CSCI 2200 — Foundations of Computer Science (FoCS)
Problem Set 1 (document version 1.1)

Overview

- This problem set is due at your Wednesday, September 7 recitation
- You may work on this problem set in a group of no more than four students; **each of your teammates must be in your recitation section**
- Please start this problem set early and ask questions during office hours and at your recitation section; also ask (and answer) questions on the Discussion Forum
- **(v1.1)** You can type or hand-write (or both) your solutions to the required graded problems

Problems

These problems are generally good practice problems to work on. Those marked with an asterisk (*) are required and will be reviewed/graded in recitation.

- All refresher questions in Chapter 0
- **Problem 1.1**
- **Problem 1.6**
- **Problem 1.7**
- **Problem 2.1**
- **Problem 2.2**
- ***Problem 2.3**
- **Problem 2.6**
- **Problem 2.7**
- **Problem 2.8**
- ***Problem 2.11**
- **Problem 2.12**
- **Problem 2.13**
- ***Problem 2.14**
- **Problem 2.15**
- **Problem 2.20**
- **Problem 2.22**
- ***Problem 2.24**
- **Problem 2.29**

(v1.1) The above problems are transcribed in the pages that follow.

- **Problem 1.1** The parity of an integer is 0 if it is even and 1 if it is odd. Which of the following operations preserve parity:

  (a) Multiplying by an even.

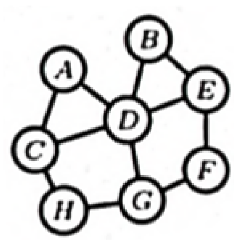
  (b) Multiplying by an odd.

  (c) Raising to a positive integer power.

- **Problem 1.6** Students $A, \ldots, H$ form a friendship network (below). To advertise a new smartphone, you plan to give some students free samples. Here are two models for the spread of phone-adoption.

  Model 1 (Weak Majority): People buy a phone if at least as many friends have the phone as don't.

  Model 2 (Strong Majority): People buy a phone if more friends have the phone than don't.

  (a) Use your intuition and determine the most "central" of the people in this friend-network.

  (b) If you give a phone only to this central node, who ultimately has a phone in: (i) Model 1 (ii) Model 2?

  (c) How many phones must you distribute, and to whom, so that everyone switches to your phone in Model 2?

  (d) Repeat part (c), but now you cannot give a phone to the central node.

  (A slight change to a model can have a drastic impact on the conclusions. A good model is important.)

- **Problem 1.7** Five radio stations (red stars) broadcast to different regions, as shown. The FCC assigns radio-frequencies to stations. Two radio stations with overlapping broadcast regions must use different radio-frequencies so that the common listeners do not hear garbled nonsense.

What is the minimum number of radio-frequencies that the government needs?

- **Problem 2.1** What is the difference between a Theorem, a Conjecture and an Axiom?

- **Problem 2.2** List the elements in the following sets (E is the set of even numbers).

(a) $A = \{n | -4 \leq n \leq 15; n \in E\}$.

(b) $A = \{x | x^2 = 9; x \in \mathbb{Z}\}$.

(c) $A = \{x | x^2 = 6; x \in \mathbb{Z}\}$.

(d) $A = \{x | x = x^2 - 1; x \in \mathbb{R}\}$.

- ***Problem 2.3** Give "formal" definitions of these sets using a variable.

(a) $A = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$.

(b) $B = \{0, 4, 16, 36, 64, 100, \ldots\}$.

(c) $C = \{1, 2, 4, 7, 11, 16, 22, \ldots\}$.

(d) $D = \{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$.

- **Problem 2.6** Give two sets A, B for which $A \nsubseteq B$ and $B \nsubseteq A$.

- **Problem 2.7** Complement depends on the universal set $\mathcal{U}$. Let $X = \{a, e\}$. What is $\overline{X}$ when:

(a) $\mathcal{U} = \{\text{lower case vowels}\}$.

(b) $\mathcal{U} = \{\text{lower case letters}\}$.

- **Problem 2.8** True of False: (a) $\mathbb{N} \subseteq \mathbb{Z}$ (b) $\mathbb{N} \subset \mathbb{Z}$ (c) $\mathbb{Z} \subseteq \mathbb{Q}$ (d) $\mathbb{Z} \subset \mathbb{Q}$

- ***Problem 2.11** Let $B = \{\{a, b\}, a, b, c\}$. List the power set $\mathcal{P}(B)$ (it has 16 elements).

- **Problem 2.12** List all subsets of $\{a, b, c, d\}$ that contain c but not d.

- **Problem 2.13**
  (a) What are $|M \cap V|$ and $\mathcal{P}(|M \cap V|)$ for $M = \{m, a, l, i, k\}$, $V = \{a, e, i, o, u\}$?
  (b) What is $|\mathbb{N}|$?

- ***Problem 2.14** $|A| = 7$ and $|B| = 4$. What are the possible values for $|A \cap B|$ and $|A \cup B|$?

- **Problem 2.15** What is the set $\mathbb{Z} \cap \overline{\mathbb{N}} \cap S$, where $S = \{z^2 | z \in \mathbb{Z}\}$ is the set of perfect squares?

- **Problem 2.20** A sequence $s_0, s_1, s_2, s_3, \ldots$ is described below. Give a "simple" formula for the nth term s_n in the sequence, for $n = 0, 1, 2, 3, \ldots$. Your answer should be of the form $s_n = f(n)$ for some function $f(n)$.
  (a) $0, 1, 2, 3, 4, 5, 6, \ldots$
  (b) $1, -1, 1, -1, 1, -1, \ldots$
  (c) $0, 1, -2, 3, -4, 5, -6, \ldots$
  (d) $2, 0, 2, 0, 2, 0, \ldots$
  (e) $1, 2, 4, 8, 16, \ldots$
  (f) $1, 3, 5, 7, 9, \ldots$
  (g) $1, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, \ldots$
  (h) $0, 3, 8, 15, 24, 35, 48, 63, \ldots$
  (i) $1, 2, 1/3, 4, 1/5, 6, 1/7, 8, 1/9, 10, 1/11, 12, \ldots$
  (j) $1, 1/2, 4, 1/8, 16, 1/32, 64, 1/128, \ldots$

- **Problem 2.22** Draw a picture of each graph representing friendships among our 6 friends $V = \{A, B, C, D, E, F\}$.
  (a) $E = \{(A, B), (B, C), (C, D), (D, E), (E, F), (F, A)\}$.
  (b) $E = \{(A, B), (A, C), (A, D), (A, E), (A, F)\}$.
  (c) $E = \{(A, D), (B, D), (C, D), (A, E), (B, E), (C, E), (A, F), (B, F), (C, F)\}$.
  (d) $E = \{(A, B), (B, C), (A, C), (D, E), (E, F), (D, F)\}$.

  You should recognize familiar social structures in your pictures.

- ***Problem 2.24** Model the relationship between radio-stations in Problem 1.7 using a graph.
  (a) Would you use friendship networks, affiliation graphs or conflict graphs?
  (b) Draw a picture of your graph for the 5 radio stations.
  (c) Show that 3 radio frequencies (1,2,3) suffice for no listener to hear garbled nonsense.

- **Problem 2.29** Mimic the method we used to prove $\sqrt{2}$ is irrational and prove $\sqrt{3}$ is irrational. Now use the same method to try and prove $\sqrt{9}$ is irrational. What goes wrong?