1. Simplify the following expressions using Boolean algebraic laws. Give each step of your simplification and denote which laws you're using for each step. Do not skip or combine steps!

(a) 
$$A \cdot (\overline{A} + BB) + \overline{(B+A)} \cdot (\overline{A} + B)$$

$$A \cdot (\overline{A} + BB) + \overline{(B + A)} \cdot (\overline{A} + B) \qquad \text{DeMorgan's Law}$$

$$A \cdot (\overline{A} + BB) + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B) \qquad \text{Distributive Law}$$

$$(A\overline{A} + ABB) + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B) \qquad \text{Inverse Law}$$

$$0 + ABB + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B) \qquad \text{Identity Law}$$

$$ABB + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B) \qquad \text{Distributive Law}$$

$$ABB + \overline{B} \cdot \overline{A} \cdot \overline{A} + \overline{B} \cdot \overline{A} \cdot B \qquad \text{Inverse Law}$$

$$ABB + \overline{B} \cdot \overline{A} \cdot \overline{A} + \overline{A} \quad \text{Distributive Law}$$

$$ABB + \overline{B} \cdot \overline{A} \cdot \overline{A} \qquad \text{Distributive Law}$$

$$A(BB) + \overline{B} \cdot \overline{A} \cdot \overline{A} \qquad \text{Identity Law}$$

$$A(BB + 0) + \overline{B} \cdot \overline{A} \cdot \overline{A} \qquad \text{Inverse Law}$$

$$A(BB + B\overline{B}) + \overline{B} \cdot \overline{A} \cdot \overline{A} \qquad \text{Distributive Law}$$

$$A(B \cdot (B + \overline{B})) + \overline{B} \cdot \overline{A} \cdot \overline{A} \qquad \text{Inverse Law}$$

$$A(B \cdot 1) + \overline{B} \cdot \overline{A} \cdot \overline{A} \qquad \text{Identity Law}$$

$$AB + \overline{B}(\overline{A} \cdot \overline{A}) \qquad \text{Inverse law}$$

$$AB + \overline{B}(\overline{A} \cdot \overline{A} + A\overline{A}) \qquad \text{Distributive law}$$

$$AB + \overline{B}(\overline{A} \cdot \overline{A} + A\overline{A}) \qquad \text{Distributive law}$$

$$AB + \overline{B}(\overline{A} \cdot \overline{A} + A\overline{A}) \qquad \text{Distributive law}$$

$$AB + \overline{B}(\overline{A} \cdot \overline{A}) \qquad \text{Inverse law}$$

$$AB +$$

$$AB + \overline{B} \cdot \overline{A}$$

(b) 
$$\overline{C \cdot B} + A \cdot B \cdot C + \overline{A + C + \overline{B}}$$

$$\overline{B} + \overline{C} + A$$

(c) 
$$(A+B)\cdot(\overline{A}+C)\cdot(\overline{C}+B)$$

$$(\overline{\mathbf{A}} + \mathbf{C})(\overline{\mathbf{C}} + \mathbf{B})\mathbf{A} + (\overline{\mathbf{A}} + \mathbf{C})(\overline{\mathbf{C}} + \mathbf{B})\mathbf{B} \\ (\overline{\mathbf{C}} + \mathbf{B})(\overline{\mathbf{A}}\mathbf{A} + \mathbf{A}\mathbf{C}) + (\overline{A} + C)(\overline{C} + B)B \\ (\overline{C} + B)(\mathbf{0} + AC) + (\overline{A} + C)(\overline{C} + B)B \\ (\overline{C} + B)(AC) + (\overline{A} + C)(\overline{C} + B)B \\ (\overline{C} + B)(AC) + (\overline{A} + C)(\overline{C} + B)B \\ (\overline{C} + B)(AC) + (\overline{A} + C)(\overline{C} + B)B \\ (\overline{C} + B)(\overline{A}C) + (\overline{A} + C)(\overline{C} + B)B \\ (\overline{C} + B)B \\ (\overline{C} + ABC) + (\overline{A} + C)(\overline{C} + B)B \\ (\overline{C} + B)B \\ (\overline{C}$$

