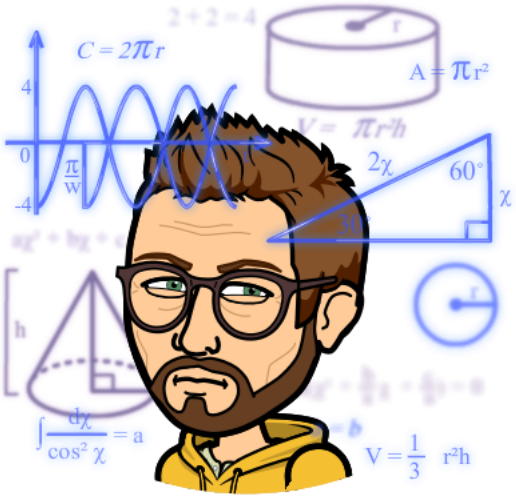
CSCI 2200 — Foundations of Computer Science (FoCS)
Homework 1 (document version 1.0)

Overview

- This homework is due by 11:59PM on Thursday, September 15
- You may work on this homework in a group of no more than four students; unlike recitation problem sets, **your teammates may be in any section**
- You may use at most **two** late days on this assignment
- Please start this homework early and ask questions during office hours and at your September 14 recitation section; also ask (and answer) questions on the Discussion Forum
- Please be concise in your answers; even if your solution is correct, if it is not well-presented, you may still lose points
- You can type or hand-write (or both) your solutions to the required graded problems below; **all work must be organized in one PDF that lists all teammate names**
- You are strongly encouraged to use LaTeX, in particular for mathematical symbols; see references in Course Materials
- **EARNING LATE DAYS:** for each homework that you complete using LaTeX (including any tables, graphs, etc., i.e., no hand-written anything), you earn one additional late day; you can draw graphs and other diagrams in another application and include them as image files



Warm-up exercises

The problems below are good practice problems to work on. Do not submit these as part of your homework submission. **These are ungraded problems.**

- **Problem 1.26**
- **Problem 2.19**
- **Problem 3.4**
- **Problem 3.13**
- **Problem 3.14**
- **Problem 3.22**
- **Problem 3.24**
- **Problem 3.43**
- **Problem 3.47**

Graded problems

The problems below are required and will be graded.

- **Problem 2.16 (Cartesian Product).**
- **Problem 2.29**
- **Problem 3.20 (DNF). Parts (a) and (b) only.**
- **Problem 3.23**
- **Problem 3.31**
- **Problem 3.44**
- **Problem 3.56**
- **Problem 4.7. Part (a) only.**

All of the above problems (both graded an ungraded) are transcribed in the pages that follow.

Graded problems are noted with an asterisk (*).

If any typos exist below, please use the textbook description.

- **Problem 1.26.** Two players alternately pick numbers without replacement from the set $\{1, 2, 3, \ldots, 9\}$. The first player to obtain three numbers that sum to 15 wins. What is your strategy?

- ***Problem 2.16 (Cartesian Product).** Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$. The Cartesian product $A \times B$ is the set of pairs formed from elements of A and elements of B,

$$A \times B = \{(a, b) \mid a \in A,\ b \in B\}$$

  (a) List the elements in $A \times B$. What is $|A \times B|$?
  (b) List the elements in $B \times A$. What is $|B \times A|$?
  (c) List the elements in $A \times A = A^2$. What is $|A \times A|$?
  (d) List the elements in $B \times B = B^2$. What is $|B \times B|$?

  Generalize the definition of $A \times B$ to a Cartesian product of three sets $A \times B \times C$.

- **Problem 2.19.** How many binary sequences are of length $1, 2, 3, 4, 5$? Guess the pattern.

- ***Problem 2.29.** Mimic the method we used to prove $\sqrt{2}$ is irrational and prove $\sqrt{3}$ is irrational. Now use the same method to try and prove $\sqrt{9}$ is irrational. What goes wrong?

