

Arda

Math

Problem 2 continuity

$$\lim_{x \rightarrow 2} f(x) = \begin{cases} 2x-2 & \text{if } x \neq 2 \\ -2x+7 & \text{if } x = 2 \end{cases}$$

~~discontinuous at x=2~~

left side $\lim_{x \rightarrow 2} \text{case 1: } 2 \cdot 2 - 2 = 2$

so $\lim_{x \rightarrow 2} = 2 \cdot 2 - 2 = 2$

right $\lim_{x \rightarrow 2} = -2 \cdot 2 + 7 = 2$

left $\lim_{x \rightarrow 2} = -2$

lim $\sqrt{x+2}$ $x \rightarrow 4$ only casesible

$$\lim_{x \rightarrow 4} = 4 \text{ since } \sqrt{4+2} = 2/2$$

so $\lim_{x \rightarrow 4} \sqrt{x+2} = 4$

$$\lim_{x \rightarrow 4} = 4 \quad \text{so}$$

$$f(x) = ax + b$$

$$f(x) = 2ax + b \quad \begin{cases} 2a+b=2 \\ 4a+b=4 \end{cases} \quad \text{-subtraction}$$

$$2a = 2/2$$

$$a = 1/2$$

$f(x) = \begin{cases} 1/2x+2 & x < 2 \\ 7 & 2 \leq x < 5 \\ \sqrt{x+2} & x \geq 5 \end{cases}$

$$b = 0 \text{ so}$$

Problem 3

i) is not linearly independent as they are
are not linearly independent linearly
for

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \xrightarrow{\text{R}_2 - R_1, \text{R}_3 - R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{\text{R}_3 - R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{\text{R}_3 \cdot \frac{1}{2}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x_3 = 1$$

$$x_2 = 0$$

$$x_1 = 0$$

$$x_1 = 0$$

Thus the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ are linearly

independent as the only way to get
the zero vector is by multiplying
by 0. And the condition for that
it's shouldn't be 0.

iii) $\left(\begin{array}{c} 3 \\ 0 \end{array} \right), \left(\begin{array}{c} 0 \\ 1 \end{array} \right), \left(\begin{array}{c} -1 \\ 1 \end{array} \right)$ * Thus the three
vectors are linearly
dependent on
each other.

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array} \right) \xrightarrow{\text{R}_2 + \text{R}_1, \text{R}_3 - \text{R}_1}$$

$$\begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & -1 & 3 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{\text{R}_3 + \text{R}_2} \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{aligned} 1 & 0 & -1 & = 0 & (x_3) \\ 0 & 1 & -1 & = 0 & (x_3) \\ 0 & 0 & 0 & = 0 & (x_3) \end{aligned}$$

$$\hookrightarrow \text{Test } x_3 = ?$$

* Thus $-1 \neq 0$ $x_2 = x_3 = 1$
Thus the vectors are
linearly dependent on each other.

$$\begin{aligned} -7 \cdot 1 + x_2 &= 0, \dots -1 & x_1 = k, 1 & x_1 - x_3 = 0 \end{aligned}$$