# $BC + \overline{A}B$

2. Find all solutions of the following Boolean equations without using the truth tables:

(a) 
$$(\overline{A} + C) \cdot (\overline{B} + D + A) \cdot (D + A \cdot \overline{C}) \cdot (\overline{D} + A) = 1$$

$$(\overline{A} + C) \cdot (\overline{B} + D + A) \cdot (D + A \cdot \overline{C}) \cdot (\overline{D} + A) = 1$$

$$(\overline{A} \cdot \overline{B} + \overline{A}D + \overline{A}A + C\overline{B} + CD + AC)(D + A\overline{C})(\overline{D} + A) = 1$$

$$(\overline{A} \cdot \overline{B} + \overline{A}D + C\overline{B} + CD + AC)(D + A\overline{C})(\overline{D} + A) = 1$$

$$(\overline{A} \cdot \overline{B} + \overline{A}D + C\overline{B}D + CDD + ACD + \overline{A} \cdot \overline{B}A\overline{C} + \overline{A}DA\overline{C} + C\overline{B}A\overline{C} + \dots$$

$$CDA\overline{C} + ACA\overline{C})(\overline{D} + A) = 1$$

$$AA = A \rightarrow AA = AA + 0 = AA + A\overline{A} = A(A + \overline{A}) = A(1) = A$$

$$(\overline{A} \cdot \overline{B}D + \overline{A}D + C\overline{B}D + CD + ACD + \overline{A} \cdot \overline{B}A\overline{C} + \overline{A}DA\overline{C} + C\overline{B}A\overline{C} + \dots$$

$$CDA\overline{C} + ACA\overline{C})(\overline{D} + A) = 1$$

$$(\overline{A} \cdot \overline{B}D + \overline{A}D + C\overline{B}D + CD + ACD + \overline{A} \cdot \overline{B}A\overline{C} + \overline{A}DA\overline{C} + C\overline{B}A\overline{C} + \dots$$

$$CDA\overline{C} + ACA\overline{C})(\overline{D} + A) = 1$$

$$(\overline{A} \cdot \overline{B}D + \overline{A}D + C\overline{B}D + CD + ACD)(\overline{D} + A) = 1$$

$$(\overline{A} \cdot \overline{B}D + \overline{A}D\overline{D} + C\overline{B}D\overline{D} + CD\overline{D} + ACD\overline{D} + A\overline{A} \cdot \overline{B}D + \overline{A}AD + \dots$$

$$AC\overline{B}D + ACD + AACD = 1$$

$$(AC\overline{B}D) + ACD + AACD = 1$$

$$(AC\overline{B}D) + ACD + ACD = 1$$

$$(AC\overline{B}D) + ACD = 1$$

$$(AC\overline{B}D) + ACD = 1$$

$$(AC\overline{B}D) + ACD = 1$$

$$Distributive Law$$

$$ACD(\overline{B} + 1) = 1$$

$$Distributive Law$$

$$ACD(\overline{B} + 1) = 1$$

$$Distributive Law$$

## The equation is true when ALL of ACD is true, or ACD = 1

(b) 
$$(((\overline{K} \cdot L \cdot N) \cdot (L+M)) + ((\overline{K} + L + N) \cdot (K \cdot \overline{L} \cdot \overline{M}))) \cdot (\overline{K} + \overline{N}) = 1$$

