CSCI 2200 FOUNDATIONS OF COMPUTER SCIENCE

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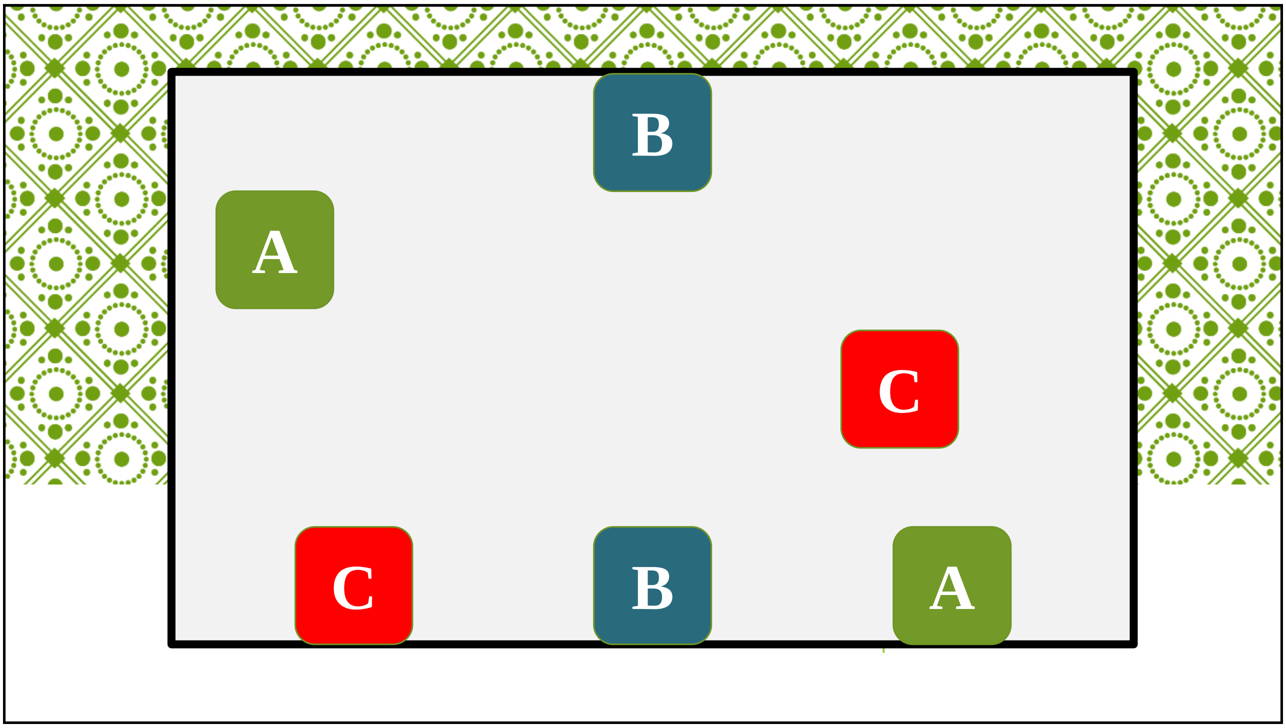
Fall 2022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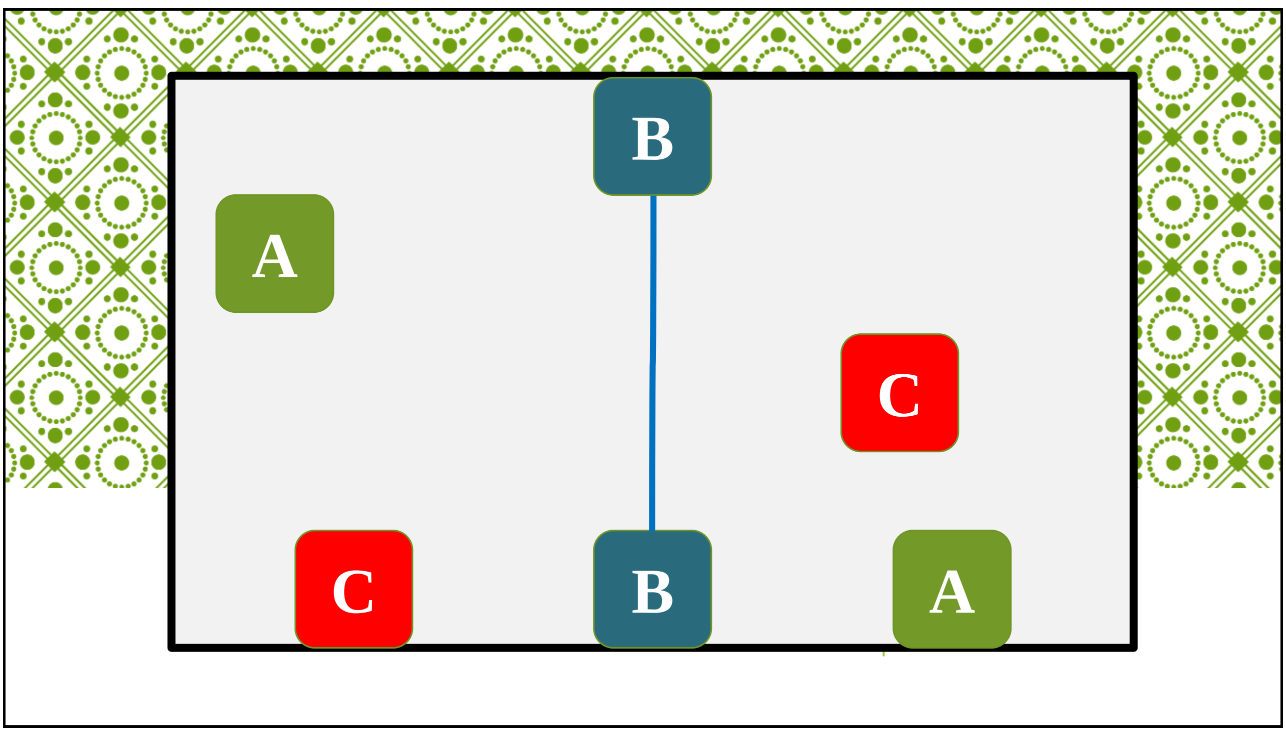
