VLSI DESIGN FLOW: RTL TO GDS

Lecture 16
Logic Optimization: Part III

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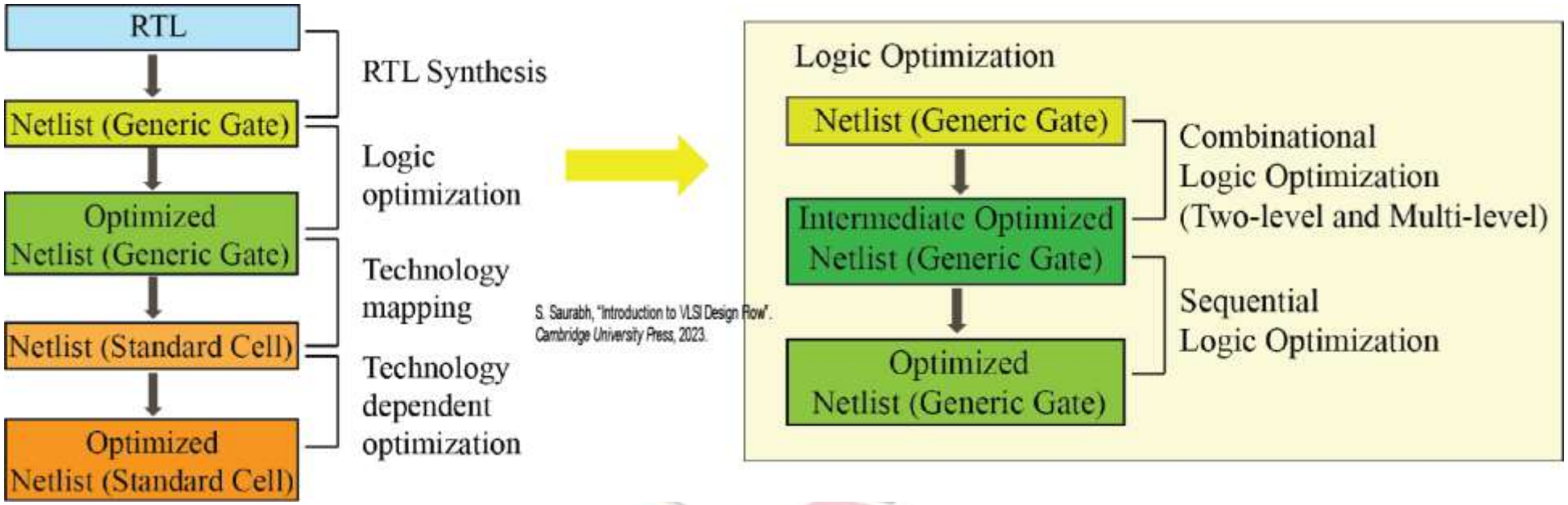
Lecture Plan



- Sequential Logic Optimization

Sequential Logic Optimization

Finite State Machine (FSM) : Definition

- FSM is an abstract mathematical model to represent wide variety of sequential circuits and systems

An FSM consists:

- Finite non-empty sets of ***states*** (S)
- Finite non-empty sets of inputs (I) and outputs (O)
- Given initial state (s_0)

An FSM consists:

- State transition function (δ)
  - Given the current state and current input, it produces the next state

Output function can be modelled in two ways in an FSM:

- Output is a function of the current state only: *Moore Machine*
- Output is a function of the current state and current input: *Mealy Machine*

