

# 1 Assignment No. 7: Dynamic Order Statistics

Allocated time: 2 hours

## 1.1 Implementation

You are required to implement **correctly** and **efficiently** the management operations of an **order statistics tree** (*chapter 14.1 from the book(?)*).

You have to use a balanced, augmented Binary Search Tree. Each node in the tree holds, besides the necessary information, also the *size* field (i.e. the size of the sub-tree rooted at the node).

The management operations of an **order statistics tree** are:

- BUILD\_TREE(*n*)
  - *builds* a **balanced** BST containing the keys 1,2,...*n* (*hint*: use a divide and conquer approach)
  - make sure you initialize the size field in each tree node
- OS\_SELECT(*tree*, *i*)
  - selects the element with the *i*-th smallest key
  - the pseudo-code is available in *chapter 14.1 from the book(?)*
- OS\_DELETE(*tree*, *i*)
  - you may use the deletion from a BST, without increasing the height of the tree (why don't you need to rebalance the tree?)
  - keep the size information consistent after subsequent deletes
  - there are several alternatives to update the size field without increasing the complexity of the algorithm (it is up to you to figure this out).

Does OS\_SELECT resemble anything you studied this semester?

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## 1.2 Requirements

### 1.2.1 BUILD\_TREE: correct and efficient implementation (5p)

You will have to prove your algorithm(s) work on a small-sized input (11)

- pretty-print the initially built tree

### 1.2.2 OS\_SELECT: correct and efficient implementation (1p)

You will have to prove your algorithm(s) work on a small-sized input (11)

- execute OS-SELECT for a few elements (at least 3) by a randomly selected index

### 1.2.3 OS\_DELETE: correct and efficient implementation (2p)

You will have to prove your algorithm(s) work on a small-sized input (11)

- execute OS-SELECT followed by OS-DELETE for a few elements (at least 3) by a randomly selected index *and pretty-print the tree after each execution.*

### 1.2.4 Management operations evaluation - BUILD, SELECT, DELETE (2p)

! Before you start to work on the algorithms evaluation code, make sure you have a **correct algorithm!**

Once you are sure your program works correctly:

- vary  $n$  from 100 to 10000 with a step of 100;
- for each  $n$  (don't forget to repeat 5 times),
  - BUILD a tree with elements from 1 to  $n$
  - perform  $n$  sequences of OS-SELECT and OS-DELETE operations using a randomly selected index based on the remaining number of elements in the BST,
  - Evaluate the number of operations needed for each management operation (BUILD, SELECT, DELETE – *resulting in a plot with 3 series*). Evaluate the computational effort as the sum of the comparisons and assignments performed by each individual management operation ofr each value of  $n$ .

### 1.2.5 Bonus: Implementation using AVL / Red black tree (1p)