Turning Up the Heat: The Discouraging Effect of Competition in Contests

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- **Incentives vs Discouragement** is the key tradeoff when increasing competition in contest theory.
- **Incentives**: Increasing competition naturally increases contestants' incentives to exert high effort.
- **(Indirect) Discouragement**:Contestants' gains from exerting effort are reduced for beating their rivals harder.

Focus

1. How to depict the competitiveness of the competition?
   1. the number of contestants
   2. the reward structure
   3. we care about increasing prize inequality, increasing contest scale, and contestant entry.
2. How does the competitiveness of the competition affect contestants' efforts?

Main Findings

1. Assumptions: **homogeneous contestants and convex effort costs**
2. Under such assumptions, the discouragement always dominates the incentives.
   That means increasing competition always reduces expected effort of individual contestants.
3. **Intuition**
   Increasing competition
   $\Rightarrow$ More spread-out distributions(extreme effort levels)
   $\Rightarrow$ Decreasing expected effort under convex costs
   P.S. The negative effect of contestant entry is obvious and profound.

Applications

- Promotion("last-place elimination systems")
- Grading scheme in college("Pooling GPA")
- Matching Online Gamers
- Policy Implications: personnel policies that feature egalitarian pay systems and dismissal of worst-performing employees.

1. Root: Barut and Kovenock (1998)
   Homo+Linear+No Info Asymmetric
   → Price structure and contest scale doesn't matter ($v_0 = 0$)
   → Contestant entrty exerts negative effects
2. Subsequent Research: modifying heterogeneity and info asymmetry
   Xiao(2018), Moldovanu and Sela(2001), Olszewski and Siegel(2018)

- $n \geq 1$: homogeneous risk-neutral contestants
- x_i: efforts
- $c(x_i) : R_+ \to R_+$: effort cost which is differentiable, strictly increasing and convex
- $v = (v_1, v_2, ..., v_n) \in R_+^n$: an ordered vector of prizes where $0 = v_1 \leq v_2 \leq ... \leq v_n$ and $v_1 < v_n$
- $F_v(x)$: distriution function
- $\pi_v(p) = \sum_{i=1}^{n} v_i \binom{n-1}{i-1} p^{i-1}(1-p)^{n-i}$: the expected reward

- a unique symmetric equilibrium in mixed strategies
- continuously randomize their efforts over $[0, c^{-1}(v_n)]$
- receive an expected payoff equal to 0
- **Theorem1**:indifference condition
  $\pi \circ F_v(x) - c(x) = 0, x \in [0, c^{-1}(v_n)]$
  $\rightarrow F_v(x) = (\pi^{-1} \circ c)(x)$
- **Corollary1**: If two prize vectors $w, v \in P^n$, satisfy that $w \geq v$, then $X^w \geq_{FSD} X^v$

Example: Setup

- Jill, a retail broker for a small investment bank who needs some assitants for cold call, whose effort cost is $c(x) = 0.001x^2$
- Jill has received a 1,000 incentive fund from the bank earmarked for awards to high-performing assistants
- Initially, Jill design a one-winner contest, which $v_1 = (0, 0, 1000)$
- How to incentize them? Compare the contest policies later!

- **Price Inequality**: $w, v \in P^n$ and $\sum_{i=1}^{k} w_i = \sum_{i=1}^{k} v_i$. Vector w is more unequal than v if w is more unequal than v in the Lorenz order.
  that is $\sum_{i=1}^{k} w_i \leq \sum_{i=1}^{n} v_i$, for all $k = \{1, .., n\}$
- Example: a two-winner contest in which the 1,000 prize is split equally between the two best-performing assistants. The prize vector is $v_2 = (0, 500, 500)$
- **Scaling**: Let $s > 1$ be an integer; $w \in P^{ns}$ is a scaling of $v \in P^n$ if $w_k = v_{[k/s]}$ for all $k \in \{1, ..., n\}$
- Example: Jill decides toscale up the size of the contest by consolidating her contest with Jack's identical one-winner contest. The prize vector is $v_3 = (0, 0, 0, 0, 1000, 1000)$

Theorem

1. **Theorem2**: Suppose vector w is more unequal than v. For any concave, strictly increasing function, u, of individual contestant effort $E[u(X^v)] \leq E[u(X^w)]$
2. **Theorem3**: Suppose vector w is a scaling of v. For any concave, strictly increasing function, u, of individual contestant effort $E[u(X^v)] \leq E[u(X^w)]$

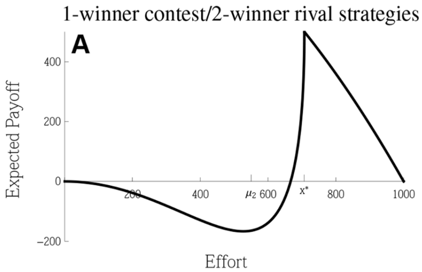
Corollary and Intuition

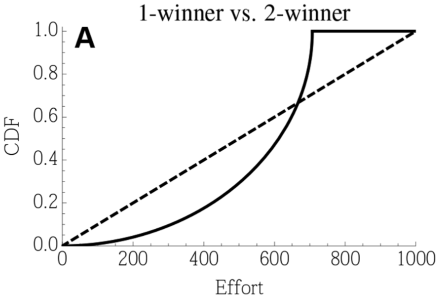
- **Corollary2/3**: Increasing price inequality/Scaling induces decreasing expected individual effort and the expected total effort
- **Intuition**: contestants face a direct incentive to increase effort but all contestants increasing effort cannot be sustained in equilibrium
  $\Rightarrow$ requiring an increase in the payoff from intermediate effort levels and a decrease in the payoff from high effort levels
  $\Rightarrow$ increasing the likelihood contestants make extreme efforts(more spread-out distributions)
  $\Rightarrow$ cost convexity comes into play

