon_i^n\}$: the matching values/membership benefits of a customer on side $i \in S = \{1, ..., s\}$ with n platforms
- $\phi_i(x^k) + \epsilon_i^k - p_i^k$: the utility of a customer on side i from joining platform k, $\phi_i : [0,1]^S \to R$
- $\sum_{i \in S}(p_i^k - c_i)x_i^k = \sum_{i \in S} p_i^k x_i^k$: platform k's profit

Assumptions

- a continuum of heterogeneous customers (of measure 1)
- ϵ_i is common knowledge and $\epsilon_i \sim_{iid} G_i(\epsilon_i)$
- single-homing and full market coverage: each customer participates in one and only one platform
- Assumptions about $G_i(\cdot)$
  1. Cross-side independence: ϵ_i is independent of ϵ_j across different sides $i \neq j \in S$
  2. Cross-platform symmetry: $G_i(\cdot)$ is symmetric across n platforms
  3. $G_i(\cdot)$ and $\phi_i(x)$ is continuously differentiable
- $\sigma_i(z) = \sum_{j \in S} \frac{\partial \phi_j(x)}{\partial x_i}|_{x=z1_s}$: the aggregate marginal externality provided by customers from side i to all sides at the symmetric allocation $x = z1_s = z(1, ..., 1)'$

Participation equilibrium

- each customer on side i joins the platform that yields the highest utility in equilibrium
- $x_i^k(P) = \int_{\epsilon_i : k \in argmax_{t \in N} \phi_i(x^k) + \epsilon_i^k - p_i^k} dG_i(\epsilon_i)$
- B_ϵ: the upper bound of the slopes of the demand functions without externalities
- B_ϕ: the upper bound of the marginal externalities

Proposition1

For any price profile P, there exists a participation equilibrium. Moreover, the participation equilibrium is unique if $B_\phi < 1/B_\epsilon$

- When the degree of externalities is small, relatively to the dispersion of customers' heterogeneity, $x_i^k(P)$ forms a contraction mapping

Prcing Equilibrium: Notations

- $H_i(\cdot; n)$ and $h_i(\cdot; n)$: the CDF and PDF of $\epsilon_i^1 - max_{k\neq 1}\epsilon_i^k$, independent of n
- the inverse hazard rate at 0: $M_i(n) = \frac{1-H_i(0;n)}{h_i(0;n)}$
- the average marginal externality:$\eta_i(n) = \frac{1}{n-1}\sigma_i(\frac{1}{n})$

Assumption1

Every stationary point of $R(\cdot)$ on $[0,1]^s$ is a global maximum point, where
$R(z) = \sum_{i\in S} z_i\{p_i^* + H_i^{-1}(1 - z_i; n) + [\phi_i(z) - \phi_i(\frac{1_s - z}{n-1})]\}$

Prcing Equilibrium

- Under Assumption 1, there exists a subgame perfect equilibrium such that
  1. $p^* = \{p_1^*, ..., p_s^*\}$
  2. $x^* = \frac{1}{n}1_s$
  3. $p_i^*(n) = M_i(n) - \eta_i(n)$
- the equilibrium price consists of two additively separable terms:
  1. $M_i(n) = \frac{1-H_i(0;n)}{h_i(0;n)}$
     market power of oligopolistic firms offering different products determined by the heterogeneity of membership benefits
  2. $\eta_i(n) = \frac{1}{n-1}\sigma_i(\frac{1}{n}) = \sum_{j \in S}(\underbrace{1 + \frac{1}{n-1}}_{\text{the loop effect}})x_j^* \frac{\partial \phi_j}{x_i}$
     subsidy term=business-stealing effect+direct externalities

Examples

Example 1 In his model of platform competition ($n=2$) in two-sided markets ($s=2$), Armstrong (2006) uses a Hotelling specification with uniform distribution of consumer location. The equilibrium prices in his setting are given by

$$p_1^* = t_1 - \alpha_2, \ \ p_2^* = t_2 - \alpha_1,$$

where t_1 and t_2 are unit transport costs, and α_1 and α_2 are the degrees of cross-group externalities enjoyed by two sides, respectively. This pricing formula is the same as ours in (7). Indeed, in this Hotelling specification, the market power effect is just t_i.[23]

