Coordinate Geometry Revisited

A Matrix approach

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Problem

The straight line $2x$ - $3y = 1$ divides the circular region $x^2 + y^2 \leqslant 6$ into two parts. If

$$\boldsymbol{S} = \left\{ \left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right) \right\}$$

then find the number of points in S lying inside the smaller part of the circle.

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- Find position of each point in $\boldsymbol{S}$ with respect to the line
- Find the points which satisfy our condition using the constraints found in above steps

Matrix Transformation

- Consider a matrix $\boldsymbol{S_o}$ with all the points in the set S stacked as follows:

$$\boldsymbol{S_o} = \begin{bmatrix} 2 & \frac{5}{2} & \frac{1}{4} & \frac{1}{8} \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

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- The equation of a line is $\boldsymbol{N} * \boldsymbol{x} = p$ where x is a 2 dimensional column vector and p is a constant.

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- Let $\boldsymbol{M} = \boldsymbol{N} * \boldsymbol{S_o}$
- Find the number of elements in $\boldsymbol{M} - \boldsymbol{P} > 0$ and $(\boldsymbol{S_o^T} * \boldsymbol{S_o})_{ii} \leqslant 6$

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- The diagonal elements of $\boldsymbol{S_o^T} * \boldsymbol{S_o}$ are:

$$(\boldsymbol{S_o^T} * \boldsymbol{S_o})_{ii} = \begin{bmatrix} \frac{73}{16} & \frac{109}{16} & \frac{1}{8} & \frac{1}{32} \end{bmatrix}$$

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- Hence the number of points lying in the smaller region of the circle is 2.

Figure