

Arda

Mathe

Problem 7 Functions

a) $f: \{1, 2, 3, 4, 5, 6\} \rightarrow \{1, \dots, 6\}$

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 8-x & \text{if } x \geq 3 \end{cases}$$

i)	$f(1) = 1$	$f(3) = 5$	$f(5) = 3$
	$f(2) = 4$	$f(4) = 4$	$f(6) = 2$

The function is not injective as the number 4 has two inputs $f(2)$ and $f(4)$.

Th ii) The function is not surjective as the number 6 does not have any input.

b) $A = \{x | x - 1 \in \mathbb{Z}, x \in \mathbb{Z} \cap [-2, 2]\}$ and

$B = \{y^2 | y \in \mathbb{N} \text{ and } y \leq 5\}$,

A is $x \in A = x \in \{-2, -1, 0, 1, 2\} \Rightarrow x - 1 =$

A is $A(-2) = 6$ $A(-1) = 4$ $A(0) = 2$ $A(1) = 0$ $A(2) = 2$

$A' = \{(-2, 6), (-1, 4), (0, 2), (1, 0), (2, 2)\} \rightarrow \{6, 4, 2\}$

B is $y = \{0, 1, 4, 9, 16\}$ $B' = \{0, 1, 4, 9, 16\}$

ii) $g: A \rightarrow B$ such that g is surjective and not injective

$g: \{6, 4\}, \{9, 16\}, \{2\}, \{0\}$

function does not exist as there are too few inputs in A to make it surjective since the cardinality is higher in B than A. A must have one input more than B.

ii) $g_2 : A \rightarrow B$ such that g_2 is injective and not surjective

$$g_2 : \{(0, 1), (2, 1), (4, 9), (6, 9)\}$$

function exists

Problem & Induction

$$\sum_{n=1}^n \frac{1}{2^n} = 2 - \frac{n+2}{2^n}$$

Base case

$$\sum_{n=1}^1 \frac{1}{2^n} = 2 - \frac{1+2}{2^1}$$

$$\frac{1}{2} = 2 - \frac{3}{2}$$

$$\frac{1}{2} = \frac{4}{2} - \frac{3}{2}$$

$$\frac{1}{2} = \frac{1}{2} \quad n=1 \text{ is true}$$

$$\frac{n+7}{2^{n+7}}$$

Induction Hypothesis ($n \rightarrow n+1$)

$$\sum_{n=1}^n \frac{1}{2^n} = 2 - \frac{n+2}{2^n}$$

$$\sum_{n=1}^{n+1} \frac{1}{2^n} = 2 - \frac{n+3}{2^{n+1}} = \sum_{n=1}^{n+1} \frac{1}{2^n} = \left(\sum_{n=1}^n \frac{1}{2^n} \right) + \frac{n+1}{2^{n+1}}$$

$$= \left(2 - \frac{n+2}{2^n} \right) + \frac{n+1}{2^{n+1}} - \sum_{n=2}^{n+1} \frac{1}{2^n}$$

$$= 2 - \frac{2n+4}{2^{n+1}} + \frac{n+1}{2^{n+1}} = 2 + \frac{-2n-4+n+1}{2^{n+1}} = 2 + \frac{-3}{2^{n+1}}$$

$$= 2 - \frac{n+3}{2^{n+1}}$$

Thus if the formula holds for n , it also holds for $n+1$. Therefore by mathematical induction, the formula is true for all positive integers.