1a

Maths

a) $x_n = \sqrt{9n^4+1} - (3n^2+1) \cdot \frac{\sqrt{9n^4+1}+(3n^2+1)}{\sqrt{9n^4+1}+3n^2+1}$

$$x_n = \frac{9n^4+1-9n^4-1-6n^2}{\sqrt{9n^4+1}+3n^2+1} = \frac{-6n^2}{\sqrt{\dots}+3n^2+1}$$

$$= \frac{n^2(-6)}{n^2\sqrt{9+\underbrace{\tfrac{1}{n^4}}_{\downarrow 0}}+n^2(3+\underbrace{\tfrac{1}{n^2}}_{\downarrow 0})} = \frac{-6}{\sqrt{9}+3} = \frac{-6}{6} = -1$$

b) $x_n = (-1)^n \sqrt[n]{n^2 2^n} \to (n^{\frac{1}{n}})^2$

$$x_n = (-1)^n \cdot \underbrace{n^{\frac{2}{n}}}_{\underset{n\to\infty}{\longrightarrow} 1} \cdot 2$$

↳ $(-1)^n \cdot 2$ = divergent ~~(b~~

bcs x_n(even) $\to 2$

and x_n(odd) $= -2$

c) $x_n = \frac{(p^{\frac{n-2}{n}} \cdot p^{\frac{2}{n}})^{n+1} \cdot q^{-n+1}}{q^{\frac{1}{n}} \cdot (q^{n-1})^{\frac{1}{n}} \cdot p^2} =$

simplification:

$$p^{\frac{n-2}{n}} \cdot p^{\frac{2}{n}} = p^{\frac{n-2}{n}+\frac{2}{n}} = p^{\frac{n}{n}} = p^{\frac{n}{n}} = p^1$$

$\cancel{(p^{\frac{n+1}{n}})^{n+1}} =$

$$x_n = \frac{p^{n+1} \cdot q^{-n+1}}{q^{\frac{1}{n}} \cdot (q^{n-1})^{\frac{1}{n}} \cdot p^2} = \;=\; \frac{p^{n+1} \cdot q^{-n+1}}{q^1 p^2}$$

↳ $q^{\frac{1}{n}} \cdot q^{1-\frac{1}{n}} = q^{\frac{1}{n}+1-\frac{1}{n}} = q^1$

$$\frac{p^{n+1}\cdot q^{-n+1}}{qp^2} = \frac{p^{n+1}}{p^2}\cdot\frac{q^{-n+1}}{q} =$$

$$p^{n-1}\cdot q^{-n} = \frac{\cancel{p^{n-1}}}{\cancel{q^n}} = \frac{p^n \to \infty}{q^n \to \infty}$$

$$p^n\cdot p^{-1}\cdot q^{-n} = \frac{1}{p}\cdot p^n\cdot q^{-n} =$$

$$\frac{1}{p}\left(\frac{p}{q}\right)^n \Rightarrow 0<p<q \Rightarrow 0<\frac{p}{q}<1$$

$$\lim_{n\to\infty} x_n = \left(\frac{p}{q}\right)^n \to 0$$

So limit is 0

$x_n = ?$
$x_1 = 1$

Problem 2 Sequences

$x_1 := 1 \quad x_{n+1} := \sqrt{6+x_n}$ for $n \in \mathbb{N}$

$$x_2 := \sqrt{7}$$

$$x_3 := \sqrt{6+\sqrt{7}}$$

$$(x_{n+1})^2 = \left(\sqrt{6+x_n}\right)^2$$

$$x_{n+1}^2 = 6+x_n$$

$$x_{n+1}^2 - x_n - 6$$

$x_n = L$

$$L^2 - L - 6 = 0$$

$L = -2 \Leftarrow$ impossible so $L=3$ is it

$L = 3$

since $L=3$ is the limit

$n+1$

$\sqrt{6+x_n} \geq 0$ for all n ~~because a square~~ so the sequence is bounded below by 3. (a square root cannot be negative)

Upper bound, since the limit is 3 the sequence must be bounded above by 3

Proof by induction (Monotonicity)

Base case

$\cancel{x_{n+1}=\sqrt{6+x_n}}$ $x_2 = \sqrt{6+1} = \sqrt{7} \overset{?}{\geq} x_1$

Inductive Hypothesis, $n+1=k$

$x_n = \sqrt{6+x_{n-1}}$

$x_{n+1} = \sqrt{6+x_n}$

$x_n \geq x_{n-1} \quad | +6$

$6+x_n \geq 6+x_{n-1} \quad ; \sqrt{}$

$$\underbrace{\sqrt{6+x_n}}_{x_{n+1}} \geq \underbrace{\sqrt{6+x_{n-1}}}_{x_n}$$

So $x_{n+1} \geq x_n$

Thus the sequence is ~~bounded~~ monotonically increasing.