B
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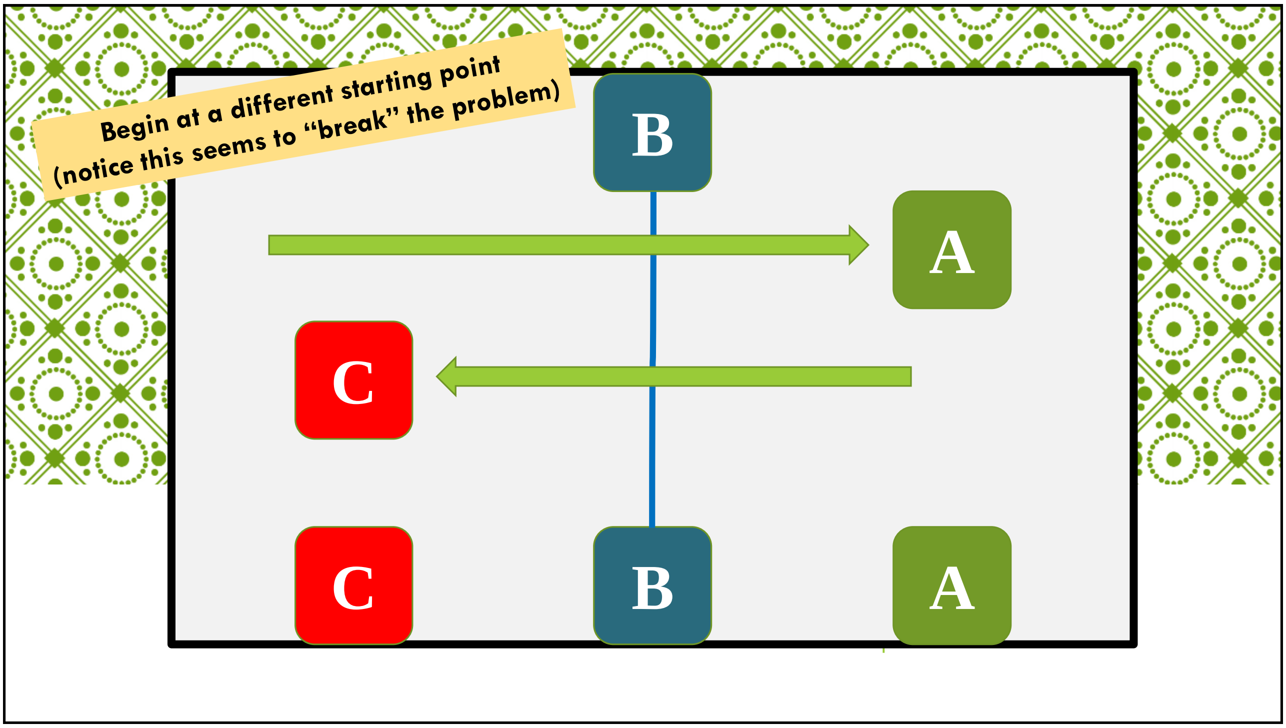