- **Problem 3.4.** Define the propositions $p =$ "Kilam is a CS major" and $q =$ "Kilam is a hockey player". Use the connectors $\vee$, $\wedge$, $\rightarrow$ to formulate these claims.

  (a) Kilam is a hockey player and CS major.
  (b) Kilam either plays hockey or is a CS major.
  (c) Kilam plays hockey, but he is not a CS major.
  (d) Kilam is neither a hockey player nor a CS major.
  (e) Kilam is a CS major or a hockey player, not both.
  (f) Kilam is not a hockey player but is a CS major.

- **Problem 3.13.** If it rains on a day, it rains the next day. Today it didn't rain. On which days must there be no rain?
  (a) Tomorrow. (b) All future days. (c) Yesterday. (d) All previous days.

- **Problem 3.14.** For $p =$ "You're sick", $q =$ "You miss the final", $r =$ "You pass FOCS", translate into English:

  (a) $q \rightarrow \neg r$.
  (b) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$.
  (c) $(p \wedge q) \vee (\neg q \wedge r)$.

- ***Problem 3.20 (DNF). Parts (a) and (b) only.** Use $\neg$, $\wedge$, $\vee$ to give compound propositions with these truth-tables.

  [Hint: You need only consider the rows which are T and use OR of AND's.]

(a)

q	r	
T	T	F
T	F	T
F	T	F
F	F	F

(b)

q	r	
T	T	F
T	F	T
F	T	F
F	F	T

  (AND-OR-NOT formulas use only $\neg$, $\wedge$, $\vee$. Any truth-table can be realized by an AND-OR-NOT formula. Even more, one can construct an OR or AND's, the *disjunctive normal form (DNF).*)

- **Problem 3.22.** How many rows are in the truth table of $\neg(p \vee q) \wedge \neg r$? Give the truth table.

- ***Problem 3.23.**

  (a) Give the truth-table for these compound propositions.

$$p \wedge \neg p; \quad p \vee \neg p; \quad p \to (p \vee q); \quad ((p \to q) \wedge (\neg q)) \to \neg p$$

  (b) How many rows are in the truth-table of the proposition $(p \vee q) \to (r \to s)$?

  (c) Show that $(p \to q) \vee p$ is ALWAYS true. This is called a tautology.

- **Problem 3.24.** Let $q \to p$ be F and $q \to r$ be T. Answer T/F: (a) $p \vee q$ (b) $p \to q$ (c) $p \wedge q \wedge r$.

- ***Problem 3.31.** Use truth tables to determine the logical equivalence of the compound statements.

  (a) $(p \to q) \to r$ and $p \to (q \to r)$

  (b) $(p \wedge \neg q) \vee q$ and $p \vee q$

- **Problem 3.43.** For $x \in \{1, 2, 3, 4, 5\}$ and $y \in \{1, 2, 3\}$, determine T/F with short justifications.

  (a) $\exists x : x + 3 = 10$

  (b) $\forall y : y + 3 \leq 7$

  (c) $\exists x : (\forall y : x^2 < y + 1)$

  (d) $\forall x : (\exists y : x^2 + y^2 < 12)$

- ***Problem 3.44.** For $x, y \in \mathbb{Z}$, determine T/F with short justifications.

  (a) $\forall x : (\exists y : x = 5/y)$

  (b) $\forall x : (\exists y : y^4 - x < 16)$

  (c) $\forall x : (\exists y : \log_2 x \neq y^3)$

- **Problem 3.47.** Use quantifiers to precisely formulate the associative laws for multiplication and addition and the distributive law for multiplication over addition.

- ***Problem 3.56.** In which (if any) of the domains $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are these claims T? (x and y can have different domains.)

  (a) $\exists x : x^2 = 4$

  (b) $\exists x : x^2 = 2$

  (c) $\forall x : (\exists y : x^2 = y)$

  (d) $\forall y : (\exists x : x^2 = y)$

- ***Problem 4.7. Part (a) only.** Give direct proofs:

  (a) $x, y \in \mathbb{Q} \to xy \in \mathbb{Q}$.