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ML Homework 4 Report

Instructor: Prof. Kaliappan Gopalan

Time: 20/03/2019

1. Files

a) main_part_1.m – Includes the main program of the homework 4 part 1.

b) main_part_2.m – Includes the main program of the homework 4 part 2.

c) computeCost.m – This function is used to compute the cost between hypothesis and input y for part 1.

d) computeCost_new.m – This function is used to compute the cost between hypothesis and input y for part 1.

e) computeCost_part2.m – This function is used to compute the cost between hypothesis and input y for part 2.

f) normalization.m – This function is used in part 2 to normalize the input x.

g) gradientDescent.m – This function is used in part 1 to execute the gradient descent to find proper parameters.

h) gradientDescent_part2.m – This function is used in part 2 to execute the gradient descent to find proper parameters.

i) mapFeatures.m – This function is used in part 2 to map the x into polynomial form.

2. Logistic Regression

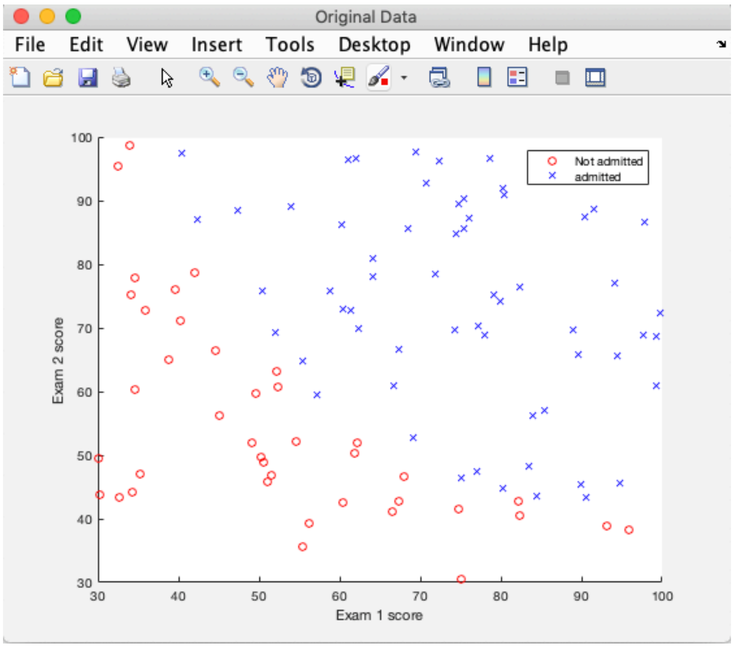
In this case, I found that without normalization, the proper theta could not be found by gradient descent. Therefore, I made two versions of logistic regression, which the first one is logistic regression with gradient descent and the second one is logistic regression by using the build-in function *optimset* and *fminunc*.

The second version referenced some materials online:

https://www.youtube.com/watch?v=bQqtZyav6K8

https://github.com/j-pandeirada/mlcourse/blob/master/Class%203/ex2.m

1. Scatter plot of the two features with y = 1 and y = 0 distinguished (similar to Fig. 1).



2. Cost at initial theta of all zeros.

```
------------------------------------------------------------
Starting logistic regression version 1...
With theta = [0; 0; 0]
Cost computed = 0.673345
```

3. Parameter values for theta.

```
After 400 iterations with alpha = 0.1,
Theta found by gradient descent:
-3.061089
3.717335
-1.274227
```

