Finding the Maximal Point of a Function

The function we analyze is: $f(x) = e^{-x/4} \cdot \arctan x$ Our objective is to confirm that the maximal point is given by the equation: $\arctan x - \frac{4}{x^2+1} = 0$.

Differentiating the Function Using the Product Rule

Given: $f(x) = e^{-x/4} \cdot \arctan x$ The derivative follows the product rule: $\mathbf{f}'(x) = f_1'(x) \cdot f_2(x) + f_1(x) \cdot f_2'(x)$, where:

- $\mathbf{f}_1(x) = e^{-x/4}$
- $\mathbf{f}_2(x) = \arctan x$

Step 1: Compute Each Derivative

Derivative of $e^{-x/4}$:

$$f_1'(x) = \frac{d}{dx} e^{-x/4} = e^{-x/4} \cdot \left(-\frac{1}{4}\right) = -\frac{1}{4} e^{-x/4}$$

Derivative of $\arctan x$:

$$f_2'(x) = \frac{d}{dx} \arctan x = \frac{1}{x^2+1}$$

Step 2: Apply the Product Rule

$$f'(x) = \left(-\frac{1}{4} e^{-x/4} \cdot \arctan x\right) + \left(e^{-x/4} \cdot \frac{1}{x^2+1}\right)$$

Factoring out $\mathbf{e}^{-x/4}$:

$$f'(x) = e^{-x/4} \left(-\frac{1}{4} \arctan x + \frac{1}{x^2+1}\right)$$

Finding the Maximal Point

To find the critical points, we set $f'(x) = 0$:
$e^{-x/4} \left(-\frac{1}{4} \arctan x + \frac{1}{x^2+1} \right) = 0$

Since $\mathbf{e}^{-x/4}$ is never zero, we solve: $-\frac{1}{4} \arctan x + \frac{1}{x^2+1} = 0$ Multiplying both sides by 4: $\arctan x - \frac{4}{x^2+1} = 0$ Thus, the maximal point of the function is given by: $\arctan x - \frac{4}{x^2+1} = 0$