Social Connectedness and Local Contagion

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Outline

Introduction

Introduction

- Setting: a coordination game among agents with private values and binary actions in a network
- Equilibrium Characterization: global games – **private noisy signal** and **dominance region** (Carlsson and van Damme,1993; Frankel et al.,2003)
  $\Rightarrow$ Classic equilibrium selection: the equilibrium selected in the noiseless limit comes in the form of **cut-off strategies**

Main Works

- Core contributions:
  1. explore the role of the network's architecture in determining **who coordinates their adoption choices with whom** ("coordination set" partition)
  2. develop an algorithm called the **Sequential Average Network Density (SAND)** to fully characterize the equilibrium partitions
- Other discussions: coordination under specific network topology; optimal intervention (values,network structure); comparisons with BCZ(2006) and SS(2012)

Setup

Setup

- A finite set of agents N simultaneously choose technology adoption ($a_i = 1$) or reject it ($a_i = 0$).
- Agents are connected via a network $\mathcal{G} = (N, E)$, E defines the set of edges between unordered pairs ij taken from N
  1. a connected and undirected graph
  2. $N_i \equiv \{j : (i,j) \in E\}$ the set of i 's neighbours, and $d_i \equiv |N_i|$ her degree

Setup

- Payoffs from adopting the technology depend on the action profile $\mathbf{a} = (a_1, \ldots, a_{|N|}) \in \{0,1\}^N$ and the underlying fundamental state $\theta \in \Theta$, where Θ is a bounded interval in $\mathbb{R}$
- $u_i(\mathbf{a}, \theta) = \begin{cases} v_i + \theta + \phi \sum_{j \in N_i} a_j & \text{if } a_i = 1 \\ 0 & \text{if } a_i = 0 \end{cases}.$

where $v_i \in \mathbb{R}$ and $\phi > 0$

Setup: Dominance regions and multiplicity of equilibria

- In the stage game where θ is commonly known among agents $\Rightarrow$ a dominant strategy to adopt (not to adopt) when θ is sufficiently high (low)
- Assume there exist $\underline{\theta}_i$ and $\bar{\theta}_i$ in the interior of Θ for each i
- Dominant regions: $[\min \Theta, \underline{\theta}]$ and $[\bar{\theta}, \max \Theta]$, with $\underline{\theta} \equiv \min_{i \in N} \{\underline{\theta}_i\}$ and $\bar{\theta} \equiv \max_{i \in N} \{\bar{\theta}_i\}$ such that not adopting and adopting the technology (respectively) are dominant strategies for all players.
- In this article, we adopt the global game approach for equilibrium selection (see e.g. Carlsson and van Damme, 1993; Frankel et al., 2003).

Setup: Limiting equilibrium

- Agents share a common prior $\theta \sim H(\cdot)$, which is denoted by the with continuously differentiable density $h(\cdot) > 0$
- Private information: $s_i = \theta + \nu\epsilon_i$, where $\nu > 0$ and $\epsilon_i \sim G$ with support [-1,1], independently drawn across agents conditional on θ
- Interested in the perturbed global game Γ^ν for ν close to zero
- (Frankel et al., 2003) As ν close to zero, the equilibrium selected in the noiseless limit uniquenessly comes in the form of **cut-off strategies**

  i.e. there exists a vector of limiting state cutoffs $\boldsymbol{\theta}^* = \left(\theta_1^*, \ldots, \theta_{|N|}^*\right)$ fully determines the limiting equilibrium $\pi^* = \left(\pi_1^*, \ldots, \pi_{|N|}^*\right)$, with $\pi_i^* = \mathbb{I}\left(\theta \geq \theta_i^*\right)$.

