1. Problem PS 3.1 $\rightarrow$ Use induction to prove the following claims

   (a) $P(n) = L_0 + L_1 + L_2 + ... + L_n = L_{n+2} - 1$

   - BASE CASE: $L_0 = L_2 - 1$
   - $L_0 = 2$, $L_2 = 3$, $2 = 3 - 1 \rightarrow 2 = 2$, base case is true
   - INDUCTION: If $L_0 + L_1 + ... + L_n + L_{n+1} = L_{n+1+2} - 1$
   - LHS: $L_{n+2} - 1 + L_{n+1}$
   - We know that $L_n = L_{n-1} + L_{n-2}$
   - Then, $L_{n+3} = L_{n+3-1} + L_{n+3-2} = L_{n+2} + L_{n+1}$
   - We can substitute the LHS, it becomes $L_{n+3} - 1$, which is the same as the RHS.
   - With induction, we proved that the following statement is **T** for all n. $\blacksquare$

   (b) $L_n = F_{n-1} + F_{n+1}$

   - BASE CASE: $L_1 = F_0 + F_2$
   - $L_1 = 1$, $F_0 = 0$, $F_2 = 1$
   - $1 = 1$, base case is true
   - INDUCTION: $L_{n+1} = F_n + F_{n+2}$
   - We know that $L_{n+1} = L_n + L_{n-1}$
   - We can substitute L_n from above, $L_n = F_{n-1} + F_{n+1}$ and $L_{n-1} = F_{n-2} + F_n$
   - With this, we get $F_{n-1} + F_{n+1} + F_{n-2} + F_n$
   - Reorganize to get $F_{n-2} + F_{n-1} + Fn + Fn + 1$
   - We can get $F_n + Fn + 2$ from this.
   - With induction, we proved that the following statement is **T** for all n. $\blacksquare$

2. Problem 3.2 $\rightarrow$ Use induction to prove

   (a) $2 + 6 + 12 + ... + (n^2 - n) = \frac{n(n^2-1)}{3}$

   - BASE CASE: for n = 0: $0^2 - 0 = \frac{0(0^2-1)}{3} = 0$, base case is true

- INDUCTION: $2+6+12+...+(n^2-n)+((n+1)^2-(n+1)) = \frac{(n+1)((n+1)^2-1)}{3}$
- work with LHS, simplify RHS as necessary, but don't change anything
- $\frac{n^3-n}{3} + (n^2+n) = \frac{(n+1)(n^2+2n)}{3}$
- simplify to get $\frac{n^3-n+3(n^2+n)}{3}$ = RHS $\frac{n^3+3n^2+2n}{3}$
- simplify futher to get LHS $\frac{n^3-n+3n^2+3n}{3} = \frac{n^3+3n^2+2n}{3}$, which is the same as the RHS
- With induction, we proved that the following statement is **T** for all n. ■

3. Prove that for $n \geq 1$, there is $k \geq 0$ and ℓ odd such that $n = 2^k \ell$

- BASE CASE: $n = 1$ and $k = 0$, therefore plugging into $Q(n) = P(n) = 2^k \ell$ becomes $1 = 2^0 \ell$
- in this case, $\ell = 1$, which makes it odd, base case passed
- INDUCTION STEP: $n = n+1$ and $k = k+1$
- $n + 1 = 2^{k+1}\ell \to$ plug in n for $2^k \ell$
- $2^k \ell + 1 = 2^{k+1}\ell$, knowing that ℓ will be odd, we can substitute it for $2w+1$
- $2^k(2w+1) + 1 = 2^{k+1}\ell$
- $2^k 2w + 2^k + 1 = 2^k \times 2 \times \ell$