Part 2

Report Biomedical Signal Processing

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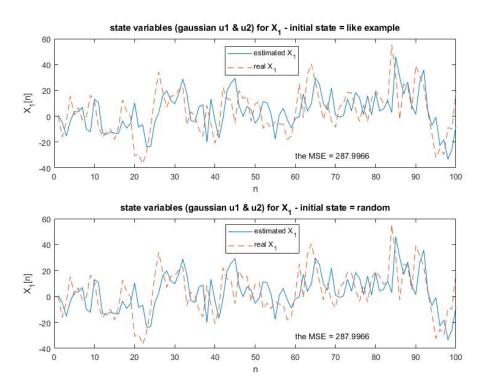
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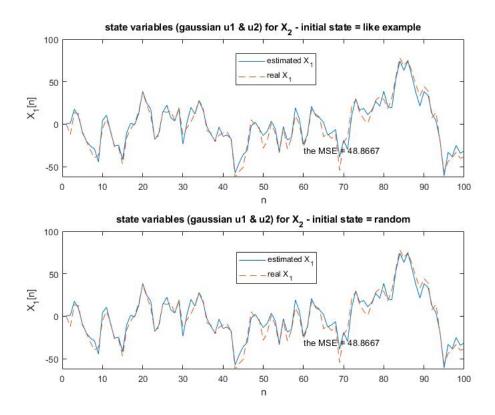
1 Question 1

- -Above all I should mention that I've answered the questions of part d and e in earlier three parts.
- -Also the evaluation criteria is shown on the plot.
- -At the end of the question I've explained the answers.

1.1 part a,d,e

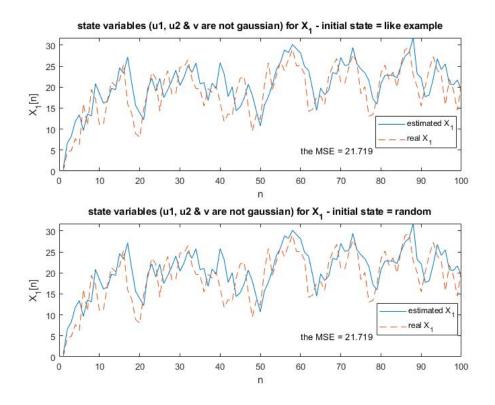
- The estimation of state variables by initial states like "example 2" and random initial states = [0.65,0.78], MSE as Evaluation criteria:

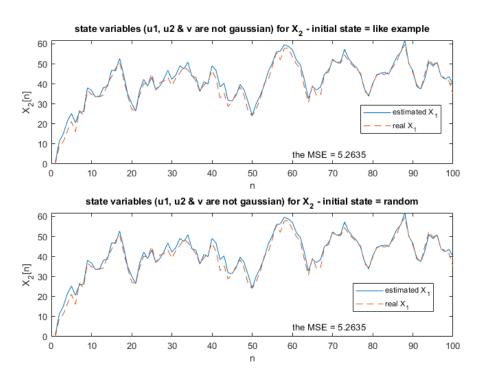




1.2 part b,d,e

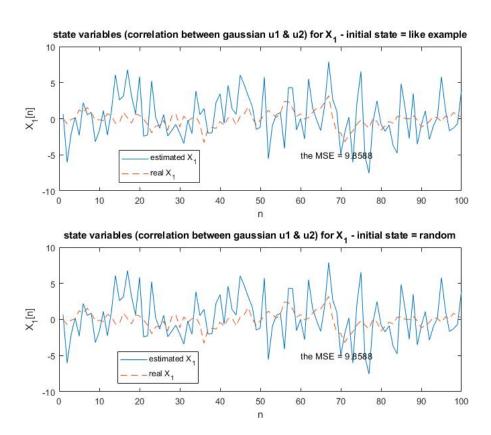
- The estimation of state variables by initial states like "example 2" and random initial states = [0.65, 0.78], MSE as Evaluation criteria when noises are not gaussian:

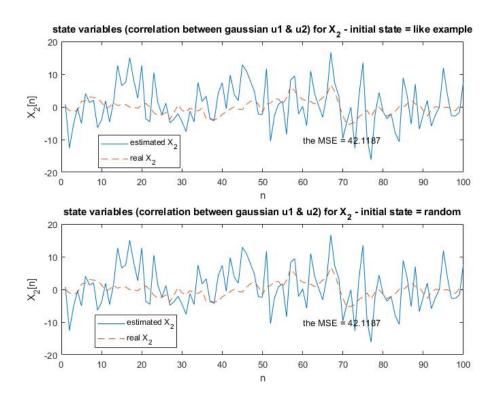




1.3 part c,d,e

- The estimation of state variables by initial states like "example 2" and random initial states = [0.65,0.78], MSE as Evaluation criteria when u1,u2 noises are correlated:





1.4 Explanation

Except for part C, where the noises u_1 and u_2 are correlated, the estimation of the state variables is excellent, and our Kalman filter's estimations closely follow the real values. The estimation of X_2 is particularly accurate, likely due to the fact that two filters are applied.