Charaterization of Coordination Set and Algorithm

Coordination Set

- The cutoffs $\boldsymbol{\theta}^* = \left(\theta_1^*, \ldots, \theta_{|N|}^*\right)$ represent the prospensity to adopt

Definition 1 (Coordination sets)

$\boldsymbol{\theta}^*$ map to a unique partition called coordination sets $\mathcal{C}^* = \{C_1^*, \ldots C_M^*\}$ of N:

1. $\forall m \in \{1, \ldots M\}, \forall i,j \in C_m^*$, we have $\theta_i^* = \theta_j^*$;
2. For all$(i,j) \in E$ satisfying $\theta_i^* = \theta_j^*$, there is some $m \in \{1, \ldots, M\}$, such that $i,j \in C_m^*$.

- Item (i) guarantees that agents with distinct cutoffs cannot coexist in the same coordination set,
- Item (ii) eliminates the fact that one could have singletons for reasons other than everybody adopting a different threshold

SAND Algorithm

- $F(\cdot) : 2^N \to \mathbb{R}, F(S) \equiv v(S) + \phi e(S),\quad$ for any $S \subset N$

  where $v(S) \equiv \sum_{i \in S} v_i$, and $e(S) \equiv \frac{1}{2} \sum_{i \in S} d_i(S)$, where

  $d_i(S) \equiv |N_i \cap S|$ denotes the within-degree of i

Sequential Average Network Density (SAND)

Step 1.

$$A_1^* = \underset{S \supsetneq \emptyset}{\operatorname{argmax}} \frac{F(S)}{|S|}.$$

(If there are multiple maximizers, we set A_1^* to the largest maximizer. Step k .

$$A_k^* = \underset{S \supsetneq A_{k-1}^*}{\operatorname{argmax}} \frac{F(S) - F\left(A_{k-1}^*\right)}{|S| - \left|A_{k-1}^*\right|}.$$

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Continue until $A_k^* = N$

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- $\emptyset \subsetneq A_1^* \subsetneq A_2^* \subsetneq \cdots \subsetneq A_K^* = N$
- $t_{[1]}^* > t_{[2]}^* > \cdots > t_{[K]}^*$

  $t_{[k]}^* = \frac{F\left(A_k^*\right) - F\left(A_{k-1}^*\right)}{\left|A_k^*\right| - \left|A_{k-1}^*\right|}$ denote the maximum value obtained in Step k

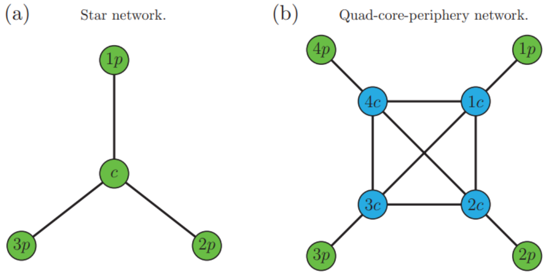

FIGURE 1

Coordination and network structure.

Example 1 *For the star network in Figure 1(a), the SAND algorithm terminates in step 1:*[25]

S	$\{c\}$	$\{c,1p\}$	$\{c,1p,2p\}$	$\mathbf{N}$
$e(S)/\|S\|$	0	1/2	2/3	**3/4**

Example 2 *For the quad-core-periphery network in Figure 1(b), the SAND algorithm terminates in two steps. In Step 1, $A_1^*=\{1c,2c,3c,4c\}$, as shown in the table below:*

S	$\{1c,2c,3c\}$	$\{\mathbf{1c},\mathbf{2c},\mathbf{3c},\mathbf{4c}\}$	$\{1c,2c,3c,4c,1p\}$	N
$e(S)/\|S\|$	3/3	**6/4**	7/5	10/8

In Step 2, the algorithm terminates with $A_2^*=N$.[26]

SAND Algorithm

Thm 1 (Main Equilibrium Characterization)

Suppose agent $i \in A_k^* \backslash A_{k-1}^*$ (*defining* $A_0^* = \emptyset$) in the SAND algorithm. Then, her equilibrium cutoff θ_i^* satisfies

$$\theta_i^* = -t_{[k]}^*.$$

- Remark1: low computational complexity; confirmation of uniqueness of limiting equilibrium and its independence of noise distribution
- Remark2: A similar alternative equilibrium characterization using the potential function in Leister et al.(2019) – the global game machinery selects the maximizer of the potential $P(a, \theta)$ for generic .