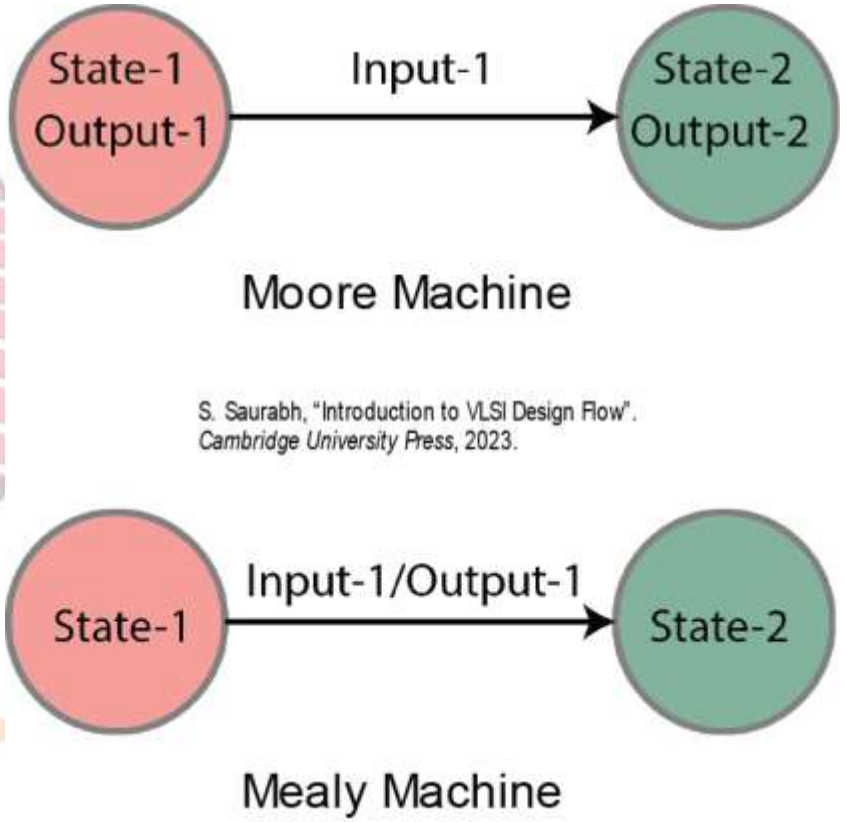
Finite State Machine (FSM) : State Diagram

- We can represent an FSM pictorially using a directed graph called ***state diagram***

State diagram is a directed graph:

- A state corresponds to a vertex in the graph
- Transition from one state to another state is represented as an ***edge*** in the graph
  - Edge starts from the current state and ends in the next state
  - Mark inputs at the edge

- Mark output value:
  - Inside the state for Moore machine
  - At the edges for the Mealy machine.



Finite State Machine (FSM): Example-1

A fan regulator can be described as an FSM:

- Four settings of regulator described as four states
  - $S = \{s0, s1, s2, s3\}$
  - $s0$ can be taken as an initial state

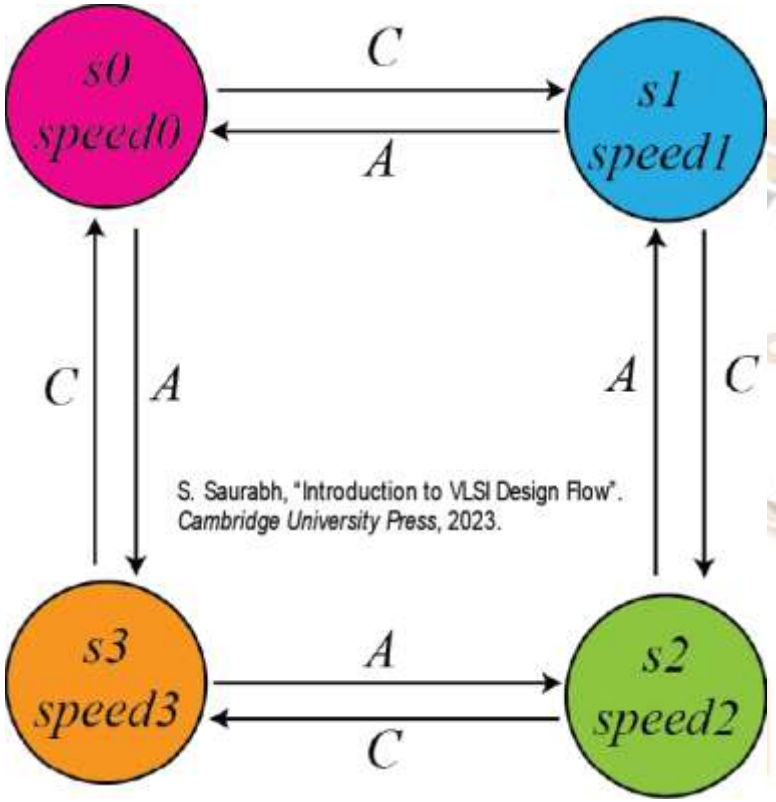
- Inputs to the regulator (rotate clockwise and rotate anti-clockwise) are inputs of the FSM
  - $I = \{C, A\}$
- Four different speeds at four settings are outputs of the FSM
  - $O = \{speed0,\ speed1,\ speed2,\ speed3\}$

Transitions:

- Rotate clockwise successively: $s0 \rightarrow s1 \rightarrow\ s2 \rightarrow s3 \rightarrow s0$ …
- Rotate anti-clockwise successively: $s0 \rightarrow s3 \rightarrow\ s2 \rightarrow s1 \rightarrow s0$ …

We can draw the state diagram for this FSM.

