Suspense and Surprise

Jeffrey Ely, Alexander Frankel, Emir Kamenica('15 JPE)

Presenter: Renjie Zhong
2020200977@ruc.edu.cn

Renmin University of China

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Motivation

- Demand for noninstrumental information due to entertainment value
- Two distinct issues
  1. industries where provision of entertainment is a crucial service
  2. substantial social consequences
- The noninstrumental information is suspense and surprise in this paper, the former is **the variance of next period's beliefs** and the latter is **the distance between current belief and last period's belief**
- What is the nature of so-called noninstrumental information?
- **The stochastic path of his beliefs matters!**

Main Findings

- **Suspense is constant across periods** and there is no variability in ex post suspense, leading to decreasing residual uncertainty over time.
- Under Maximizing Expected Surprise Policy, **surprise in each period is variable**, as is the ex post total surprise:
  1. Residual uncertainty may go up or down over time
  2. When there are many periods, the beliefs shift only a small amount in each period.
  3. There is a positive probability that the state is fully revealed before the final period.

Realted Literature

1. Information Design
2. Microfoundations of Preferences
3. Preferences over the Timing of Resolution of Uncertainty
4. Suspense and Surprise

Model

Setup:Preferences, Beliefs and Technology

- a finite state space Ω and state ω
- a typical belief μ, where μ^{ω} designates the probability of ω
- the prior belief μ_0 and the current belief μ_t in period t, where $t \in \{1, 2, ..., T\}$
- a signal $\pi : \Omega \to \Delta S$
- an information policy $\tilde{\pi}$ maps the current period and the current belief into a signal and $\tilde{\Pi}$ denote the set of all information policies
- a belief martingale $\tilde{\mu}$ is a sequence $(\tilde{\mu}_t)_{t=0}^{T}$:
  1. $\tilde{\mu}_t \in \Delta(\Delta\Omega)$ for all t and $\tilde{\mu}_0$ is degenerated ar μ_0
  2. $E[\tilde{\mu}_t|\mu_0, ..., \mu_{t-1}] = \mu_{t-1}$ for all $t \in \{1, 2, ..., T\}$
- the belief martingale induced by information policy $\tilde{\pi}$ (given the prior μ_0) by $\langle \tilde{\pi} \mid \mu_0 \rangle$.

Setup: Suspense

- Agent has preferences over **his belief path and the belief martingale**
- a preference for suspense is induced by **variance over the next period's beliefs**

$$U_{\mathsf{susp}}(\eta, \tilde{\mu}) = \sum_{t=0}^{T-1} u\left(E_t \sum_{\omega} \left(\tilde{\mu}_{t+1}^{\omega} - \mu_t^{\omega}\right)^2\right)$$

for some increasing, strictly concave function $u(\cdot)$ with $u(0) = 0$.

- the principal's problem: $max_{\tilde{\pi} \in \tilde{\Pi}} E_{\langle \tilde{\pi} | \mu_0 \rangle} U_{susp}(\eta, \langle \tilde{\pi} \mid \mu_0 \rangle)$

Setup: Surprise

- a preference for surprise is induced by **change from the previous belief to the current one**.

$$U_{\textsf{surp}}(\eta) = \sum_{t=1}^{T} u\left(\sum_{\omega} \left(\mu_t^{\omega} - \mu_{t-1}^{\omega}\right)^2\right)$$

for some increasing, strictly concave function $u(\cdot)$ with $u(0) = 0$.

- the principal's problem: $max_{\tilde{\pi} \in \tilde{\Pi}} E_{\langle \tilde{\pi} | \mu_0 \rangle} U_{susp}(\eta)$

Lemma1

Given any Markov belief martingale $\tilde{\mu}$, there exists an information policy $\tilde{\pi}$ such that $\tilde{\mu} = \langle \tilde{\pi} \mid \mu_0 \rangle$.(KG,2011)

Implication: the principal's choice of an information policy as equivalent to the choice of a martingale

Extensions

the agent may:

1. value both suspense and surprise
2. experience additional utility from the realization of a particular state
   state-dependent significance

$$U_{\mathsf{susp}}(\eta, \tilde{\mu}) = \sum_{t=0}^{T-1} u\left(E_t \sum_{\omega} \alpha^{\omega}\left(\tilde{\mu}_{t+1}^{\omega} - \mu_t^{\omega}\right)^2\right)$$

3. have a distinct preference for learning the outcome by the end
4. be invested only in some aspect of her belief such as the expectation