$$((\overline{K}LN)(L+M)+(\overline{K}+L+N)(K\overline{L}\cdot\overline{M}))(\overline{K}+\overline{N})=1 \qquad \text{Distributive Law} \\ (\overline{K}LNL+\overline{K}LNM+K\overline{L}M\overline{K}+K\overline{L}L\overline{M}+K\overline{L}N\overline{M})(\overline{K}+\overline{N})=1 \qquad \text{Distributive } + \text{Associative Law} \\ (\overline{K}N(L(1+1))+\overline{K}LNM+(K\overline{K})\overline{L}M+K\overline{M}(\overline{L}L)+K\overline{L}\cdot\overline{M}N)... \\ (\overline{K}+\overline{N})=1 \qquad \qquad \text{Zero/One} + \text{Inverse} + \text{Identity} \\ (\overline{K}LN+\overline{K}LNM+K\overline{L}\cdot\overline{M}N)(\overline{K}+\overline{N})=1 \qquad \qquad \text{Distributive Law} \\ \overline{K}\cdot\overline{K}LN+\overline{K}LNM\overline{K}+K\overline{K}\cdot\overline{L}\cdot\overline{M}N+\overline{K}LN\overline{N}+... \\ KLNM\overline{N}+K\overline{L}MN\overline{N}=1 \qquad \qquad \text{Distributive} + \text{Associative Law} \\ (\overline{K}(1+1))LN+(\overline{K}(1+1))LNM+(K\overline{K})L\overline{M}N+\overline{K}L(N\overline{N})+... \\ KLM(N\overline{N})+K\overline{L}M(N\overline{N})=1 \qquad \qquad \text{Distributive} + \text{Associative Law} \\ \overline{K}LN+\overline{K}LNM=1 \qquad \qquad \text{Distributive Law} \\ \overline{K}LN+\overline{K}LNM=1 \qquad \qquad \text{Distributive Law} \\ \overline{K}LN=1 \qquad \qquad \text{Distributive Law$$

# The equation is true when LN is true and K is false, or $\overline{K}LN=1$

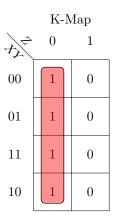
3. Simplify the following expression by first constructing a truth table, using that truth table to construct a K-map, and then using that K-map to simplify.

$$Q = \overline{X} \cdot \overline{Y} \cdot Z + X \cdot Y \cdot \overline{Z} + \overline{X} \cdot Y \cdot \overline{Z} + X \cdot \overline{Y} \cdot \overline{Z}$$

ACD = 1

Truth Table

x	у	z	result
0	0	0	Τ
0	0	1	$\mathbf{F}$
0	1	0	Т
0	1	1	$\mathbf{F}$
1	0	0	Т
1	0	1	F
1	1	0	Т
1	1	1	F
			1



# This k-map simplifies the equation to $\overline{Z}$

4. Convert the following truth table into its sum of products representation:

A B C Output 
0 0 0 1 
0 1 0 
1 0 
1 0 
1 0 
1 0 
1 0 
1 1 
1 Sums of Product: 
$$\overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A}B\overline{C} + \overline{A}BC + \overline{A}BC$$

5. Draw a logical circuit diagram that represents the above sum of products expression using OpenCircuits (https://opencircuits.io/). Clearly label all inputs/outputs and all components. Make sure you connect appropriate input components (e.g., buttons, switches, clocks, etc.) and output components (e.g., LEDs, displays, etc.) to facilitate testing of your circuit. Download your diagram using OpenCircuits' "Download" feature, rename it to hw4\_SOP.circuit, and submit on Submitty along with your hw4.pdf file.

#### The answer to this question is in file hw4\_SOP.circuit.

6. Test you circuit by supplying appropriate inputs and observing the expected values of the output. Explain why your set of tests is sufficient to prove that your logical circuit does in fact implement the required Boolean function. For each test, provide a picture (snapshot) of your circuit. Insert all such pictures in the hw4.pdf PDF file. You can download pictures (PNG, JPEG, or PDF) of your circuit diagram using OpenCircuits' "Export Image" feature.

## The images of the circuit diagram is submitted as A\_B\_C.png

0\_0\_0.png is true as seen in the truth table, the LED light is on 0\_1\_0.png is true as seen in the truth table, the LED light is on 0\_1\_1.png is true as seen in the truth table, the LED light is on 1\_1\_1.png is true as seen in the truth table, the LED light is on 0\_0\_1.png is false as seen in the truth table, the LED light is off 1\_0\_0.png is false as seen in the truth table, the LED light is off 1\_0\_1.png is false as seen in the truth table, the LED light is off 1\_1\_0.png is false as seen in the truth table, the LED light is off

7. Given inputs A and B, show that NOR (A + B) is functionally complete by giving logical circuits equivalent to AND (A \* B), OR (A + B), and NOT A gates using only NOR gates in their construction.

