(1)

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$$L = \int_0^{2\pi} \sqrt{(x')^2 \cdot +(y')^2}\; dt$$

$$L = \int_0^{2\pi} \sqrt{4\left(\sin^2 t + \cos^2 t - \cancel{2\sin t \cos t}\right) + 4\left(\cos^2 t + \sin^2 t + \cancel{2\sin t \cos t}\right)}$$

$$= \int_0^{2\pi} \sqrt{4\left(2\sin^2 t + 2\cos^2 t\right)}\, dt = 2\sqrt{2} \int_0^{2\pi} \sqrt{(\underbrace{\sin^2 t + \cos^2 t}_{1})}\, dt$$

$$= 2\sqrt{2} \int_0^{2\pi} dt = 2\sqrt{2} \cdot (2\pi - 0) = \underline{\underline{4\sqrt{2}\pi}}$$

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(8)

$$2x = u \qquad 2dx = du \qquad dx = \frac{du}{2}$$

$$A = \int_0^{\pi/4} (\cos^2 x - \sin^2 x)\, dx = \int_0^{\pi/4} (\cos 2x)\, dx$$

$$= \int_0^{\pi/4} \cos u \cdot \frac{du}{2} \Rightarrow \frac{1}{2} \int_0^{\pi/4} \cos u \,.\, du$$

$$= \frac{1}{2}\left(\sin u \Big|_0^{\pi/4}\right) = \frac{1}{2}\left(\sin 2x \Big|_0^{\pi/4}\right) = \frac{1}{2}\left(\sin\frac{\pi}{2} - \underbrace{\cancel{\sin 0}}_{0}\right)$$

$= \frac{1}{2}(1) = \frac{1}{2}$ (A, sadece tüm alanın çeyreği yani tüm alan: $4A$)

$$4A = \text{Tüm alan} = \frac{1}{2} \cdot 4 = \underline{\underline{2}}$$

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(2)

$$I_n = \int_0^{\pi/4} \tan^n(x)\,dx = \int \tan^{(n-2)}(x) \cdot \tan^2(x)\,dx$$

$$\tan^2 x = \sec^2(x) - 1$$

$$\int_0^{\pi/4} \tan^{(n-2)} x \ \tan^2(x)\,dx = \int_0^{\pi/4} \tan^{(n-2)}(x)\left(\sec^2(x) - 1\right) dx$$

$$= \int_0^{\pi/4} \tan^{(n-2)}(x) \cdot \sec^2(x) - \tan^{(n-2)}(x)\,dx$$

$u = \tan x$

$du = \sec^2(x)\,dx$

$$\Rightarrow \int_0^{\pi/4} u^{(n-2)} \cdot du = \frac{u^{n-1}}{n-1}\Bigg|_0^{\pi/4}$$

$$I_n = \int_0^{\pi/4} \tan^n(x)\,dx = \frac{u^{n-1}}{n-1}\Bigg|_0^{\pi/4} - \int_0^{\pi/4} \tan^{(n-2)}(x)\,dx$$

$$I_n = \int_0^{\pi/4} \tan^n(x)\,dx = \frac{\tan^{n-1}(x)}{n-1}\Bigg|_0^{\pi/4} - \int_0^{\pi/4} \tan^{(n-2)}(x) \cdot dx$$

$$= \frac{\tan^{\pi/4 - 1}(x)}{\pi/4 - 1} - \int_0^{\pi/4} \tan^{(n-2)}(x) \cdot dx$$

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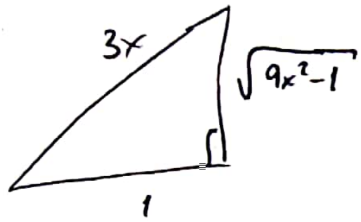
$$\int \frac{1}{(9x^2-1)^{\frac{3}{2}}}\,dx$$

$3x = \sec u$ dersek

$x = \frac{1}{3}\sec u \Rightarrow x^2 = \frac{1}{9}\sec^2 u$

$dx = \frac{1}{3}\sec u \tan u \, du$ olur.



Bu durumda:

$$(9x^2-1)^{3/2} = \left(9\left(\tfrac{1}{3}\sec u\right)^2-1\right)^{3/2}$$

$$= (\sec u^2-1)^{3/2} = \left(\sqrt{\sec^2 u-1}\right)^3$$

$$= \left(\sqrt{\frac{1}{\cos^2 u}-1}\right)^3 = \left(\sqrt{\frac{1-\cos^2 u}{\cos^2 u}}\right)^3 \Rightarrow \left(\sqrt{\frac{\sin^2 u}{\cos^2 u}}\right)^3 = \tan^3 u$$

$$\int \frac{\frac{1}{3}\sec u \tan u}{\tan^3 u}\,du = \frac{1}{3}\int \frac{du}{\cos u \cdot \frac{\sin^2 u}{\cos^2 u}}$$

$$= \frac{1}{3}\int \frac{1}{\sin u}\cdot\frac{\cos u}{\sin u}\,du = \frac{1}{3}\int \operatorname{cosec} u \cdot \cot u \, du$$

$$= -\frac{1}{3}\cdot\operatorname{cosec} u + c$$ (üstteki üçgene göre $\operatorname{cosec} u = \frac{3x}{\sqrt{9x^2-1}}$)

$$= -\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}x}{\sqrt{9x^2-1}} + c \Rightarrow \frac{-x}{\sqrt{9x^2-1}} + c$$

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(11)

$$\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\dots+\frac{1}{1+\frac{n}{n}}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+\frac{k}{n}}$$

$\frac{b-a}{n}=\frac{1}{n} \Rightarrow b-a=1$ olur.

