Finding the Maximal Point of a Function

The function we analyze is: $f(x) = e^{-x/4} \cdot \arctan x$ Our objective is to confirm that the maximal point is given by the equation: $\arctan x - \frac{4}{x^2+1} = 0$.

Differentiating the Function Using the Product Rule

Given: $f(x) = e^{-x/4} \cdot \arctan x$ The derivative follows the product rule: $\mathbf{f}'(x) = f_1'(x) \cdot f_2(x) + f_1(x) \cdot f_2'(x)$, where:

- $\mathbf{f}_1(x) = e^{-x/4}$
- $\mathbf{f}_2(x) = \arctan x$

Step 1: Compute Each Derivative

Derivative of $e^{-x/4}$:

$$f_1'(x) = \frac{d}{dx} e^{-x/4} = e^{-x/4} \cdot \left(-\frac{1}{4}\right) = -\frac{1}{4} e^{-x/4}$$

Derivative of $\arctan x$:

$$f_2'(x) = \frac{d}{dx} \arctan x = \frac{1}{x^2+1}$$

Step 2: Apply the Product Rule

$$f'(x) = \left(-\frac{1}{4} e^{-x/4} \cdot \arctan x\right) + \left(e^{-x/4} \cdot \frac{1}{x^2+1}\right)$$

Factoring out $\mathbf{e}^{-x/4}$:

$$f'(x) = e^{-x/4} \left(-\frac{1}{4} \arctan x + \frac{1}{x^2+1}\right)$$

Finding the Maximal Point

To find the critical points, we set $f'(x) = 0$:
$e^{-x/4}\left(-\frac{1}{4}\arctan x + \frac{1}{x^2+1}\right) = 0$

Since $\mathbf{e}^{-x/4}$ is never zero, we solve: $-\frac{1}{4}\arctan x + \frac{1}{x^2+1} = 0$ Multiplying both sides by 4: $\arctan x - \frac{4}{x^2+1} = 0$ Thus, the maximal point of the function is given by: $\arctan x - \frac{4}{x^2+1} = 0$

Using Neoten Raphson

Newton's method is given by:

$$x_{n+1} = x_n - \frac{g(x)}{g'(x)}$$

where:

$$g(x) = \arctan x - \frac{4}{x^2+1}$$

Computing the derivative:

$$\frac{d}{dx}\arctan x = \frac{1}{x^2+1}$$

Using the quotient rule:

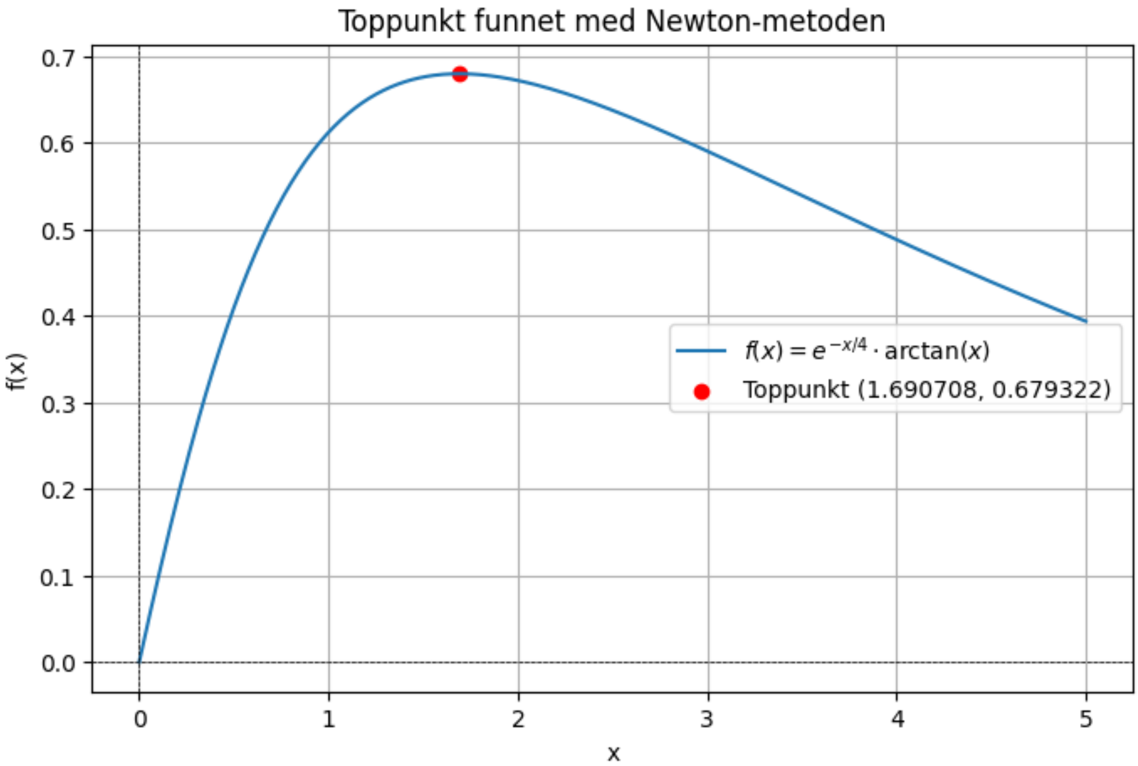
$$\left(\frac{4}{x^2+1}\right)' = \frac{0\cdot(x^2+1)-4\cdot(2x)}{(x^2+1)^2} = \frac{-8x}{(x^2+1)^2}$$