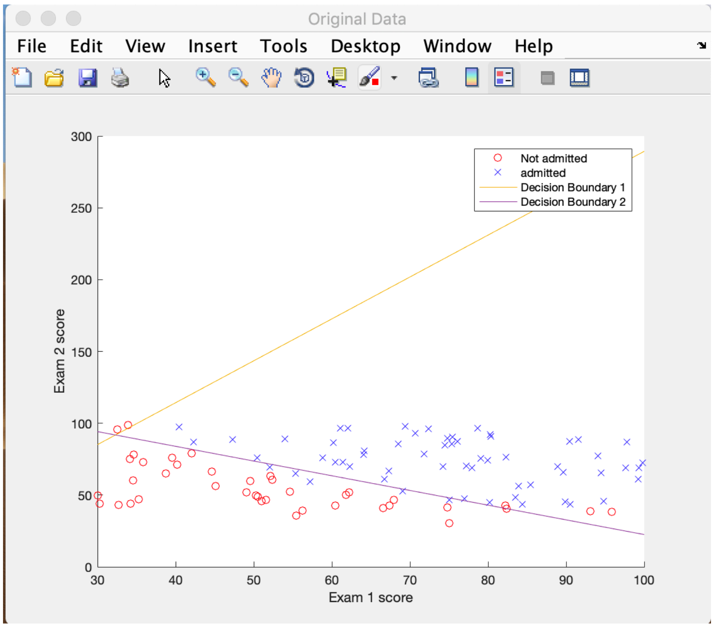
```
Theta found by optimization algorithm:
-24.933085
0.204408
0.199619
```

4. Cost after 400 iterations of logistic regression algorithm with $\alpha = 0.1$.

```
With theta = [-3.061089 ; 3.717335; -1.274227]
Cost computed = 1.744516
```

```
With theta_v2 = [-24.933085 ; 0.204408; 0.199619]
Cost computed = 0.189333
```

5. Decision boundary for logistic regression superimposed on the scatter plot.



6. Training accuracy – No. of correct decisions/Total No. of students, in percentage.

```
------------------------------------------------------------
For version 1, the accuracy is 62.000000
For version 2, the accuracy is 89.000000
```

7. Probability of getting admitted for a student with Exam 1 score of 45 and Exam 2 score of 85.

```
------------------------------------------------------------
The probability of this student getting admitted is 0.774324
>>
```

3. Polynomial logistic regression

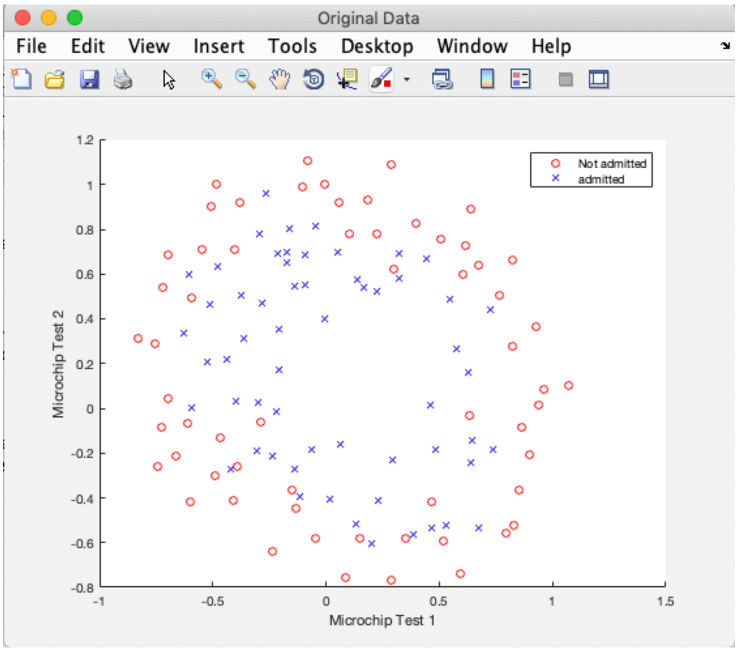
In this case, I got the theta through gradient descent.

The plotting the decision boundary part conferenced the material:

https://github.com/j-

pandeirada/mlcourse/blob/master/Class%203/plotDecisionBoundary.m

1. Scatter plot of the two features with y = 1 and y = 0 distinguished.



2. Cost at initial theta of all zeros.

```
Starting logistic regression...
With theta = zeros(28, 1)
Cost computed = 0.693147
```

3. Parameter values for theta.

```
After 500 iterations with alpha = 0.1 and lambda = 1.000000,
Theta found by gradient descent:
4.598634
5.234325
13.122253
-17.972881
-8.371846
-7.518103
-0.713802
-3.254186
-3.059647
-2.750799
-15.954125
-0.798885
-2.612156
-12.098203
-4.652524
-1.122244
-2.867930
-3.166634
-0.477166
-7.994915
-12.686870
-0.294773
-0.032822
-0.683576
-1.938114
-0.018939
-0.698735
-12.682028
```

