CSCI 2200 — Foundations of Computer Science (FoCS)
Problem Set 3 (document version 1.1)

Overview

- This problem set is due at your Wednesday, September 28 recitation
- You may work on this problem set in a group of no more than four students; **each of your teammates must be in your recitation section**
- Please start this problem set early and ask questions during office hours and at your recitation section; also ask (and answer) questions on the Discussion Forum
- You can type or hand-write (or both) your solutions to the required graded problems

Problems

These problems are generally good practice problems to work on. Those marked with an asterisk (*) are required and will be reviewed/graded in recitation.

- ***Problem PS3.1 below.**
- ***Problem PS3.2 below.**
- ***Problem 6.21.** (Strong induction will be covered on Tuesday, September 27.)
- **Problems 4.37 and 4.38.**
- **Problem 5.21.**
- **Problem 5.28.**
- **Problem 5.60.**

(v1.1) Some of the above problems are transcribed below.

- ***Problem PS3.1.** Named after mathematician Édouard Lucas (1842-1891), the *Lucas numbers* are similar to the well-known *Fibonacci numbers* in how we determine the next number in the sequence.

  Specifically, the Lucas numbers are fully defined by $L_n = L_{n-1} + L_{n-2}$, where $n \geq 2$, $L_0 = 2$, and $L_1 = 1$. The Fibonacci numbers are fully defined by $F_n = F_{n-1} + F_{n-2}$, where $n \geq 2$, $F_0 = 0$, and $F_1 = 1$.

  Use induction to prove the following claims:

  (a) $L_0 + L_1 + L_2 + \ldots + L_n = L_{n+2} - 1$.
  (b) $L_n = F_{n-1} + F_{n+1}$.

- ***Problem PS3.2.** Use induction to prove $2 + 6 + 12 + \ldots + (n^2 - n) = \frac{n(n^2-1)}{3}$.

- ***Problem 6.21.** Prove that for $n \geq 1$, there is $k \geq 0$ and ℓ odd such that $n = 2^k \ell$.

- **Problem 5.21.** Let A be a finite set of size $n \geq 1$. Prove by induction that $|\mathcal{P}(A)| = 2^n$.

- **Problem 5.28.** Prove each claim by induction for $n \geq 3$.

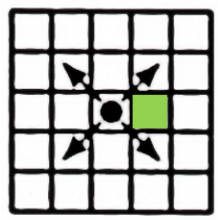
  (a) There is a set with n numbers $x_1, \ldots, x_n$ such that each x_i divides the sum $s = x_1 + \ldots + x_m$.

  (b) There is a convex polygon with at least 3 acute internal angles.

  (c) There are n distinct positive numbers whose reciprocals sum to 1.

- **Problem 5.60.** A robot has a repetoire of moves on an infinite grid as shown in the figures.

  (a) The robot moves one diagonal step at a time. Prove that no sequence of moves takes the robot to the shaded square. [Hint: Let (x, y) be the robot's position. Consider $x + y$.]



  (b) One of the moves changed (i.e., see revised figure below). Now prove that any square (m, n) can be reached by a finite sequence of moves.