Building Blocks of Potential Flow

1.1 Governing Equations of Doublet

Once source and sink are viewed from such far distance that the distance between them approaches to zero, we will observe a pattern so called doublet.
For stream-function associated to the source-sink pair evaluated at point $P = (x, y)$ we use the following equation.

$$\psi\ x,y\ = \frac{\sigma}{2\pi}\ \theta_1 - \theta_2\ = -\frac{\sigma}{2\pi}\varDelta\theta \tag{1}$$

The strength of the doublet pattern is κ and it can be derived from the equation below.

$$\kappa = \sigma\ell$$

And finally the stream-function of the doublet evaluated at point $P(x, y)$, is given by

$$\psi\ x,y\ = \underset{l\to 0}{\ell im}\left(-\frac{\sigma}{2\pi}\right) d\theta \tag{2}$$

And

$$\sigma\ell = cte$$

In Cartesian coordinates a doublet located at the origin has the stream function

$$\psi\ x,y\ = -\frac{\kappa}{2\pi}\left(\frac{y}{x^2+y^2}\right) \tag{3}$$

From which the velocity components can be derived.

$$u\ x,y\ = \frac{\partial\psi}{\partial y} = -\frac{\kappa}{2\pi}\frac{x^2-y^2}{x^2+y^2\ ^2} \tag{4}$$

$$v\ x,y\ = -\frac{\partial\psi}{\partial x} = -\frac{\kappa}{2\pi}\frac{2xy}{x^2+y^2\ ^2} \tag{5}$$