Example 2 Consider a one-sided market with linear form of within-side externalities and Gumbel distribution of matching values studied in Anderson *et al.* (1992). Under the assumption that $\beta > \frac{8}{27}\frac{n}{n-1}\gamma$, the symmetric equilibrium price is given by

$$p^* = \frac{n}{n-1}\beta - \frac{\gamma}{n-1},$$

where β is the scale parameter of Gumbel distribution and γ is the constant network effect parameter. This pricing formula is the same as ours in (7).[24]

Analysis

Price

- restrict our attention to IID matching values

Lemma1

If $\frac{z}{1-z}\sigma_i(z)$ increases (decreases) in z for $z \in (0,1)$, the equilibrium cross-subsidy $\eta_i(n)$ decreases (increases) with n.
Moreover, if $lim_{z\to 0} z\sigma_i(z) = 0$, then $lim_{n\to\infty}\eta_i(n) = 0$.

- Indeed, we can decompose the subsidy term into 3 effects:
  1. the loop effect $1+\frac{1}{n-1}$ decreases with n
  2. the market share $\frac{1}{n}$ decreases with n
  3. aggregate marginal externality $\sigma_i(\frac{1}{n})$ may increase or decrease with n

Price

- Typically $M_i(n)$ is monotonically decreasing in n (under the log-concavity of f_i (Zhou,2017))
- If $\eta_i(n)$ also increases with n, then the price decreases with n, as in the standard single-sided market setting.
- $\eta_i(n)$ typically decreases with n as well, which can easily offset the monotonicity of the product differentiation effect
- Examples:
  Suppose linear externalities $\phi_i(x) = \sum_{j \in S} \gamma_{ij} x_j$, thus $\sigma_i = \tilde{\gamma}_i = \sum_{j \in S} \gamma_{ij}$
  1. an exponential distribution with $\lambda_i > 0$, $p_i^*(n) = \frac{1}{\lambda_i} - \frac{1}{n-1}\tilde{\gamma}_i$
  2. the Gumbel distribution with $\beta_i > 0$, $p_i^*(n) = \frac{n}{n-1}\beta_i - \frac{1}{n-1}\tilde{\gamma}_i$

Price

- $\tau = -lim_{\theta \to \bar{\theta}} \frac{d}{d\theta}(\frac{1-F(\theta)}{f(\theta)})$: the tail index of a CDF F, with density f and support $[\underline{\theta}, \overline{\theta}]$
- $r = 1 + lim_{z \to 0} z \frac{\sigma_i'(z)}{\sigma_i(z)}$: the elasticity of $\frac{z}{1-z}\sigma_i(z)$ w.r.t z at $z = 0$

Theorem 1

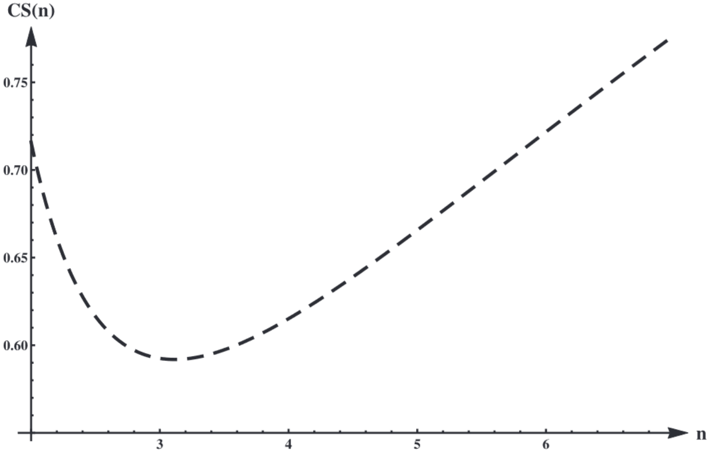
For each $i \in S$, assume f_i is log-concave. Then the following hold:

1. If $0 < \tau_i < r_i$, there exists a positive n_0 such that $p_i^*(n)$ decreases with n for any $n \geq n_0$;
2. If $0 < r_i < \tau_i$, there exists a positive n_0 such that, for any $n \geq n_0$, $p_i^*(n)$ increases (decreases) with n whenever $\eta_i(n) > (<) 0$

