

有限差分法笔记

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主要是针对空间离散进行隐式构造，元式

$$\frac{\partial^4 d_1(x_j, t_k)}{\partial x^4} = \frac{1}{h_x^4} (u_{j+2}^k - 4u_{j+1}^k + 6u_j^k - 4u_{j-1}^k + u_{j-2}^k) \quad (1)$$

记号

$$d_1(x, t) = u \quad (2)$$

$$d_2(x, t) = \partial u / \partial t = v \quad (3)$$

方程：

$$u_{tt} + u_t = f - u_{xxxx} \quad (4)$$

方程组：

$$v_t = f - v - u_{xxxx} \quad (5)$$

$$u_t = v \quad (6)$$

0.0.1 方程组隐格式

(5) 显格式：

$$\frac{v_j^{k+1} - v_j^k}{h_t} = f_j^k - v_j^k - \left(\frac{u_{j+2}^k - 4u_{j+1}^k + 6u_j^k - 4u_{j-1}^k + u_{j-2}^k}{h_x^4} \right) \quad (7)$$

令右端中空间差分项为 $u^k = (u^{k+1} + u^k)/2$ 得到一个可能的隐格式：

$$\frac{v_j^{k+1} - v_j^k}{h_t} = f_j^k - v_j^k - \quad (8)$$

$$\frac{1}{2h_x^4} (u_{j+2}^{k+1} + u_{j+2}^k - 4u_{j+1}^{k+1} - 4u_{j+1}^k + 6u_j^{k+1} + 6u_j^k - 4u_{j-1}^{k+1} - 4u_{j-1}^k + u_{j-2}^{k+1} + u_{j-2}^k) \quad (9)$$

$$\frac{u_j^{k+1} - u_j^k}{h_t} = v_j^k \quad (10)$$

写作矩阵：

$$Q_v \mathbf{v}^{k+1} + Q_u \mathbf{u}^{k+1} = \mathbf{f}^k + \mathbf{u}^k + \mathbf{v}^k \quad (11)$$

0.0.2 方程隐格式

(4) 的隐格式

$$\frac{u_j^{k+1} - 2u_j^k + u_j^{k-1}}{h_t^2} + \frac{u_j^{k+1} - u_j^k}{h_t} \quad (12)$$

$$= f_j^k - \frac{1}{2h_x^4} (u_{j+2}^{k+1} + u_{j+2}^k - 4u_{j+1}^{k+1} - 4u_{j+1}^k + 6u_j^{k+1} + 6u_j^k - 4u_{j-1}^{k+1} - 4u_{j-1}^k + u_{j-2}^{k+1} + u_{j-2}^k) \quad (13)$$

$$= f_j^k + \frac{1}{2h_x^4} (\delta^4 u^{k+1} + \delta^4 u^k) \quad (14)$$

化简：

$$u_j^{k+1} - 2u_j^k + u_j^{k-1} + h_t u_j^{k+1} - h_t u_j^k = h_t^2 f_j^k + \frac{h_t^2}{2h_x^4} (\delta^4 u^{k+1} + \delta^4 u^k) \quad (15)$$

$$(1 + h_t) u_j^{k+1} - \frac{h_t^2}{2h_x^4} \delta^4 u^{k+1} = h_t^2 f_j^k + (2 + h_t) u_j^k + \frac{h_t^2}{2h_x^4} \delta^4 u^k - u_j^{k-1} \quad (16)$$

写作矩阵：

$$Q \mathbf{u}^{k+1} = \mathbf{f}^k + \tilde{Q} \mathbf{u}^k - \mathbf{u}^{k-1} \quad (17)$$

左侧展开：

$$(1 + h_t) u_j^{k+1} - \frac{h_t^2}{2h_x^4} (u_{j+2}^{k+1} - 4u_{j+1}^{k+1} + 6u_j^{k+1} - 4u_{j-1}^{k+1} + u_{j-2}^{k+1}) \quad (18)$$

右侧展开：

$$h_t^2 f_j^k + (2 + h_t) u_j^k + \frac{h_t^2}{2h_x^4} (u_{j+2}^k - 4u_{j+1}^k + 6u_j^k - 4u_{j-1}^k + u_{j-2}^k) - u_j^{k-1} \quad (19)$$