Universiteti Publik “Kadri Zeka”, Gjilan

Fakulteti i Shkencave Kompjuterike

Lënda: Teoria e Grafeve

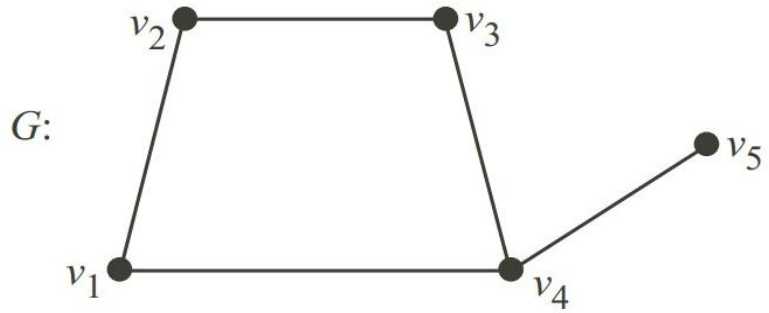
Tema:

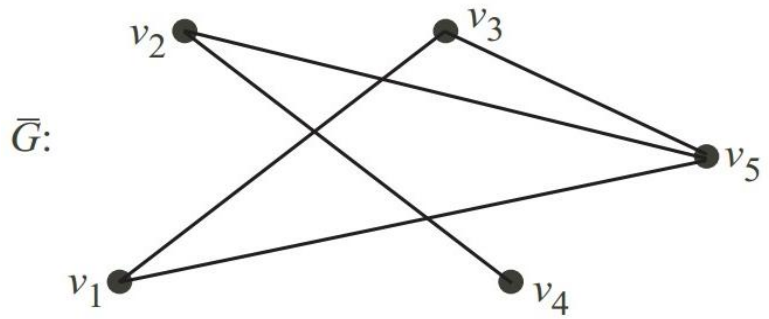
Veprimet me grafe

(Komplementi, Unioni, Prerja, Diferenca simetrike, Pema dhe pyjet, Vetitë e pemëve)

Perkufizim. Komplementi i grafit te thjeshte $G = (V, E)$ eshte grafi i thjeshte $G\text{Þ} = (V, E(\text{Ø}))$, ku brinjet ne E jane brinje qe saktesishte nuk jane ne G.

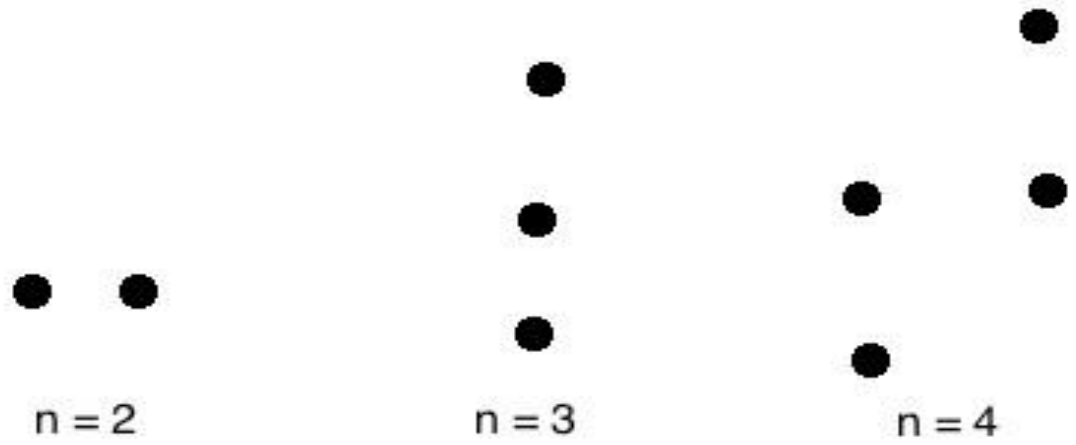
Shembull.





- Grafi që përbëhet vetëm nga n kulme të izoluara dhe pa brinje quhet **graf i zbrazët** (bosh).

- Grafi i zbrazët me 0 kulme quhet **grafi zero**, ai me një kulm quhet **graf singleton**.
- Grafi i zbrazët me n kulme është grafi komplement i grafit të plotë K_n, dhe zakonisht ai K_n.
- Tipe grafesh te zbrazeta, shiko figuren.



Shembull. Komplementi i grafit te plote K_n është grafi i zbrazet me n kulme.

(Ilustro me një shembull)

Shihet qartë se, $G\check{\mathrm{E}} = G$.

(Ilustro me një shembull).