Matlab Code

```
1 clc; clear
2 %% inputs
3 a = 0.6;
4 b = 0.4;
5 u1_std = sqrt(12)*4;
6 u2_std = sqrt(12)*3;
```

```
_{7} v std = sqrt(12)*2;
_{8} k = 100;
9 u1 a = u1 std * randn(1, k);
u2 \ a = u2 \ std * randn(1, k);
v = v \text{ std } * randn(1, k);
u1 b = u1 std * rand(1, k);
u2 b = u2 std * rand(1, k);
v b = v std * rand(1, k);
15 \text{ rho} = 0.8;
x1 = randn(k, 1);
x2 = randn(k, 1);
_{18} R = [1 \text{ rho}; \text{ rho } 1];
_{19} L = chol(R, 'lower');
20 correlated noises = L * [x1 x2]';
u1 c = correlated noises(1, :);
u2_c = correlated_noises(2, :);
v_c = v_std * randn(1, k);
est_x0_handout = [0, 0];
25 est x0 random = [0.65, 0.78];
26 % part a
_{27} [~,~,est_x_a,x1_a,x2_a, ~, ~, ~, ~] = kalmanmanf_part(
     a, b, u1 std, u2 std, v std, u1 a, u2 a, v a,
     est x0 handout);
[\sim,\sim,est_x_da,x1_da,x2_da,\sim,\sim,\sim,\sim] =
     kalmanmanf_part(a, b, u1_std, u2_std, v_std, u1_a,
     u2 a, v a, est x0 random);
29 figure;
subplot(2,1,1)
_{31} plot(est x a(1,:))
32 hold on
33 plot(x1_a, '--')
```

```
legend ('estimated X 1', 'real X 1')
35 title ('state variables (gaussian ul & u2) for X 1 -
     initial state = like example')
36 ylabel('X 1[n]')
37 xlabel('n')
38 MSE = mean((est x a(1,:)-x1 a).^2,2);
_{39} text (60, -30, ['the MSE = ', num2str(MSE)])
subplot(2,1,2)
plot(est_x_da(1,:))
42 hold on
43 plot (x1 da, '---')
44 legend ('estimated X 1', 'real X 1')
45 title ('state variables (gaussian ul & u2) for X_1 -
     initial state = random')
46 ylabel('X 1[n]')
47 xlabel('n')
48 MSE = mean((est_x_da(1,:)-x1_da).^2,2);
49 text(60, -30, ['the MSE = ', num2str(MSE)])
50 % part d
51 figure;
subplot(2,1,1)
_{53} plot(est x a (2,:))
54 hold on
55 plot(x2 a, '--')
s6 legend('estimated X_1', 'real X_1')
57 title ('state variables (gaussian ul & u2) for X 2 -
     initial state = like example')
58 ylabel('X 1[n]')
59 xlabel('n')
60 MSE = mean((est x a(2,:)-x2 a).^2,2);
_{61} text (60, -30, ['the MSE = ', num2str(MSE)])
```

```
subplot(2,1,2)
plot(est_x_da(2,:))
64 hold on
65 plot (x2 da, '--')
66 legend('estimated X_1', 'real X_1')
67 title ('state variables (gaussian ul & u2) for X 2 -
     initial state = random')
68 ylabel('X 1[n]')
69 xlabel('n')
70 MSE = mean((est_x_da(2,:)-x2_da).^2,2);
_{71} text(60,-30,['the MSE = ', num2str(MSE)])
72 %% part b
[\sim, \sim, est_x_b, x1_b, x2_b, \sim, \sim, \sim, \sim] = kalmanmanf_part(
     a, b, u1_std, u2_std, v_std, u1_b, u2_b, v_b,
     est x0 handout);
[\sim, \sim, est_x_db, x1_db, x2_db, \sim, \sim, \sim, \sim] =
     kalmanmanf_part(a, b, u1_std, u2_std, v_std, u1_b,
     u2_b, v_b, est_x0_random);
75 figure;
_{76} subplot (2,1,1)
plot(est x b(1,:))
78 hold on
79 plot (x1 b, '--')
80 legend ('estimated X 1', 'real X 1')
81 title ('state variables (u1, u2 & v are not gaussian)
     for X 1 - initial state = like example')
82 ylabel('X 1[n]')
83 xlabel('n')
MSE = mean((est x b(1,:)-x1 b).^2,2);
_{85} text (60,5,['the MSE = ', num2str (MSE)])
subplot(2,1,2)
```

```
plot(est x db(1,:))
  hold on
  plot(x1 db, '--')
90 legend ('estimated X 1', 'real X 1')
91 title ('state variables (u1, u2 & v are not gaussian)
     for X 1 - initial state = random')
  ylabel('X 1[n]')
  xlabel('n')
94 MSE = mean((est_x_db(1,:)-x1 db).^2,2);
  text(60,5,['the MSE = ', num2str(MSE)])
  %
96
  figure;
  subplot(2,1,1)
  plot(est_x_b(2,:))
  hold on
100
  plot(x2 b, '--')
  legend('estimated X_1', 'real X_1')
  title ('state variables (u1, u2 & v are not gaussian)
103
     for X 2 - initial state = like example')
  ylabel('X 2[n]')
  xlabel('n')
105
  MSE = mean((est x b(2,:)-x2 b).^2,2);
  text(60,5,['the MSE = ', num2str(MSE)])
  subplot(2,1,2)
108
  plot(est x db(2,:))
  hold on
  plot (x2 db, '--')
  legend('estimated X_1', 'real X_1')
  title ('state variables (u1, u2 & v are not gaussian)
     for X 2 - initial state = random')
114 ylabel('X_2[n]')
```

```
xlabel('n')
  MSE = mean((est x db(2,:)-x2 db).^2,2);
  text(60,5,['the MSE = ', num2str(MSE)])
118 % part c
[\sim, \sim, \text{est x c}, \text{x1 c}, \text{x2 c}, \sim, \sim, \sim] = \text{kalmanmanf partc}
      (a, b, u1 std, u2 std, v std, u1 c, u2 c, v c,
      est x0 handout, rho);
[\sim, \sim, est \ x \ dc, x1 \ dc, x2 \ dc, \sim, \sim, \sim, \sim] =
      kalmanmanf partc(a, b, u1 std, u2 std, v std, u1 c,
       u2 c, v c, est x0 random, rho);
  figure;
121
  subplot(2,1,1)
122
  plot(est_x_c(1,:))
  hold on
  plot(x1 c, '--')
  legend('estimated X_1', 'real X_1')
  title ('state variables (correlation between gaussian
      ul & u2) for X 1 - initial state = like example')
  ylabel('X_1[n]')
  xlabel('n')
  MSE = mean((est x c(1,:)-x1 c).^2,2);
  text(60, -5, ['the MSE = ', num2str(MSE)])
  subplot(2,1,2)
  plot(est x dc(1,:))
  hold on
  plot(x1 dc, '--')
  legend('estimated X 1', 'real X 1')
  title ('state variables (correlation between gaussian
     u1 & u2) for X 1 - initial state = random')
  ylabel('X 1[n]')
  xlabel('n')
```

```
MSE = mean((est x dc(1,:)-x1 dc).^2,2);
  text(60, -5, ['the MSE = ', num2str(MSE)])
  %
142
  figure;
143
  subplot(2,1,1)
  plot(est x c(2,:))
  hold on
  plot (x2 c, '--')
  legend('estimated X 2', 'real X 2')
  title ('state variables (correlation between gaussian
     u1 & u2) for X 2 - initial state = like example')
  ylabel('X_2[n]')
150
  xlabel('n')
  MSE = mean((est_x_c(2,:)-x2_c).^2,2);
  text(60, -5*2, ['the MSE = ', num2str(MSE)])
153
  subplot(2,1,2)
  plot(est_x_dc(2,:))
  hold on
156
  plot (x2 dc, '--')
  legend('estimated X 2', 'real X 2')
  title ('state variables (correlation between gaussian
159
     u1 & u2) for X 2 - initial state = random')
  ylabel('X 2[n]')
  xlabel('n')
161
  MSE = mean((est_x_dc(2,:)-x2_dc).^2,2);
  text(60, -5*2, ['the MSE = ', num2str(MSE)])
  % the function for kalman filter
  function [estimated_z, real_output, est_x, x1, x2,
     est xminus, est p, est pminus, est G] =
     kalmanmanf part(a, b, std1, std2, stdv, u1, u2, v,
     est x0)
```

```
k = 100;
166
       H1 = tf([1], [1 -a], 1);
167
       H2 = tf([1], [1 -b], 1);
168
       x1 = filter(H1.Numerator\{1\}, H1.Denominator\{1\}, u1
          );
       x2 = filter(H2. Numerator \{1\}, H2. Denominator \{1\}, x1
170
           + u2);
       real output = x2 + v;
171
       est x = zeros(2, k);
172
       est xminus = zeros(2, k);
173
       est p = zeros(2, 2, k);
174
       est pminus = zeros(2, 2, k);
175
       est_G = zeros(2, 1, k);
176
       est p(:, :, 1) = [std1^2/(1-a^2), std1^2/((1-a^2))]
177
          *(1-a*b)); std1^2/((1-a^2)*(1-a*b)), (std1
          ^2+(1-a^2)*std2^2)/((1-a^2)*(1-b^2));
       est x(:, 1) = est x0;
178
       est x(:, 1) = [0.65; 0.78];
179
       F = [a, 0; a, b];
180
       Q = [std1^2, std1^2, std1^2, std1^2 + std2^2];
181
       R = stdv^2;
182
       H = [0,1];
183
       estimated z = zeros(1,k);
184
       estimated z(1,1) = H*est x(:,1) + v(1);
185
       for i = 2:k
186
           est xminus(:, i) = F * est x(:, i-1);
187
           est pminus (:, :, i) = F * est p(:, :, i-1) * F
              ' + Q;
           Gk = est pminus(:, :, i) * H' / (H *
189
               est pminus (:, :, i) * H' + R;
```

```
est x(:, i) = est xminus(:, i) + Gk * (
190
               real_output(i) - H * est_xminus(:, i));
           est_p(:, :, i) = est_pminus(:, :, i) - Gk * H
191
               * est pminus(:, :, i);
           est G(:, :, i) = Gk;
192
            estimated z(1,i) = H*est x(:,i) + v(i);
193
       end
  end
195
196
   function [estimated z, real output, est x, x1, x2,
      est_xminus, est_p, est_pminus, est_G] =
      kalmanmanf partc(a, b, std1, std2, stdv, u1, u2, v
      , est_x0 , rho)
       k = 100;
198
       H1 = tf([1], [1 -a], 1);
199
       H2 = tf([1], [1 -b], 1);
200
       x1 = filter(H1. Numerator \{1\}, H1. Denominator \{1\}, u1
201
          );
       x2 = filter(H2. Numerator \{1\}, H2. Denominator \{1\}, x1
202
           + u2);
       real output = x2 + v;
203
       est x = zeros(2, k);
204
       est xminus = zeros(2, k);
205
       est p = zeros(2, 2, k);
206
       est pminus = zeros(2, 2, k);
207
       est G = zeros(2, 1, k);
208
       est p(:, :, 1) = [std1^2/(1-a^2), std1*std2*rho]
209
          /((1-a^2)*(1-a*b)); std1*std2*rho/((1-a^2)*(1-a)
          *b)), (std1^2+std2^2)/((1-b^2));
       est x(:, 1) = est x0;
210
       est x(:, 1) = [0.65; 0.78];
211
```

```
F = [a, 0; a, b];
212
       Q = [std1^2, std1*std2*rho; std1*std2*rho, std1^2]
213
          + std2^2];
       R = stdv^2;
214
       H = [0,1];
215
       estimated z = zeros(1,k);
216
       estimated z(1,1) = H*est x(:,1) + v(1);
       for i = 2:k
218
           est_xminus(:, i) = F * est_x(:, i-1);
219
           est_pminus(:, :, i) = F * est_p(:, :, i-1) * F
220
              ' + Q;
           Gk = est pminus(:, :, i) * H' / (H *
221
              est_pminus(:, :, i) * H' + R);
           est_x(:, i) = est_xminus(:, i) + Gk * (
222
              real_output(i) - H * est_xminus(:, i));
           est_p(:, :, i) = est_pminus(:, :, i) - Gk * H
223
              * est_pminus(:, :, i);
           est_G(:, :, i) = Gk;
224
           estimated_z(1,i) = H*est_x(:,i) + v(i);
225
       end
226
227 end
```