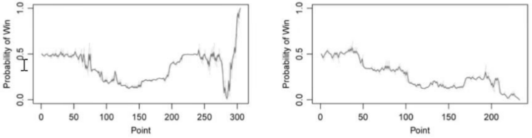
- **Example:** Publishing house as the principal and reader of mystery novels as agent. In this case, a writer (Mrs. X) is associated with a belief martingale.
- **Technology:** In the opening pages of the novel, a dead body is found at a remote country house where n guests and staff are present. In every novel by Mrs. X, one of these n individuals single-handed committed the murder. The opening pages establish a prior μ_0 over the likelihood that each individual $\omega \in \Omega$ is the culprit. There are then T chapters each revealing some information about the identity of the murderer. Mrs. X explicitly randomizes the plot of each chapter on the basis of her information policy and her current belief and learns whodunit only when she completes the novel.
- **Implicit Assumption:**
  - The model captures both settings in which the state is realized ex ante and those in which it is realized ex post.
  - The principal and the agent share a common language for conveying the informational content of a signal.
  - Commitment problems may arise.

Intepretations Time

- In some settings, time is naturally discrete, and a period in our model is determined by the frequency with which the agent receives new information (e.g. tennis, soccer). How to derive results on continuous time limits through a discrete-time model?
  - First, the very passage of time may be necessary for the enjoyment of suspense and surprise.
  - Second, the choice of the period will also be determined by data availability.
- In situations in which the choice of a period is arbitrary, our model has an unappealing feature that increasing T generates more utility. However, a longer T is undesirable (e.g. novel, game). This paper formalizes this question by adding an opportunity cost of time.

Intepretations Preference

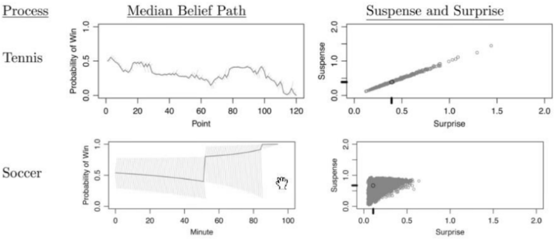
- The key feature of **suspense**: the belief about the state of the world is about to change (e.g. college applicant, soccer player). For the purpose of aggregating suspense over multiple periods, it seems most plausible to assume that $u(\cdot)$ is strictly concave.
- The key feature of **surprise**: the belief about the state of the world changed dramatically.
- Two crucial distinction between suspense and surprise:
  - Suspense is experienced ex ante whereas surprise is experienced ex post.
  - The overall surprise depends solely on the belief path realized. In contrast, suspense depends on the belief martingale as well as on the belief path.



(a) Likelihood that Djoković beats Federer (b) Likelihood that Murray beats Nadal

Intepretations Preference

- Suspense and surprise are important in many contexts.
- Sports fans enjoy the drama of the shifting fortunes between players. Playing blackjack at the casino, a gambler knows the odds are against her but derives pleasure from the ups and downs of the game itself. Politicos and potential voters enjoy following the news when there is an exciting race for political office such as the 2008 Clinton-Obama primary.



Analysis

Two Key Observations

- Notations:
  1. $\sigma_t^2 = E_t \sum_\omega \left(\tilde{\mu}_{t+1}^\omega - \mu_t^\omega\right)^2$
  2. rewrite the principal's problem: $E_\omega \sum_{t=0}^{T-1} u(\sigma_t^2)$

Two Observations(Lemma 2)

1. be fully revealing by the end (Cauchy Inequality)
2. yield the same expected sum of variances
   i.e. a "budget of variance" equal to $\Phi(\mu_0) = \sum_\omega \mu^\omega(1-\mu^\omega)$

- the principal decides how to allocate this variance across periods:

$$max \quad E_\omega \sum_{t=0}^{T-1} u(\sigma_t^2) \quad s.t. \quad E_{\tilde{\mu}} \sum_t \sigma_t^2 = \Phi(\mu_0)$$

- By concavity of $u(\cdot)$, it would be ideal to **dole out variance evenly over time(Jensen's inequality)**

Suspense-Optimal Information Policies

Proposition1

A belief martingale maximizes expected suspense if and only if $\mu_t \in M_t = \{\mu | \Phi(\mu) = \frac{T-t}{T}\Phi(\mu_0)\}$ for all t. The agent's expected suspense from such a policy is $Tu(\frac{\Phi(\mu_0)}{T})$

Geometric Characterization

- Uniform Belief: $\mu_* = (\frac{1}{|\Omega|}, \ldots, \frac{1}{|\Omega|})$
- Some Simple Algebra:
  $M_t = \{\mu | |\mu - \mu_*| = |\mu - \mu_0| + \frac{t}{T}\Phi(\mu_0)\}$