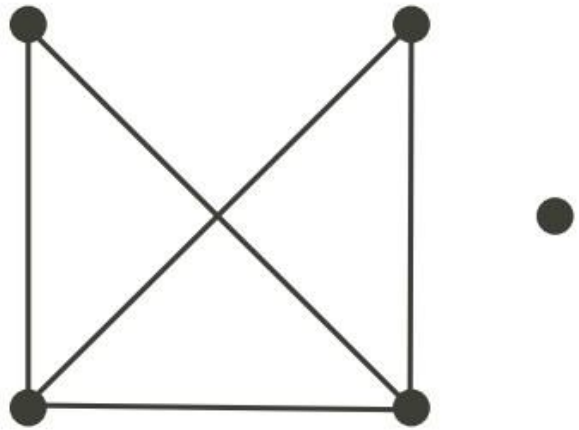
Nese grafet $G = (V, E)$ dhe $G' = (V', E')$ jane te thjeshtë dhe $V' \subseteq V$, atëherë diferenca e grafeve është $G - G' = (V, E'')$, ku E'' përmban brinjët që janë në G dhe që nuk janë në G' .

Shembull. Janë dhënë grafet $G\ dhe\ G'$

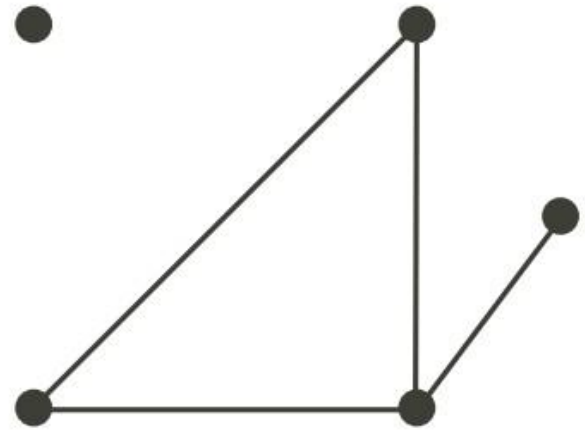
Gjeni grafin $G'' = G - G'$?

Shembull. Janë dhënë grafet $G\ dhe\ G'$

G:



G':



$G - G'$:

Ketu kemi veprime binare ne mes dy grafeve te thjeshte $G_1 = (V_1, E_1)$dhe $G_2 = (V_2, E_2)$:

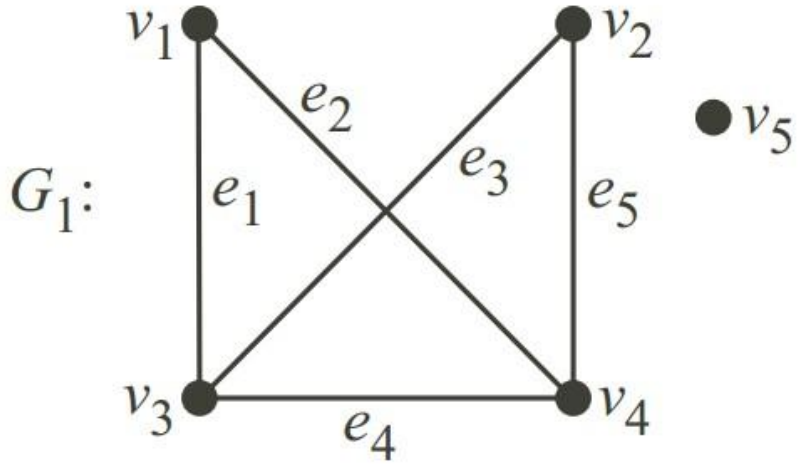
- Unioni është $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$(grafe i thjeshte)
- Prerja është $G_1 \cap G_2 = (V_1 \cap V_2, E_1 \cap E_2)$(grafe i thjeshte)
- Shuma rreth $G_1 \oplus G_2$ është nëngraf i $G_1 \cup G_2$ i indukuar (nxitur) nga brinjët $E_1 \oplus E_2$

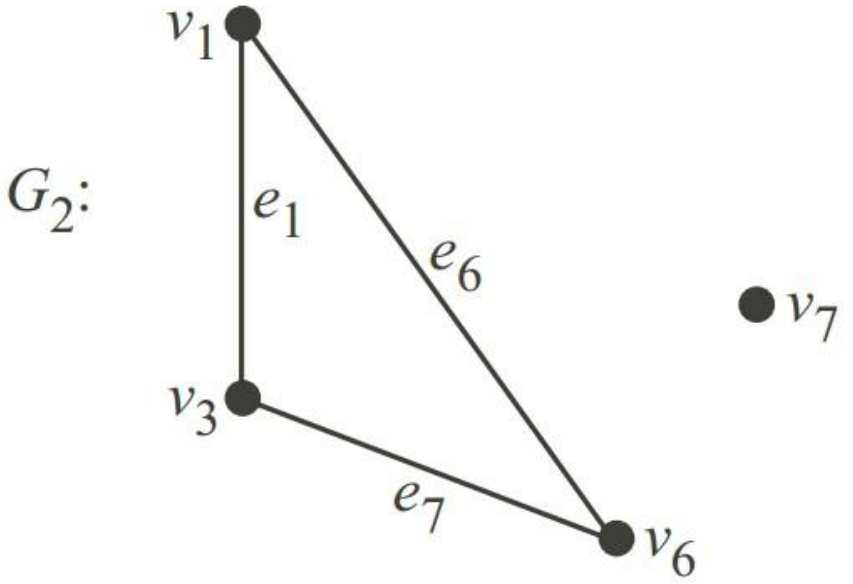
Verejtje: Veprimi i bashkesive $\oplus$ eshte diference simetrike,