4. Cost after 500 iterations of regularized logistic regression algorithm.

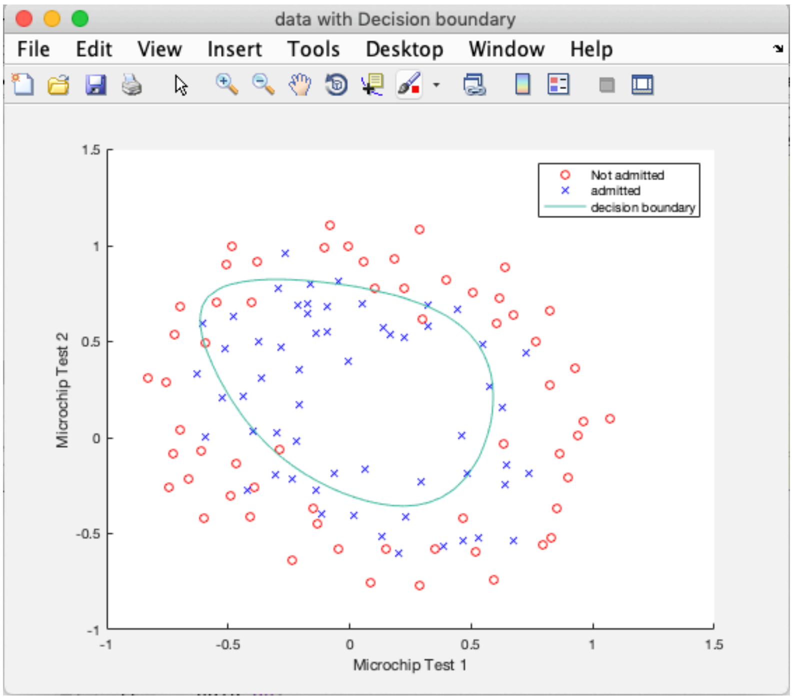
```
With this theta,
Cost computed = 0.815212
```

5. Training accuracy.

```
-----------------------------------------------------------
The accuracy is 72.881356
-----------------------------------------------------------
```

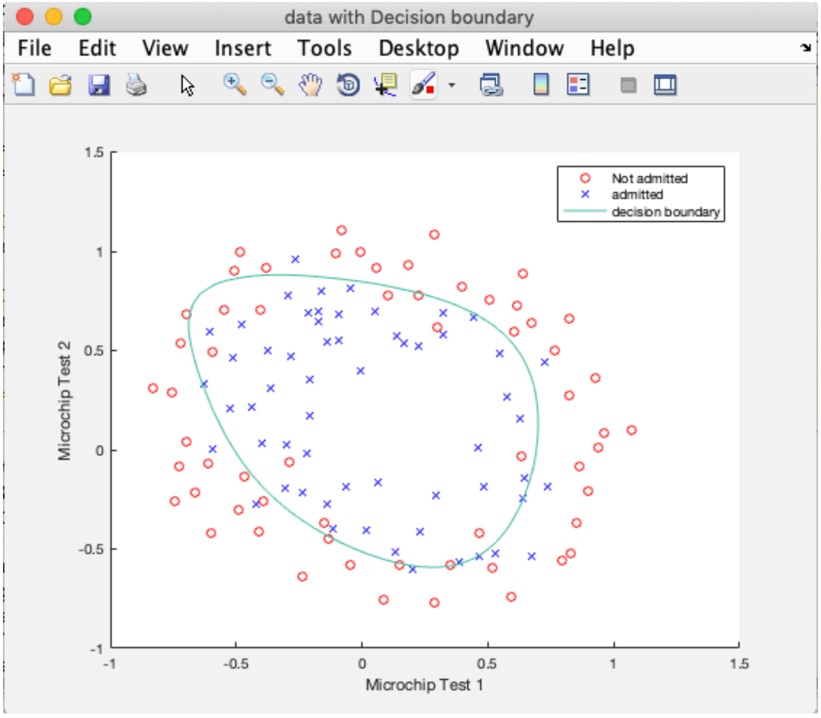
6. Plots of data along with decision boundary for at least three cases of λ.

a) With lambda = 1:



