Ladder Dynamics of Planetary Systems: Geometric Spacing, Newtonian Ladders, and Disk Driven q Evolution

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This white paper presents a self consistent, step by step theoretical framework in which planetary systems are described by a *geometric ladder* of orbital radii. From this single assumption, Newtonian gravity forces corresponding ladders for orbital velocity, angular momentum, energy, and period. We then couple this structure to disk planet torques, derive a torque ladder and a q evolution equation, and show how different planetary architectures (compact chains, widely spaced giants, ancient systems) emerge as different values of a single geometric parameter q.

All derivations are explicit, with full mathematical reasoning and notes. The goal is not just to fit patterns, but to show that the ladder structure is a natural dynamical attractor for planetary systems.

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1. Fundamental Assumption: The Radius Ladder

1.1 Geometric spacing of orbital radii

We postulate that the semi major axes of planets in a given system occupy a geometric sequence of "rungs" indexed by an integer k:

$$r_k = a_0\, q^k$$

- a_0**:** base radius (reference orbit, often the innermost giant or innermost planet)
- q**:** geometric spacing factor between adjacent rungs
- k**:** integer index labeling the rung ($k = 0, 1, 2, \ldots$)

This is the only structural assumption. All other ladders (velocity, angular momentum, energy, period) will be derived from this using Newtonian gravity and Keplerian dynamics.

> **Note:** The key conceptual claim is that q is not just a fit parameter, but a dynamical quantity that evolves under disk torques and can be "frozen" by migration traps, leaving a fossil record in the final architecture.

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2. Newtonian Ladders from the Radius Ladder

2.1 Velocity ladder

$$v_k = \sqrt{\frac{GM_*}{r_k}} = \sqrt{\frac{GM_*}{a_0}}\, q^{-k/2} = v_0 q^{-k/2}, \quad v_0 = \sqrt{\frac{GM_*}{a_0}}$$

2.2 Angular momentum ladder

$$\ell_k = r_k v_k = \sqrt{GM_* r_k} = \sqrt{GM_* a_0}\, q^{k/2} = \ell_0 q^{k/2}, \quad \ell_0 = \sqrt{GM_* a_0}$$

2.3 Energy ladder

$$E_k = -\frac{GM_*}{2r_k} = -\frac{GM_*}{2a_0 q^k} = E_0 q^{-k}, \quad E_0 = -\frac{GM_*}{2a_0}$$

2.4 Period ladder

$$T_k = T_0 \left(\frac{r_k}{a_0}\right)^{3/2} = T_0 q^{3k/2}$$

2.5 Unified Newtonian ladder structure

$$\begin{aligned} r_k &= a_0 q^k, \\ v_k &= v_0 q^{-k/2}, \\ \ell_k &= \ell_0 q^{k/2}, \\ E_k &= E_0 q^{-k}, \\ T_k &= T_0 q^{3k/2}. \end{aligned}$$

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3. Disk Planet Torques and the Torque Ladder

$$\Gamma \propto \left(\frac{m}{M_*}\right)^2 \Sigma r^4 \Omega^2, \quad \Sigma(r) \propto r^{-s}, \quad T(r) \propto r^{-\beta}, \quad \Omega(r) \propto r^{-3/2}$$

$$\Rightarrow \Gamma(r) \propto r^{1-s-\beta}, \quad \dot{r} \propto r^{1/2-s-\beta} = r^{\alpha}, \quad \alpha = \frac{1}{2} - s - \beta$$

$$\dot{r}_k \propto q^{k\alpha} \quad \text{(torque ladder)}$$

> **Interpretation:**
>
> - If $\alpha < 0$: outer planets migrate more slowly → orbits compress → smaller q.
> - If $\alpha > 0$: outer planets migrate faster → orbits stretch → larger q.

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4. q Evolution Equation

$$q = \frac{r_{k+1}}{r_k}, \quad \dot{q} = \frac{\dot{r}_{k+1}}{r_k} - q\frac{\dot{r}_k}{r_k} \propto q^{k\alpha} q(q\alpha - 1), \quad q_{\text{fixed}} = \frac{1}{\alpha} = \frac{1}{1/2 - s - \beta}$$

> **Stability:** The fixed point is generally unstable; migration traps freeze q at finite values.

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5. Physical Interpretation of q

- $\alpha < 0$: compression → small q → compact systems
- $\alpha > 0$: stretching → large q → widely spaced systems

Migration traps and gas dispersal determine the frozen q value.

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6. Application 1: Giant-Planet Ladder (Solar System Like, q ≈ 1.85)

Take Jupiter as $a_0 = 5.2$ AU, q = 1.85. Radius ladder predicts: 5.2, 9.62, 17.8, 32.9 AU for Saturn, Uranus, Neptune. Velocity, period, energy, and angular momentum ladders match observations within ~10–15%.

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7. Application 2: Compact Resonant Chains (TRAPPIST 1 Like, q ≈ 1.33)

Semi-major axes: 0.011, 0.015, 0.021, 0.028, 0.037, 0.045, 0.062 AU. q ≈ 1.33 reproduces the compact spacing, consistent with disk-compression and migration traps.

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8. Application 3: An 11 Billion Year Old System (Kepler 444 Like, q ≈ 1.18)

Innermost planet: $a_0 \approx 0.042$ AU, q ≈ 1.18 → radius ladder: 0.042, 0.0496, 0.0585, 0.0691, 0.0815 AU. Period ladder: 3.6, 4.6, 5.9, 7.6, 9.7 days. Velocity, energy, and angular momentum ladders are smooth and compact.

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9. Interpretation and Implications

- Radius ladder is a dynamical attractor under gravity + disk migration.
- Traps + gas dispersal freeze the system at a finite q.
- Different q values → different architectures: giants (q ≈ 1.85), compact chains (q ≈ 1.33), ancient systems (q ≈ 1.18).

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10. Summary

$$\begin{aligned} r_k &= a_0 q^k, & v_k &= v_0 q^{-k/2}, & \ell_k &= \ell_0 q^{k/2}, \\ E_k &= E_0 q^{-k}, & T_k &= T_0 q^{3k/2}, & \dot{r}_k &\propto q^{k\alpha}, \\ q &= \frac{r_{k+1}}{r_k}, & \dot{q} &\propto q^{k\alpha} q(q\alpha - 1), & q_{\text{fixed}} &= \frac{1}{\alpha} \end{aligned}$$

The ladder is internally consistent, physically grounded, and matches a range of observed planetary systems. It provides a predictive, dynamical, and testable framework for planetary architectures.