Finite State Machine (FSM): Example-1



Which kind of FSM is this?

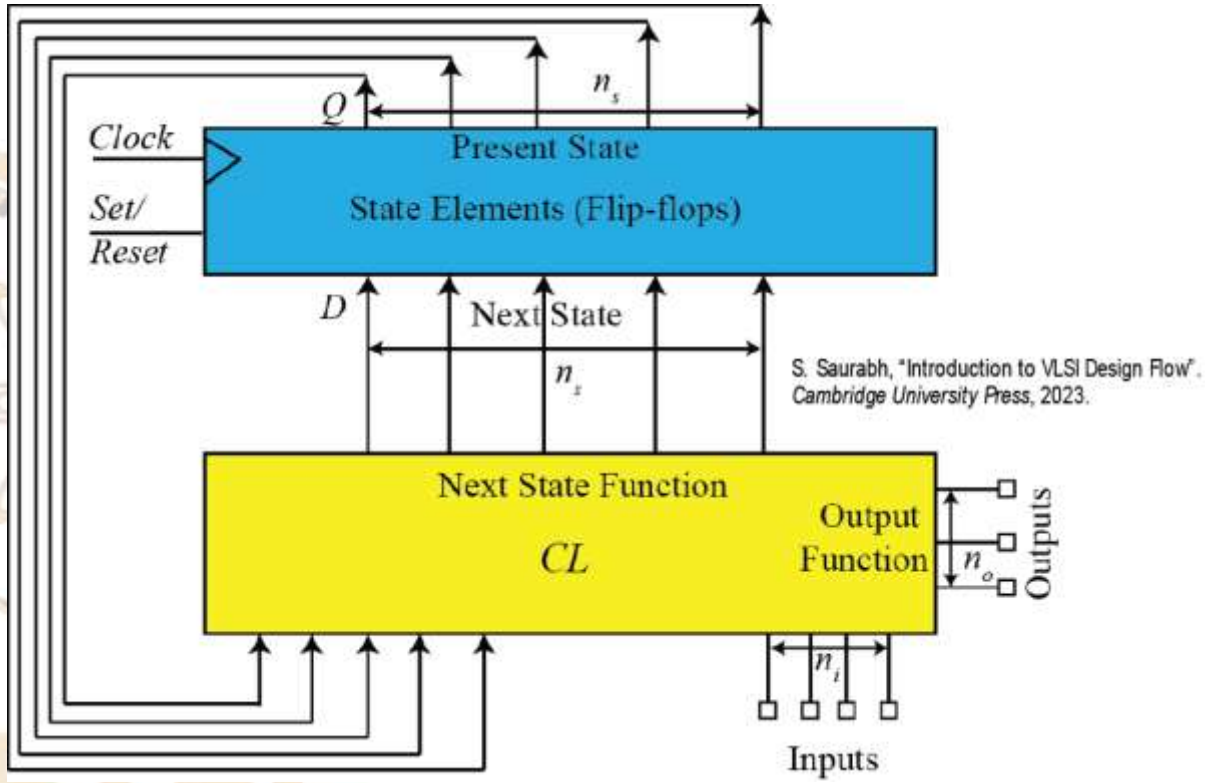
Ans: Moore FSM.

Finite State Machine (FSM) Implementation

- FSM is implemented using flip-flops and combinational circuit elements

- States are represented by the combination of the Q-pin value of flip-flops.
- Transition function is implemented using combinational logic

- Cost (area) of FSM implementation can be reduced by:
  - Reducing number of flip-flops (state minimization)
  - Reducing logic complexity of combinational circuit elements (state encoding)



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State Minimization

State Minimization: Basics

Motivation for state minimization:

- To represent a state we use a sequence of bits
  - For n_s states, minimum $[log_2 n_s]$ bits required
- By reducing number of states n_s, we can reduce number of bits in FSM representation
  - Reduce number of flip-flops in FSM implementation

Objective of state minimization:

- To derive an FSM that has the minimum number of states and exhibits the same behavior as the original FSM
- Relies on determining **equivalent states**

Two states of an FSM are equivalent :