2 Question 2

- Explanation of each part located at the end of them.

2.1 part a

I've done it by below code and the vector at each feature : **code**

```
feature1 = zeros(1, train num);
feature2 = zeros(1, train num);
 feature3 = zeros(1, train num);
  mean train = zeros(1, train num);
  for i = 1: train num
       if i \le 9
           field_name = ['ECGO' num2str(i)];
       e1se
           field_name = ['ECG' num2str(i)];
      end
10
      mean train(i) = mean(ecg.(field name));
11
      lower_than_m= [];
12
      higher_than_m = [];
13
      k = 0;
      1 = 0;
15
       for j = 1:length(ecg.(field_name))
16
           if ecg.(field_name)(j)<mean_train(i)</pre>
              k = k+1;
18
              lower_than_m(k) = ecg.(field_name)(j);
19
           e1se
20
              1 = 1+1;
              higher than m(1) = ecg.(field name)(j);
22
           end
23
      end
```

28 end

feature1

feature2

```
Teature2 =

Columns 1 through 19

-0.002 -0.0210 -0.0264 -0.0176 -0.0154 -0.0161 -0.0506 -0.0343 -0.0169 -0.0186 -0.0114 -0.0229 -0.0184 -0.0164 -0.0176 -0.0164 -0.0174 -0.0188 -0.0066

Columns 20 through 38

-0.0550 -0.0089 -0.0062 -0.0039 -0.0069 -0.0035 -0.0049 -0.0049 -0.0046 -0.0036 -0.0072 -0.0038 -0.0064 -0.0062 -0.0046 -0.0040 -0.0049 -0.0064

Columns 39 through 40

-0.0126 -0.0056
```

feature3

```
Execute3 =

Columns 1 through 19

0.1337 0.1451 0.1627 0.1843 0.1491 0.1482 0.1698 0.1706 0.1464 0.1412 0.1756 0.2098 0.1503 0.1235 0.1473 0.1466 0.1493 0.1559 0.1947

Columns 10 through 38

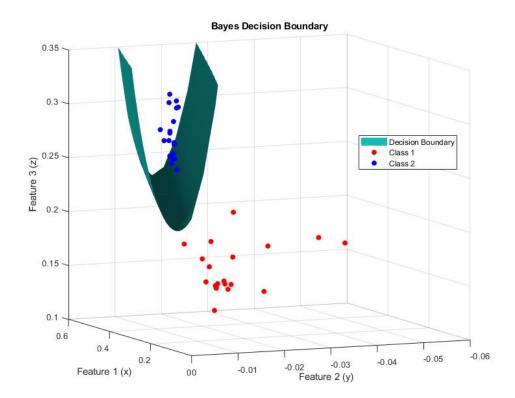
0.1677 0.2089 0.2027 0.2669 0.3183 0.2741 0.2055 0.2063 0.3255 0.2060 0.2666 0.2066 0.3271 0.3030 0.2757 0.2724 0.2942 0.2507 0.3174

Columns 39 through 40

0.3068 0.2928
```

2.2 part b

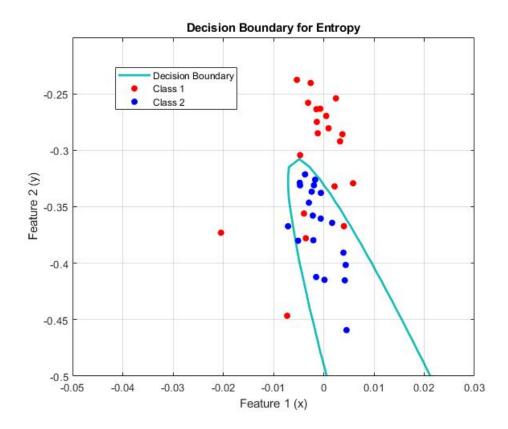
I have done classification by bayes decision boundary and using the equation from handout:



As we can see they are classified correctly.

2.3 part c

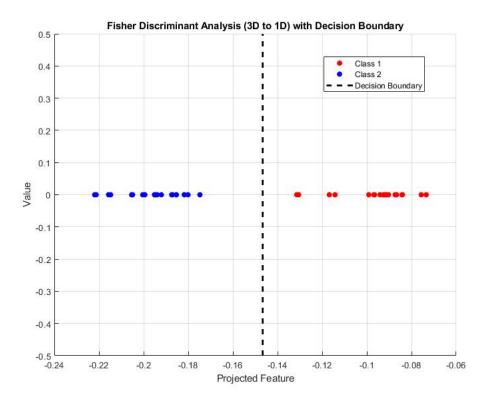
I have done classification by entropy reduction and using the equation from handout:



As we can see they are classified correctly but there are some features which wrongly labeled.

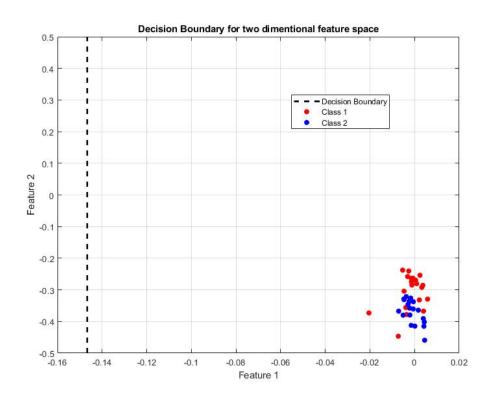
2.4 part d

I have done classification by FLD and using the equation from handout:



As we can see they are classified correctly.