Result Characterization

- Indifferent between adopting or not at her cutoff for i:

$$\theta_i^* + v_i + \phi \sum_{j\in N_i} w_{ij}^* = 0, \quad \forall i \in N$$

where $w_{ij}^* = \lim_{\nu\to 0} \mathbb{E}\left[\mathbb{I}\left(s_j \geq c_j^{*\nu}\right) \mid s_i = c_i^{*\nu}\right]$

Lemma1 (classical result in global game)

For each $(i,j) \in E$,

1. The following identity holds:

$$w_{ij}^* + w_{ji}^* = 1, .$$

2. If, in addition, $\theta_i^* < \theta_j^*$, then

$$w_{ij}^* = 0, \text{ and } w_{ji}^* = 1.$$

Result Characterization

- Thm1: $\theta^* = -t^*_{[k]}$.
- $$0 = \underbrace{\left(\sum_{i\in S}\theta_i^*\right)}_{\geq |S|\theta_{[1]}} + \underbrace{\left(v(S) + \phi\sum_{i\in S}\sum_{j\in N_i} w_{ij}^*\right)}_{\geq v(S)+\phi e(S)=F(S)} \Rightarrow \frac{F(S)}{|S|} \leq -\theta_{[1]}$$
  where t cutoff $\theta_{[1]} \equiv \min_{j\in N}\theta_j^*$
- In fact, the upper bound $-\theta_{[1]}$ is achievable for $S = \mathcal{A}_1 = \left\{i \in N \mid \theta_i = \theta_{[1]}\right\}$ (the set of agents with the lowest cutoff): $\frac{F(\mathcal{A}_1)}{|\mathcal{A}_1|} = -\theta_{[1]} \geq \frac{F(S)}{|S|}, \forall S$.
- The process then continues until it stops when all the players are included.

New Insights

- Katz-Bonacich Centrality (interconnectedness) determines the propensity to adopt (BCZ,2006)
- New insights: interconnectedness + embeddedness jointly determine the propensity

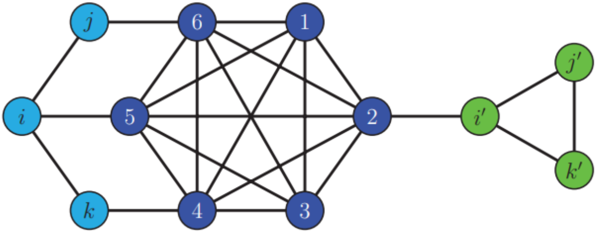

FIGURE 2
Interconnectedness versus Embeddedness.

- The SAND stops in three steps with:

$$A_1^* = S_1 = \{1,2,3,4,5,6\}, A_2^* = S_1 \cup S_2, S_2 = \{i,j,k\}, A_3^* = S_1 \cup S_2 \cup S_3 = N, S_3 = \{i',j',k'\}$$

- What is notable is that agents in S_2 , instead of S_3 , are found in Step 2.
- This implies, in particular, that agent $j \in S_2$ has a higher propensity to adopt than agent $i' \in S_3$, although j has fewer links than i' .

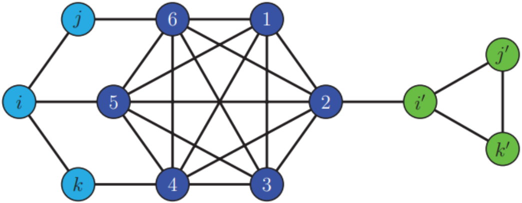

FIGURE 2

Interconnectedness versus Embeddedness.