$$E_1 \oplus E_2 = (E_1 - E_2) \cup (E_2 - E_1)$$

Pasi qe shuma rrethe është nengraf i indukuar nga brinje te nje bashkesie, ateherë nuk ka kulme te izoluara. Te tri veprimet jane associative dhe komutative.

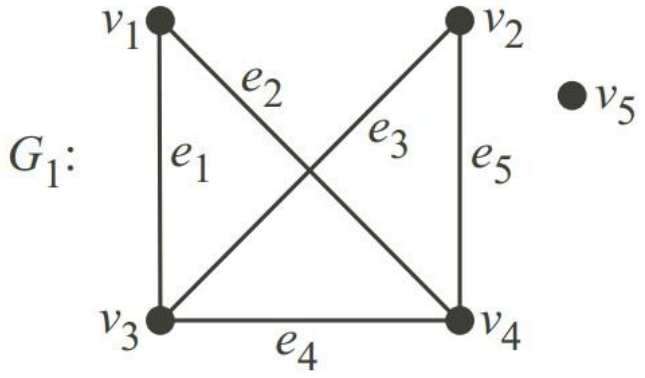
Shembull. Janë dhënë grafet.

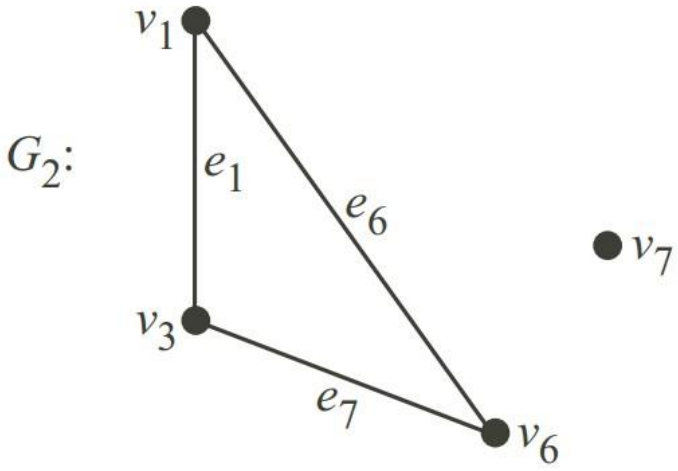


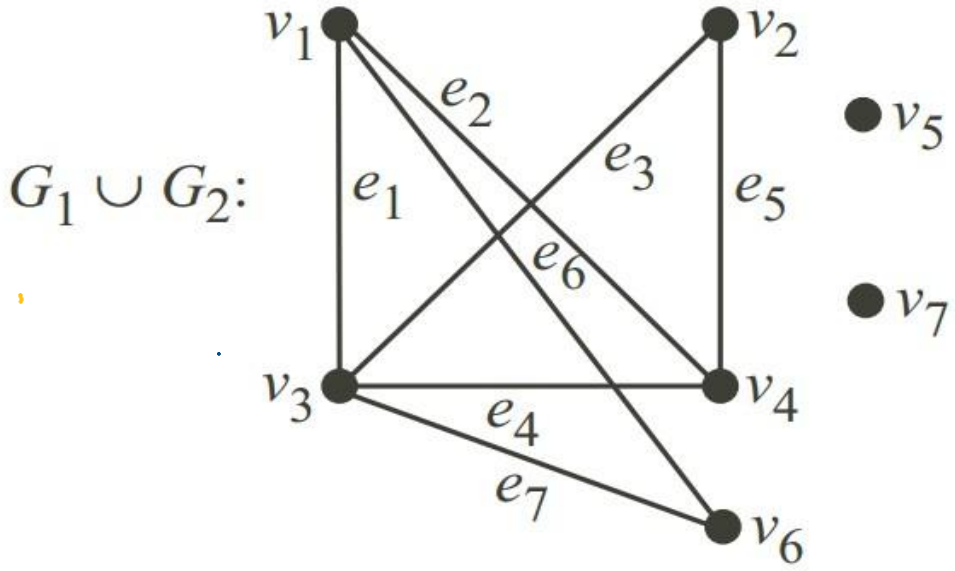


Gjeni $G_1 \cup G_2$, $G_1 \cap G_2$ dhe $G_1 \oplus G_2$?

