

• **Problem 3.59**

1. addition: a, b, c, d
2. subtraction: b, c, d
3. multiplication: a, b, c, d
4. division: c, d
5. exponential: a

• **Problem 4.7**

1. Assume $n \in \mathbb{Z}$ is true
2. We then divide it into two cases, where n is odd and n is even
3. Case 1: $n = 2k + 1$, n is odd $\rightarrow n^2 + n$ is even
 - then $(2k + 1)^2 + (2k + 1) = 4k^2 + 4k + 1 + 2k + 1$
 - then, $4k^2 + 4k + 2k + 2$
 - then, $2(2k^2 + 2k + k + 1)$
 - We prove by direct proof that if n is odd, then $n^2 + n$ is even ■
4. Case 2: $n = 2k$ is even $\rightarrow n^2 + n$ is even
 - subbing in, we get $(2k)^2 + 2k = 4k^2 + 2k$, which is $2(2k^2 + k)$
 - We prove by direct proof that when n is even, $n^2 + n$ is even ■
5. We only have two cases of what n could be, which is odd or even, by direct proof, we prove that $n \in \mathbb{Z} \rightarrow n^2 + n$ is true for any positive or negative integer

• **Problem 4.10**

- (k) contrapositive claim: if n is a perfect square, then 3 does not divide $n - 2$
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- (l) contrapositive claim: if $p^2 + 1$ is prime, then for $p > 2$ is composite
- Any number p can be written as $k + 2$ or $k + 3$
 - if k is even, $k + 2$ is even, $k + 3$ will be even when k is odd

- Case 1: k is even, $k = 2w$ for some integer w

$$\begin{aligned}
 p^2 + 1 &\rightarrow (k + 2)^2 + 1 = (2w + 2)^2 + 1 \\
 &= 4w^2 + 4w + 4w + 4 + 1 \\
 &= 4(w^2 + 2w + 1) + 1
 \end{aligned}$$

for any value w , it will always be a prime number ■

- Case 2: k is odd, $k = 2w + 1$

$$\begin{aligned}
 p^2 + 1 &\rightarrow (k + 3)^2 + 1 = (2w + 1 + 3)^2 + 1 \\
 &= (2w + 4)^2 + 1 \\
 &= 4w^2 + 8w + 8w + 16 + 1 \\
 &= 4(w^2 + 2w + 2w + 4) + 1 \\
 &= 4(w^2 + 4w + 4) + 1
 \end{aligned}$$

for any value w , it will be a prime number ■

• **Problem 5.20**

1. Base Case: $n = 1$, $2^0 = 1$, base case proved
2. Induction step: n becomes $n + 1$
3. We can divide it into two cases, where n is even and n is odd
4. Case 1: $n + 1$ is even

$$\begin{aligned}
 n + 1 &= 2k \\
 k &= 2^{w_1} + 2^{w_2} + 2^{w_3} + 2^{w_4} + \dots + 2^{w_i} \\
 n + 1 &= 2(2^{w_1} + 2^{w_2} + 2^{w_3} + 2^{w_4} + \dots + 2^{w_i})
 \end{aligned}$$

By induction, we proved that even numbers are created by distinct powers of 2 ■

5. Case 2: $n + 1$ is odd

$$n + 1 = 2k + 2^0$$

$$k = 2^{w_1} + 2^{w_2} + 2^{w_3} + 2^{w_4} + \dots + 2^{w_i}$$

$$n + 1 = 2(2^{w_1} + 2^{w_2} + 2^{w_3} + 2^{w_4} + \dots + 2^{w_i}) + 2^0$$

By induction, we proved that odd numbers are created by distinct powers of 2 ■

• **Problem 5.39**

1. Base Cases:

$$P(n) = 4a + 5b$$

$$P(12) = 4(3) + 5(0)$$

$$P(13) = 4(2) + 5(1)$$

$$P(14) = 4(1) + 5(2)$$

$$P(15) = 4(0) + 5(3)$$

2. We create a variable k , where $k \geq 15$

3. To prove that $P(k)$ is true:

$$k - 3 \geq 12$$

$$k - 3 = 4a + 5b$$

$$\text{Induction Step: } k + 1 = (k - 3) + 4$$

$$= 4a + 5b + 4$$

$$= 4(a + 1) + 5b$$

4. By leaping induction, we proved that for any $k + 1$, it is simply a variant of $4a + 5b$ ■