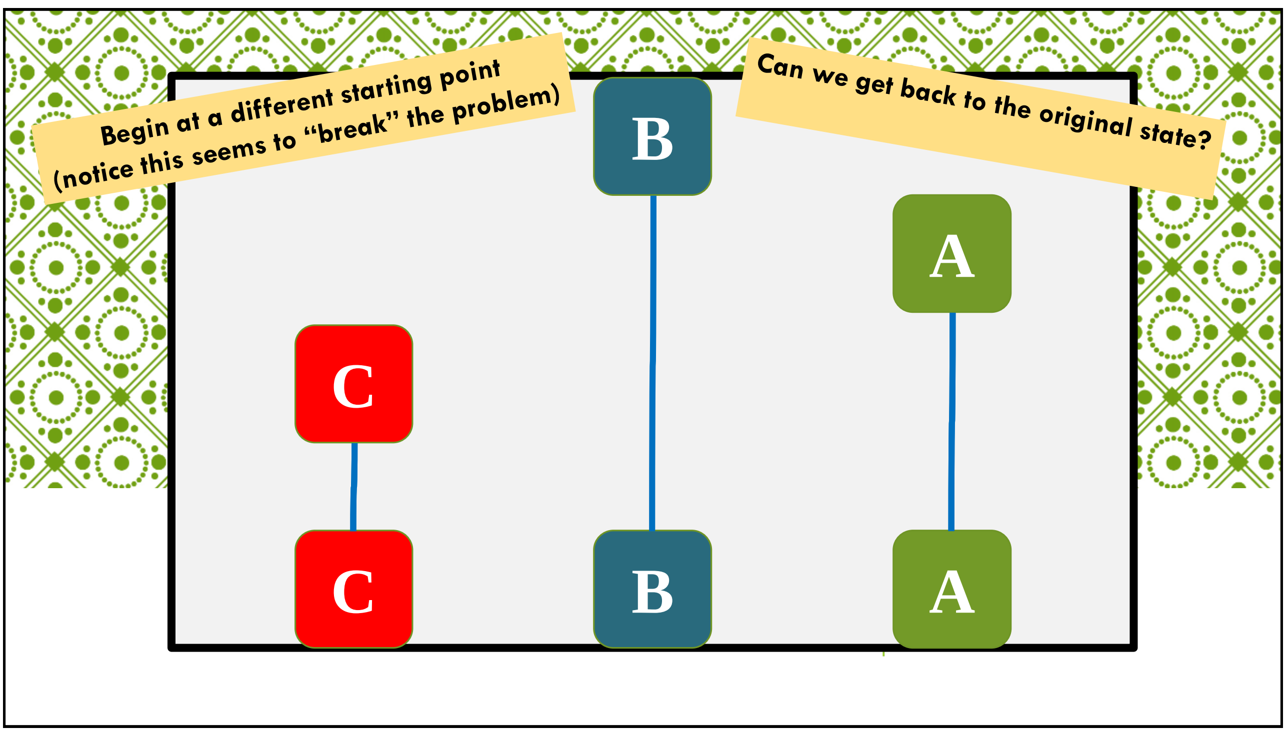
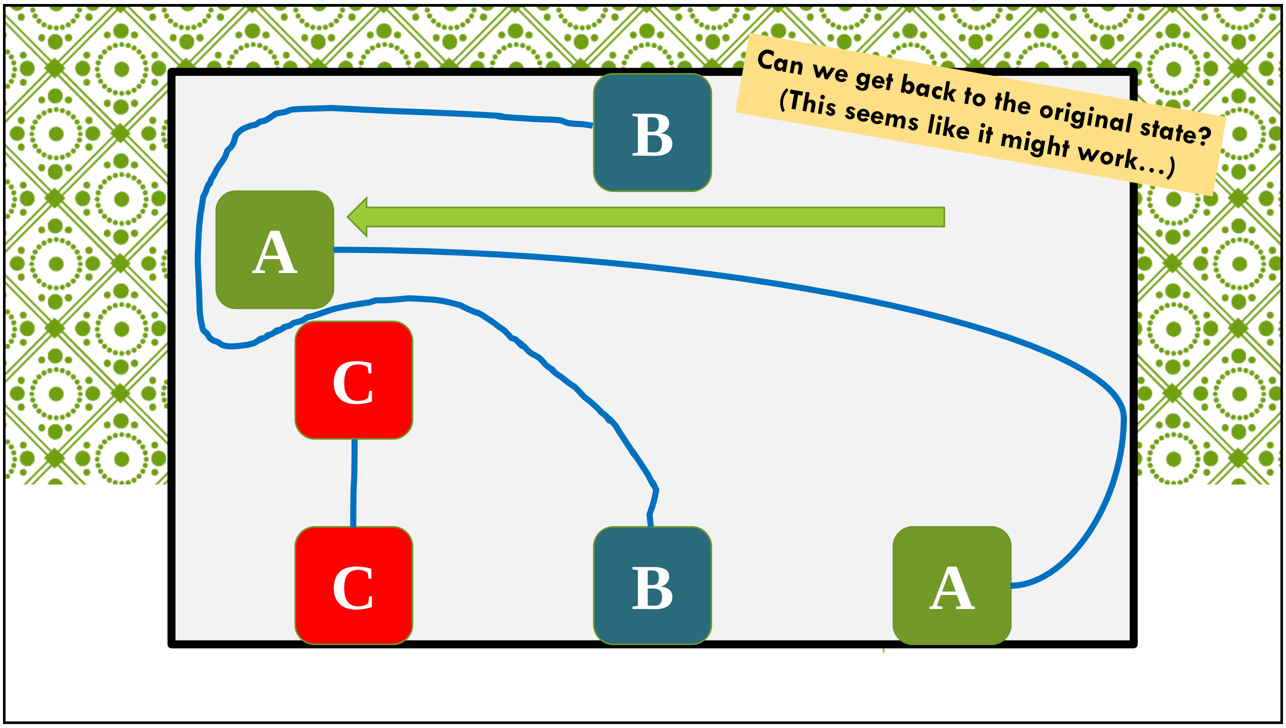
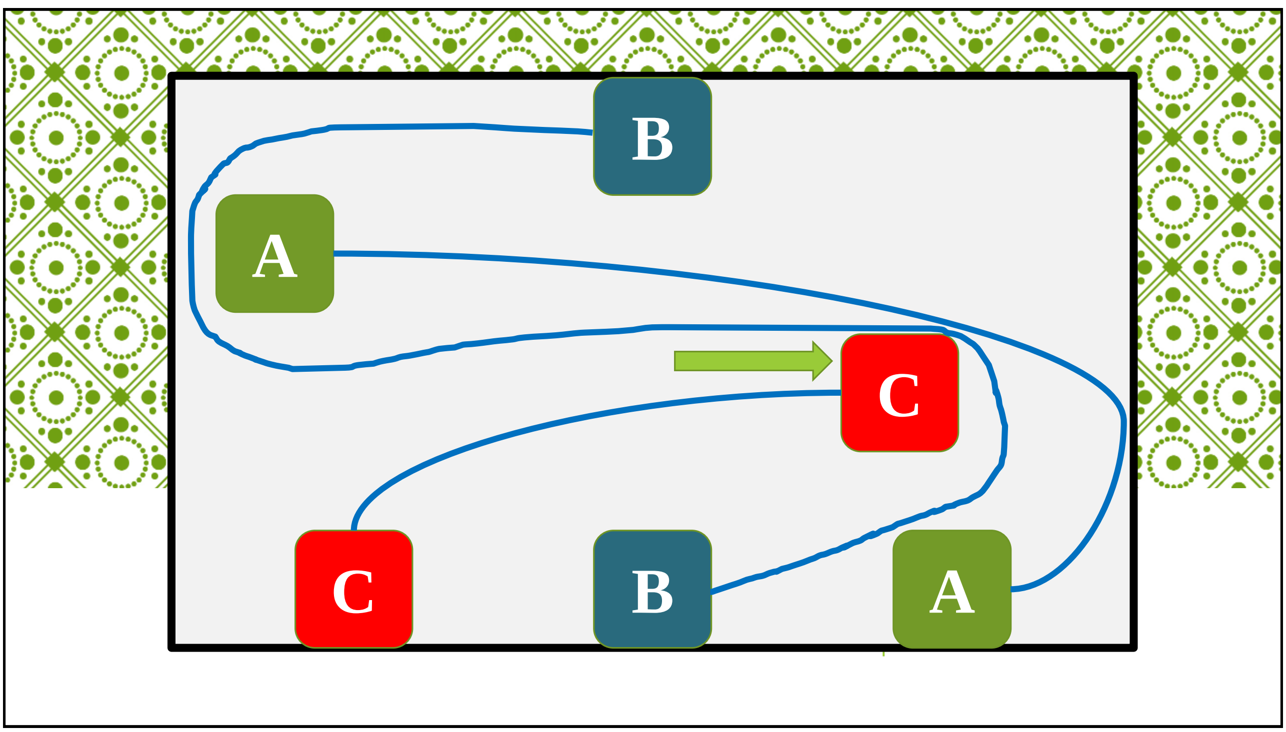