New Insights

- Define $L(S', S'')$ for $S' \cap S'' = \emptyset$ to be the number of links from agents in $S' \subset N$ to agents in $S'' \subset N$.
- Then, for $S \supsetneq A^*_{k-1}$, we have

$$e(S) - e\left(A^*_{k-1}\right) = e\left(S \backslash A^*_{k-1}\right) + L\left(S \backslash A^*_{k-1}, A^*_{k-1}\right),$$

- Step k in SAND can be written in the following manner by setting $T = S \backslash A^*_{k-1}$:

$$\max_{\emptyset \neq T \subseteq N \backslash A^*_{k-1}} \overbrace{\frac{e(T)}{|T|}}^{\text{embeddedness}} + \overbrace{\frac{L\left(T, A^*_{k-1}\right)}{|T|}}^{\text{interconnectedness}}.$$

$$\max_{\emptyset \neq T \subseteq N \setminus A_{k-1}^*} \overbrace{\frac{e(T)}{|T|}}^{\text{embeddedness}} + \overbrace{\frac{L\left(T, A_{k-1}^*\right)}{|T|}}^{\text{interconnectedness}} .$$

- In BCZ, what matters the most is the complementarity in actions between agents,
  $\Rightarrow$ the ones who generate and receive the largest spillovers from their neighbours (same coordination set).
- LZZ additionally has a coordination problem since agents do not know with certainty whether certain neighbours adopt
  $\Rightarrow$ the level of spillovers to which they are exposed (different coordination sets).

Further Characterizations and Discussions

Single Coordination Set

- Assumption 1 (Homogeneous intrinsic valuations) $v_i = v$ for each $i \in N$

  Under Assumption 1, $\mathcal{C}^*$ is independent of v and of ϕ.
- Single Coordination Set:
  1. if and only if it is balanced, in the sense that for every non-empty $S \subset N$, $\frac{e(S)}{|S|} \leq \frac{e(N)}{|N|}$
  2. if and only if there exists $\mathbf{w} = \{w_{ij}, (i,j) \in E\}$,such that for all $i,j \in N : (i) w_{ij} \geq 0, (ii) w_{ij} + w_{ji} = 1$, and $(iii) \sum_{k \in N_i} w_{ik} = \frac{e(N)}{|N|}$. $e(S) = \frac{1}{2} \sum_{i,j \in S:(i,j) \in E} (w_{ij} + w_{ji}) \leq \sum_{i \in S} \sum_{j \in N_i} w_{ij} = \sum_{i \in S} \frac{e(N)}{|N|} = |S| \frac{e(N)}{|N|}$, for all $S \subset N$
- A regular network / tree network / regular bipartite network / has a unique cycle / has a maximum of four agents $\Rightarrow$ single coordination set

The Agents with Highest Propensity

- A_1^* is the unique nonempty set A that simultaneously satisfies the following conditions:
  1. For any non-empty subset $\underline{A}$ of A, the average density of $\underline{A}$ is no greater than that of A , that is,

$$\frac{e(\underline{A})}{|\underline{A}|} \leq \frac{e(A)}{|A|}, \quad \forall \emptyset \subsetneq \underline{A} \subseteq A.$$

  2. For any nonempty subset T of $N \backslash A$, the average number of links across T and A is smaller than the difference in average densities between A and T , that is,

$$\frac{L(T,A)}{|T|} < \frac{e(A)}{|A|} - \frac{e(T)}{|T|}, \quad \forall \emptyset \subsetneq T \subseteq N \backslash A.$$

Local Contagion

Proposition (Local contagion/ "risk-sharing islands")

1. For a generic $\mathbf{v} = (v_1, ..., v_n)$, there exists a nonempty open neighbourhood $\mathcal{N}(\mathbf{v})$ around $\mathbf{v}$, such that, for any $\mathbf{v}', \mathbf{v}'' \in \mathcal{N}(\mathbf{v})$, the coordination sets under $\mathbf{v}'$ are the same as those under $\mathbf{v}''$.
2. The mapping $\boldsymbol{\theta}^*(\mathbf{v})$ is piecewise linear, Lipschitz continuous, and monotone. For generic $\mathbf{v}$, for each $i, j \in C_m^*$, and $k \notin C_m^*$:

$$\frac{\partial \theta_j^*}{\partial v_i} = \frac{-1}{|C_m^*|}, \quad \text{and} \quad \frac{\partial \theta_k^*}{\partial v_i} = 0$$

