EE5609 Assignment 1

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Abstract—**This document contains the solution to find the angle between two lines.**

Download all python codes from

https://github.com/saipranavkr/EE5609/codes

and latex-tikz codes from

https://github.com/saipranavkr/EE5609

1 Problem

Find the angle between the pair of lines

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$$
$$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$$

2 Solution

Consider the equation of the lines to be

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} = \lambda_1$$
$$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2} = \lambda_2$$

Using these equations, the matrix form of the lines can be expressed as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\lambda_1 - 3 \\ 5\lambda_1 + 5 \\ 4\lambda_1 - 3 \end{pmatrix}$$

and

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda_2 - 1 \\ \lambda_2 + 4 \\ 2\lambda_2 + 5 \end{pmatrix}$$

Which can be expressed as follows

$$\mathbf{x} = \begin{pmatrix} -3 \\ 5 \\ 3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{x} = \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.2}$$

Using the definition of a line in co-ordinate geometry, we see from the above two equations, the direction vectors **a** and **b** of the two lines are

$$\mathbf{a} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

respectively. In order to find the angle between the two direction vectors, we use the definition of dot product,

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \, \|\mathbf{b}\|} \tag{2.0.3}$$

Which gives us,

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^T \mathbf{b} = 16$$
$$\|\mathbf{a}\| = \sqrt{50}, \|\mathbf{b}\| = \sqrt{6}$$

Which gives us

$$\cos\theta = \frac{8}{5\sqrt{3}} \tag{2.0.4}$$

$$\implies \theta = \arccos\frac{8}{5\sqrt{3}}$$
$$\implies \theta = 22.517^\circ \tag{2.0.5}$$