Solving $Ax = b$ by Cramer's Rule.

Evaluating Cramer's and finding a unique solution the function should fulfill the following criteria:

Conditions for Using Cramer's Rule

- The Matrix is squared
- The Matrix A column and variable vector should be equal
- The Matrix column should be the same for all records

When the following criteria are obtained then the Cramer's Rule can be calculated.
Finding a unique solution for $Ax = b$ **should** $\det(A) \neq 0$

Determinant Calculation for $\det(A)$

Calculation $\det(A)$ can be made using several methods when matrix size extends 2×2 in size.

- Using **Leibniz formula** is effectivc, finding upper and lower triangular matrices then multiply $L \cdot U$ manually by hand.

- In this code, we use **Laplace expansion**, given by the formula:

$$\det(A) = \sum_{j=1}^{n} (-1)^{1+j} \cdot a_{1j} \cdot \det(M_{1j})$$

In [46]:
```
import numpy as np


def determinant(A):
    n = len(A)
    # hvis matrise er 2x2
    if n == 2:
        return A[0][0]*A[1][1] - A[0][1]*A[1][0]
    det = 0
    for j in range(n):

        minor = [row[:j] + row[j+1:] for row in A[1:]]
        cofactor = (-1) ** j * A[0][j] * determinant(minor)
        det += cofactor
```

```
    return det


def cal_cramers(A_matrix, b_variabel):
    # sjekke om det er liste
    if not (isinstance(A_matrix, list) and
    all(isinstance(row, list) for row in A_matrix)):
        return "Error: A_matrisk bør være liste av liste (matrix)."
    # sjekker om all sølyer har samme lengde
    row_lengths = [len(row) for row in A_matrix]
    if len(set(row_lengths)) != 1:
        return "Error: Martise kan ikke ha lengde forskjell."
    # Sjkke om det er squard martise
    n_rows = len(A_matrix)
    n_cols = len(A_matrix[0])
    if n_rows != n_cols:
        return f"Error: Matrise A_matrix er ikke square.  {n_rows}
    # Step 4: sjekke om det lende rader tilsvarer antall variabeler
    if len(b_variabel) != n_rows:
        return f"Error: Vector b bør ha samme antall som: ({n_rows}

    #calculating carmers det(A)
    detA = determinant(A_matrix)
    if detA == 0:
        return "Error: ingen eintydig løsning finnes fordi det(A) =

    # 3. Cramer's Rule
    solutions = []
    for i in range(n_cols):
        A_i = [row[:] for row in A_matrix]
        for row_index in range(n_rows):
            A_i[row_index][i] = b_variabel[row_index]

        detA_i = determinant(A_i)
        x_i = detA_i / detA
        solutions.append(x_i)

    solution_str = f"det(A) = {detA}\nLøsning med Cramer's rule:\n"
    for idx, value in enumerate(solutions):
        solution_str += f"x{idx+1} = {value}\n"
    return solution_str


A1 = [[1,2,3],[1,2,3],[1,2,3]]
B1 = [1,2,3]
A2 = [[1,2,3],[1,2,3]]
B2 = [1,2,3]
A3 = [[1,2,3],[1,2,3],[1,2,3]]
B3 = [1,2]
A4 = [[1,2,3],[1,2,3],[1,2]]
B4 = [1,2, 2]

#Validation testing
print("Validation Testing: ")
print("--------------------------------------------------------------
```

```
print(cal_cramers(A1, B1))
print(cal_cramers(A2, B2))
print(cal_cramers(A3, B3))
print(cal_cramers(A4, B4))
print("\n")
#Testing theory
print("Testing theory Ax = b")
print("------------------------------------------------------------
print("[1 2 3 5\n3 4 4 5\n1 2 3 4\n2 3 4 5]\tb = [1 2 4 5]\n")
print(cal_cramers([[1,2,3,5],[3,4,4,5], [1,2,3,4],
                   [2,3,4,5]], [1,2,4,5]))
print("************************************************************
print("[8 2 3\n3 4 4\n1 2 3]\nb = [1 2 4]\n")
print(cal_cramers([[8,2,3],[3,4,4], [1,2,3]], [1,2,4]))
```

```
Validation Testing:
-----------------------------------------------------------------
Error: ingen eintydig løsning finnes fordi det(A) = 0
Error: Matrise A_matrix er ikke square.  2 rader og 3 søyler.
Error: Vector b bør ha samme antall som: (3).
Error: Martise kan ikke ha lengde forskjell.


Testing theory Ax = b
-----------------------------------------------------------------
[1 2 3 5
3 4 4 5
1 2 3 4
2 3 4 5]        b = [1 2 4 5]

det(A) = -1
Løsning med Cramer's rule:
x1 = -1.0
x2 = -2.0
x3 = 7.0
x4 = -3.0

*******************************************************************
[8 2 3
3 4 4
1 2 3]
b = [1 2 4]

det(A) = 28
Løsning med Cramer's rule:
x1 = -0.42857142857142855
x2 = -1.9642857142857142
x3 = 2.7857142857142856
```

$Ax = b$

ex 2

$$\begin{bmatrix} 8 & 2 & 3 \\ 3 & 4 & 4 \\ 1 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$A = L \cdot U$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\Rightarrow U = \begin{bmatrix} 8 & 2 & 3 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \qquad L \cdot U$$

$$l_{21} = \frac{a_{21}}{u_{11}} = \frac{3}{8} = 0{,}37$$

$$l_{31} = \frac{a_{31}}{a_{11}} = \frac{1}{8} = 0{,}12$$

$$u_{22} = a_{22} - l_{21} \cdot u_{12} = 4 - 0{,}37 \cdot 2 = 3{,}2$$

$$u_{23} = a_{23} - l_{21} \cdot u_{13} = 4 - 0{,}37 \cdot 3 = 2{,}8$$

$$l_{32} = \frac{1}{u_{22}} (a_{32} - l_{31} \cdot u_{12})$$

$$l_{32} = \frac{1}{3{,}2} (2 - 0{,}1 \cdot 2) = \frac{1{,}7}{3{,}2} = 0{,}5$$

$$u_{33} = u_{33} - (l_{31} \cdot u_{13} + l_{32} \cdot u_{23})$$

$$u_{33} = 3 - (0{,}12 \cdot 3 + 0{,}5 \cdot 2{,}8)$$

$$= 3 - (0{,}3 + 1{,}5) = 3 - 19 = 1{,}0$$

$$\det(A) = \det(L) \cdot \det(U)$$

$$\det(L) = 1$$

$$\det(A) = u_{11} \cdot u_{22} \cdot u_{33} =$$

$$8 \cdot 3{,}5 \cdot 1{,}0 = \underline{\underline{28}}$$