1. They produce identical outputs
2. The corresponding next states are the same or equivalent.

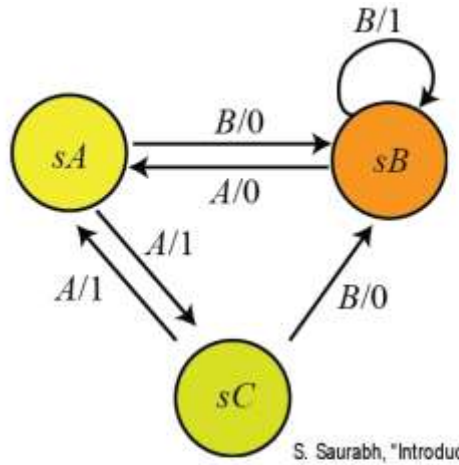
- Among set of equivalent states, retain any one of the equivalent states and remove others
- Update the transition function to maintain the same behavior as the original FSM.

State Minimization: Illustration

Consider an FSM shown: inputs $I = \{A, B\}$ and outputs $O = \{0, 1\}$.

Consider the states sA and sC :

- Outputs:
  - When input is A, both states produce 1
  - When input is B, both states produce 0.
- Next States:
  - When input is A, sA transitions to sC (equivalent state) and sC transitions to sA (equivalent state).
  - When the input is B, both sA and sC transitions to sB.
- The states sA and sC are equivalent



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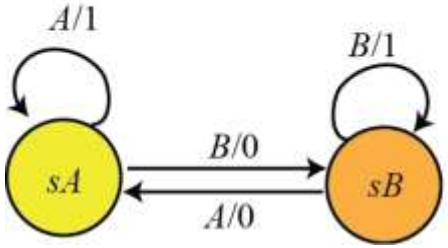


- Efficient algorithms exist to find set of equivalent states.

State Encoding

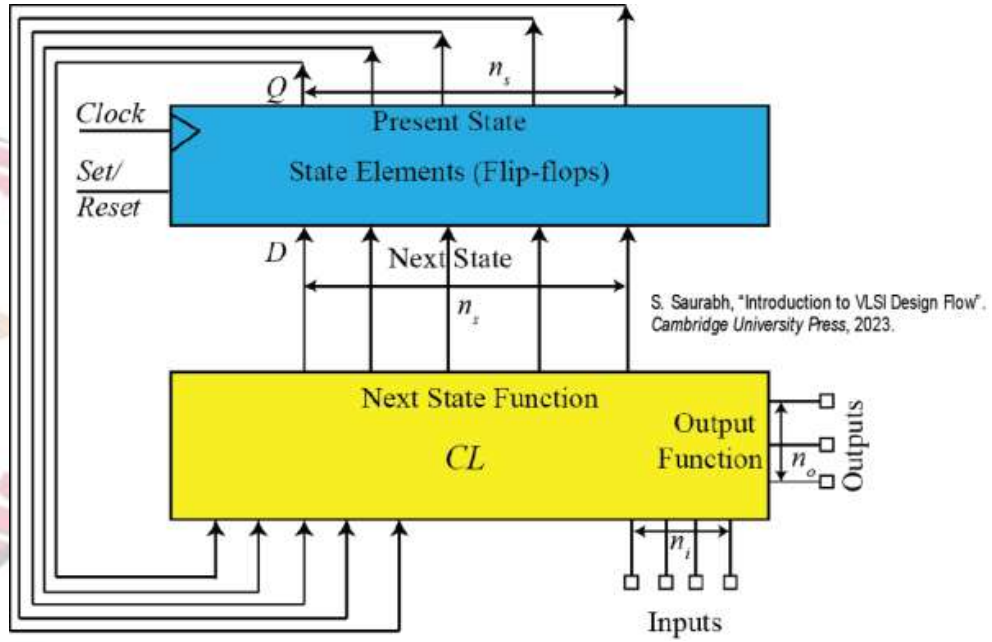
State Encoding

State encoding:

- It assigns a binary representation to each state of an FSM.

Impact of state encoding:

- Changing the state encoding, can change the next state function and the output function
- Combinational circuit block CL and the associated QoR can change.