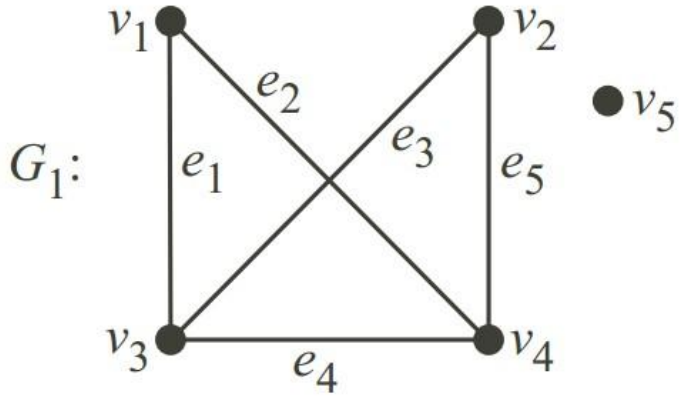
Zgjidhje.

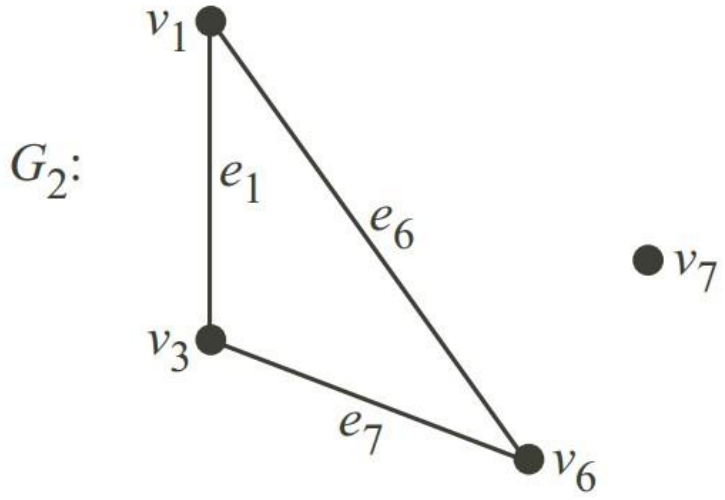

G_1:
v_1
v_2
v_3
v_4
v_5
e_1
e_2
e_3
e_4
e_5

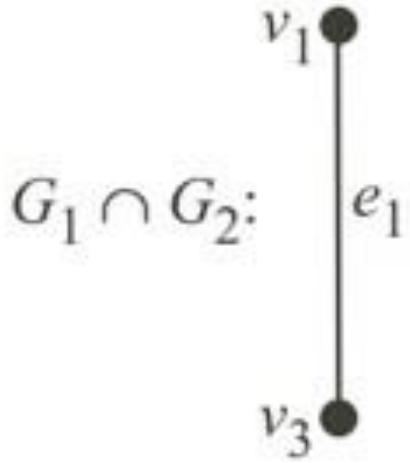

G_2:
v_1
v_3
v_6
v_7
e_1
e_6
e_7


$G_1 \cup G_2$:
v_1
v_2
v_3
v_4
v_5
v_6
v_7
e_1
e_2
e_3
e_4
e_5
e_6
e_7

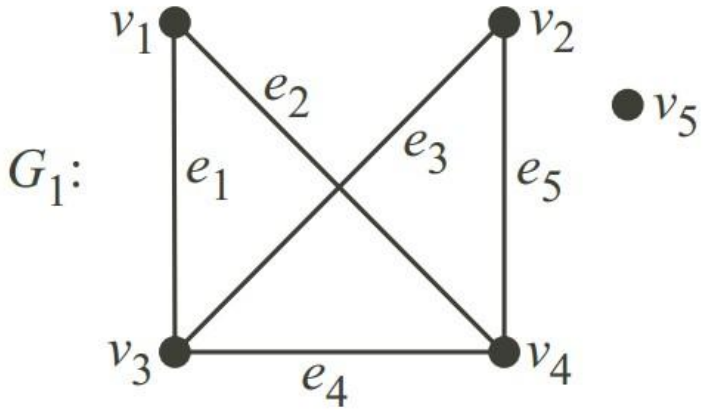
Zgjidhje.

