- **Problem 7.9.** $G^0 = 0$. $G_1 = 1$ and $G_n = 7G_{n-1} - 12G_{n-2}$ for $n > 1$. Compute $G_5$. Show $G_n = 4^n - 3^n$ for $n \geq 0$.

- **Problem 7.12(c). (See Problem 7.28 for hints.)** Tinker to guess a formula for each recurrence and prove it. In each case $A_1 = 1$ and for $n > 1$:

  (c) $A_n = 10nA_{n-1}/(n-1) + n$

| $n$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $A_n$ | 22 | 333 | 4444 | 55555 |

i. formula found:

$$\frac{10^n - 1}{9} n$$

ii. prove the base case:

$$\begin{aligned} A(2) &= \frac{100-1}{9}(2) \\ &= \frac{99}{9}(2) = 22 \end{aligned}$$

base case proven

iii. prove using direct proof

$$10n\frac{A(n-1)}{n-1} + n = \frac{10^n - 1}{9}(n)$$

with with LHS

$$\begin{aligned} 10n\frac{A(n-1)}{n-1} + n &= 10n\frac{\frac{10^{n-1}-1}{9}\cancel{(n-1)}}{\cancel{n-1}} + n \\ &= \cancel{(10)}n(\frac{10^n - 10}{\cancel{10}}\frac{1}{9}) + n \\ &= n\frac{10^n - 10}{9} + n \\ &= n\frac{10^n - 10}{9} + \frac{9n}{9} \\ &= \frac{10^n n - 10n + 9n}{9} \\ &= \frac{10^n n - n}{9} \\ \frac{10^n - 1}{9}n &= \frac{10^n - 1}{9}n \end{aligned}$$

iv. we prove by direct proof that the statement is true for all $n > 1$ ■

- **Problem 7.13(a).** Analyze these very fast growing recursions. [Hint: Take logarithms.]

  (a) $M_1 = 2$ and $M_n = aM_{n-1}^2$ for $n > 1$. Guess and prove a formula for $M_n$. Tinker, tinker.

| $n$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $A_n$ | a4 | a16 | a256 | a65536 |

(i) formula found:

$$M(n) = 2^{2^{n-1}}$$

(ii) base case:

$$\begin{aligned} M(2) &= a(2^{2^1}) \\ &= a(2^2) \\ &= a4 \end{aligned}$$

base case proven

(iii) prove using direct proof

$$\begin{aligned} aM(n-1)^2 &= a2^{2^{n-1}} \\ M(n-1)^2 &= 2^{2^{n-1}} \text{ simplify} \\ \log_2(M(n-1)^2) &= \log_2(2^{2^{n-1}}) \text{ log both sides} \\ \log_2(M(n-1)^2) &= 2^{2^{n-1}} \end{aligned}$$

work with LHS

$$\begin{aligned} \log_2(M(n-1)^2) &= 2\log_2(M(n-1)) \\ &= 2\log_2(2^{2^{(n-1)-1}}) \\ &= 2(2^{n-2}) \\ &= 2^{n-1} \end{aligned}$$

(iv) we prove by direct proof that the statement is true for all $n > 1$ ■

- **Problem 7.19(d).** Recall the Fibonacci numbers: $F_1$, $F_2 = 1$; and, $F_n = F_{n-1} + F_{n-2}$ for $n > 2$

  (d) Prove that every third Fibonacci number, $F_{3n}$, is even

- **Problem 7.42.**
- **Problem 7.45(c).**
- **Problem 7.49.**
- **Problem 8.12(d).**
- **Problem 8.14.**