1. (1) states that, the solution is locally invariant in $\mathbf{v}$
2. (2) indicates that, increasing the intrinsic value of agent i, locally reduces the common cutoff value for its coordination set

Changes to intrinsic valuations

- A planner who shares prior $H(\cdot)$ over θ and holds no private information.
- The planner has a fixed budget B to subsidy the users.

  feasible set:

  $$K(\mathbf{v}, B) \equiv \left\{ \tilde{\mathbf{v}} \in \mathbb{R}^{|N|} : \tilde{v}_j \geq v_j, \forall j \in N, \text{ and } \sum_{j \in N} \tilde{v}_j - v_j \leq B \right\}$$

- two types of problems:
  1. The expected aggregate adoption:

     $TA(\mathbf{v}) \equiv \sum_{j \in N} \mathbb{E}\left[\pi_i^*(\theta)\right] = \sum_{j \in N} \left(1 - H\left(\theta_j^*\right)\right)$
  2. The expected aggregate welfare: $TW(\mathbf{v}) \equiv \mathbb{E}\left[W\left(\boldsymbol{\pi}^*(\theta), \theta\right)\right]$, where

     $W(\mathbf{a}, \theta) = \sum_{i \in N} u_i(\mathbf{a}, \theta)$.

Planner/s Optimal Policies

Optimal Policies

For sufficiently small B

1. The set of solutions to the A-planner's problem is given by the set of expended $\tilde{\mathbf{v}}$ satisfying $\tilde{v}_i > v_i$, if and only if i maximizes $H'\left(\theta_i^*\right)$
2. The set of solutions to the W-planner's problem is given by the set of expended $\tilde{\mathbf{v}}$ satisfying $\tilde{v}_i > v_i$ if and only if $i \in C_m^* \subseteq A_k^*$ maximizes:

$$1 - H\left(\theta_i^*\right) + \phi\left(\frac{L\left(C_m^*, A_{k-1}^*\right) + e\left(C_m^*\right)}{\left|C_m^*\right|}\right) H'\left(\theta_i^*\right)$$

Planner/s Optimal Policies

Optimal Policies (for sufficiently small B)

1. A-planner's taregt if and only if i maximizes $H'(\theta_i^*)$
2. W-planner's taregt if and only if $i \in C_m^* \subseteq A_k^*$ maximizes:

$$1 - H(\theta_i^*) + \phi\left(\frac{L\left(C_m^*, A_{k-1}^*\right) + e(C_m^*)}{|C_m^*|}\right) H'(\theta_i^*)$$

- A-planner cares the direct effect: a subsidy to member i's adoption increases adoption among other members in the same coordination set, while having no influence on members of other coordination sets
- W-planner also values the additional externalities among members of the targeted coordination set

Changes to the network structure

- Two possible changes: removing key player and adding links
- The key players are determined using the SAND algorithm (remove agent i and compare)
- $\mathcal{G}_{+ij}$ is defined as the network created by adding the additional link ij in $\mathcal{G}$
  1. $\mathcal{C}^*_{+ij}$ is the limit partition under $\mathcal{G}_{+ij}$
  2. Let θ^*_k and $\theta^*_{k,+ij}$ correspond to the cutoffs of agent k under networks $\mathcal{G}$ and $\mathcal{G}_{+ij}$, respectively.

Changes to the network structure

linkage

Take i, j with $i \in C_m^*$, $ij \notin E$.