The accuracy has shown in section 3.5

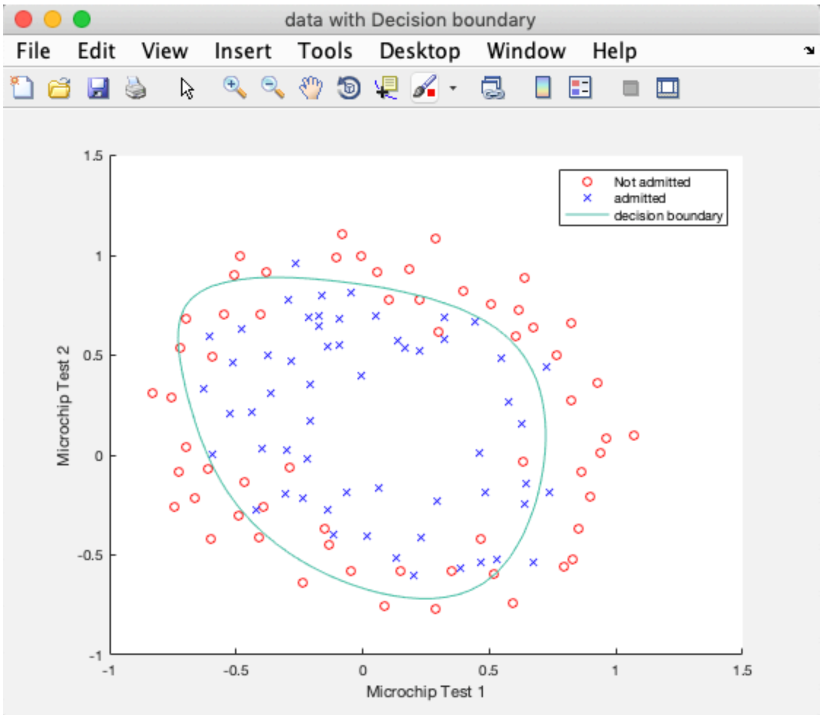
b) With lambda = 0.5



The cost and the accuracy is:

```
With this theta,
Cost computed = 0.392187
-------------------------------------------------------
The accuracy is 80.508475
-------------------------------------------------------
```

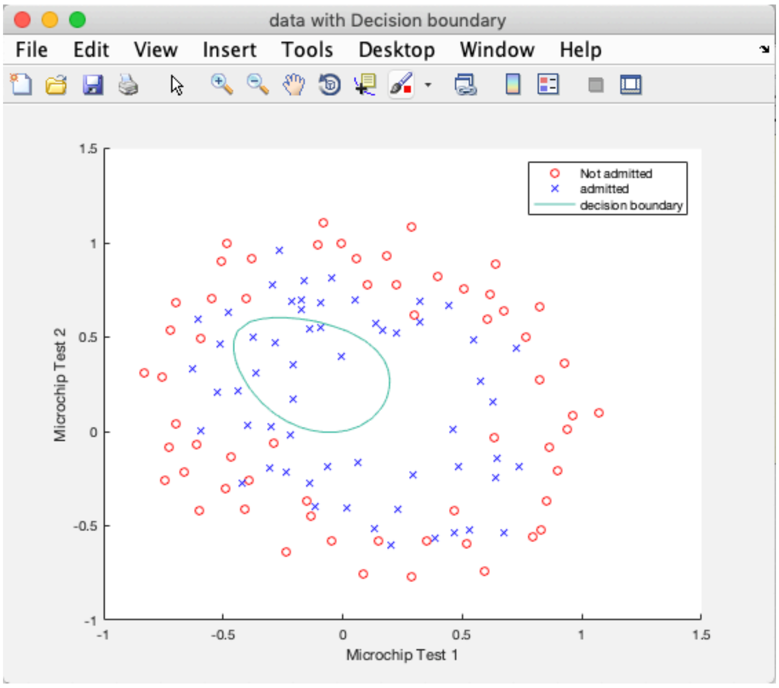
c) With lambda = 0



The cost and the accuracy is:

```
With this theta,
Cost computed = 0.497538
-----------------------------------------------------------
The accuracy is 82.203390
-----------------------------------------------------------
```

d) With lambda = 5



The cost and the accuracy is:

```
With this theta,
Cost computed = 2.062943
-----------------------------------------------------------
The accuracy is 55.084746
-----------------------------------------------------------
```

Analysis:

lambda	cost	accuracy
0	0.497538	82.203390
0.5	0.392187	80.508475
1	0.815212	72.881356
5	2.062943	55.084746

We can see as the lambda decreasing, the accuracy is increasing. However, I got least cost at lambda = 0.5.