$\left.\begin{matrix} b=1 \\ a=0\end{matrix}\right\}$ alırsak; $f\left(a+\frac{b-a}{n}.k\right)=\frac{1}{1+\frac{k}{n}}$

$f\left(\frac{k}{n}\right)=\frac{1}{1+\frac{k}{n}}$ $\quad \frac{k}{n}=x$ desek $\quad f(x)=\frac{1}{1+x}$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{1+\frac{k}{n}}=\int_0^1\frac{1}{1+x}\,dx \text{ olur.}$$

$$\int_0^1\frac{1}{1+x}\,dx=\ln(|1+x|)\Big|_0^1=\ln(2)-\underbrace{\ln(1)}_{0}$$

$$=\underline{\underline{\ln(2)}}$$

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(6)

$$\int \frac{\sqrt{1+\cos x}\, dx \cdot \sqrt{1-\cos x}}{\sqrt{1-\cos x}} = \int \frac{\sin x\, dx}{\sqrt{1-\cos x}}$$

$1-\cos x = t^2$ $\Rightarrow$ iki tarafında türevini alırsak

$$\sin x\, dx = 2t\, dt \Rightarrow \int \frac{\sin x\, dx}{\sqrt{1-\cos x}} = \int \frac{2t\, dt}{t} = 2\int dt$$

$= 2t + c$

$t = \sqrt{1-\cos x} \Rightarrow \underline{\underline{2\sqrt{1-\cos x} + c}}$

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(5) $\displaystyle\int \frac{dx}{(x+\frac{1}{2})^2 - \frac{5}{4}}$ $\qquad (x+\frac{1}{2}) = u$

$\qquad dx = du$

$$= \int \frac{du}{u^2 - \underbrace{\tfrac{5}{4}}_{\left(\frac{\sqrt{5}}{2}\right)^2}} = \frac{1}{2 \cdot \frac{\sqrt{5}}{2}} \cdot \ln\left(\frac{u - \sqrt{5}/2}{u + \sqrt{5}/2}\right) + c$$

$\underline{u = x + \frac{1}{2}}$

$$= \frac{1}{\sqrt{5}} \cdot \ln\left(\frac{x+\frac{1}{2} - \frac{\sqrt{5}}{2}}{x+\frac{1}{2} + \frac{\sqrt{5}}{2}}\right) + c$$

$$= \frac{1}{\sqrt{5}} \cdot \ln\left(\frac{2x+1-\sqrt{5}}{2x+1+\sqrt{5}}\right) + c$$

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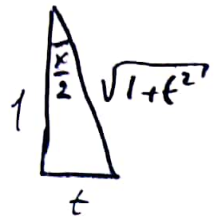
(4)

$\tan\frac{x}{2} = t$ dönüşümü yaparsak



$$\sin\frac{x}{2} = \frac{t}{\sqrt{1+t^2}}$$

$$\cos\frac{x}{2} = \frac{1}{\sqrt{1+t^2}}$$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} \Rightarrow 2\cdot\frac{t}{\sqrt{1+t^2}}\cdot\frac{1}{\sqrt{1+t^2}} \Rightarrow \sin x = \frac{2t}{1+t^2}$$

Bu durumda $\cos x = \frac{1-t^2}{1+t^2}$, $\tan\frac{x}{2} = t \Rightarrow \frac{x}{2} = \arctan t$

$$\frac{1}{2}dx = \frac{1}{1+t^2}dt \Rightarrow dx = \frac{2dt}{1+t^2}$$

$$\int\frac{dx}{4\sin x - 3\cos x + 5} = \int\frac{\frac{2dt}{1+t^2}}{4\cdot\frac{2t}{1+t^2} - 3\cdot\frac{1-t^2}{1+t^2} + 5}$$

$$= 2\cdot\int\frac{\frac{dt}{\cancel{1+t^2}}}{\frac{8t+8t^2+2}{\cancel{1+t^2}}} = \frac{2}{2}\int\frac{dt}{(2t+1)^2} \Rightarrow \begin{matrix} 2t+1 = u \\ 2dt = du \\ dt = \frac{du}{2} \end{matrix}$$

$$= \frac{1}{2}\int\frac{du}{u^2} \Rightarrow \frac{1}{2}\cdot\frac{-1}{u} + c$$

$$= \frac{1}{2}\cdot\frac{-1}{2\tan\frac{x}{2}+1} + c$$

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(10)

x=1 için integral sınırları aynı değer olduğundan integralin değeri sıfırdır

$$\int_1^1 e^{\sin\frac{\pi t}{2}}\,dt = 0$$

$$\lim_{x\to 1}\frac{\int_1^x e^{\sin\frac{\pi t}{2}}\,dt}{\ln x} \Rightarrow \frac{0}{0}$$ belirsizliği olur.

