Coordinate Geometry Revisited

A Matrix approach

Raghav Girgaonkar Sagar Jain

Indian Institute of Technology Hyderabad

February 2019

Problem

The straight line $2x$ - $3y = 1$ divides the circular region $x^2 + y^2 \leqslant 6$ into two parts. If

$$S = \left\{ \left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right) \right\}$$

then find the number of points in S lying inside the smaller part of the circle.

Approach

The steps we took to solve this problem were:

- Find the centre of the given circle(in this case it is (0,0))

Approach

The steps we took to solve this problem were:

- Find the centre of the given circle(in this case it is (0,0))
- Find position of centre with respect to the line given

Approach

The steps we took to solve this problem were:

- Find the centre of the given circle(in this case it is (0,0))
- Find position of centre with respect to the line given
- Find position of each point in S with respect to the line

Approach

The steps we took to solve this problem were:

- Find the centre of the given circle(in this case it is (0,0))
- Find position of centre with respect to the line given
- Find position of each point in S with respect to the line
- Find the points which satisfy our condition using the constraints found in above steps

Matrix Transformation

- Consider a matrix S_o with all the points in the set S stacked as follows:

$$S_o = \begin{bmatrix} 2 & \frac{5}{2} & \frac{1}{4} & \frac{1}{8} \\ \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

Matrix Transformation

- Consider a matrix S_o with all the points in the set S stacked as follows:

$$S_o = \begin{bmatrix} 2 & \frac{5}{2} & \frac{1}{4} & \frac{1}{8} \\ \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

- Now the normal vector to the given line is as follows:

$$N = \begin{bmatrix} 2 & -3 \end{bmatrix}$$

Matrix Transformation

- Consider a matrix S_o with all the points in the set S stacked as follows:

$$S_o = \begin{bmatrix} 2 & \frac{5}{2} & \frac{1}{4} & \frac{1}{8} \\ \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

- Now the normal vector to the given line is as follows:

$$N = \begin{bmatrix} 2 & -3 \end{bmatrix}$$

- The matrix

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

Matrix Transformation

- Consider a matrix S_o with all the points in the set S stacked as follows:

$$S_o = \begin{bmatrix} 2 & \frac{5}{2} & \frac{1}{4} & \frac{1}{8} \\ \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

- Now the normal vector to the given line is as follows:

$$N = \begin{bmatrix} 2 & -3 \end{bmatrix}$$

- The matrix

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

- and the matrix

$$O = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Matrix Transformation

- Consider a matrix S_o with all the points in the set S stacked as follows:

$$S_o = \begin{bmatrix} 2 & \frac{5}{2} & \frac{1}{4} & \frac{1}{8} \\ \frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

- Now the normal vector to the given line is as follows:

$$N = \begin{bmatrix} 2 & -3 \end{bmatrix}$$

- The matrix

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

- and the matrix

$$O = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

- The equation of a line is $N * x = p$ where x is a 2 dimensional column vector and p is a constant.

Matrix Transformation Contd

Our original problem now transforms as follows:

- Let $M = N * S_o$

Matrix Transformation Contd

Our original problem now transforms as follows:

- Let $M = N * S_o$
- We can see that $N * O^T - 1 = -1$ this is the relative position of the centre with respect to the line.

Matrix Transformation Contd

Our original problem now transforms as follows:

- Let $M = N * S_o$
- We can see that $N * O^T - 1 = -1$ this is the relative position of the centre with respect to the line.
- Find the number of elements in $M - P > 0$ and $(S_o^T * S_o)_{ii} \leqslant 6$

Solution

- $S_o^T * S_o$ will give us a 4*4 matrix with the diagonal elements being the values of $x^2 + y^2$ for the 4 points, hence we will consider only the diagonal entries.

Solution

- $S_o^T * S_o$ will give us a 4*4 matrix with the diagonal elements being the values of $x^2 + y^2$ for the 4 points, hence we will consider only the diagonal entries.
- Calculating $M - P$ we get:

$$M - P = \begin{bmatrix} \frac{3}{4} & \frac{7}{4} & \frac{1}{4} & -\frac{3}{2} \end{bmatrix}$$

Solution

- $S_o^T * S_o$ will give us a 4*4 matrix with the diagonal elements being the values of $x^2 + y^2$ for the 4 points, hence we will consider only the diagonal entries.
- Calculating $M - P$ we get:

$$M - P = \begin{bmatrix} \frac{3}{4} & \frac{7}{4} & \frac{1}{4} & -\frac{3}{2} \end{bmatrix}$$

- The diagonal elements of $S_o^T * S_o$ are:

$$(S_o^T * S_o)_{ii} = \begin{bmatrix} \frac{73}{16} & \frac{109}{16} & \frac{1}{8} & \frac{1}{32} \end{bmatrix}$$

Solution

- $S_o^T * S_o$ will give us a 4*4 matrix with the diagonal elements being the values of $x^2 + y^2$ for the 4 points, hence we will consider only the diagonal entries.
- Calculating $M - P$ we get:

$$M - P = \begin{bmatrix} \frac{3}{4} & \frac{7}{4} & \frac{1}{4} & -\frac{3}{2} \end{bmatrix}$$

- The diagonal elements of $S_o^T * S_o$ are:

$$(S_o^T * S_o)_{ii} = \begin{bmatrix} \frac{73}{16} & \frac{109}{16} & \frac{1}{8} & \frac{1}{32} \end{bmatrix}$$

- Now the only points which satisfy the conditions $M - P > 0$ and $(S_o^T * S_o)_{ii} \leqslant 6$ are $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$

Solution

- $S_o^T * S_o$ will give us a 4*4 matrix with the diagonal elements being the values of $x^2 + y^2$ for the 4 points, hence we will consider only the diagonal entries.
- Calculating $M - P$ we get:

$$M - P = \begin{bmatrix} \frac{3}{4} & \frac{7}{4} & \frac{1}{4} & -\frac{3}{2} \end{bmatrix}$$

- The diagonal elements of $S_o^T * S_o$ are:

$$(S_o^T * S_o)_{ii} = \begin{bmatrix} \frac{73}{16} & \frac{109}{16} & \frac{1}{8} & \frac{1}{32} \end{bmatrix}$$

- Now the only points which satisfy the conditions $M - P > 0$ and $(S_o^T * S_o)_{ii} \leqslant 6$ are $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$
- Hence the number of points lying in the smaller region of the circle is 2.

Figure