4. Codes

1. main_part_1.m

```
%% Machine Learning Homework 4 part 1

% Author: Xinrun Zhang

% Time: 03/15/2019 14:39

% ======================================================================


%% Initialization

clear ; close all; clc


% Import the data;

fprintf('Initializing...\n')

fprintf('Reading the data...\n');

data = load('ex2data1.txt');


X = data(:, [1, 2]);

y = data(:, 3);  % y has values of 0 and 1.


% Initialize the theta vector;

theta = zeros(3, 1);


% Initialize the gradient descent parameters

iteration = 400;

alpha = 0.1;

% ======================================================================


%% Plot the original data;

data_sorted = sortrows(data,3);

X_0 = data_sorted(1:40, [1, 2]);

X_1 = data_sorted(41:100, [1, 2]);
```

```
fprintf('Visualizing the original data...\n\n');
figure('Name','Original Data','NumberTitle','off');
scatter(X_0(:, 1), X_0(:, 2), 'o', 'r');
hold on;
scatter(X_1(:, 1), X_1(:, 2), 'x', 'b');
hold off;
xlabel('Exam 1 score')
ylabel('Exam 2 score')
legend('Not admitted', 'admitted');
% ======================================================================

%% Data processing
fprintf('Data processing...\n\n')
% Add a column of ones to x;
x = [ones(100, 1), X(:,1:2)];
fprintf('----------------------------------------------------------\n');
% ======================================================================

%% Logistic regression version 1
fprintf('Starting logistic regression version 1...');
J = computeCost(x, y, theta); %compute the cost
fprintf('\nWith theta = [0; 0; 0]\nCost computed = %f\n', J);

% Running gradient descent
theta = gradientDescent(x, y, theta, iteration, alpha);
% Print the output, including new theta and J;
fprintf('\nAfter 400 iterations with alpha = 0.1, ')
fprintf('\nTheta found by gradient descent:\n');
fprintf('%f\n', theta);

J = computeCost(x, y, theta);
```

```
fprintf('\nWith theta = [%f ; %f; %f]\nCost computed = %f\n', theta(1),theta(2),theta(3),
J);
fprintf('----------------------------------------------------------\n');
% ======================================================================

%% Analysis
% I can't get proper desicion boundary from gradient descent algorithm
% which I wrote in the function gradientDescent.m.
% Therefore, I searched online and found another way to get the optimal
% The computeCost_new.m is created to generate gradient.
% ======================================================================

%% Logistic regression version 2
fprintf('Starting logistic regression version 2...');
theta_new = zeros(3, 1);
[~, grad] = computeCost_new(theta_new, x, y);

% run the function optimization algorithm
options = optimset('GradObj', 'on', 'MaxIter', 400);
theta_new= fminunc(@(t)computeCost_new(t, x, y), theta_new, options);

fprintf('\nTheta found by optimization algorithm:\n');
fprintf('%f\n', theta_new);

J = computeCost(x, y, theta_new);
fprintf('\nWith theta_v2 = [%f ; %f; %f]\nCost computed = %f\n',
theta_new(1),theta_new(2),theta_new(3), J);
fprintf('----------------------------------------------------------\n');
% ======================================================================

%% Plot the decision boundary
a = 30:0.1:100;
db_1 = (-1./theta(3)).*(theta(2).*a + theta(1));
```

```
db_2 = (-1./theta_new(3)).*(theta_new(2).*a + theta_new(1));
figure('Name','Original Data','NumberTitle','off');
scatter(X_0(:, 1), X_0(:, 2), 'o', 'r');
hold on;
scatter(X_1(:, 1), X_1(:, 2), 'x', 'b');
hold on;
plot(a, db_1);
hold on;
plot(a,db_2);
xlabel('Exam 1 score')
ylabel('Exam 2 score')
legend('Not admitted', 'admitted', 'Decision Boundary 1', 'Decision Boundary 2');
% =====================================================================


%% Compute the accuracy
predict_v1 = round(logsig(x * theta));
predict_v2 = round(logsig(x * theta_new));

accuracy_v1 = mean( double(predict_v1 == y) * 100);
accuracy_v2 = mean( double(predict_v2 == y) * 100);

fprintf('For version 1, the accuracy is %f\n', accuracy_v1);
fprintf('For version 2, the accuracy is %f\n', accuracy_v2);
fprintf('---------------------------------------------------------\n');
% =====================================================================


%% Compute the probability
prob = logsig(theta_new(1) + theta_new(2)*45 + theta_new(3)*85);
fprintf('The probability of this student getting admitted is %f\n', prob);
% =====================================================================
```

