1. Simplify the following expressions using Boolean algebraic laws. Give each step of your simplification and denote which laws you're using for each step. Do not skip or combine steps!

(a)
$$A \cdot (A + B \cdot B) + (B + A) \cdot (A + B)$$

$$A \cdot (\overline{A} + BB) + \overline{(B + A)} \cdot (\overline{A} + B)$$

$$A \cdot (\overline{A} + BB) + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B)$$

$$A \cdot (\overline{A} + B) + (\overline{B} \cdot \overline{A}) \cdot (\overline{A} + B)$$

$$(\overline{A} + B)(A + \overline{B} \cdot \overline{A})$$

$$(\overline{A} + B)(A + \overline{B} \cdot A + \overline{A})$$

$$(\overline{A} + B)(A + \overline{B} \cdot 1)$$

$$(\overline{A} + B)(A + \overline{B})$$

$$(\overline{A} + B)(A + \overline{B})$$

$$(\overline{A} + B)A + (\overline{A} + B)\overline{B}$$

$$A\overline{A} + AB + \overline{A} \cdot \overline{B} + \overline{B}B$$

$$0 + AB + \overline{A} \cdot \overline{B} + 0$$

$$AB + \overline{A} \cdot \overline{B} + 0$$

$$AB + \overline{A} \cdot \overline{B}$$

Demorgan's Law
Idempotent Law
Distributive Law
Distributive Law
Inverse Law
Identity Law
Distributive Law
Distributive Law
Inverse Law
Identity Law
Identity Law
Identity Law

(b)
$$\overline{C \cdot B} + A \cdot B \cdot C + \overline{A + C + \overline{B}}$$

$$\overline{CB} + ABC + \overline{A + C + \overline{B}}$$

$$(\overline{C} + \overline{B}) + ABC + \overline{A} + C + \overline{B}$$

$$\overline{C} + \overline{B} + ABC + \overline{A} \cdot \overline{C}B$$

$$\overline{C} + \overline{B} + ABC + \overline{A}B\overline{C}$$

$$(\overline{C} + \overline{B}) + ABC$$

$$(\overline{C} + \overline{B}) + AB$$

$$(\overline{C} + \overline{B}) + A$$

$$(c) (A + B) \cdot (\overline{A} + C) \cdot (\overline{C} + B)$$

Demorgan's Law Commutative Law Absorption Law Absorption Law Absorption Law

Demorgan's Law

$$(A+B)(\overline{A}+C)(\overline{C}+B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}+C)\overline{C} + (A+B)(\overline{A}+C)B \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+B)(C\overline{C}) + (A+B)(\overline{A}+C)B \qquad \text{Inverse Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+B)(0) + (A+B)(\overline{A}+C)B \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+B)(\overline{A}+C)B \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(BB) \qquad \text{Idempotent Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+C)(AB) + (A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+B)(A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+B)(A+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}\cdot\overline{C}) + (A+B)(\overline{A}+C)(B) \qquad \text{Distributive Law} \\ (A+B)(\overline{A}+C)(B) \qquad \text{Distributive Law} \\$$

2. Find all solutions of the following Boolean equations without using the truth tables: