1. assume n is an integer, give direct and contraposition proofs

   (a) ($n^3 + 5$ is odd) => (n is even)

      i. direct proof

         - $n^3 + 5$ is odd, $n^3 + 5 = 2k + 1$, $n^3 = 2k - 4$
         - $n = \sqrt[3]{2k - 4}$
         - a cube root of an even number is always even by definition, statement claimed in p is true

      ii. contraposition proof

         - assume n is not even, n is odd, $n = 2k + 1$
         - $n^3 + 5 = (2k + 1)^3 + 5 => 8k^3 + 12k^2 + 6k + 6$
         - $8k^3 + 12k^2 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3)$
         - we have shown that p is even when n is odd, the statement claimed is true

   (b) (3 does not divide n) => (3 divides $n^2 + 2$)

      i. direct proof

         - if 3 does not divide n, then $n \neq 3k$
         - assuming p is true, 3 divides $n^2 + 2$, so $n^2 + 2 = 3k$
         - $n^2 = 3k - 2$, then $n = \sqrt{3k - 2}$
         - plugging in $n = 3k$, we get $n = \sqrt{n - 2}$
         - this is not true for all cases, therefore, this statement is false

      ii. contraposition proof

         - 3 does not divide by $n^2 + 2$
         - therefore, $n^2 + 2 \neq 3k$, for some integer k

- 3 does not divide n, $n \neq 3k$
- $(3k)^2 + 2 \neq 3k$

2. prove by contradiction

   (a) $(x, y) \in \mathbb{Z}^2 \to x^2 - 4y - 3 \neq 0$

   - assume $x^2 - 4y - 3 = 0$
   - rearranging variables to isolate x, we get $x^2 = 4y + 3$
   - we now know that x^2 is odd, therefore $x^2 = 2k + 1$
   - substitute x^2 in, $2k + 1 - 4y = 3$
   - isolate y, we get $\frac{2k+1-3}{4} = y$, $y = \frac{2k-2}{4}$
   - simplify to get $y = \frac{k-1}{2}$, uh oh, there are integers k that makes y not a positive integer
   - the statement is true due to proof by contradiction

3. prove these if and only if, prove two implications

   (a) prove: 4 divides $n \in \mathbb{Z}$ IF AND ONLY IF $n = 1 + (-1)^k(2k - 1)$ for $k \in \mathbb{N}$. (Try $n < 0$, $n = 0$, $n > 0$; k is even/odd.)

   i. p $\to$ q

   - direct proof: $4k = n \to n = 1 + (-1)^k(2k - 1)$

   ii. q $\to$ p

   - hi

4. determine the type of proof and prove

   (a) If n is odd, then $n^2 - 1$ is divisible by 8.

   - this statement can be proven by a direct proof

- if n is odd, then $n = 2k + 1$
- then with $n^2 - 1$, you can substitute n for $2k + 1$
- becomes $(2k + 1)^2 - 1 = 4k^2 + 4k + 1 - 1$
- at its simplest form, it is $4(k^2 + k) \neq 8k$
- by direct proof, the statement is false