Begin at a different starting point
(notice this seems to "break" the problem)
B
A
C
C
B
A


Begin at a different starting point
(notice this seems to "break" the problem)
Can we get back to the original state?
B
A
C
C
B
A


Can we get back to the original state?
(This seems like it might work…)
B
A
C
C
B
A


B
A
C
C
B
A

REMINDERS…

Get the textbook

Read the Preface and Chapters 0 and 1 of the textbook

Tinker with the problems from last class and from the textbook

Turn on Submitty email notifications

Re-read the syllabus slides and post any questions on the Discussion Forum

Email me directly (goldsd3@rpi.edu) about any registration/SIS issues

DISCRETE OBJECTS

A *discrete object* refers to something with clearly definable and knowable boundaries

Collections (or containers) of discrete objects include *sets* and *sequences*

A *graph* models and helps visualize relationships between discrete objects

DISCRETE OBJECTS – SETS

A *set* is a collection of discrete objects with no repetition (i.e., no duplicate objects)

$V = \{ a, e, i, o, u \}$ $G = \{ g, o, l, d, y \}$ $R = \{ r, p, i \}$

Set operations include *intersection*, *union*, and *complement* — new sets are formed

$G \cap V = \{ o \}$ $G \cup V = \{ g, o, l, d, y, a, e, i, u \}$ $G \cap R = \{ \} = \varnothing$

$G \cup V \cup R = \{ a, d, e, g, i, l, o, p, r, u, y \}$ $\overline{G} = ???$

(we need a universal set to describe this)

Order does not matter…

$G = \{ o, l, d, y, g \}$ $G = \{ d, g, l, o, y \}$ $R = \{ r, i, p \}$

DISCRETE OBJECTS – SETS

Order does not matter…

$G = \{ o, l, d, y, g \}$ $G = \{ d, g, l, o, y \}$ $R = \{ r, i, p \}$

Order <u>does</u> matter when we use ellipses

(universal set of lowercase letters)

$U = \{ a, b, c, \ldots, x, y, z \}$ $X = \{ p, e, r, \overset{?}{\ldots}, i, o, n \}$

$U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z \}$

See how much order can matter — which set below makes sense?