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Encoding Length

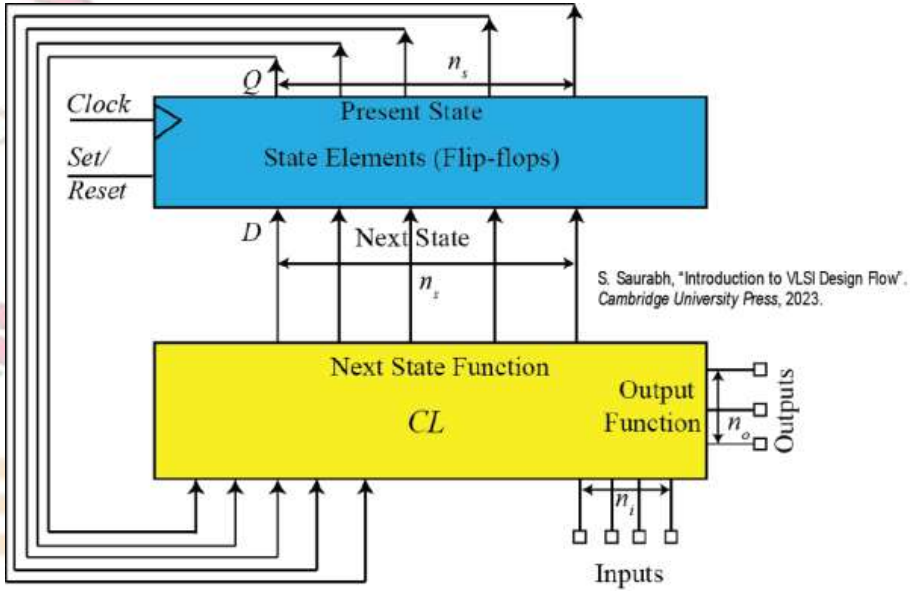
Encoding length:

- To represent an FSM with n_s states, we need at least $\lceil log_2 n_s \rceil$ bits
- Longer encoding lengths can also be chosen
  - Example: one-hot encoding reserves one bit for each state (encoding length is n_s)

Impact of encoding length:

- Number of flip-flops
- Inputs/Outputs of CL

- Sometimes CL can be simplified by changing encoding length.
  - ➢ One-hot encoding: identifying a state requires examining only one state bit.
  - ➢ One-hot encoding can have fewer logic levels between flip-flops and be faster



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Choosing Encoding Scheme

Challenges:

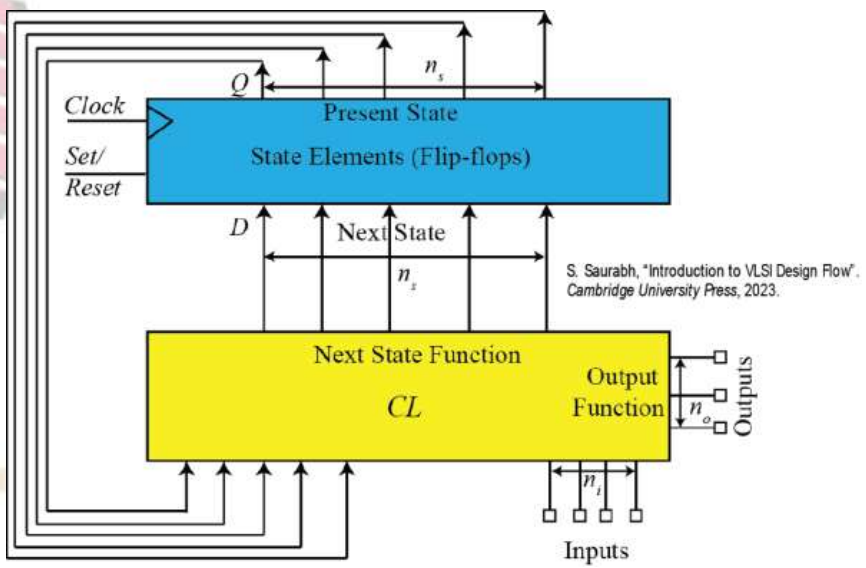
- Exponential number of possibilities
  - ➢ Brute force cannot be applied to examine all solutions.
- During encoding, CL implementation is not done
  - ➢ Need to assess the encoding quality based on the expected FSM implementation.

Approach:

- CL needs to produce the next state functions and output functions
  - ➢ Encode such that they share logic
  - ➢ Allow more common cubes and common sub-expressions

Heuristics-based Algorithms:

- ➢ Quantify the possibility of common cube extraction as "gain" of an encoding scheme
- ➢ Determine encoding that maximizes the "gain"



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- S. Devadas, H.-K. Ma, A. R. Newton, and A. Sangiovanni-Vincentelli. “MUSTANG: State assignment of finite state machines targeting multilevel logic implementations.” *IEEE Transactions on Computer-aided Design of Integrated Circuits and Systems* 7, no. 12 (1988), pp. 1290–1300.