-Now if i classify the 2D feature space we can see they are not classified correctly at all:



2.5 part e

-I have done classification by features ,which are described in question, for test dataset and these are the accuracy of them:

```
accuracy by mahlanobis (part b) :
  97.5000
accuracy by Euclidean (part b) :
  97.5000
accuracy by bayes decision boundary (part b) :
  97.5000
accuracy by mahlanobis (part c) :
  82.5000
accuracy by Euclidean (part c) :
    85
accuracy by bayes decision boundary (part c) :
   92.5000
accuracy by mahlanobis (part d) :
  97.5000
accuracy by Euclidean (part d) :
  97.5000
accuracy by bayes decision boundary (part d) :
```

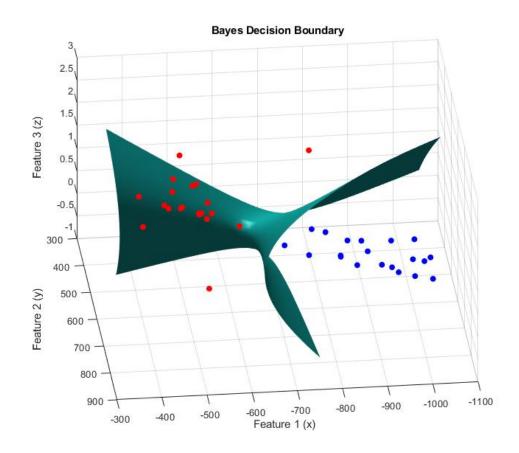
2.6 part f

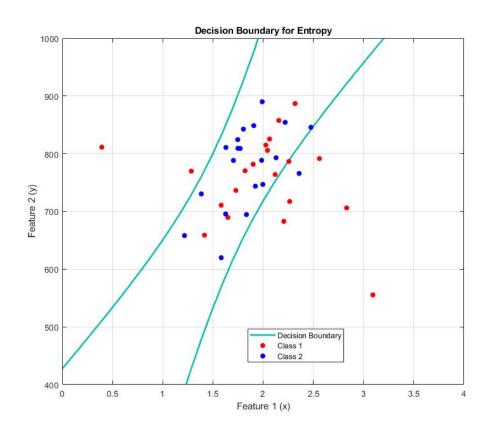
Now I repeat the classification for three new features:

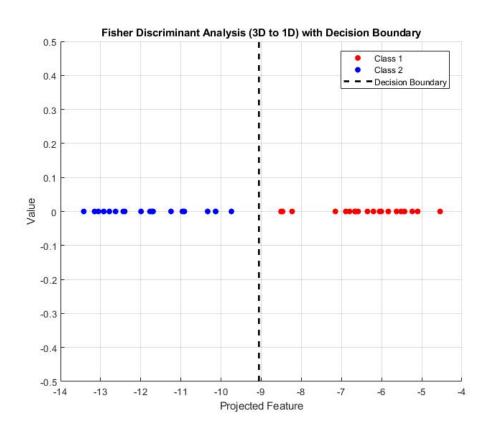
feature1: T amp wave **feature1:** P amp wave

feature1: std

Now I have repeated all parts ass below:







```
accuracy by mahlanobis (part b) :
   95
accuracy by Euclidean (part b) :
   95
accuracy by bayes decision boundary (part b) :
  97.5000
accuracy by mahlanobis (part c) :
  72.5000
accuracy by Euclidean (part c) :
   95
accuracy by bayes decision boundary (part c) :
  82.5000
accuracy by mahlanobis (part d) :
  100
accuracy by Euclidean (part d) :
  100
accuracy by bayes decision boundary (part d) :
  100
______
```