$E = \{ 2, 4, 6, 8, 10, \ldots \}$ $E' = \{ 2, 4, 6, 8, 11, \overset{?}{\ldots} \}$

DISCRETE OBJECTS – PREDEFINED SETS

We will use a few important predefined sets

$\mathbb{N} = \{ 1, 2, 3, 4, 5, \ldots \}$	(natural numbers)
$\mathbb{Z} = \{ 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots \}$	(integers)
$\mathbb{N}_0 = \{ 0, 1, 2, 3, 4, \ldots \}$	(natural numbers and zero)
$\mathbb{Q} = \{ r \mid r = \frac{a}{b} ; a \in \mathbb{Z}, b \in \mathbb{N} \}$	(rational numbers)

How can we better define our set $E = \{ 2, 4, 6, 8, 10, \ldots \}$?

$$E = \{ n \mid n = 2k \; ; k \in \mathbb{N} \}$$

DISCRETE OBJECTS – SUBSETS

We describe sets within sets using the *subset relation*

$A \subseteq B$	"A is a subset of B"	(everything in A is in B)
$A \subset B$	"A is a proper subset of B"	(everything in A is in B…
		…but something in B is <u>not</u> in A)

We can define *set equality* using subsets

$A = B$ "A equals B" (both $A \subseteq B$ and $B \subseteq A$ are true)

The empty set $\varnothing$ is a subset of any set, i.e., $\varnothing \subseteq A$ for any set A

DISCRETE OBJECTS — SET CARDINALITY

The number of elements in a set may be infinite, e.g., for sets $\mathbb{N}$ or $\mathbb{Z}$

For any finite set, the *cardinality* is simply the number of elements in the set

$V = \{ a, e, i, o, u \}$ $G = \{ g, o, l, d, y \}$ $R = \{ r, p, i \}$

$|V| = 5$ $|G| = 5$ $|R| = 3$

$U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z \}$

$|U| = 26$

The cardinality of the empty set $\varnothing$ is simply $|\varnothing| = 0$

DISCRETE OBJECTS — POWER SETS

Sets can contain other sets — remember that sets contain discrete objects

The *power set* of set A is a set consisting of all subsets of A

$A = \{ a, b \}$ $\mathcal{P}(A) = \{ \varnothing, \{ a \}, \{ b \}, \{ a, b \} \}$

What is the power set of set $F = \{ a, b, f \}$?

$\mathcal{P}(F) = \{ \varnothing, \{ a \}, \{ b \}, \{ f \}, \{ a, b \}, \{ a, f \}, \{ b, f \}, \{ a, b, f \} \}$

Do you see a pattern for power set cardinality?

$|\mathcal{P}(X)| = 2^{|X|}$

$|A| = 2$ $|\mathcal{P}(A)| = 4$ $|F| = 3$ $|\mathcal{P}(F)| = 8$

DISCRETE OBJECTS – SEQUENCES

A *sequence* is a list of discrete objects for which order and repetition matter

Think of a sequence as a *string* of characters or symbols

The following sequences are all distinct from one another

g *tg* *ac* *tgac* *atgattgacattgagga* *cat*

david *goldschmidt* *tdimhcsdlog* *divad*

What does the binary sequence below seem to indicate…?

0110110101100101011011010110010 1

DISCRETE OBJECTS – GRAPHS

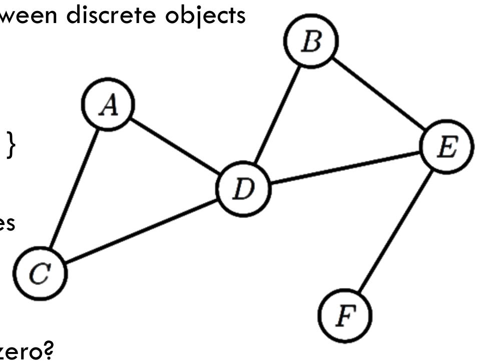
A *graph* models and helps visualize relationships between discrete objects

