1 Assignment No. 8: Disjoint Sets

Allocated time: 2 hours

1.1 Implementation

You are required to implement **correctly** and **efficiently** the base operations for **disjoint set** (*chapter 21.1* from [1]) and the **Kruskal's algorithm** (searching for the minimum spanning tree, *chapter 23.2* from [1]) using disjoint sets.

You have to use a tree as the representation of a disjoint set. Each tree holds, besides the necessary information, also the *rank* field (i.e. the height of the tree).

The base operations on **disjoints sets** are:

- MAKE_SET (x)
  - creates a set with the element x
- UNION (x, y)
  - makes the union between the set that contains the element x and the set that contains the element y
  - the heuristic *union by rank* takes into account the height of the two trees so as to make the union
  - the pseudo-code can be found in *chapter 21.3*[1]
- FIND_SET (x)
  - searches for the set that contains the element x
  - the heuristic *path compression* links all nodes that were found on the path to x to the root node

1.2 Requirements

1.2.1 Correct implementation of MAKE_SET, UNION and FIND_SET (5p)

The correctness of the algorithm must be proved on a small-sized input

- create (MAKE) 10 sets + show the contents of the sets
- execute the sequence UNION and FIND_SET for 5 elements + show the contents of the sets

1.2.2 Correct and efficient implementation for Kruskal's algorithm (2p)

The correctness of the algorithm must be proved on a small-sized input

- create a graph of 5 nodes and 9 edges + **print the edges**
- apply Kruskal's algorithm + **print the chosen edges**

1.2.3 Evaluate the disjoint sets operations (MAKE, UNION, FIND) using Kruskal's algorithm (3p)

! Before you start to work on the algorithms evaluation code, make sure you have a correct algorithm!

Once you are sure your program works correctly:

- vary n from 100 to 10000 with a step of 100
- for each n
  - build an **undirected**, **connected**, and **random** graph with random weights on edges (**n** nodes, **n*4** edges)
  - find the minimum spanning tree using Kruskal's algorithm
    - evaluate the computational effort of each individual base operation (MAKE, UNION, FIND– *resulting in a plot with 3 series*) on disjoint sets as the sum of the comparisons and assignments performed; thus, there should be **3 series in the plot**, one for each operation.

References

[1] Thomas H. Cormen et al. *Introduction to Algorithms*. 2nd. The MIT Press, 2001. ISBN: 0262032937.