Matlab Code

```
clc; clear
clc; clear
ceg = load('ECG.mat');
train_num = 40;
test_num = 40;
part a
feature1 = zeros(1, train_num);
feature2 = zeros(1, train_num);
feature3 = zeros(1, train_num);
```

```
mean train = zeros(1, train num);
  for i = 1:train_num
       if i \le 9
11
           field name = ['ECG0' num2str(i)];
12
       e1se
13
           field_name = ['ECG' num2str(i)];
14
      end
       mean train(i) = mean(ecg.(field name));
16
      lower than m = [];
17
      higher_than_m = [];
18
      k = 0;
19
      1 = 0;
20
       for j = 1:length(ecg.(field_name))
21
           if ecg.(field_name)(j)<mean_train(i)</pre>
22
              k = k+1;
23
              lower_than_m(k) = ecg.(field_name)(j);
24
           e1se
25
              1 = 1+1;
26
              higher than m(1) = ecg.(field name)(j);
27
           end
28
      end
29
       feature1(i) = mean(lower than m) / min(ecg.(
30
          field name));
       feature2(i) = mean(lower than m)/var(lower than m)
31
       feature 3 (i) = (mean(higher than m)-mean(
32
          lower_than_m))/(max(ecg.(field_name))-min(ecg.(
          field name)));
33 end
34 %% part b
class1_featurematrix = [feature1(1:20), feature2(1:20)
```

```
' feature3 (1:20) '];
mean1 mat = mean(class1 featurematrix);
 cov1 mat = cov(class1 featurematrix);
 class2 featurematrix = [feature1(21:40)' feature2
     (21:40) ' feature3 (21:40) '];
 mean2 mat = mean(class2 featurematrix);
 cov2 mat = cov(class2 featurematrix);
cov mat train = cov([feature1(1:40)' feature2(1:40)']
     feature3 (1:40) ']);
_{42} x range = linspace(0, 0.6, 40);
_{43} y range = linspace(0, -0.1, 40);
z_{range} = linspace(0, 0.35, 40);
45 [X, Y, Z] = meshgrid(x_range, y_range, z_range);
D1 = zeros(size(X));
D2 = zeros(size(X));
x_{1} = X(:);
y_flat = Y(:);
_{50} z flat = Z(:);
  for i = 1:length(x flat)
      beta = [x flat(i); y flat(i); z flat(i)];
      D1(i) = -0.5 * log(det(cov1 mat)) - 0.5 * (beta -
         mean1_mat')' * inv(cov1_mat) * (beta -
         mean1 mat');
      D2(i) = -0.5 * log(det(cov2 mat)) - 0.5 * (beta -
54
         mean2 mat') ' * inv(cov2 mat) * (beta -
         mean2 mat');
55 end
_{56} D1 = reshape(D1, size(X));
D2 = reshape(D2, size(X));
_{58} Difference = D1 - D2;
59 figure;
```

```
isosurface (X, Y, Z, Difference, 0);
61 hold on;
  scatter3 (class1 featurematrix (:,1),
     class1 featurematrix (:,2), class1 featurematrix
     (:,3), 'r', 'filled');
63 hold on;
64 scatter3 (class2 featurematrix (:,1),
     class2 featurematrix (:,2), class2 featurematrix
     (:,3), 'b', 'filled');
s xlabel('Feature 1 (x)');
of ylabel('Feature 2 (y)');
zlabel('Feature 3 (z)');
68 title ('Bayes Decision Boundary');
69 grid on;
70 legend({'Decision Boundary', 'Class 1', 'Class 2'}, '
     Location', 'best')
71 %% part c
_{72} [evc1, eva1] = eig(cov1_mat);
[evc2, eva2] = eig(cov2_mat);
74 [evc, eva] = eig(cov mat train);
u = evc(:,1:2);
u1 = evc1(:,1:2); % Assuming these are the principal
     components
u2 = evc2(:,1:2);
78 class1 featurematrix entropy = class1 featurematrix *
     u;
79 class2_featurematrix_entropy = class2_featurematrix *
 mean1 mat entropy = mean(class1 featurematrix entropy)
mean2_mat_entropy = mean(class2_featurematrix_entropy)
```

```
82 cov1 mat entropy = cov(class1 featurematrix entropy);
83 cov2 mat entropy = cov(class2 featurematrix entropy);
_{84} x range = linspace(-0.05, 0.03, 40);
 y_range = linspace(-0.5, -0.2, 40);
 [Xent, Yent] = meshgrid(x range, y range);
D1 = zeros(size(Xent));
D2 = zeros(size(Xent));
 x flat = Xent(:);
  y flat = Yent(:);
  for i = 1:length(x flat)
      beta = [x flat(i); y flat(i)];
92
      D1(i) = -0.5 * log(det(cov1_mat_entropy)) - 0.5 *
         (beta - mean1 mat entropy)' * inv(
         cov1 mat entropy) * (beta - mean1 mat entropy);
      D2(i) = -0.5 * log(det(cov2_mat_entropy)) - 0.5 *
         (beta - mean2_mat_entropy)' * inv(
         cov2_mat_entropy) * (beta - mean2_mat_entropy);
95 end
_{96} D1 = reshape(D1, size(Xent));
 D2 = reshape(D2, size(Xent));
 Difference_ent = D1 - D2;
 figure;
  contour (Xent, Yent, Difference ent, [0, 0], 'LineWidth
     ', 2);
  hold on;
  scatter(class1 featurematrix entropy(:, 1),
     class1 featurematrix entropy(:, 2), 'r', 'filled');
  hold on;
  scatter(class2 featurematrix entropy(:, 1),
     class2_featurematrix_entropy(:, 2), 'b', 'filled');
```

```
xlabel('Feature 1 (x)');
  ylabel('Feature 2 (y)');
  title ('Decision Boundary for Entropy');
107
  grid on;
  legend({'Decision Boundary', 'Class 1', 'Class 2'}, '
     Location', 'best');
  % part d
111
  class1 featurematrix = [feature1(1:20)' feature2(1:20)
112
     ' feature3 (1:20) '];
  class2 featurematrix = [feature1(21:40)' feature2
     (21:40) ' feature3 (21:40) '];
  mean1 = mean(class1 featurematrix);
  mean2 = mean(class2 featurematrix);
  SW = cov(class1 featurematrix) + cov(
     class2 featurematrix);
  mean diff = mean1 - mean2;
  S_B = mean_diff * mean_diff';
  [V, D] = eig(SW \setminus SB);
  [\sim, idx] = max(diag(D));
  w = V(:, idx);
121
  projected_class1 = class1_featurematrix * w;
  projected class2 = class2 featurematrix * w;
  mean proj1 = mean(projected class1);
124
  mean proj2 = mean(projected class2);
125
  var proj1 = var(projected class1);
  var proj2 = var(projected class2);
  pooled variance = ((length(projected class1) - 1) *
     var proj1 + (length (projected class2) - 1) *
     var proj2) / (length (projected class1) + length (
     projected_class2) - 2);
```

```
decision boundary = (mean proj1 + mean proj2) / 2;
  figure;
130
  hold on;
131
  scatter(projected class1, zeros(size(projected class1)
     ), 'r', 'filled');
  scatter (projected class2, zeros (size (projected class2)
     ), 'b', 'filled');
  plot([decision_boundary decision_boundary], [-0.5
     0.5], 'k--', 'LineWidth', 2);
  xlabel('Projected Feature');
  ylabel('Value');
  title ('Fisher Discriminant Analysis (3D to 1D) with
137
     Decision Boundary');
  legend({'Class 1', 'Class 2', 'Decision Boundary'}, '
     Location', 'best');
  grid on;
  figure;
140
  plot ([decision boundary decision boundary], [-0.5]
     0.5], 'k--', 'LineWidth', 2);
  hold on;
  scatter(class1 featurematrix entropy(:, 1),
     class1 featurematrix entropy(:, 2), 'r', 'filled');
  hold on;
  scatter(class2 featurematrix entropy(:, 1),
     class2 featurematrix entropy(:, 2), 'b', 'filled');
  xlabel('Feature 1');
  ylabel('Feature 2');
  title ('Decision Boundary for two dimentional feature
     space');
  grid on;
  legend({'Decision Boundary', 'Class 1', 'Class 2'}, '
```

```
Location', 'best')
  % part e
  feature1test = zeros(1, test num);
   feature2test = zeros(1, test num);
                 = zeros(1, test num);
  feature3test
154
  mean test = zeros(1, test num);
155
   for i = 41:80
       field name = ['ECG' num2str(i)];
157
       mean test (i-40) = mean(ecg.(field name));
158
       lower than mtest= [];
159
       higher than mtest = [];
160
       k = 0;
161
       1 = 0;
162
       for j = 1:length(ecg.(field name))
163
           if ecg. (field name) (j) < mean test (i-40)
164
              k = k+1;
165
               lower than mtest(k) = ecg. (field name)(j);
           e1se
167
               1 = 1+1;
168
               higher than mtest(1) = ecg.(field name)(j);
169
           end
170
       end
171
       feature ltest (i-40) = mean(lower than mtest) / min
172
          (ecg.(field name));
       feature2test (i-40) = mean(lower than mtest)/var(
173
          lower than mtest);
       feature3 test (i-40) = (mean(higher than mtest) -
174
          mean(lower than mtest))/(max(ecg.(field name))-
          min(ecg.(field name)));
  end
   classltest_featurematrix = [featureltest(1:20)'
```

```
feature2test(1:20) ' feature3test(1:20) '];
   class2test featurematrix = [feature1test(21:40)]
      feature2test(21:40)' feature3test(21:40)'];
  cov1 mat test = cov(class1test featurematrix);
  cov2 mat test = cov(class2test featurematrix);
179
  cov mat test = cov([feature1test(1:40)' feature2test
180
     (1:40) 'feature3test(1:40)']);
  % for part b
  labels class1 mah = zeros(1,20);
182
  labels class2 mah = zeros(1,20);
  labels class1 o = zeros(1,20);
  labels class2 o = zeros(1,20);
185
  labels class1 bayes = zeros(1,20);
  labels\_class2\_bayes = zeros(1,20);
  temp o = 0;
188
  temp mah = 0;
  temp bayes = 0;
  for i = 1:20
191
       d1 mah = (class1test featurematrix(i,1:3)-
          mean1 mat)*inv(cov1 mat)*(
          class1test featurematrix(i,1:3)-mean1 mat);
       d2 mah = (class1test featurematrix(i,1:3)-
193
          mean2 mat)*inv(cov2 mat)*(
          class1test featurematrix(i,1:3)-mean2 mat);
       if d1 mah>d2 mah
194
           labels class1 mah(1,i) = 2;
195
           temp mah = temp mah +1;
196
       else
197
           labels class1 mah(1,i) = 1;
198
       end
       dl_mah = (class2test_featurematrix(i,1:3)-
200
```

```
mean1 mat)*inv(cov1 mat)*(
           class2test featurematrix(i,1:3)-mean1 mat);
       d2 mah = (class2test featurematrix(i,1:3)-
201
          mean2 mat)*inv(cov2 mat)*(
           class2test featurematrix(i,1:3)-mean2 mat);
       if d1 mah>d2 mah
202
            labels class2 mah(1,i) = 2;
       e1se
204
            labels class 2 \operatorname{mah}(1, i) = 1;
205
            temp mah = temp mah +1;
206
       end
207
   end
208
   for i = 1:20
       d1 o = (class1test feature matrix (i, 1:3) - mean1 mat)
210
           *(classitest featurematrix(i,1:3)-mean1 mat);
       d2_o = (classltest_featurematrix(i,1:3)-mean2_mat)
211
           *(class1test_featurematrix(i,1:3)-mean2_mat)';
       if d1 o>d2 o
212
            labels class1 o(1,i) = 2;
213
            temp o = temp o +1;
214
       e1se
215
            labels class 1 \circ (1, i) = 1;
216
       end
217
       d1 o = (class2test featurematrix(i,1:3)-mean1 mat)
218
           *(class2test featurematrix(i,1:3)-mean1 mat);
       d2 o = (class2test featurematrix(i,1:3)-mean2 mat)
219
           *(class2test featurematrix(i,1:3)-mean2 mat)';
       if d1 o>d2 o
220
            labels class 2 \circ (1,i) = 2;
221
       e1se
222
            labels_class2_o(1,i) = 1;
223
```

```
temp o = temp o +1;
224
       end
225
   end
226
   for i = 1:20
227
       d1 o = +0.5*log(det(cov1 mat))+0.5*(
228
           classitest featurematrix (i, 1:3)-mean1 mat)*inv(
          cov1 mat)*(class1test featurematrix(i,1:3)-
          mean1 mat);
       d2 \text{ o} = +0.5*\log(\det(\text{cov2 mat}))+0.5*(
229
           classitest featurematrix (i, 1:3)-mean2 mat)*inv(
          cov2 mat)*(class1test featurematrix(i,1:3)-
          mean2 mat);
       if d1 o>d2 o
            labels class1 bayes (1,i) = 2;
231
            temp bayes = temp bayes +1;
232
       e 1 s e
233
            labels_class1_bayes(1,i) = 1;
234
       end
235
       d1 o = +0.5*log(det(cov1 mat))+0.5*(
236
           class2test featurematrix(i,1:3)-mean1 mat)*inv(
          cov1 mat)*(class2test featurematrix(i,1:3)-
          mean1 mat);
       d2 \text{ o} = +0.5*\log(\det(\text{cov}2 \text{ mat}))+0.5*(
237
           class2test featurematrix(i,1:3)-mean2 mat)*inv(
           cov2 mat)*(class2test featurematrix(i,1:3)-
          mean2 mat);
       if d1 o>d2 o
238
            labels class2 bayes (1,i) = 2;
239
       e1se
240
            labels class 2 bayes (1, i) = 1;
            temp_bayes = temp_bayes +1;
242
```

```
end
243
  end
244
   disp('accuracy by mahlanobis (part b) :')
245
   disp((40-temp \ o)/40*100)
   disp('accuracy by Euclidean (part b):')
247
   disp((40-temp mah)/40*100)
248
   disp('accuracy by bayes decision boundary (part b) :')
   disp((40-temp bayes)/40*100)
250
   disp('
251
      ')
  % for part c
  [evcl test b, eval test b] = eig(covl mat test);
  [evc2 test b, eva2 test b] = eig(cov2 mat test);
  [evc test b, eva test b] = eig(cov mat test);
255
  u1 test b = evc1 test b(:,1:2);
  u2 \text{ test } b = evc2 \text{ test } b(:,1:2);
257
  u_test_b = evc_test_b(:,1:2);
258
  class1 featurematrix test entropy =
      class1 featurematrix * u test b;
   class2 featurematrix test entropy =
260
      class2 featurematrix * u test b;
  labels class1 mah ent = zeros(1,20);
  labels class 2 mah ent = zeros(1,20);
262
   labels class1 o ent = zeros(1,20);
263
  labels class2 o ent = zeros(1,20);
264
  labels class1 bayes ent = zeros(1,20);
265
  labels class2 bayes ent = zeros(1,20);
  temp o = 0;
267
  temp mah = 0;
  temp bayes = 0;
```

```
for i = 1:20
       d1 mah = (class1 featurematrix test entropy(i,1:2)
271
          -mean1 mat entropy)*inv(cov1 mat entropy)*(
          class1 featurematrix test entropy(i,1:2)-
          mean1 mat entropy)';
       d2 mah = (class1 featurematrix test entropy(i,1:2)
272
          -mean2 mat entropy)*inv(cov2 mat entropy)*(
          class1 featurematrix test entropy(i,1:2)-
          mean2 mat entropy)';
       if d1 mah>d2 mah
           labels class1 mah ent(1,i) = 2;
274
           temp mah = temp mah +1;
275
       e 1 s e
276
           labels class 2 mah ent(1,i) = 1;
277
       end
278
       dl_mah = (class2_featurematrix_test_entropy(i,1:2)
          -mean1_mat_entropy)*inv(cov1_mat_entropy)*(
          class2 featurematrix test entropy(i,1:2)-
          mean1 mat entropy)';
       d2 mah = (class2 featurematrix test entropy(i,1:2)
280
          -mean2 mat entropy)*inv(cov2 mat entropy)*(
          class2 featurematrix test entropy(i,1:2)-
          mean2 mat entropy);
       if d1 mah>d2 mah
281
           labels class1 mah ent(1,i) = 2;
282
       e1se
283
           labels class 2 mah ent(1,i) = 1;
284
           temp mah = temp mah +1;
285
       end
286
  end
  for i = 1:20
```

```
d1 o = (class1 feature matrix test entropy(i,1:2) -
289
          mean1 mat entropy)*(
          class1 featurematrix test entropy(i,1:2)-
          mean1 mat entropy);
       d2 o = (class1 featurematrix test entropy(i,1:2)-
290
          mean2 mat entropy)*(
          class1 featurematrix test entropy(i,1:2)-
          mean2_mat_entropy) ';
       if d1 o>d2 o
291
           labels_classl_o_ent(1,i) = 2;
292
           temp o = temp o +1;
293
       e1se
294
           labels class1_o_ent(1,i) = 1;
295
       end
296
       d1 o = (class2 featurematrix test entropy(i,1:2)-
297
          mean1_mat_entropy)*(
          class2 featurematrix test entropy(i,1:2)-
          mean1 mat entropy);
       d2 o = (class2 featurematrix test entropy(i,1:2)-
298
          mean2 mat entropy)*(
          class2 featurematrix test entropy(i,1:2)-
          mean2 mat entropy);
       if d1 o>d2 o
299
           labels_class2_o_ent(1,i) = 2;
300
       e1se
301
           labels class 2 o ent(1,i) = 1;
302
           temp o = temp o +1;
303
       end
304
  end
305
  for i = 1:20
       d1_mah = 0.5*log(det(cov1_mat_entropy))+0.5*(
307
```

```
class1 featurematrix test entropy(i,1:2)-
          mean1 mat entropy)*inv(cov1 mat entropy)*(
          class1 featurematrix_test_entropy(i,1:2)-
          mean1 mat entropy);
       d2 mah = 0.5*\log(\det(\text{cov2 mat entropy}))+0.5*(
308
          class1 featurematrix test entropy(i,1:2)-
          mean2 mat entropy)*inv(cov2 mat entropy)*(
          class1 featurematrix test entropy(i,1:2)-
          mean2 mat entropy)';
       if d1 mah>d2 mah
           labels class1 bayes ent(1,i) = 2;
310
           temp bayes = temp bayes +1;
311
       e1se
312
           labels class1 bayes ent(1,i) = 1;
313
       end
314
       d1 mah = 0.5*\log(\det(\text{cov1 mat entropy}))+0.5*(
315
          class2 featurematrix test entropy(i,1:2)-
          mean1 mat entropy)*inv(cov1 mat entropy)*(
          class2 featurematrix test entropy(i,1:2)-
          mean1 mat entropy);
       d2 mah = 0.5*\log(\det(\text{cov2 mat entropy}))+0.5*(
316
          class2 featurematrix test entropy(i,1:2)-
          mean2 mat entropy)*inv(cov2 mat entropy)*(
          class2 featurematrix test entropy(i,1:2)-
          mean2 mat entropy);
       if d1 mah>d2 mah
317
           labels class 2 bayes ent(1,i) = 2;
318
       e1se
319
           labels class 2 mah ent(1,i) = 1;
320
           temp bayes = temp bayes +1;
321
       end
322
```

```
end
323
  disp('accuracy by mahlanobis (part c) :')
324
  disp((40-temp mah)/40*100)
325
  disp('accuracy by Euclidean (part c) :')
  disp((40-temp \ o)/40*100)
327
  disp('accuracy by bayes decision boundary (part c):')
328
  disp((40-temp bayes)/40*100)
  disp ('
      ')
  % for part d
  mean1 test fisher = mean(class1test featurematrix);
  mean2 test fisher = mean(class2test featurematrix);
  S W test = cov(class1test featurematrix) + cov(
      class2test featurematrix);
  mean_diff = mean1_test_fisher - mean2_test_fisher;
  S B = mean diff * mean diff';
  [V_{test}, D_{test}] = eig(S_{W_{test}} \setminus S_{B});
337
  [\sim, idx test] = max(diag(D test));
  w test = V test(:, idx test);
  projected class1 test = class1test featurematrix *
340
  projected class2 test = class2test featurematrix *
     w test;
  mean proj1 test = mean(projected class1 test);
  mean proj2 test = mean(projected class2 test);
343
  var proj1 test = var(projected class1 test);
344
  var proj2 test = var(projected class2 test);
  labels class1 mah fisher = zeros(1,20);
  labels class 2 mah fisher = zeros (1,20);
  labels_class1_o_fisher = zeros(1,20);
```

```
labels class2 o fisher = zeros(1,20);
   labels class1 o bayes = zeros(1,20);
350
   labels class2 o bayes = zeros(1,20);
351
  temp o = 0;
  temp mah = 0;
353
   temp\_bayes = 0;
354
   for i = 1:20
       d1 mah = (projected class1 test(i)-mean proj1 test
356
          )*inv(var_proj1_test)*(projected_class1_test(i)
          -mean projl test);
       d2 mah = (projected class1 test(i)-mean proj2 test
357
          )*inv(var proj2 test)*(projected class1 test(i)
          -mean proj2 test);
       if d1 mah>d2 mah
358
           labels class 1 mah fisher (1, i) = 2;
359
           temp_mah = temp_mah + 1;
360
       e 1 s e
361
           labels class 2 mah fisher (1, i) = 1;
362
       end
363
       d1 mah = (projected class2 test(i)-mean proj1 test
364
          )*inv(var proj1 test)*(projected class2 test(i)
          -mean projl test);
       d2 mah = (projected class2 test(i)-mean proj2 test
365
          )*inv(var proj2 test)*(projected class2 test(i)
          -mean proj2 test);
       if d1 mah>d2 mah
366
           labels class 1 mah fisher (1, i) = 2;
367
       e1se
368
           labels class 2 mah fisher (1, i) = 1;
369
           temp mah = temp mah +1;
370
       end
371
```

```
end
372
   for i = 1:20
373
       d1 o = (projected class1 test(i)-mean proj1 test)
374
           *(projected class1 test(i)-mean proj1 test);
       d2 o = (projected class1 test(i)-mean proj2 test)
375
           *(projected class1 test(i)-mean proj2 test);
       if d1_o > d2 o
            labels class1 o fisher (1,i) = 2;
377
            temp o = temp o +1;
378
       e 1 s e
379
            labels class1 o fisher (1,i) = 1;
380
       end
381
       dl_o = (projected_class2_test(i)-mean_proj1_test)
382
           *(projected class2 test(i)-mean proj1 test);
       d2 o = (projected class2 test(i)-mean proj2 test)
383
           *(projected_class2_test(i)-mean_proj2_test)';
       if d1 o>d2 o
384
            labels class2 o fisher (1,i) = 2;
385
       e1se
386
            labels class2 o fisher (1,i) = 1;
387
            temp o = temp o +1;
388
       end
389
   end
   for i = 1:20
391
       d1 mah = 0.5*\log(\det(\text{var proj 1 test}))+0.5*(
392
           projected class1 test(i)-mean proj1 test)*inv(
           var proj1 test)*(projected class1 test(i)-
           mean projl test)';
       d2 \text{ mah} = 0.5 * \log(\det(\text{var proj 2 test})) + 0.5 * (
           projected class1 test(i)-mean proj2 test)*inv(
           var_proj2_test)*(projected_class1_test(i)-
```