1. Assume that $\mathcal{C}_{+ij}^* = \mathcal{C}^*$. If
    1. $j \notin C_m^*$ with $\theta_i^* > \theta_j^*$, then $\theta_i^* - \theta_{i,+ij}^* = \phi \frac{1}{|C_m^*|}$, and $\theta_{j,+ij}^* = \theta_j^*$;
    2. $j \in C_m^*$, then $\theta_i^* - \theta_{i,+ij}^* = \phi \frac{1}{|C_m^*|}$
2. Assume that $\mathcal{C}_{+ij}^* \neq \mathcal{C}^*$. If
    1. $j \notin C_m^*$ with $\theta_i^* > \theta_j^*$, then $\theta_i^* > \theta_{i,+ij}^* \geq \theta_{j,+ij}^*$, where $\theta_{j,+ij}^* = \theta_j^*$ if $\theta_{i,+ij}^* \neq \theta_{j,+ij}^*$;
    2. $j \in C_m^*$, then i and j are in the same coordination set in $\mathcal{C}$

Ballester et al. (2006) (BCZ)

- Each agent i chooses effort a_i to maximize the following payoff:

$$u_i(\mathbf{a}) = v_i a_i - \frac{1}{2}a_i^2 + \phi \sum_{j\in\mathcal{N}} g_{ij} a_i a_j,$$

where $v_i = r - p$ for each i, where p is the (common) price and $r > 0$ is the marginal private returns of exerting a_i .

- Contrary to LZZ(2022), effort a_i is continuous, i.e., $a_i \in \mathbf{R}_+$.

BCZ(2006)

When $\phi\lambda_{\max}(\mathbf{G}) < 1$, there exists a unique Nash equilibrium given by:

$$\mathbf{a}^*(p) = (a_1^*(p), \ldots, a_n^*(p))' = \begin{cases} [\mathbf{I} - \phi\mathbf{G}]^{-1}(r-p)\mathbf{1} & \text{if } p \le r \\ \mathbf{0} & \text{if } p > r, \end{cases}$$

Thus, the aggregate demand is equal to:

$$D^{BCZ}(p) = \sum_{i \in \mathcal{N}} a_i^*(p) = \begin{cases} b(\mathbf{G}, \phi)(r-p) & \text{if } p \leq r \\ 0 & \text{if } p > r, \end{cases}$$

where $b(\mathbf{G}, \phi) = \mathbf{1}'[\mathbf{I} - \phi\mathbf{G}]^{-1}\mathbf{1}$ is the (unweighted) aggregate Katz-Bonacich centrality of $\mathbf{G}$ with parameter ϕ .

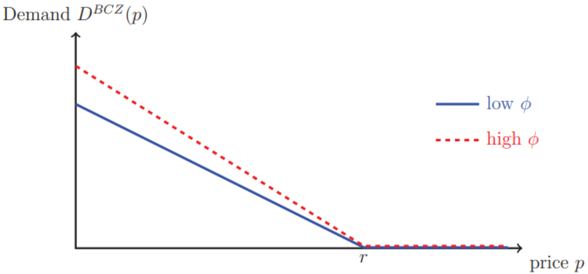

FIGURE 3

Aggregate demand in Ballester et al. (2006)

Sákovics and Steiner (2012) (SS)

- The utility of agent i:

$$u_i(a,\theta,p)=\begin{cases}\phi_i+v_i-p & \text{if } a\geq 1-\theta\\ v_i-p & \text{if } a<1-\theta\end{cases},$$

where $a=\int_0^m w_i a_i di$, $v_i<0$, $\phi_i+v_i-p>0$ for all p , and $w_i>0$.

- Each group g of players has measure m_g, $\sum_g m_g=m$,$\sum_g w_g m_g=1$(a normalization), and each $v_i=v_j$, $\phi_i=\phi_j$ and $w_i=w_j$ for $i,j\in g$.
- Players are endowed with signals $s_i=\theta+\nu\epsilon_i$, $\nu\in(0,1]$ and ϵ_i follows cdf $F(\cdot)$ with support $[-1,1]$.
- Each player i 's group membership is private information with $\Pr(g_i=g)=m_g/m$.