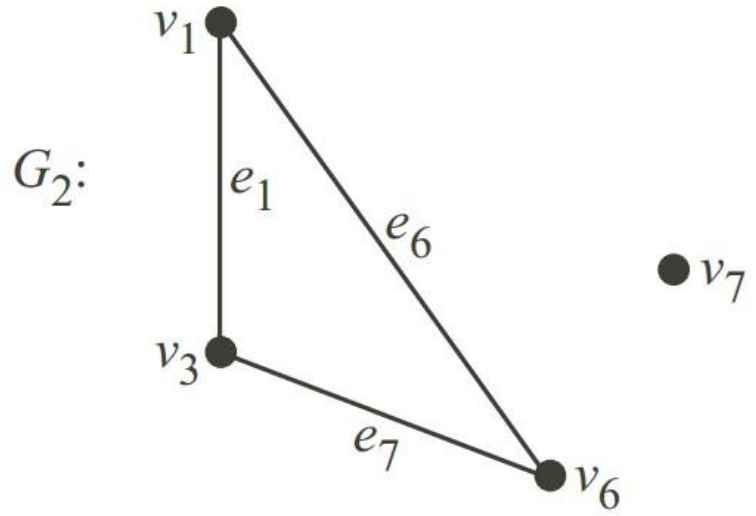

G_1:
v_1
v_2
v_3
v_4
v_5
e_1
e_2
e_3
e_4
e_5

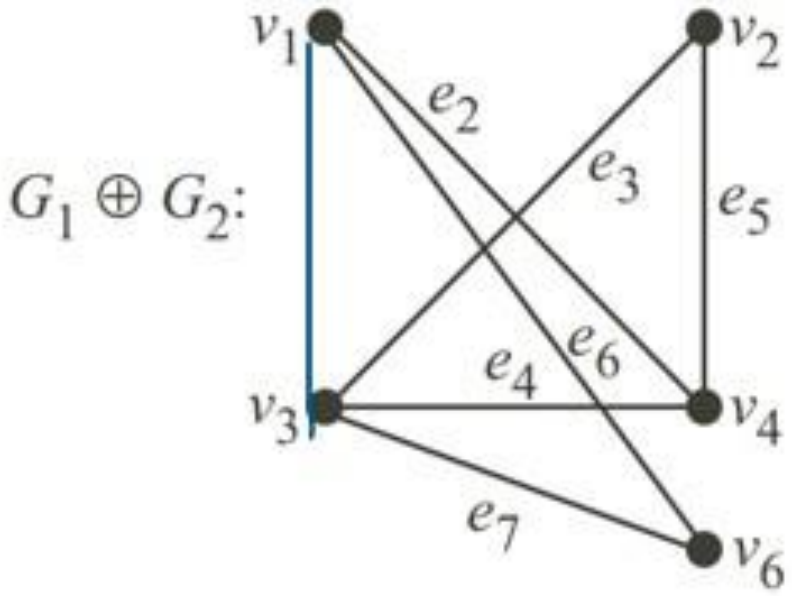

G_2:
v_1
v_3
v_6
v_7
e_1
e_6
e_7


$G_1 \cap G_2$:
v_1
v_3
e_1

Zgjidhje.

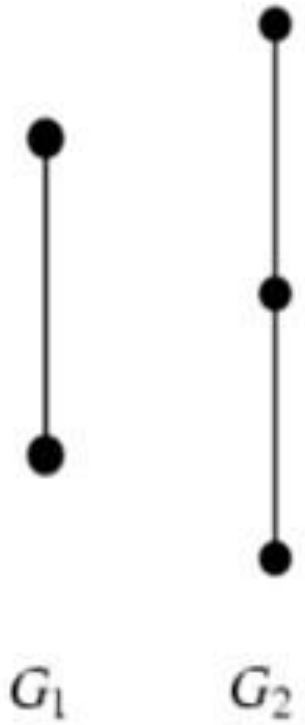

G_1:
v_1
v_2
v_3
v_4
v_5
e_1
e_2
e_3
e_4
e_5


G_2:
v_1
v_3
v_6
v_7
e_1
e_6
e_7


$G_1 \oplus G_2$:
v_1
v_2
v_3
v_4
v_6
e_2
e_3
e_4
e_5
e_6
e_7

Shembull.

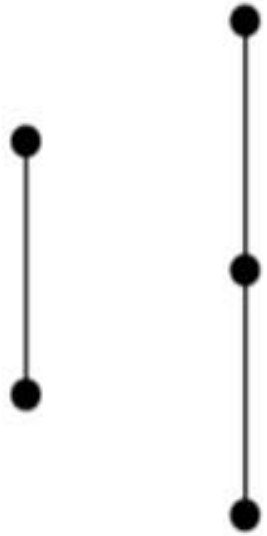
Janë dhënë grafet



Shembull.

Gjeni $G_1 \cup G_2$?

