CSCI 2200 — Foundations of Computer Science (FoCS)
Problem Set 5 (document version 1.1)

Overview

- This problem set is due at your Wednesday, October 26 recitation
- You may work on this problem set in a group of no more than four students; **each of your teammates must be in your recitation section**
- Please start this problem set early and ask questions during office hours and at your recitation section; also ask (and answer) questions on the Discussion Forum
- You can type or hand-write (or both) your solutions to the required graded problems

Problems

These problems are generally good practice problems to work on. Those marked with an asterisk (*) are required and will be reviewed/graded in recitation.

- **Problem 9.1.**
- **Problem 9.2(a-f).**
- ***Problem 9.2(g-h).**
- **Problem 9.3(a-i).**
- ***Problem 9.3(j-k).**
- **Problem 9.10.**
- ***Problem 9.14.**
- **Problem 9.18.**
- ***Problem 9.31(a-b).**
- **Problem 9.69-9.72.**

(v1.1) The above problems are transcribed in the pages that follow.

- **Problem 9.1.** Compute the following sums.

(a) $\sum_{i=1}^{5} 1$

(b) $\sum_{i=1}^{5} i$

(c) $\sum_{i=1}^{5} i^2$

(d) $\sum_{i=1}^{5} (4-i)$

(e) $\sum_{i=1}^{5} 2$

(f) $\sum_{i=1}^{5} 2^i$

(g) $\sum_{i=1}^{5} (2^i)^2$

(h) $\sum_{i=1}^{5} 2^{i^2}$

(i) $\sum_{i=1}^{5} \ln i$

(j) $\sum_{i=1}^{5} \ln i^2$

(k) $\sum_{i=1}^{5} (\ln i)^2$

(l) $\sum_{i=1}^{5} 2^{\log_2 i}$

(m) $\sum_{i=1}^{3} \sum_{j=1}^{3} 2$

(n) $\sum_{i=1}^{3} \sum_{j=1}^{3} (i-j)$

(o) $\sum_{i=1}^{3} \sum_{j=1}^{i} 2$

(p) $\sum_{i=1}^{3} \sum_{j=1}^{i} (i-j)$

(q) $\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} 2$

(r) $\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} 2^{i+j+k}$

(s) $\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} (i+j)$

(t) $\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} ijk$

- **Problem 9.2(a-f).** Tinker and then compute formulas that do not contain a sum for the following:

(a) $\sum_{i=1}^{n} 3i$

(b) $\sum_{i=1}^{n} (3i+2j)$

(c) $\sum_{i=1}^{2n} (1+2i)$

(d) $\sum_{i=1}^{n} (3i+2i^2)$

(e) $\sum_{i=1}^{n} (i+1)^2$

(f) $\sum_{i=0}^{n} 2^{3+i}$

- ***Problem 9.2(g-h).** Tinker and then compute formulas that do not contain a sum for the following:

(g) $\sum_{i=1}^{n} ij$

(h) $\sum_{i=0}^{n} (i+j)^2$

- **Problem 9.3(a-i).** Compute formulas that do not contain a sum for the following:

(a) $\sum_{i=1}^{n}\sum_{j=1}^{m}(i+j)$ (d) $\sum_{i=1}^{n}\sum_{j=1}^{i}(i+j)^2$ (g) $\sum_{i=0}^{n}\sum_{j=0}^{i}2^{i+j}$

(b) $\sum_{i=1}^{n}\sum_{j=1}^{i}(i+j)$ (e) $\sum_{i=0}^{n}\sum_{j=0}^{m}2^{i+j}$ (h) $\sum_{i=0}^{n}\sum_{j=i}^{n}(i+j)$

(c) $\sum_{i=0}^{n}\sum_{j=0}^{n}(2^i+2^j)^2$ (f) $\sum_{i=0}^{n}\sum_{j=0}^{i}(2^i+2^j)^2$ (i) $\sum_{i=0}^{n}\sum_{j=0}^{i}(2^j+i)^2$

- ***Problem 9.3(j-k).** Compute formulas that do not contain a sum for the following:

(j) $\sum_{i=0}^{n}\sum_{j=0}^{i}2^i$ (k) $\sum_{i=0}^{n}\sum_{j=0}^{i}i2^j$

- **Problem 9.10.** Here are errors in the use of asymptotic notation. Explain why they are errors.

(a) $2n^2+n=\Theta(n^2)$.

(b) $4^n \in \Theta(2^n)$ because 4 is a constant factor bigger than 2, and we ignore constants.

(c) $O(1)+O(1)=O(1)$.

(d) Look! Your runtime $T \in O(n^2)$, so your algorithm is slower than linear.

(e) Look! My runtime $T \in o(n^3)$, so your algorithm is super fast (linear).

(f) $f \in O(g)$ (i.e., "$f \leq g$"). Taking exponents on both sides, we conclude $2^f \in O(2^g)$.

- ***Problem 9.14.** Determine which of these functions is in $\Theta(n)$, in $\Theta(n^2)$, or neither.

(a) 10 (c) $\lfloor n \rfloor$ (e) n^2+n+1 (g) $5n\log n$ (i) n^2+3n

(b) $3n+9$ (d) $\lceil n/2 \rceil$ (f) $\lfloor n \rfloor \cdot \lceil n/2 \rceil$ (h) 2^n (j) $n^2\log n$

- **Problem 9.18.** Answer true or false.

(a) $\sqrt{n}\ln n \in O(n)$
(b) $2n^2 + 1 \in O(n^2)$
(c) $\sqrt{n} \in O(\ln n)$
(d) $\ln n \in O(\sqrt{n})$
(e) $3n^3 + \sqrt{n} \in \Theta(n^3)$
(f) $\ln n \in \Theta(\log_2 n)$
(g) $2^{n+1} \in O(2^n)$
(h) $2^{2n} \in O(2^n)$
(i) $3^n \in O(2^n)$
(j) $3n^3(1 + \sqrt{n}) \in \Theta(n^3)$
(k) $\ln n^2 \in \Theta(\ln n)$
(l) $2^n \in \Theta(3^n)$
(m) $\ln^2 n \in \Theta(\ln n)$
(n) $2^n \in O(3^n)$
(o) $\ln(2^n) \in \Theta(\ln(3^n))$
(p) $2^{2\log_2 n} \in \Theta(n^2)$
(q) $2^{2\ln n} \in \Theta(n^2)$
(r) $2^{2\ln n} \in O(n^2)$
(s) $n! \in \Theta(n^n)$
(t) $n! \in O(n^n)$
(u) $\sum_{i=1}^{n} i^2 \in \Theta(n^3)$
(v) $\sum_{i=1}^{n} \sqrt{i} \in \Theta(n^2)$
(w) $\sum_{i=1}^{n} 2^i \in \Theta(2^n)$
(x) $\sum_{i=1}^{n} 3^i \in \Theta(3^n)$
(y) $\sum_{i=1}^{n} 3^i \in \Theta(2^n)$

- ***Problem 9.31(a-b).** Give the asymptotic big-Theta behavior of the runtime T_n, where

(a) $T_0 = 1$; $T_n = T_{n-1} + n^2$ for $n \geq 2$.
(b) $T_0 = 1$; $T_1 = 2$; $T_n = 2T_{n-1} - T_{n-2} + 2$ for $n \geq 2$.