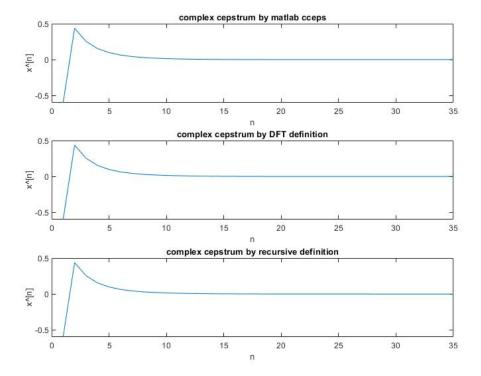
```
mean proj2 test);
       if d1 mah>d2 mah
394
            labels class1 mah fisher (1, i) = 2;
395
            temp bayes = temp bayes +1;
       e1se
397
            labels class 2 mah fisher (1, i) = 1;
398
       end
       d1 \text{ mah} = 0.5 * \log (\det (\text{var proj 1 test})) + 0.5 * (
400
           projected class2 test(i)-mean proj1 test)*inv(
           var proj1 test)*(projected class2 test(i)-
          mean projl test)';
       d2 mah = 0.5*\log(\det(\text{var proj 2 test}))+0.5*(
401
           projected class2 test(i)-mean proj2 test)*inv(
           var_proj2_test)*(projected_class2_test(i)-
          mean proj2 test);
       if d1 mah>d2 mah
402
            labels\_class1\_mah\_fisher(1,i) = 2;
403
       e1se
404
            labels class2 mah fisher (1, i) = 1;
405
            temp bayes = temp bayes +1;
406
       end
407
   end
408
   disp('accuracy by mahlanobis (part d):')
   disp((40-temp \ o)/40*100)
410
   disp('accuracy by Euclidean (part d):')
411
   disp((40-temp mah)/40*100)
   disp('accuracy by bayes decision boundary (part d):')
413
   disp((40-temp bayes)/40*100)
   disp('
```