Consumer Suprlus

- $CS_i(n) = \delta_i(n) - p_i^*(n) + \phi_i(1_s q^*(n))$
  quad where $\delta_i(n) = E[max_{k \in N} \epsilon_i^k]$, $q^*(n) = \frac{1}{n}$
- we can decompse the competition effect into 3 effects:
  1. the platform/product variety effect, $\frac{\partial \delta_i(n)}{\partial n}$
  2. the price effect, $-\frac{\partial p_i^*(n)}{\partial n}$
  3. the network consolidation effect, $-\frac{\partial \phi_i(1_s q^*(n))}{\partial n}$
- In absence of any externalities, competition increases CS
- The presence of (within-side and/or cross-side) externalities affects CS in two ways:
  1. the perverse competition-price pattern emerges(-)
  2. the network consolidation effect(?)=aggregate marginal externality(+)+the business-stealing effect(-)

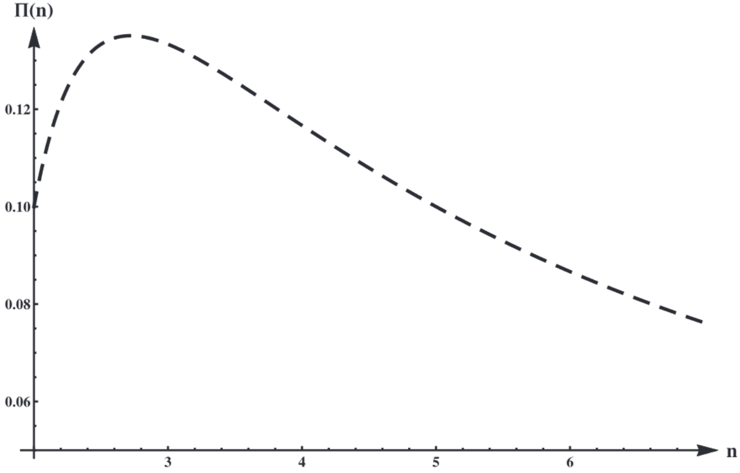
U-shaped Consumer Suprlus Curve



Platform Profits

- $\Pi(n) = q^*(n) \sum_{i \in S} p^*(n)$
- we can decompse the competition effect into 2 effects:
  1. the market share effect(-), $q^*(n)$
  2. the price effect(?), $p^*(n)$

Inverted U-shaped Platform Profit Curve



Asymptotic Impact of Competition

- Suppose n is sufficiently large (goes to infinite)
- In CS: $CS_i(n) = \delta_i(n) - p_i^*(n) + \phi_i(1_s q^*(n))$
  the product variety term is unbounded for a distribution with unbounded support, while the other two terms are bounded under fairly weak conditions.
  $\Rightarrow$ the variety effect dominates the two other effects
  $\Rightarrow$ the ultimate monotonicity of consumer surplus
- In profit: $\Pi(n) = q^*(n) \sum_{i \in S} p^*(n)$
  market share diminishes to zero while the price generally is bounded (under the log-concavity and the assumption1),
  $\Rightarrow$ platform profit converges to zero
  $\Rightarrow$ platform profit is most likely to be decreasing when n is sufficiently large.

Notations

- the total welfare: $W(n) = n\Pi(n) + \sum_{i \in S} CS_i(n)$
- the fixed cost of entry: K
- the socially optimal: $n^* = argmax_{n \geq 2}(W(n) - nK)$
- the free-entry equilibrium number of platforms: $n^e = max_{n \geq 2}\Pi(n) - K$
- entry excessive (insufficient) if $n^e > (<) n^*$

Notations

- the total welfare:
  $W(n)' = \Pi^*(n) + \sum_{i \in S}[\delta_i'(n) + \frac{\partial q^*(n)}{\partial n}(np_i^* + \sigma_i(q_n))]$
- we can decompose the term in the bracket into 2 effects:
  1. the platform/product variety effect(+) $\delta_i'(n)$
  2. the business-stealing effect $\frac{\partial q^*(n)}{\partial n}(np_i^* + \sigma_i(q_n))]$
- Indeed, the business-stealing effect consists of two distinct components:
  1. the mark-up(?) $\times$ the reduction in market shares(-)
  2. the network consolidation effect(?)
     (a reduction in market share(-) on the aggregate marginal externality generated(?))