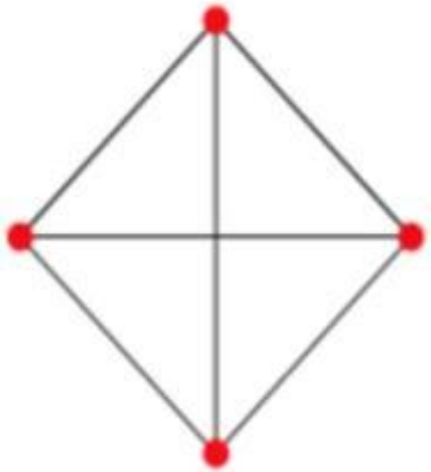
Janë dhënë grafet



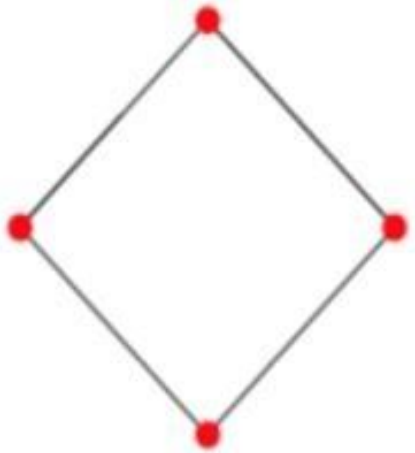
$G_1 \cup G_2$

Janë dhënë grafet G dhe H.

Shembull.



