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Problem 2

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n} = (-1)^n \underbrace{\left(1 - \frac{1}{n}\right)}_{\to 1}$$

is ~~monotonic~~ monotonically increasing ~~falling~~ as $1 - \frac{1}{n} < 1 - \frac{1}{n+1}$ so according to the Leibniz criteria the series ~~converges to 0~~ diverges.

b) $\sum_{n=1}^{\infty} 5q^{2n}$ with $q \in (-1;1)$

→ geometric series

$5 \cdot \left(\frac{q^2}{1-q^2}\right) =$ converses to $\frac{5q^2}{1-q^2}$

c) Use comparison test:

$\frac{n^3+1}{n^5+1} < \frac{2k^3}{k^5}$ $\quad \frac{2n^3}{n^5} \to \frac{2}{n^2} \to 2 \cdot \frac{1}{n^2}$

$\frac{1}{n^2}$ converges to $\frac{\pi^2}{6}$ so we know that according to the comparison test the series $\frac{n^3+1}{n^5+1}$ must converge.

d) $\sum_{n=1}^{\infty} \left(\sqrt[n]{n} - 1\right)^n =$ root test $\sqrt[n]{\left(\sqrt[n]{n}-1\right)^n}$

$= \sqrt[n]{n} - 1 \to \lim_{n \to \infty} \sqrt[n]{n} - 1 \to 0$

since $0 < 1$ the series converges.

Problem 3

$$\begin{pmatrix} 0 & 1 & 0 & 2 \\ -2 & 0 & 4 & 2 \\ 1 & -3 & -2 & -7 \end{pmatrix} \quad \begin{pmatrix} 1 & -3 & -2 & -7 \\ -2 & 0 & 4 & 2 \\ 0 & 1 & 0 & 2 \end{pmatrix} + 2 I$$

3×4

$$\begin{pmatrix} 1 & -3 & -2 & -7 \\ 0 & -6 & 0 & -12 \\ 0 & 1 & 0 & 2 \end{pmatrix} + \cdot 6\, III \quad + \text{switch } \cancel{\text{rows}} \quad \cdot 6$$

2 pivots $\begin{pmatrix} \boxed{1} & -3 & -2 & -7 \\ 0 & \boxed{1} & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$x_2 = -2x_4$

$x_1 = \cancel{6x_4}\ x_4 + 2x_3$

$$\begin{pmatrix} x_4 + 2x_3 \\ -2x_4 \\ x_3 \\ x_4 \end{pmatrix}$$

$\text{rank}(A) = 2$

$\text{kern}(A) = \text{span}\left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \\ 1 \end{pmatrix} \right\}$

b) Any b not linear combination ~~outside~~ the ran(A)

which is $R(A) = \text{Span}\left\{ \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix} \right\}$
$v_1 \quad v_2$

is not a solution.

~~For example~~ cross product of the v_1 and v_2

$\begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}$ ~~then~~ $6b_1 + 1b_2 + 2b_3 = 0$

take $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} =$

$6 + 0 + 0 = 0$

$6 \neq 0$

so $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = b$ is not lin. comb