2. main_part_2.m

```
%% Machine Learning Homework 4 part 2

% Author: Xinrun Zhang

% Time: 03/19/2019 17:28

% ======================================================================

%% Initialization

clear ; close all; clc


% Import the data;

fprintf('Initializing...\n')

fprintf('Reading the data...\n');

data = load('ex2data2.txt');


x = data(:, [1, 2]);

y = data(:, 3);  % y has values of 0 and 1.


% Initialize the theta vector;

theta = zeros(28, 1);


% Initialize the gradient descent parameters

iteration = 500;

alpha = 0.1;

lambda = 0.5;

% ======================================================================


%% Plot the original data;

data_sorted = sortrows(data,3);

X_0 = data_sorted(1:60, [1, 2]);

X_1 = data_sorted(61:118, [1, 2]);


fprintf('Visualizing the original data...\n\n');

figure('Name','Original Data','NumberTitle','off');
```

```
scatter(X_0(:, 1), X_0(:, 2), 'o', 'r');

hold on;

scatter(X_1(:, 1), X_1(:, 2), 'x', 'b');

hold off;

xlabel('Microchip Test 1')

ylabel('Microchip Test 2')

legend('Not admitted', 'admitted');

% ======================================================================

%% Data processing

fprintf('Data processing...\n\n')

% rebuild the x

% 1 1

% 2 11 2

% 3 21 12 3

% 4 32 23 4

% 5 43 42 24 34 5

% 6 54 53 52 25 35 45 6

x_2 = [x(:,1).^2, x(:,1).*x(:,2), x(:,2).^2];

x_3 = [x(:,1).^3, (x(:,1).^2).*x(:,2), x(:,1).*(x(:,2).^2), x(:,2).^3 ];

x_4 = [x(:,1).^4, (x(:,1).^3).*(x(:,2).^2), (x(:,1).^2).*(x(:,2).^3), x(:,2).^4];

x_5 = [x(:,1).^5, (x(:,1).^4).*(x(:,2).^3), (x(:,1).^4).*(x(:,2).^2),

(x(:,1).^2).*(x(:,2).^4), (x(:,1).^3).*(x(:,2).^4), x(:,2).^5];

x_6 = [x(:,1).^6, (x(:,1).^5).*(x(:,2).^4), (x(:,1).^5).*(x(:,2).^3),

(x(:,1).^5).*(x(:,2).^2), (x(:,1).^2).*(x(:,2).^5), (x(:,1).^3).*(x(:,2).^5),

(x(:,1).^4).*(x(:,2).^5), x(:,2).^6];

x = [x(:, 1:2), x_2(:, 1:3), x_3(:, 1:4), x_4(:, 1:4), x_5(:, 1:6), x_6(:, 1:8)];



% normalization

%x = normalization(x);


% Add a column of ones to x;

X = [ones(118, 1), x(:,1:27)];
```

```
fprintf('-----------------------------------------------------------\n');
% =====================================================================


%% Logistic regression
fprintf('Starting logistic regression...');
J = computeCost_part2(X, y, theta, lambda); %compute the cost
fprintf('\nWith theta = zeros(28, 1)\nCost computed = %f\n', J);


% Running gradient descent
theta = gradientDescent_part2(X, y, theta, iteration, alpha, lambda);


% Print the output, including new theta and J;
fprintf('\nAfter 500 iterations with alpha = 0.1 and lambda = %f, ', lambda);
fprintf('\nTheta found by gradient descent:\n');
fprintf('%f\n', theta);


J = computeCost(X, y, theta);
fprintf('\nWith this theta, \nCost computed = %f\n',J);
fprintf('-----------------------------------------------------------\n');
% =====================================================================


%% Plot the decision boundary
% define two arrays
u = linspace(-1, 1.5, 50);
v = linspace(-1, 1.5, 50);


% g_2 = [u.^2, u.*v, v.^2];
% g_3 = [u.^3, (u.^2).*v, u.*(v.^2), v.^3 ];
% g_4 = [u.^4, (u.^3).*(v.^2), (u.^2).*(v.^3), v.^4];
% g_5 = [u.^5, (u.^4).*(v.^3), (u.^4).*(v.^2), (u.^2).*(v.^4), (u.^3).*(v.^4), v.^5];
% g_6 = [u.^6, (u.^5).*(v.^4), (u.^5).*(v.^3), (u.^5).*(v.^2), (u.^2).*(v.^5),
(u.^3).*(v.^5), (u.^4).*(v.^5), v.^6];
% g = [u, v, g_2(:, 1:3), g_3(:, 1:4), g_4(:, 1:4), g_5(:, 1:6), g_6(:, 1:8)];
```

```
z = zeros(length(u), length(v));

% Evaluate z = theta*x over the grid

for i = 1:length(u)

    for j = 1:length(v)

        z(i,j) = mapFeature(u(i), v(j))*theta;

    end

end

z = z'; % important to transpose z before calling contour


% Plot z = 0

% need to specify the range [0, 0]

figure('Name','data with Decision boundary','NumberTitle','off');

scatter(X_0(:, 1), X_0(:, 2), 'o', 'r');

hold on;

scatter(X_1(:, 1), X_1(:, 2), 'x', 'b');

hold on;

contour(u, v, z, [0, 0])

hold off;

xlabel('Microchip Test 1')

ylabel('Microchip Test 2')

legend('Not admitted', 'admitted', 'decision boundary');

% ======================================================================


%% compute the accuracy

predict = round(logsig(X * theta));


accuracy = mean( double(predict == y) * 100);


fprintf('The accuracy is %f\n', accuracy);

fprintf('----------------------------------------------------------\n');

% ======================================================================
```