NOR\_and.circuit represents AND NOR\_or.circuit represents OR NOR\_not.circuit represents NOT

8. What is  $5\mathrm{ED4}$  -  $07\mathrm{A4}$  when these values represent unsigned 16-bit hexadecimal numbers? The result should be written in hexadecimal. Show your work.

Answer: 5730

9. What is 5ED4 - 07A4 when these values represent signed 16-bit hexadecimal numbers stored in sign-magnitude format? The result should be written in hexadecimal. Show your work.

# Answer: 5730

#### **Explanation:**

In sign magnitude, the most significant bit is the sign bit. In our given equations, none of the leading bits are 1: 5 = 0101, 0 = 0000, The answer to this problem will be the same as the one prior.

10. Assume 185 and 122 are unsigned 8-bit decimal integers. Calculate 185 - 122. Is there overflow, underflow, or neither?

#### Decimal to binary conversion

```
185 = 1011 1001
122 = 0111 1010
```

To perform the binary subtraction, we must convert the number being subtracted to a negative binary number using two's complement, and perform binary addition

#### Find 122 Two's Complement Number

```
122 = 0111 1010
= 1000 0101 <- one's complement (flip all 1's and 0's)
= 1000 0110 <- add 1 to the end (carried over)
```

#### Calculate 185 + 122 (Two's Complement Number)

The answer is represented in 9 bits, results in an:



11. Assume 151 and 214 are signed 8-bit decimal integers stored in two's complement format. Calculate 151 - 214 using saturating arithmetic. The result should be written in decimal. Show your work.

## Decimal to binary conversion

```
151 = 1001 \ 0111
214 = 1101 \ 0110
```

151 in Two's Complement to obtain original value

214 in Two's Complement to obtain original value

Convert numbers back to decimal

$$(151)$$
 0110 1001 = 105  $(214)$  0010 1010 = 42

# Perform operation

$$105 - 42 = 63$$

For 8-bit signed integers, saturation limits are from -128 to 127, since  $-128 \le 63 \le 127$ , no saturation is required.

12. Assume 151 and 214 are unsigned 8-bit integers. Calculate 151 + 214 using saturating arithmetic. The result should be written in decimal. Show your work.

#### Simple Addition

$$151 + 214 = 365$$

#### Saturation

For 8 bit unsigned integers, saturation limits are from 0 to 256, since 365 > 256, the final answer will be saturated to the bounds of 256.

$$151 + 214 = 256$$

13. What decimal number does the bit pattern  $0 \times 0C000000$  represent if it is a two's complement integer? An unsigned integer?

Two's Complement: 201326592
Unsigned integer: 201326592

Since the binary representation's most significant bit is a 0, both will have the same value

14. If the bit pattern 0×0C000000 is placed into the Instruction Register, what MIPS instruction will be executed

MIPS opcode takes in 6 bits, from the most significant onwards 0000 11 <- opcode 0000 11 is MIPS's jal instruction

jal 0

15. Give a reason why we use two's complement representation for negative numbers in computer arithmetic. Give an example of its usage.

We use Two's Complement representation for negative numbers because it simplifies addition and subtraction with negative numbers easier. To take Two's Complement, you must take One's Complement first, simply take the positive version of the negative number, and flip over all 0's and 1's.

However, One's Complement works, but for the number 0, it has two representation(0000 and 1111), to remove this. We take Two's Complement. We take One's Complement of the number, then we add 1 to the end.

Normal signed int

```
Assume we want to do 5 + (-5):

5 = 0101

-5 = 1101
```

when we perform the addition, we are expecting a 0, right...?

we get a value of 10010, getting rid of overflow, we get 0010, but 0010 = 2?

# Two's Complement

```
Using Two's Complement:

5 = 0101

flip 1's and 0's

5 = 1010

add 1 to the end

-5 = 1011

1110 (carry)

0101

+ 1011

-------

10000
```

here, we get a value of 10000, getting rid of the overflow, we get 0000, and 0000 = 0.