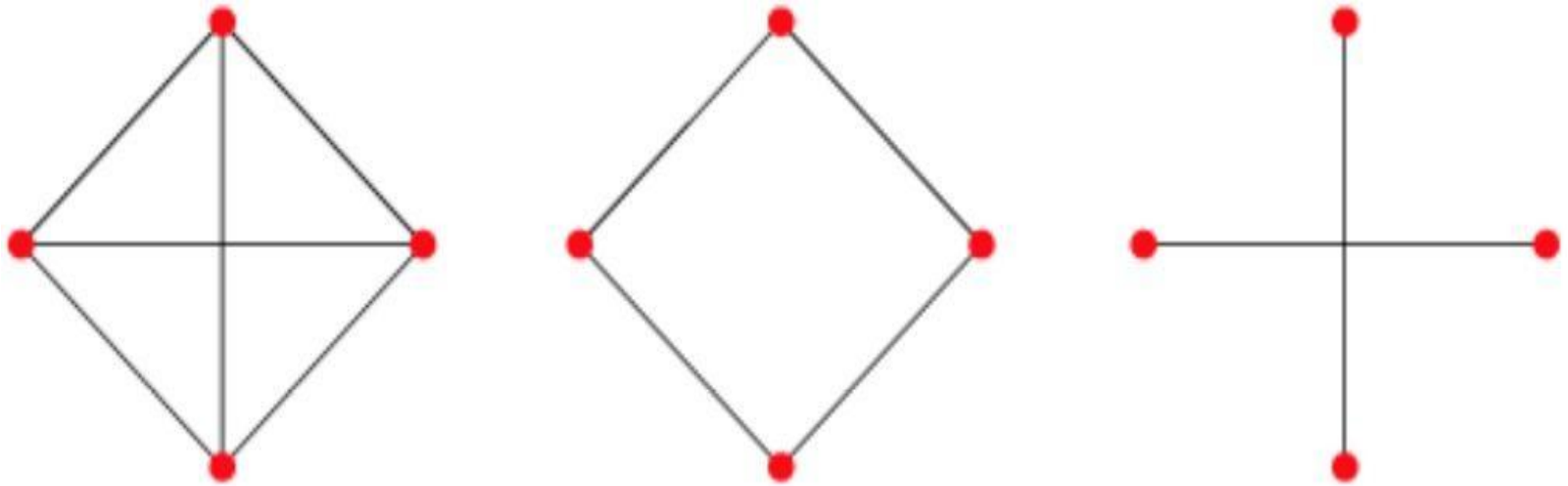
G



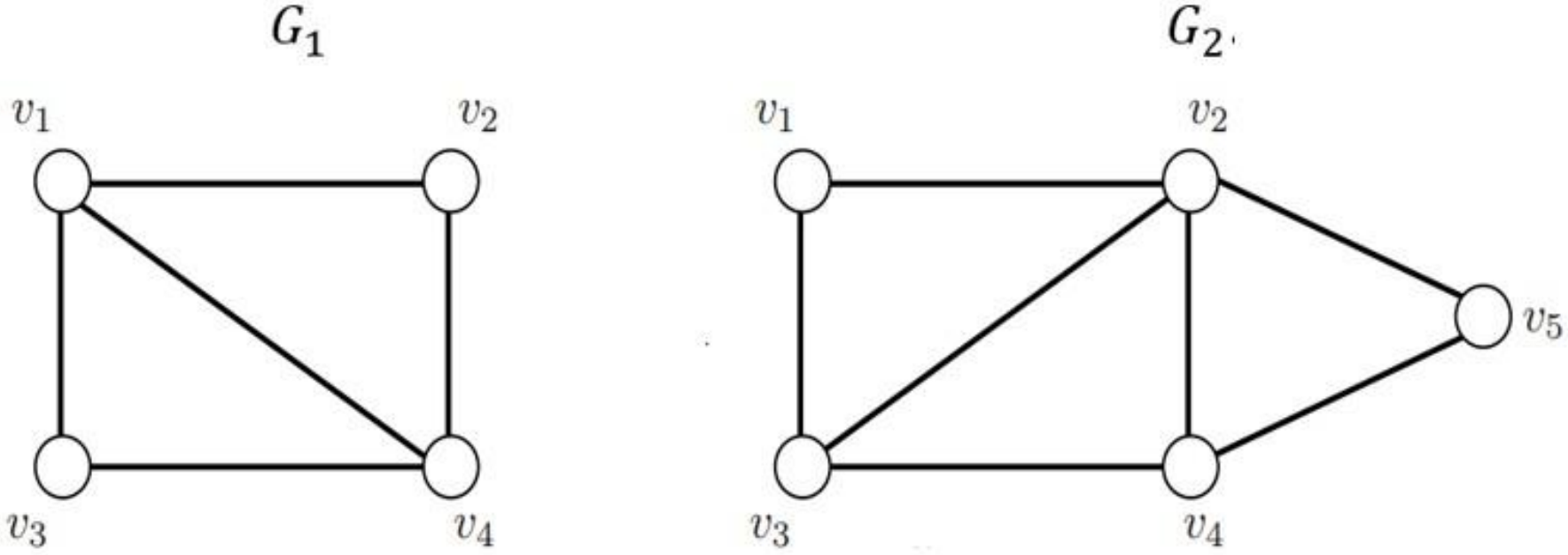
H

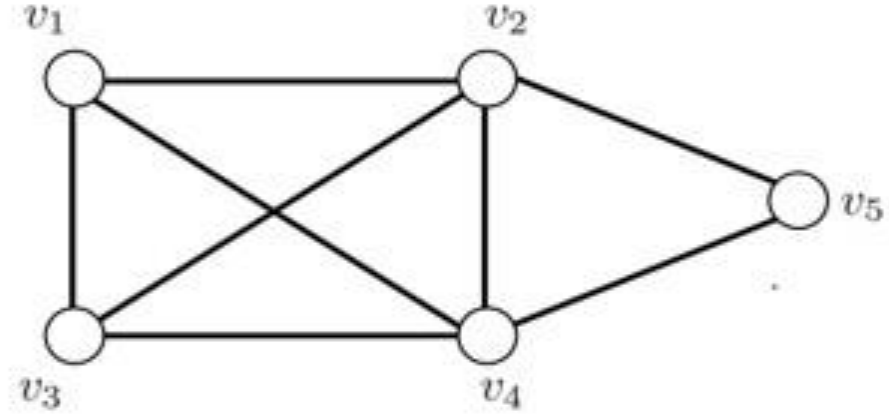
Gjeni *G-H* ?

Zgjidhje



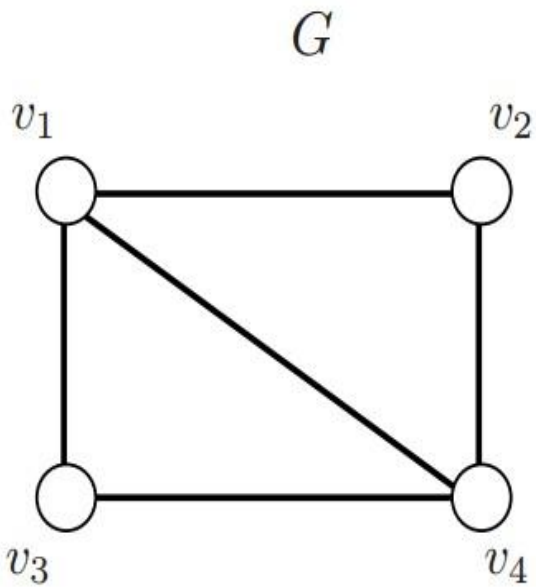
Shembull. Le te jene dhene grafet G_1 dhe G_2. Gjeni grafin union, prerjen, diferencen dhe diferencen simetrike (mbledhjen me rrethe):

