Arda Math

Problem 2 Series

a) $= \sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \dots$

$\frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{1}{1} \cdot \frac{3}{2} = \frac{3}{2} = 1.5$

The series converges to 1.5.

b) $\sum_{n=1}^{\infty} \frac{n+1}{n+2} = \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} \dots$

$\geq 1 \quad > 1$

Since the series is constantly increasing the function diverges to $+\infty$ $\left(\frac{n+1}{n+2} > 1\right)$

$\frac{11}{12}$ ... $\frac{8}{12} + \frac{9}{12}$ $\frac{4}{12} + \frac{9}{12} = \frac{13}{12}$ $\frac{24}{30} + \frac{20}{30}$ $\frac{44}{30}$

c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{4n-1}} = \frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{11}}$

decreases to 0 and is positive and converges to 0 as $n \to \infty$

(it is an alternating series)

Problem 4 System of linear equations

x_1	x_2	x_3		
5	3	1	2	
20	15	6	12	$\mid -4I$
15	3	1	-1	$\mid -3I$

5	3	1	2	
0	3	2	4	
0	-6	-2	-7	$\mid +II$

5	3	1	2
0	3	2	4
0	-3	0	-3

$x_2 = 1$ since $-3x_2 = -3$

insert in II

$3 + 2x_3 = 4$

$2x_3 = 1$

$x_3 = 0.5$ insert in I $5x_1 + 3 + 0.5 = 2$

$5x_1 = -1.5$ is

$x_1 = -0.3 = -\frac{3}{10}$

$x_1\ x_2\ x_3\ x_4$

$$\left(\begin{array}{cccc|c} 2 & 3 & -3 & 7 & 7 \\ 8 & 12 & -12 & 4 & 4 \\ -4 & 6 & 15 & 9 & 7 \\ 6 & 10 & -12 & 1 & 3 \end{array}\right) \begin{array}{l} \\ I-3I \\ 2.II \\ -3II \end{array} \rightarrow \left(\begin{array}{cccc|c} 2 & 3 & -3 & 7 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 6 & -3 \\ 0 & 1 & -3 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & 3 & -3 & 7 & 7 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 0 & 9 & 6 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

= pivot elements x_4 = free variable

$$9x_3 + 6x_4 = -7$$

$$x_3 = -\frac{7}{9} - \frac{6}{9}x_4$$

$$x_2 + \frac{7}{3} - 2x_4 - 2x_4 = 0$$

$$x_2 = -\frac{7}{3} + 4x_4$$

$$2x_1 - 7 + 12x_4 + \frac{7}{3} - 2x_4 + x_4 = 7$$

$$2x_1 + 11x_4 = 2 - \frac{7}{3}$$

$$x_1 = \frac{5}{3} + \frac{11}{2}x_4$$

$$x_2 + \frac{7}{3} + 2x_4 - 2x_4 = 0$$

$$x_2 = -\frac{7}{3}$$

insert in II

$$2x_1 + 3 - 7 + 2x_4 + x_4 = 7$$

$$2x_1 + 2 + 3x_4 = 7$$

$$2x_1 = -7 - 3x_4$$

$$x_1 = -\frac{7}{2} + \frac{3}{2}x_4$$

$$\left(\begin{array}{c} \frac{-7+3x_4}{2} \\ -\frac{7}{3} - 2x_4 \\ x_4 \end{array}\right) + V_0 \left(\begin{array}{c} -\frac{1}{2} \\ -1 \\ 1 \\ 0 \end{array}\right)$$

$$+ V_1 \left(\begin{array}{c} -\frac{3}{2} \\ 0 \\ -2 \\ 1 \end{array}\right) \Bigg\}$$

$$9x_3 + 6 = 0$$

$$x_3 = -\frac{6}{9}$$

$$= -\frac{2}{3}$$

$$x_2 + 2 - 2 = 0$$

$$x_2 = 0$$