Entry Effects

- further decomposition:
$\frac{\partial q^*(n)}{\partial n}(np_i^* + \sigma_i(q_n)) = \frac{\partial q^*(n)}{\partial n}[nM_i(n) - \frac{1}{n-1}\sigma_i(q_n)]$

Theorem 2(Entry effects)

The difference between the marginal welfare and per-platform profit equals: $W(n)' - \Pi(n) = [\delta_i'(n) - \frac{M_i(n)}{n} - \frac{\sigma(1/n)}{(n-1)n^2}]$

Suffcient Conditions for Excessive Entry

- further decomposition:
$\frac{\partial q^*(n)}{\partial n}(np_i^* + \sigma_i(q_n)) = \frac{\partial q^*(n)}{\partial n}[nM_i(n) - \frac{1}{n-1}\sigma_i(q_n)]$

Theorem 3(Entry effects)

Assume that for each $i \in S$, f_i is log-concave. Excessive entry occurs when either one of the following three conditions holds:

1. $\sum_{i \in S} \sigma_i(z) \leq 0$ for any $z \in [0,1]$
2. $\phi_i(x)$ is linear or convex in $x \in [0,1]^s$ for each $i \in S$
3. $(\frac{n}{n-1})\delta_i'(n) \leq \frac{M_i(n)}{n}$ for each $i \in S$ and $n \geq 2$

Intuition behind 3 conditions

- Condition 1: $\sum_{i \in S} \sigma_i(z) \leq 0$
  $\Rightarrow$ the aggregate marginal externality over all sides ≤ 0
- Condition 2: $\phi_i(x)$ is linear or convex in $x \in [0,1]^s$
  excess entry occurs $\Leftrightarrow$ $W(n+1) - W(n) - \Pi(n+1) \leq 0$
  $\Leftrightarrow \sum_i \phi_i(\frac{1}{n+1} 1_s) - \sum_i \phi_i(\frac{1}{n} 1_s) + \sum_{i,j} \frac{1}{n(n+1)} \frac{\partial \phi_i(x)}{\partial(x_j)}|_{x=\frac{1}{n+1}1_s} \leq 0$
  $\Leftrightarrow$ a reduction in network benefits$>$the aggregate subsidy paid by the entrant
  $\Leftrightarrow$ convex(Jensen's Inequality) or linear
- Condition 3: $(\frac{n}{n-1})\delta_i'(n) \leq \frac{M_i(n)}{n}$
  $(\frac{n}{n-1})\, W'(n) - \Pi(n) = \sum_{i \in S}\{(\frac{n}{n-1})\delta_i'(n) - \frac{M_i(n)}{n}\}$
  the variety effect is dominated by the business-stealing effect

An Equivalent Condition to C3 in Theorem 3

Lemma 2

For any $i \in S$,fi satisfies Condition 3 if and only if the corresponding quantile density function $L(z) = f_i(F_i^{-1}(z))$ satisfies the following:

$$n^3(\int_0^1 z^{n-2}L(z)dz)(\int_0^1 \frac{z^n ln(1/z)}{L(z)}dz) \leq 1 \text{ for any } n \geq 2$$

- A large class of distributions, including commonly used ones like the Gumbel, uniform and reversed Weibull distributions, satisfies it.

Not Necessary: An Example

Example 5 (Insufficient entry) Consider a two-sided market with distribution $F_i(\theta_i) = 1 - \exp(-\theta_i)$ on both sides and externality function $\phi_1(x_1, x_2) = \alpha_1 x_2^{\rho}$ and $\phi_2(x_1, x_2) = \alpha_2 x_1^{\rho}$. Under parameter values $\alpha_1 = \alpha_2 = 11.9015$, $\rho = 0.05515$, and entry cost $K = 0.0978$, the free entry equilibrium number is shown to be 6, while the socially optimal number is 7.[53] The combination of the first two effects in (15) of Theorem 2 is negative, which is consistent with Anderson *et al.* (1995), but it is dominated by the third positive term generated by the concave externality functions (*i.e.* decreasing returns to scale).

In the example, none of the sufficient conditions in Theorem 3 is applicable, as ϕ_i is monotonically increasing and strictly concave, and the exponential distribution violates the inequality in Lemma 2. Under the exponential distribution (which has a log-concave density) and without externalities, there is excessive entry. However, in the presence of cross-side externalities with decreasing returns of scale, free entry is socially insufficient. This example indicates that multi-sidedness and the nature of externalities may help provide a plausible explanation for industry concentration in many high-technology markets involving platforms.