

Coordinate Geometry Revisited

A Matrix approach

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Problem

The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts. If

$$S = \left\{ \left(2, \frac{3}{4} \right), \left(\frac{5}{2}, \frac{3}{4} \right), \left(\frac{1}{4}, -\frac{1}{4} \right), \left(\frac{1}{8}, \frac{1}{4} \right) \right\}$$

then find the number of points in S lying inside the smaller part of the circle.

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- ▶ Find position of each point in S with respect to the line
- ▶ Find the points which satisfy our condition using the constraints found in above steps

Matrix Transformation

Consider a matrix S_o with all the points in the set S stacked as follows:

$$S_o = \begin{bmatrix} 2 & \frac{5}{2} & \frac{1}{4} & \frac{1}{8} \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

Now the normal vector to the given line is as follows:

$$N = [2 \quad -3]$$

The matrix

$$P = [1 \quad 1 \quad 1 \quad 1]$$

and the matrix

$$O = [0 \quad 0]$$

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- ▶ Let $M = N * S_o$
- ▶ We can see that $N * O^T - 1 = -1$ this is the relative position of the centre with respect to the line.
- ▶ Find the number of elements in $M - P > 0$ and $(S_o^T * S_o)_{ii} \leq 6$

Solution

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$$M - P = \begin{bmatrix} \frac{3}{4} & \frac{7}{4} & \frac{1}{4} & -\frac{3}{2} \end{bmatrix}$$

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- ▶ The diagonal elements of $S_o^T * S_o$ are:

$$(S_o^T * S_o)_{ii} = \begin{bmatrix} \frac{73}{16} & \frac{109}{16} & \frac{1}{8} & \frac{1}{32} \end{bmatrix}$$

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- ▶ Hence the number of points lying in the smaller region of the circle is 2.

Figure

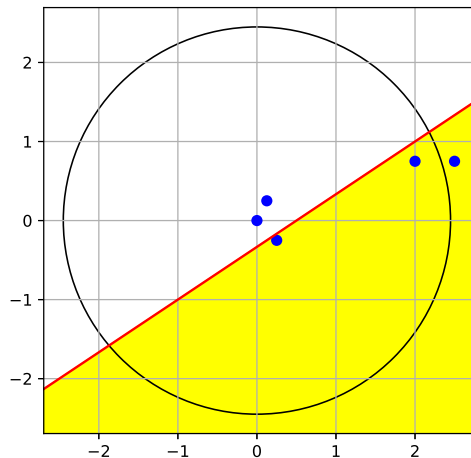


Figure: Shaded part denotes the smaller side of the circle.