We define a graph using two sets V and E

Set V contains the vertices, e.g., $V = \{ A, B, C, D, E, F \}$

Set E contains the edges, in this case undirected edges

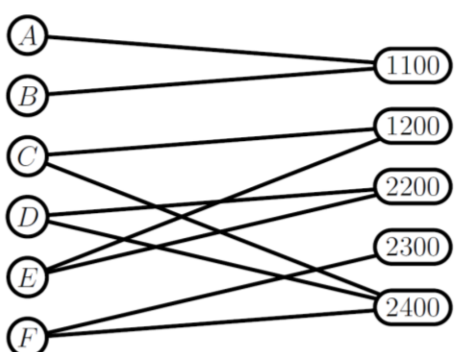
$E = \{ (A,C), (A,D), (C,D), (B,D), (B,E), (D,E), (E,F) \}$



What does $|V|$ and $|E|$ represent? Can either be zero?

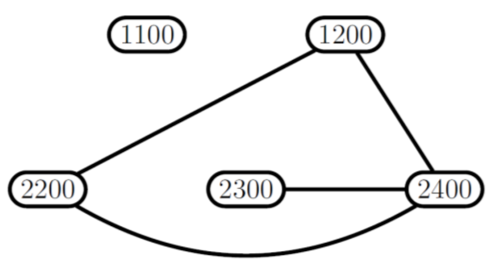
DISCRETE OBJECTS – GRAPHS

Affiliation graphs map discrete objects to groups those objects are affiliated with



Graph A: students in CSCI courses

Conflict graphs model contention or competition between discrete objects

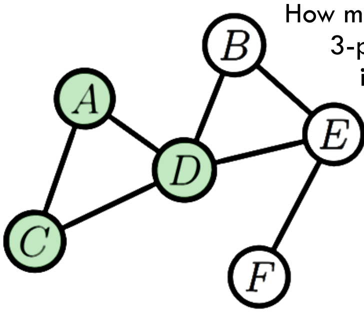


Graph B: CSCI courses with edges showing students registered for both courses

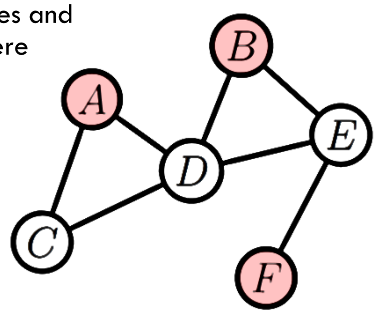
DISCRETE OBJECTS – CLIQUES AND WARS!

A *3-person-clique* is a set of three vertices connected with three edges, e.g., { *A*, C, *D* }

A *3-person-war* is a set of three vertices with no shared edges, e.g., { *A*, *B*, *F* }



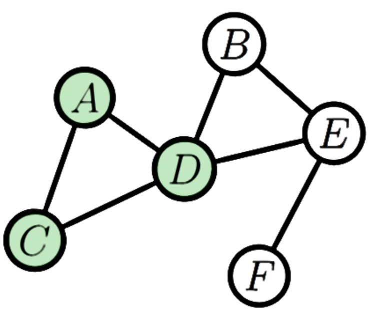
How many 3-person-cliques and 3-person-wars are there in the given graph?

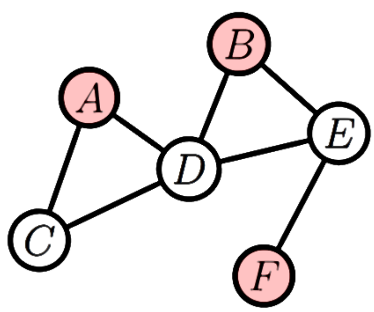


DISCRETE OBJECTS – CLIQUES AND WARS!

Does every graph with six vertices have both a 3-person-clique and a 3-person-war?

Is it possible to construct a graph with neither of these structures?



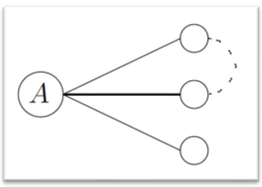


DISCRETE OBJECTS — CLIQUES AND WARS!

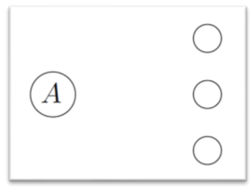
Is it possible to construct a graph with neither of these structures?