')

3 Question 3

I have used an ECG dataset which contain healthy controls, cardiac patients and Myocardial infarction patients which these diseases are exactly Heart failure as question said. Also explanation of each part located at the end of them.

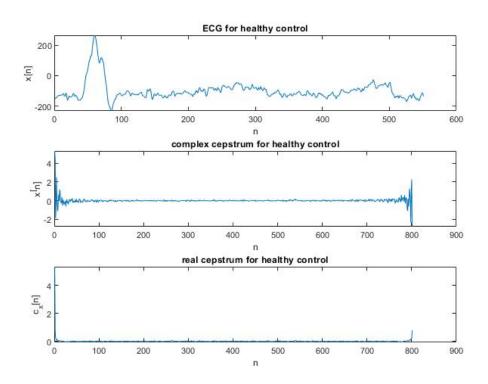
3.1 part a



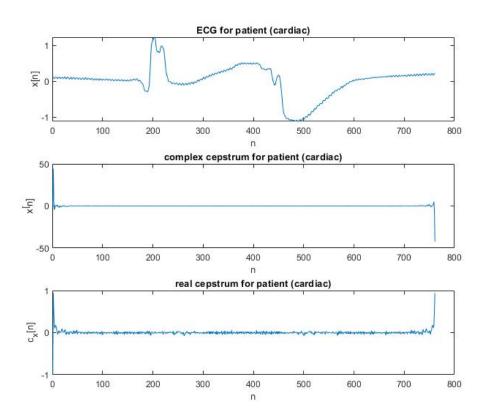
^{*} **Explanation:** As we can see there is no difference between three methods and they have same answers.

