1 Assignment No. 7: Dynamic Order Statistics

Allocated time: 2 hours

1.1 Implementation

You are required to implement **correctly** and **efficiently** the management operations of an **order statistics tree** (*chapter 14.1 from [1]*).

You have to use a balanced, augmented Binary Search Tree. Each node in the tree holds, besides the necessary information, also the *size* field (i.e. the size of the sub-tree rooted at the node).

The management operations of an **order statistics tree** are:

- BUILD_TREE(n)
  - *builds* a **balanced** BST containing the keys 1,2,...n (*hint:* use a divide and conquer approach)
  - make sure you initialize the size field in each tree node
- OS-SELECT(tree, i)
  - selects the element with the i-th smallest key
  - the pseudo-code is available in *chapter 14.1 from the book[1]*
- OS-DELETE(tree, i)
  - you may use the deletion from a BST, without increasing the height of the tree (why don't you need to rebalance the tree?)
  - keep the size information consistent after subsequent deletes
  - there are several alternatives to update the size field without increasing the complexity of the algorithm (it is up to you to figure this out).

Does OS-SELECT resemble anything you studied this semester?

1.2 Requirements

1.2.1 BUILD_TREE: correct and efficient implementation (5p)

You will have to prove your algorithm(s) work on a small-sized input (11)

- pretty-print the initially built tree

1.2.2 OS_SELECT: correct and efficient implementation (1p)

You will have to prove your algorithm(s) work on a small-sized input (11)

- execute OS-SELECT for a few elements (at least 3) by a randomly selected index

1.2.3 OS_DELETE: correct and efficient implementation (2p)

You will have to prove your algorithm(s) work on a small-sized input (11)

- execute OS-SELECT followed by OS-DELETE for a few elements (at least 3) by a randomly selected index *and pretty-print the tree after each execution.*

1.2.4 Management operations evaluation - BUILD, SELECT, DELETE (2p)

! Before you start to work on the algorithms evaluation code, make sure you have a **correct algorithm**!

Once you are sure your program works correctly:

- vary n from 100 to 10000 with a step of 100;
- for each n (don't forget to repeat 5 times),
  - BUILD a tree with elements from *1 to n*
  - perform n sequences of OS-SELECT and OS-DELETE operations using a randomly selected index based on the remaining number of elements in the BST,
  - Evaluate the number of operations needed for each management operation (BUILD, SELECT, DELETE – *resulting in a plot with 3 series*). Evaluate the computational effort as the sum of the comparisons and assignments performed by each individual management operation ofr each value of n.

1.2.5 Bonus: Implementation using AVL / Red black tree (1p)

References

[1] Thomas H. Cormen et al. *Introduction to Algorithms*. 2nd. The MIT Press, 2001. ISBN: 0262032937.