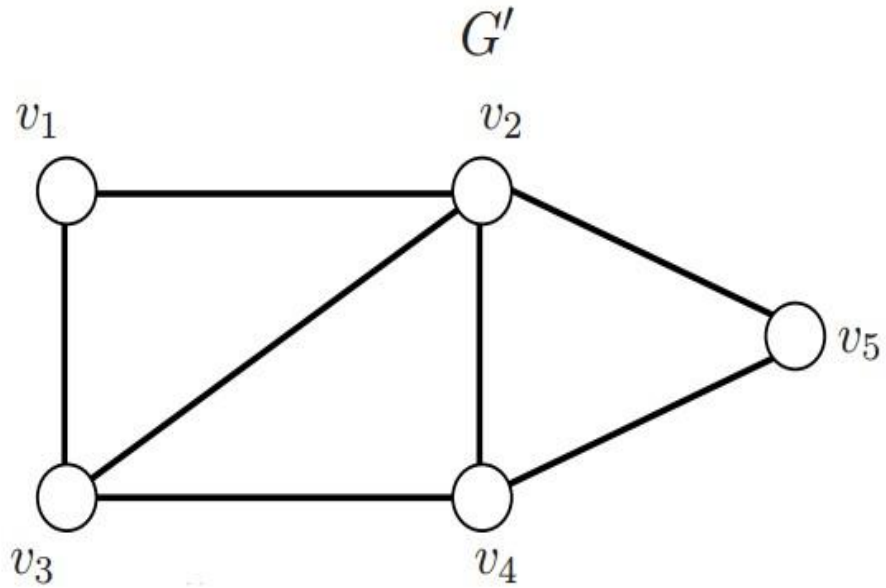


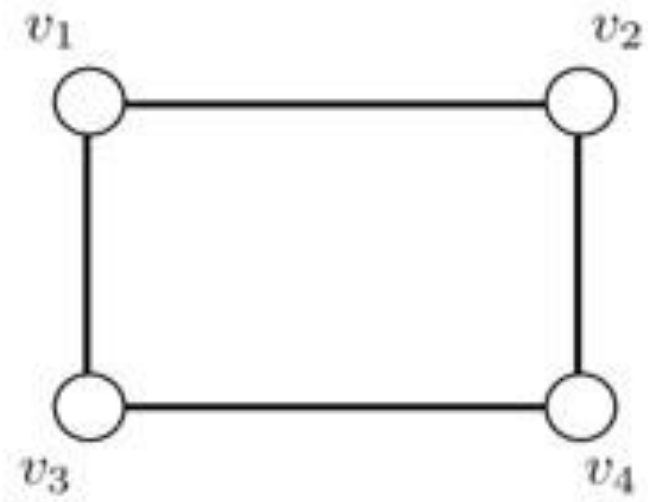


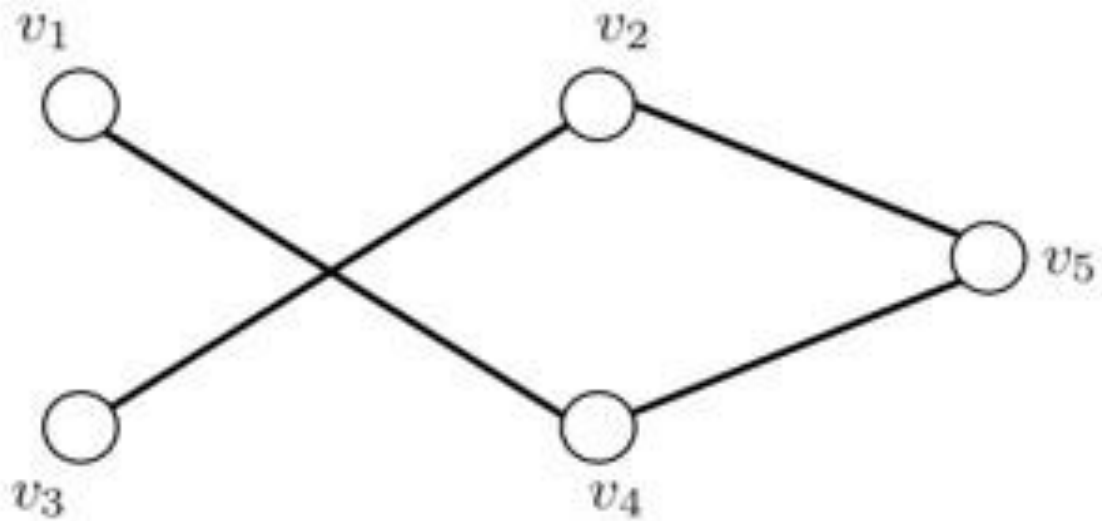
a) $G \cup G'$

G G'


G
v_1
v_2
v_3
v_4


G'
v_1
v_2
v_3
v_4
v_5


v_1
v_2
v_3
v_4



c) $G \oplus G'$

G G'