$$\lim_{x\to 1}\frac{\left(\int_1^x e^{\sin\frac{\pi t}{2}}\,dt\right)'}{(\ln x)'} = \lim_{x\to 1}\frac{e^{\sin\frac{\pi x}{2}}(x)' - e^{\sin\frac{\pi}{2}}(1)'}{\frac{1}{x}}$$

$$= \lim_{x\to 1}\frac{e^{\sin\frac{\pi x}{2}} - \cancel{e\cdot 0}}{\frac{1}{x}} = \frac{e^{\sin\frac{\pi}{2}}}{1} = e^1 = \underline{e}$$

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(12) $$\int_0^1 \frac{x}{e^x} - \int_0^1 \frac{x}{e}$$

$$= -e^{-x} - xe^{-x}\Big|_0^1 - \frac{1}{2e}x^2\Big|_0^1 \Rightarrow$$

$$\left(\left(-\frac{1}{e} - \frac{1}{e}\right) - (-1)\right) - \left(\frac{1}{2e} - 0\right)$$

$$= -\frac{2}{e} + 1 - \frac{1}{2e} \Rightarrow 1 - \frac{5}{2e} = \frac{2e-5}{2e}$$

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(2) $\displaystyle\int_0^{2\pi} \sqrt{\sin^2 2x}\, dx = \int_0^{2\pi} |\sin 2x|\, dx$

$$= \int_0^{\pi/2} |\sin 2x|\, dx + \int_{\pi/2}^{\pi} |\sin 2x|\, dx + \int_0^{3\pi/2} |\sin 2x|\, dx + \int_{3\pi/2}^{2\pi} |\sin 2x|\, dx$$

$\sin 2x = 2\sin x \cos x$

$\sin x = u$

$\cos x\, dx = du$

$$2\int_0^{\pi/2} u\, du - 2\int_{\pi/2}^{\pi} u\, du + 2\int_{\pi}^{\frac{3\pi}{2}} u\, du - 2\int_{\frac{3\pi}{2}}^{2\pi} u\, du$$

$$= u^2 \Big|_0^{\pi/2} - u^2 \Big|_{\frac{\pi}{2}}^{\pi} + u^2 \Big|_{\pi}^{\frac{3\pi}{2}} - u^2 \Big|_{\frac{3\pi}{2}}^{2\pi}$$

$$= \sin^2 x \Big|_0^{\frac{\pi}{2}} - \sin^2 x \Big|_{\frac{\pi}{2}}^{\pi} + \sin^2 x \Big|_{\pi}^{\frac{3\pi}{2}} - \sin^2 x \Big|_{\frac{3\pi}{2}}^{2\pi}$$

$$= (1-0) - (0-1) + (1-0) - (0-1)$$

$$\underline{\underline{= 4}}$$

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$3\sin\phi = 2 - \sin\phi$

$4\sin\phi = 2$

$\sin\phi = \frac{1}{2}$

$\phi = \frac{\pi}{6}$

$$A_1 = \frac{1}{2}\int_0^{\pi/6} (3\sin\phi)^2 \, d\phi = \frac{9}{2}\int_0^{\pi/6} \sin^2\phi \, d\phi$$

$$A_2 = \frac{1}{2}\int_0^{\pi/6} (2-\sin\phi)^2 \, d\phi$$

$$= \frac{1}{2}\int_0^{\pi/6} (4 - 4\sin\phi + \sin^2\phi) \, d\phi$$

$$A_2 = 2\int_0^{\pi/6} d\phi - 2\int \sin\phi \, d\phi + \frac{1}{2}\int_0^{\pi/6} \sin^2\phi \, d\phi$$

$$A_1 + A_2 = 2\int_0^{\pi/6} d\phi - 2\int_0^{\pi/6} \sin\phi \, d\phi + 5\int_0^{\pi/6} \sin^2\phi \, d\phi$$ $\rightarrow 1-\cos^2\phi$

$$= 2.\phi\Big|_0^{\pi/6} - 2.\cos\phi\Big|_0^{\pi/6} + 5.\phi\Big|_0^{\pi/6} - 5.-\frac{1}{2}\sin 2\phi\Big|_0^{\pi/6}$$

$$= 7(\pi/6 - 0) - 2\left(\frac{\sqrt{3}}{2} - 1\right) - 5\left(-\frac{\sqrt{3}}{4} - 0\right)$$

$$= \frac{7\pi}{6} - \sqrt{3} + 2 + \frac{5\sqrt{3}}{4} \Rightarrow \frac{24 + 3\sqrt{3} + 14\pi}{12}$$