3. computeCost.m

```
function J = computeCost(x, y, theta)

m = length(y);


h = logsig(x * theta);

J = (-1/m) * (y' * log(h + 0.01) + (1 - y)' * log(1 - h + 0.01));


end
```

4. computeCost_new.m

```
function [ J, grad ] = computeCost_new(theta, x, y)

m = length(y);


h = logsig(x * theta);

J = (-1/m) * sum(y .* log(h) + (1 - y) .* log(1 - h));


grad = (1 / m) * ( (h - y)' * x );


end
```

5. computeCost_part2.m

```
function J = computeCost_part2(x, y, theta, lambda)


m = length(y);


h = logsig(x * theta);

J = (-1/m) * (y' * log(h) + (1 - y)' * log(1 - h)) + (lambda / (2 * m) ) * (theta' * theta);
```

```
end
```

6. normalization.m

```
function x = normalization(x)

x_mean = mean(x);
x = (x - x_mean)./ (std(x));

end
```

7. gradientDescent.m

```
function theta = gradientDescent(x, y, theta, iteration, alpha)

m = length(y);

for i = 1:iteration
    h = logsig(x * theta);
    theta = theta - (alpha / m) * (x' * (h - y));
end

end
```

8. gradientDescent_part2.m

```
function theta = gradientDescent_part2(x, y, theta, iteration, alpha, lambda)

m = length(y);
```

```
for i = 1:iteration

    h = logsig(x * theta);

    foo = [0; theta(2:28, 1)];

    theta = theta - (alpha / m) * (x' * (h - y)) + (lambda / m) .* foo ;

end


end
```

9. mapFeatures.m

```
function out = mapFeature(X1, X2)

degree = 6;

out = ones(size(X1(:,1)));

for i = 1:degree

    for j = 0:i

        out(:, end+1) = (X1.^(i-j)).*(X2.^j);

    end

end


end
```