Start with any vertex A — vertex A must have at least three friends or at least three enemies

If A has more friends than enemies, we have either a 3-person-war or, by adding the dashed edge below, a 3-person-clique with A



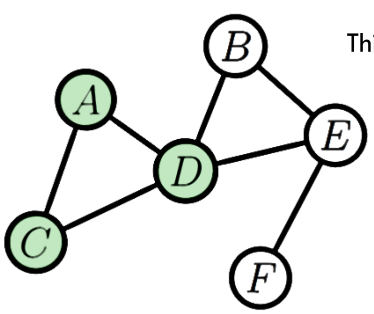
If A has more enemies than friends, any pair of A's enemies form a 3-person-war with A; to avoid this, three of A's enemies must be in a clique



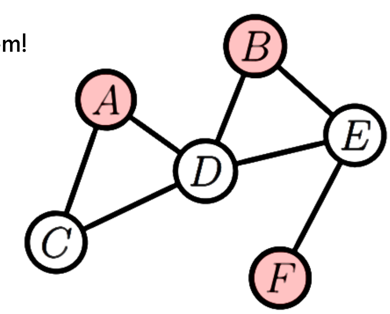
DISCRETE OBJECTS — CLIQUES AND WARS!

Every graph with six vertices has either a 3-person-clique or a 3-person-war (or both)

We used a case-by-case *exhaustive proof* to show this



This is now a theorem!



AXIOMS, CONJECTURES, AND THEOREMS

An *axiom* is a self-evident statement that is asserted as true without proof

A *conjecture* is a claim that is believed to be true but is not trusted to be true until proven

A *theorem* is a proven conjecture, i.e., a proven truth

try constructing a subset of $\mathbb{Z}$ that has no minimum element…

e.g., the *Well-Ordering Principle* is an axiom stating that any non-empty subset of $\mathbb{N}$ has a minimum element

Is there a minimum element in the following sets?

$A = \{ 2, 5, 4, 11, 7, 296, 81 \}$ $B = \{ 6, 19, 24, 18, \ldots \}$ $C = \varnothing$

$\mathbb{Q} = \{ r \mid r = \frac{a}{b}; a \in \mathbb{Z}, b \in \mathbb{N} \}$

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

Assume that $\sqrt{2}$ is rational, which means we can write it as a fraction…

$$\boxed{\sqrt{2}} = \left\{ \frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}, \ldots \right\}$$

…actually an infinite number of fractions

Each numerator is unique; and each denominator is unique

From the Well-Ordering Principle, there must be a minimum denominator b_* and a corresponding minimum numerator a_*

$$\sqrt{2} = \frac{a_*}{b_*}$$

$\mathbb{Q} = \{ r \mid r = \frac{a}{b}; a \in \mathbb{Z}, b \in \mathbb{N} \}$

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

$$\sqrt{2} = \frac{a_*}{b_*}$$

For b_* to be the minimum possible, it must be that a_* and b_* have no common factors

Square both sides to get rid of the square root, then rearrange a little bit

$$a_*^2 = 2b_*^2$$

(with $k \in \mathbb{N}$)

Aha, a_*^2 is even since it is a multiple of 2, so a_* is even and we can say $a_* = 2k$

Plug $2k$ in for a_* above and we get $(2k)^2 = 2b_*^2$ or $b_*^2 = 2k^2$ — this means b_*^2 is even!

Whoops, if a_* and b_* are both even, they have a common factor of 2

$\mathbb{Q} = \{ r \mid r = \frac{a}{b}; a \in \mathbb{Z}, b \in \mathbb{N} \}$

$\sqrt{2}$ IS IRRATIONAL – PROOF BY CONTRADICTION

we have an impossible situation, i.e., a contradiction

$$\sqrt{2} = \frac{a_*}{b_*}$$

For b_* to be the minimum possible, it must be that a_* and b_* have no common factors

Square both sides to get rid of the square root, then rearrange a little bit

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Whoops, if a_* and b_* are both even, they have a common factor of 2

WHAT NEXT…?

No classes on Monday

Tuesday follows a Monday schedule (also for office hours!)

Recitations start on Wednesday…

…and Problem Set 1 is due at your first recitation

Homework 1 will be posted early next week, too (due September 15)