

- **\*Problem 9.2(g-h).** Tinker and then compute formulas that do not contain a sum for the following:

$$\begin{aligned}
 \text{(g)} \quad & \sum_{i=1}^n ij \\
 &= j \sum_{i=1}^n i \quad \text{Constant Rule} \\
 &= j \frac{n(n+1)}{2} \quad \text{Common Sums}
 \end{aligned}$$

$$\begin{aligned}
 \text{(h)} \quad & \sum_{i=0}^n (i+j)^2 \\
 &= \sum_{i=0}^n (i^2 + 2ij + j^2) \quad \text{Distribute} \\
 &= \sum_{i=0}^n (i^2) + \sum_{i=0}^n (2ij) + \sum_{i=0}^n (j^2) \quad \text{Addition Rule} \\
 &= \frac{n(n+1)(2n+1)}{6} + \sum_{i=0}^n (2ij) + \sum_{i=0}^n (j^2) \quad \text{Common Sums} \\
 &= \frac{n(n+1)(2n+1)}{6} + 2j \sum_{i=0}^n i + j^2 \sum_{i=0}^n 1 \quad \text{Constant Rule} \\
 &= \frac{n(n+1)(2n+1)}{6} + 2j \frac{n(n+1)}{2} + j^2(n+1) \quad \text{Common Sums} \\
 &= \frac{n(n+1)(2n+1)}{6} + jn(n+1) + j^2n + j^2 \quad \text{Simplify and Distribute}
 \end{aligned}$$

- **\*Problem 9.3(j-k).** Compute formulas that do not contain a sum for the following:

$$\text{(j)} \quad \sum_{i=0}^n \sum_{j=0}^i 2^i$$

$= \sum_{i=0}^n i \sum_{j=0}^i 2^j$	Innermost Sum
$= \sum_{i=0}^n i(2^{i+1} - 1)$	Common Sums
$= \sum_{i=0}^n (i2^{i+1} - i)$	Distribute
$= \sum_{i=0}^n i2^{i+1} - \sum_{i=0}^n i$	Addition Rule
$= \sum_{i=0}^n i2^i \cdot 2^1 - \sum_{i=0}^n i$	Simplify
$= 2 \sum_{i=0}^n i2^i - \sum_{i=0}^n i$	Constant Rule
$= 2\left(\frac{n(n+1)}{2}\right)(2^{n+1} - 1) - \frac{n(n+1)}{2}$	Common Sums
$= n(n+1)(2^{n+1} - 1) - \frac{n(n+1)}{2}$	Simplify

- **\*Problem 9.14.** Determine which of these functions is in  $\Theta(n)$ , in  $\Theta(n^2)$ , or neither.

$\Theta(n)$	$\Theta(n^2)$	neither
-------------	---------------	---------

b	e	a
---	---	---

c	i	g
---	---	---

d		h
---	--	---

f		j
---	--	---

- **\*Problem 9.31(a-b).** Give the asymptotic big-Theta behavior of the runtime  $T_n$ , where,

(a)  $T_0 = 1$ ;  $T_n = T_{n-1} + n^2$ ; for  $n \geq 2$

$\Theta(n)$

(b)  $T_0 = 1$ ;  $T_1 = 2$ ;  $T_n = 2T_{n-1} - T_{n-2} + 2$  for  $n \geq 2$

$\Theta(2^n)$