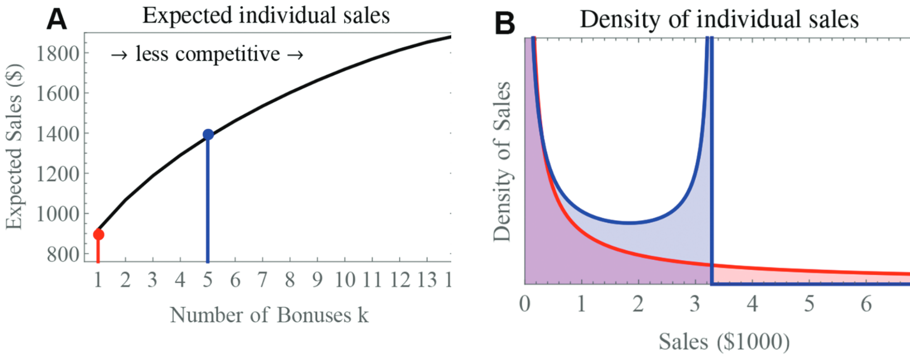
TABLE 1
EQUILIBRIUM RESULTS IN THE THREE CONTESTS UNDER CONSIDERATION

Contest	Prize Vector	Leads per assistant			Expected leads for Jill
		Mean	Minimum	Maximum	
One winner	(0, 0, 1000)	500	0	1,000	1,500
Two winners	(0, 500, 500)	555	0	707	1,665
Consolidated	(0, 0, 0, 0, 1000, 1000)	463	0	1,000	1,389

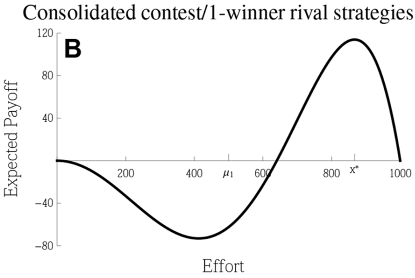
Graphical Illustration: Price Inequality

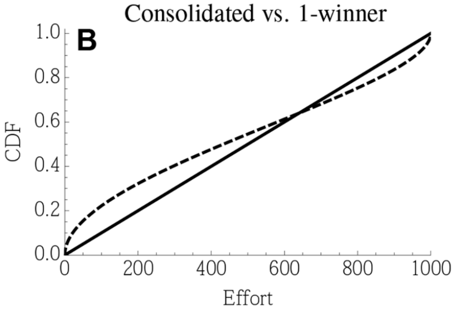




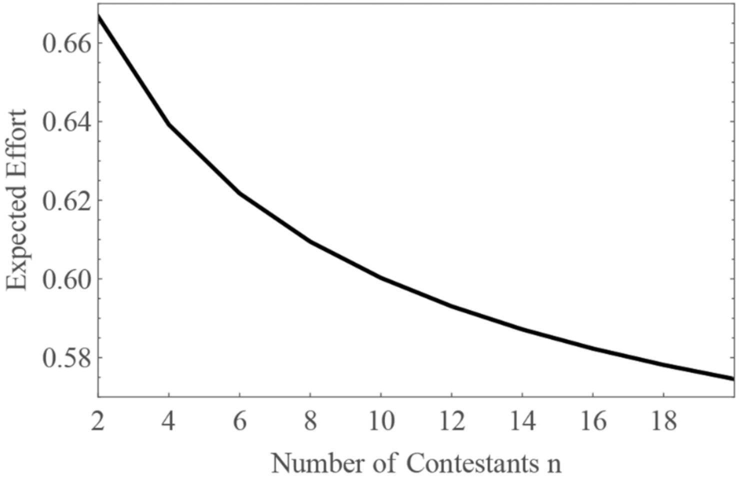

A
Expected individual sales
→ less competitive →
Expected Sales ($)
1800
1600
1400
1200
1000
800
1 2 3 4 5 6 7 8 9 10 11 12 13 1
Number of Bonuses k
B
Density of individual sales
Density of Sales
0 1 2 3 4 5 6
Sales ($1000)

Graphical Illustration: Scaling





Graphical Illustration: Scaling



- **Formal Definition**:Prize vector $w \in P^{n+1}$ is an entry transformation of of $v \in P^n$ if $w = (0, v_1, v_2, ..., v_n)$
- **Proposition 1**: Entry induces decreasing expected individual effort and $X^v \leq_{FSD} X^w$. With (strictly) convex effort-cost function, the expected total effort is (strictly) higher under w than under v
- **Intuition**:
  Reducing the average reward reduces the cost of effort a contestant is willing to incur in expectation
  Adding an entrant as having a new agent "share" a fixed total, increasing total productivity under convex effort costs

Extensions

- Contestant Heterogeneity
- Concave Effort Costs
- Risk-Averse Contestants
- Noisy Outcome