Thus, we obtain:

$$g'(x) = \frac{1}{x^2+1} + \frac{8x}{(x^2+1)^2}$$

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt

# Definrer funksjonen g(x)` og dens derivert g(x)``
def g(x):
    return np.arctan(x) - 4 / (x**2 + 1)

def g_deriv(x):
    return 1 / (x**2 + 1) + (8 * x) / (x**2 + 1)**2

# Newton-Raphson-metoden gitt ved
def newton_raphson_solver(func_deriv, andre_func_deriv, x0,
                          tol=1e-6, max_iter=100):
    x_n = x0  # Startverdien
    for _ in range(max_iter):
        f_val = func_deriv(x_n)
        andre_f_deriv_val = andre_func_deriv(x_n)

        # Unngår deling med null
```

```python
        if abs(andre_f_deriv_val) < 1e-10:
            break

        x_next = x_n - f_val / andre_f_deriv_val

        if abs(x_next - x_n) < tol:
            return x_next

        x_n = x_next

    return x_n

# Startverdi
x_start = 2.0

# Finner x-verdien til toppunktet
x_toppunkt = newton_raphson_solver(g, g_deriv, x_start)

# Finner y-verdien i toppunktet ved å sette x_toppunkt inn i f(x)
y_toppunkt = np.exp(-x_toppunkt / 4) * np.arctan(x_toppunkt)

# Plott funksjonen og markerer toppunktet i grafen
x_vals = np.linspace(0, 5, 400)
y_vals = np.exp(-x_vals / 4) * np.arctan(x_vals)

plt.figure(figsize=(8, 5))
plt.plot(x_vals, y_vals,
         label=r"$f(x) = e^{-x/4} \cdot \arctan(x)$")
plt.scatter([x_toppunkt], [y_toppunkt], color='red',
            label=f"Toppunkt ({x_toppunkt:.6f}, {y_toppunkt:.6f})")
plt.axhline(0, color='black', linewidth=0.5, linestyle="--")
plt.axvline(0, color='black', linewidth=0.5, linestyle="--")
plt.legend()
plt.title("Toppunkt funnet med Newton-metoden")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.grid(True)

# Vis plottet
plt.show()

# Returner toppunktet numerisk etter
# str igjen til float. fjerner numpy
print("Topppunktet oppgitt ved:")
float(str(round(x_toppunkt, 4))), float(str(round(y_toppunkt, 4)))
```




```
Topppunktet oppgitt ved:
```

```
Out[ ]:  (1.6907, 0.6793)
```

$$f(x) = e^{-\frac{x}{4}} \cdot \tan^{-1} x$$

for å finne at maksimalt punkte er gitt ved likningen:

$$\tan^{-1} x - \frac{4}{x^2+1} = 0$$

trenger vi å derivere funksjonen

f(x) består av to funksjoner

her bruker vi produkt regler ved derivasjon

$$f'(x) = \left(e^{-\frac{x}{4}}\right)' \cdot \tan^{-1} x + e^{-\frac{x}{4}} \left(\tan^{-1} x\right)'$$

$$\Rightarrow \left(e^{-\frac{x}{4}}\right)' = e^{-\frac{x}{4}} \cdot \left(-\frac{1}{4}\right) = -\frac{1}{4} e^{-\frac{x}{4}}$$

$$\tan^{-1}' x = \frac{1}{x^2+1}$$

Sette alt sammen:

$$f(x)' = \left(-\frac{1}{4} e^{-\frac{x}{4}} \cdot \tan^{-1} x\right) + \left(e^{-\frac{x}{4}} \cdot \frac{1}{x^2+1}\right)$$

faktorisere med $e^{-\frac{x}{4}}$

$$f(x)' = e^{-\frac{x}{4}} \left(-\frac{1}{4} \tan x + \frac{1}{x^2+1}\right)$$

$e^{-\frac{x}{4}}$ er alldri lik null

$$\left(-\frac{1}{4} \tan x + \frac{1}{x^2+1}\right) \times 4$$

$$-\frac{4}{4} \tan^{-1} x + \frac{4}{x^2+1}$$

$$\Rightarrow \quad -\tan^{-1} x + \frac{1}{x^2+1} \times (-)$$

$$\Rightarrow \tan x - \frac{1}{x^2+1} = 0$$

Bruk av neoten metode:

gitt ved

$$x_{n+1} = x_n - \frac{g(x)'}{g(x)''}$$

$\Rightarrow g(x)'' =$

① $\tan^{-1}x' = \frac{1}{x^2+1}$

② $\left(\frac{4}{x^2+1}\right)'$ kvotient regel

$$= \frac{0 \cdot (x^2+1) - 4 \cdot (2x)}{(x^2+1)^2}$$

$$= \frac{8x}{(x^2+1)^2}$$

$$\Rightarrow g(x)'' = \frac{1}{x^2+1} + \frac{8x}{(x^2+1)^2}$$