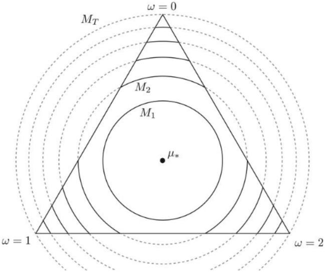


FIG. 4.—The path of beliefs with $\Omega = \{0, 1, 2\}$. The triangle represents $\Delta(\Omega)$, the two-dimensional space of possible beliefs. The M_t sets are circles centered on the uniform belief μ_*, intersected with the triangle $\Delta(\Omega)$. The belief begins at μ_0; in this picture μ_0 is at μ_*. The belief μ_t at time t will be in M_t. Given current belief $\mu_t \in M_t$, any distribution $\tilde{\mu}_{t+1}$ over next-period beliefs with mean μ_t and support contained in M_{t+1} is consistent with a suspense-maximizing policy. At time T the uncertainty is resolved, so μ_T will be on a corner of the triangle.

Summary

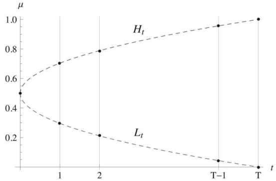
Some Key Features of Suspense-Optimal Information Policies

- The state is revealed in the last period, and not before
- Uncertainty declines over time
- Realized suspense is deterministic
- Suspense is constant over time
- The prior that maximizes suspense is the uniform belief
- The level of suspense increases in the number of periods
- Suspense-optimal information policies are independent of the stage utility function

Illustration: A Binary Example

Suppose a contest between A and B. $Pr(A) = \mu_t$ is either H_t or L_t Indeed, we can calculate that under optimal policy:

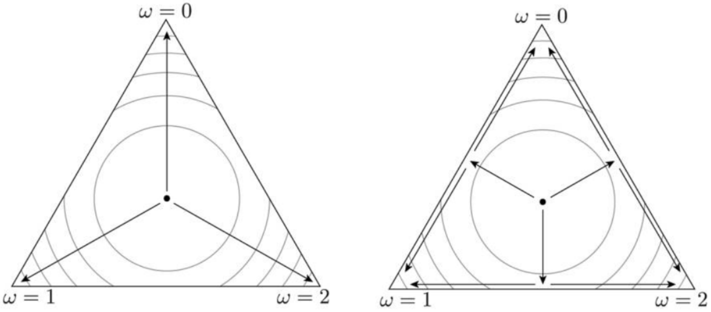
1. $H_t = \frac{1}{2} + \sqrt{(\mu_0 - \frac{1}{2})^2 + \frac{t}{T}\mu_0(1-\mu_0)}$
2. $L_t = \frac{1}{2} - \sqrt{(\mu_0 - \frac{1}{2})^2 + \frac{t}{T}\mu_0(1-\mu_0)}$



- Beliefs can jump by a large amount in a single period(twist probability is $\frac{1}{2} - \frac{1}{2}\frac{\sqrt{(\mu_0-\frac{1}{2})^2+\frac{t-1}{T}\mu_0(1-\mu_0)}}{\sqrt{(\mu_0-\frac{1}{2})^2+\frac{t}{T}\mu_0(1-\mu_0)}}$)
- Belief paths are smooth with rare discrete jumps when there are many periods

Illustration: Three or more states

We can also imagine a contest with multiple teams



(a) Alive Till the End

(b) Sequential Elimination

Extensions: State-dependent or Time-dependent Significances

Suppose the agent experiences additional utility from the realization of a particular state/time, i.e. the state-dependent or time-dependent significances

1. state-dependent

$$U_{\text{susp}}(\eta, \tilde{\mu}) = \sum_{t=0}^{T-1} u\left(E_t \sum_{\omega} \alpha^{\omega}\left(\tilde{\mu}_{t+1}^{\omega} - \mu_t^{\omega}\right)^2\right)$$

   - the M_t sets are ellipses rather than circles
   - he optimal prior is no longer necessarily uniform when there are more than two states(more weights more probability)
   - endogenize the α^{ω} and μ_0: $\mu_0^{\omega} = \frac{1}{2}$ for any $\alpha\omega > 0$

2. time-dependent significances: more weights more suspenseful

$T \leq 3$

- Notations:
  1. a binary states $\Omega = \{A, B\}$
  2. the value function of the surprise maximization problem: $W_T(\mu)$
     T:the remaining periods μ: current belief and $W_T(\mu) = 0$
- express the value function recursively

$$W_T(\mu) = max_{\tilde{\mu}' \in \Delta(\Delta(\Omega))} E_{\tilde{\mu}'}[|\mu' - \mu| + W_{T-1}(\mu')$$

$$s.t. \quad E_{\tilde{\mu}'}[\mu'] = \mu$$

- working backward from the last period

Some Observations

1. uncertainty can either increase or decrease over time
2. accept a positive probability of early resolution in return for a chance to move beliefs back to the interior and set the stage for later surprises
3. the principal requires the commitment power to follow optimal paths