SS(2012)

For each $v \in (0, 1]$, there is a unique Bayes-Nash equilibrium and each player follows the following threshold strategy:

$$a_i(s_i, g) = \begin{cases} 1 \text{ if } s_i \geq s_g^* \\ 0 \text{ if } s_i < s_g^* \end{cases}.$$

Moreover, as $v \to 0$, all thresholds s_g^* converge to a common limit θ^* , where:

$$\theta^* = \sum_g m_g \frac{w_g}{\phi_g} (p - v_g).$$

- For a given realization of θ , as $v \to 0$, the above maps to a p^* such that $p \leq p^*$ implies $a_i = 1$ and $p > p^*$ implies $a_i = 0$ for all i , with
$$p^* = \frac{\theta + \sum_g \frac{m_g w_g v_g}{\phi_g}}{\sum_g \frac{m_g w_g}{\phi_g}}$$
- Under the assumption that, for all g, $\phi_g = \phi$, p^* becomes:
$$p^* = \theta\phi + \sum_g m_g w_g v_g.$$

- When all players adopt, aggregate demand $D^{SS}(p)$ is equal to m , otherwise demand is 0 . That is, for a given realization of the state θ , as $v \to 0$,

$$D^{SS}(p) = \sum_{i \in \mathcal{N}} a_i^*(p) = \begin{cases} m \text{ if } p \leq p^* \\ 0 \text{ if } p > p^* \end{cases}$$

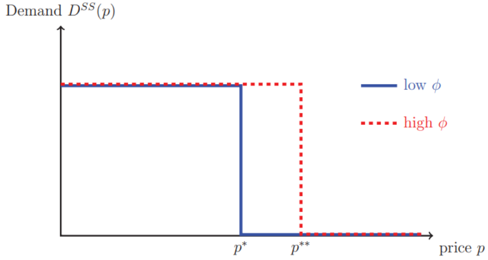

FIGURE 4
Aggregate demand in Sákovics and Steiner (2012)

Leister et al. (2022) (LZZ)

- The utility of agent i:

$$u_i(\mathbf{a}, \theta) = \left(v_i + \theta + \phi \sum_{j \in \mathcal{N}} g_{ij} a_j\right) a_i,$$

where a_i is either 0 or 1 and θ is the state.

- Assume $v_i = r - p$ for each i , where p is the price and $r > 0$ is a constant.
- Define $p^*_{[i]} = \theta + r + \phi t^*_{[i]}$ for each coordination set A^*_i .
- The aggregate demand function in this framework is given by:

$$D^{LZZ}(p) = \sum_{i \in \mathcal{N}} a^*_j(p) = \sum_{i \in \mathcal{N}} \mathbf{1}_{\left\{\theta + r - p + \phi q^*_j \geq 0\right\}},$$

Here, $\mathbf{q}^* = (q^*_1, \ldots, q^*_n)$ is defined such that $q^*_k = t^*_{[i]}$ if k is in A^*_i .

- In other words, an agent k in coordination set A^*_i will adopt if and only if $p \leq p^*_{[i]}$.

The aggregate demand function in this framework is given by:

$$D^{LZZ}(p) = \sum_{i \in \mathcal{N}} a_j^*(p) = \sum_{i \in \mathcal{N}} \mathbf{1}_{\{\theta + r - p + \phi q_j^* \geq 0\}},$$

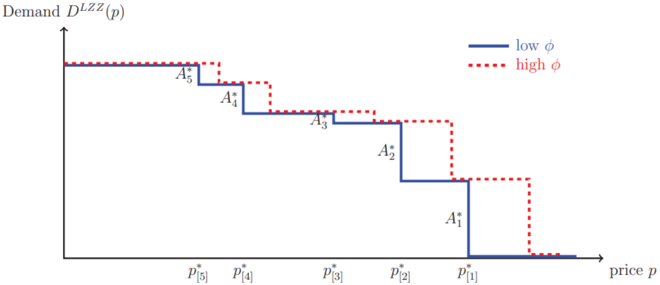

FIGURE 6
Aggregate demand in our model

Conclusion

Conclusion

- A coordination game among agents with private values and binary actions in a network
  1. limiting equilibrium (cut-off threshold strategy) in global games (equilibrium selection)
  2. SAND algorithm and coordination set partition
  3. some discussions: optimal intervetion, network topology, demand function, ...