3.2 part b

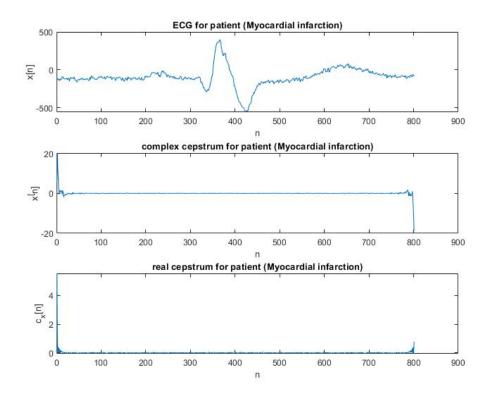
- For healthy person :



- For cardiac patient :



- For Myocardial infarction patient :



* Explanation:

-Distinct Features:

The differences in the complex and real cepstrum plots suggest that there are distinct features that can be used for classification. The cardiac and myocardial infarction patients show more pronounced peaks and variations compared to the healthy control.

-Cepstrum Analysis:

Cepstrum analysis captures the periodicity and echo-like features in the signals, which are reflective of the underlying physiological conditions. For the cardiac patient and myocardial infarction patient, the higher cepstrum values and distinct patterns can serve as markers for classification.

-Healthy Control:

The healthy control shows a relatively stable and less varying cepstrum, making

it distinguishable from the other two cases.

* Finally given the distinct differences in the cepstrum plots for the three cases, it is plausible to classify patients based on cepstrum analysis. The significant features and variations in the complex and real cepstrum values for cardiac patients and myocardial infarction patients compared to healthy controls provide a basis for differentiation. However, the exact effectiveness and accuracy of such classification would depend on the implementation of suitable machine learning algorithms and the quality of the feature extraction process.

Matlab Code

```
1 % part a
_{2} length = 35;
n = 1:length;
x(n) = 0.8. heaviside (n) -0.5.*0.8. (n-1). heaviside
     (n-1);
_{5} fft x = fft(x);
6 complex_cepstrum_recursive = [];
  complex_cepstrum_definition = ifft(log(fft_x));
s complex cepstrum = cceps(x);
  complex cepstrum recursive (1) = log(x(1));
  for i=2:length
      a = 0;
      for j = 1: i-1
          a = a + (j-1)/(i-1)*complex cepstrum recursive
             (i)*x(i-(i-1))/x(1);
      end
14
     complex cepstrum recursive(i) = x(i)/x(1) - a;
15
  end
17 figure;
18 subplot (3,1,1)
 plot(complex cepstrum)
```

```
title ('complex cepstrum by matlab cceps')
  ylabel('x \land [n]')
 xlabel('n')
subplot(3,1,2)
 plot(complex cepstrum definition )
  title ('complex cepstrum by DFT definition')
  ylabel('x \land [n]')
27 xlabel('n')
 subplot(3,1,3)
 plot(complex_cepstrum_recursive)
  title ('complex cepstrum by recursive definition')
 ylabel('x \land [n]')
xlabel('n')
33 %% part b
34 % disease : cardiac
ecg_p1 = readtable("207-try.csv");
36 % disease: Myocardial infarction
ecg p2 = load('s0001.mat');
38 % healthy control
_{39} ecg h = load('s0285.mat');
40 figure;
subplot(3,1,1)
42 plot (ecg p1. heart (640:1400))
 title ('ECG for patient (cardiac)')
 ylabel('x[n]')
45 xlabel('n')
subplot(3,1,2)
  plot(cceps(ecg p1.heart(640:1400)))
  title ('complex cepstrum for patient (cardiac)')
 ylabel ('x^{n})'
_{50} x label('n')
```

```
subplot(3,1,3)
  plot (rceps (ecg_p1 . heart (640:1400)))
  title ('real cepstrum for patient (cardiac)')
54 ylabel('c x[n]')
55 xlabel('n')
 figure;
subplot (3,1,1)
 plot(ecg p2.val(1,1750:2550))
  title ('ECG for patient (Myocardial infarction)')
  ylabel('x[n]')
61 xlabel('n')
62 subplot (3,1,2)
 plot(cceps(ecg_p2.val(1,1750:2550)))
  title ('complex cepstrum for patient (Myocardial
     infarction)')
 ylabel('x^{n}]')
 xlabel('n')
subplot (3,1,3)
 plot(rceps(ecg p2.val(1,1750:2550)))
  title ('real cepstrum for patient (Myocardial
     infarction)')
 ylabel('c x[n]')
71 xlabel('n')
72 figure;
_{73} subplot (3, 1, 1)
 plot (ecg h. val (1,1750:550+1750))
 title ('ECG for healthy control')
 ylabel('x[n]')
77 xlabel('n')
subplot(3,1,2)
79 plot (cceps (ecg_h.val(1,1750:2550)))
```

```
title('complex cepstrum for healthy control')
ylabel('x^[n]')
xlabel('n')
subplot(3,1,3)
plot(rceps(ecg_h.val(1,1750:2550)))
title('real cepstrum for healthy control')
ylabel('c_x[n]')
xlabel('n')
```

4 Question 4

First 3 parts are same as the previous homework and as Dr.Shamsollahi said I avoid from repeating them.

4.1 part d

Article: "Kalman Filtering for Improved Motion Artifact Reduction in Wearable ECG Monitors"

Summary: Published in IEEE Transactions on Biomedical Engineering in 2024, this article explored the application of Kalman filtering to reduce motion artifacts in wearable ECG monitors. The researchers developed a Kalman filter-based approach to separate true cardiac signals from motion-induced noise. The filtered signals showed significant improvements in clarity, making it easier to detect and monitor cardiac events accurately. This study emphasized the Kalman filter's role in enhancing the reliability of wearable health monitoring devices.

4.2 part e

Article: "Fuzzy Logic-Based Decision Support System for Early Diagnosis of Alzheimer's Disease"

Summary: The article, published in Expert Systems with Applications in 2023, presented a fuzzy logic-based decision support system designed for the early diagnosis of Alzheimer's disease. The system integrated various clinical and cognitive parameters to evaluate the likelihood of Alzheimer's in patients. Using fuzzy logic, the system handled uncertainties and provided interpretable diagnostic results. The study found that the fuzzy logic approach enhanced diagnostic accuracy and could serve as a valuable tool in clinical settings.

4.3 part f

Article: "Feature Extraction Using Cepstrum Analysis for Automated Speech Disorder Detection"

Summary: In a 2023 study published in Speech Communication, researchers utilized cepstrum analysis for feature extraction in automated detection of speech disorders. They analyzed the cepstral coefficients of speech recordings from patients with various speech impairments. These features were used to train a neural network classifier, which achieved high accuracy in distinguishing between different types of speech disorders. The findings highlighted the efficacy of cepstrum analysis in capturing distinctive speech characteristics, offering a valuable tool for early diagnosis and intervention.

4.4 part g

Article: "Hidden Markov Models for Real-Time Sign Language Recognition"

Summary: The 2024 article in Pattern Recognition focused on using hidden Markov models (HMMs) for real-time sign language recognition. The researchers developed an HMM-based system that could accurately interpret sign language gestures from video data. The system was tested with a diverse set of sign language gestures, demonstrating high recognition accuracy and robustness to variations in signing speed and style. This study showcased the potential of HMMs in creating effective communication tools for the hearing impaired.