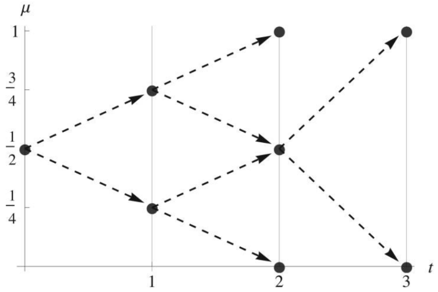


FIG. 7.—The surprise-optimal policy when $T = 3$

Two Key Features: Unbounded Payoffs

Proposition2(MZ,1977)

$$lim_{T\to\infty}\frac{W_T(\mu)}{\sqrt{T}} = \phi(\mu) = \frac{1}{2\sqrt{\pi}}e^{-(1/2)x_\mu^2}$$

with x_μ defined by $\int_{-\infty}^{x_\mu}\frac{1}{2\sqrt{\pi}}e^{-(1/2)x^2}dx = \mu$

- $\sqrt{T}\phi(\mu) - \alpha \leq W_T(\mu) \leq \sqrt{T}\phi(\mu) + \alpha$ for some constant $\alpha > 0$
- The surprise payoff—that is, the expected absolute variation—is unbounded as T goes to infinity

Two Key Features: Range of Belief Changes

Proposition3

For all $\epsilon > 0$,if $T - t$ is sufficiently large, then for any belief path in the support of any surprise-optimal martingale, $|\mu_{t+1} - \mu_t| < \epsilon$.

- Beliefs move up or down only a small amount in periods when there is a lot of time remaining.

Summary

1. The state is fully revealed, possibly before the final period
2. Uncertainty may increase or decrease over time
3. Realized surprise is stochastic
4. Surprise varies over time
5. The prior that maximizes surprise is the uniform belief
6. Beliefs change little when there are many periods remaining
7. Belief paths are spiky when there are many periods
8. Surprise-optimal information policies depend on the stage utility function

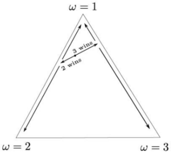
Comparisons

- Sometimes clashes for:
  1. given a martingale, belief paths with high realized suspense tend to have high realized surprise
  2. the expected suspense and surprise are highly correlated across martingales generated by "random" information policies
- maximizing a convex combination of suspense and surprise is likely to lead to belief paths that resemble the surprise optimum

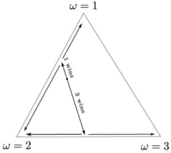
Extensions

Tournament Seeding

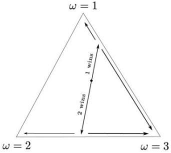
- Which team should have the bye?
- Notations
  1. $p > \frac{1}{2}$: the probability that team 1/2 defeats the team 2/3
  2. $q > p$: the probability that team 1 defeats the team 3



(a) Team 1 has the bye



(b) Team 2 has the bye.



(c) Team 3 has the bye.

- It is optimal to give the weakest team the bye.

Number of Games in a Playoff Series

- Consider two teams playing a sequence of games against each other.
- The organizer chooses odd num T to maximize suspense or surprise
- Main Findings:
  1. Maximizing suspense is equivalent to maximizing surprise here.
  2. The optimal series length is increasing in the proximity of p to one-half
- Intuition: When neither team is much better than the other, the cost of a large T becomes small while the benefit of releasing information slowly remains

Sequential Contests

- Two candidates, A and B, compete in a series of winner-take-all contests
- State i has n_i delegates and will be won by candidate A with independent probability p_i.
- A wins the nomination: gets at least $n^* \in [0, \sum_i n_i]$ delegates.
- Main Findings(Neutrality Results):
  The order of the primaries has no effect on expected suspense or surprise

Generalizations

Revisiting the Nature of Suspense and Surprise

- Suspense is anticipation of the upcoming resolution of uncertainty
  $\Rightarrow$ a measure of uncertainty $\Phi : \Delta\Omega \rightarrow R$ is strictly concave and equals zero at degenerate beliefs(a robust extension)
- Surprise is the ex post experience of a change in beliefs.
  $\Rightarrow$ an arbitrary metric d on the space of beliefs and suppose that surprise in period t is an increasing function of $d(\mu_{t-1}, \mu_t)$