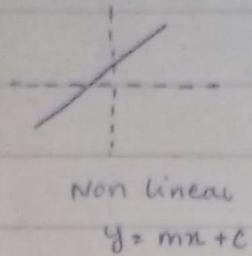
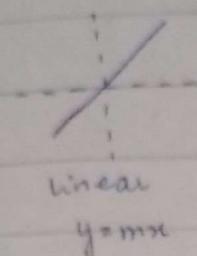


- * We will deal only with linear system in IEC.



- * Linearity is the relationship b/w input & output
- * Time is known as a third component, Graphs b/w input or output vs time won't be linear.

Capacitor \rightarrow eqn: $i_C = C \frac{dv_C}{dt}$ \rightarrow linear relationship

- * To change input mai aye wahi output mai aaye usay linear system kahenge.

$$y = 2x$$

$\therefore n = 2$

$$\boxed{y = 4}$$

Now: $n = 2+1$

$$\boxed{y = 6}$$

linear eqns mai jitna
k increase hoga utna
effect y par aya.

Input & output are
linear

$$y = 2x + c$$

$\therefore n = 2$

$$\boxed{y = 5}$$

Now: $n = 2+1$

$$\boxed{y = 13}$$

Non linear mai jitna n
increase hoa unha y mai
increase nahi aya

Input & output
are non linear.

bcz it has intercept
& 3rd component

09/3am | 2025

$$F(s) = \frac{5s^2 + 6s + 9}{(s+2)(s+8)} \rightarrow \text{Proper function} \rightarrow \text{deg order of num & denom are same}$$

$$F(s) = \frac{(s+2)}{s^2 + 9s + 2} \rightarrow \text{Strictly proper function} \rightarrow \text{deg order of num one less than denom}$$

$$F(s) = \frac{s^2 + 5s + 2}{(s+5)} \rightarrow \text{Improper function} \rightarrow \text{deg order of num is more than denominator}$$

$$F(s) = \frac{V(s)}{R(s)} \rightarrow \text{Output}$$
$$R(s) \rightarrow \text{Input}$$

Ex

$$F(s) = \frac{Y(s)}{R(s)} = \frac{1}{s^2 + 5s + 6}$$

$$Y(s) = \frac{1}{s^2 + 5s + 6} \times R(s)$$

$$Y(t) =$$

$$Z = a + bi$$

$$S = s + jw \rightarrow \text{At initial condition } (s) \text{ is zero.}$$

then $s = jw$

* Hum mech system ka mathematical model always initial condition mai find karnege.

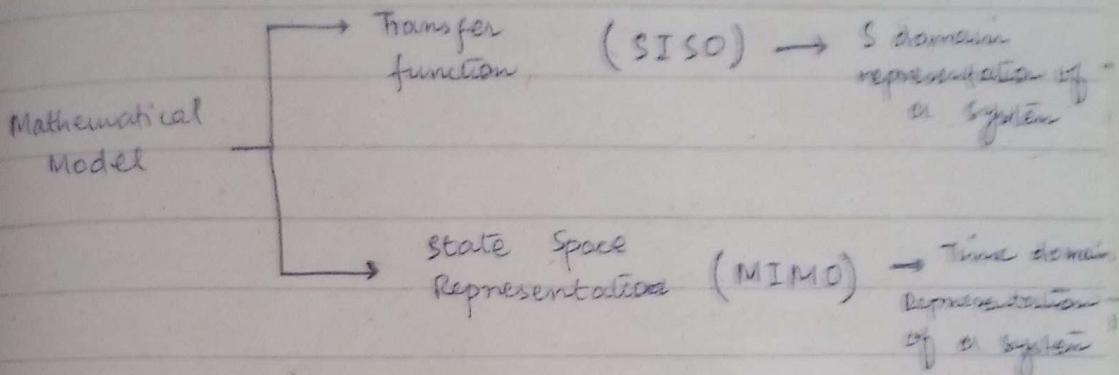
+ Roots of numerator are called zero's (0)

+ Roots of denominator are called poles

degree [no. of integrators. (1/s).
no. of poles in system is called degree.
at origin]

Transfer functions helps to find :-

- ① Order ② ~~Degree~~ Type ③ no. of locations of poles
- ④ No. of location of zeros ⑤ Stability ⑥ Response
- Curve ⑦ Speed ⑧ Dominant pole.



How to find degree

$$f(s) = \frac{(s+9)}{s^2(s+5)(s+10)(s+100)}$$

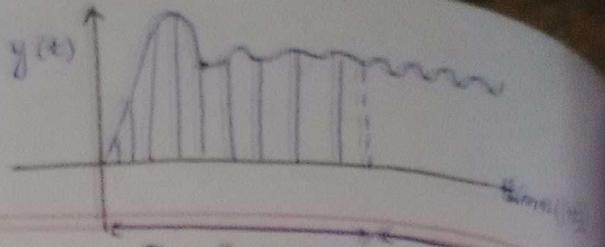
0 0 -5 -10 -100

↓ ↓ ↓ ↓

↓

2 ~~Type~~ system (Jitney zero denominator mai utna hi degree hoga).

* Degree of system ka numerator k zero se koi totaling nahi hai



$$G_1(s) = \frac{s+2}{s(s+5)(s+4)} \rightarrow Z = 0, -2, -4$$

$$s_1 = 0, s_2 = -5, s_3 = -4$$

Transient system
↓
by the time
system changes

Steady state
↑
by the time
system reaches
constant

$$G_2(s) = \frac{s(s+6)(s+8)}{s^2(s^2+9s+4)(s+10)} \rightarrow Z = 0, -6, -8$$

$$s_{1,2} = 0, s_3 = -10, s_4 = -1.5 \pm j$$

$\log = -2.3$

$$G_3(s) = \frac{1}{(s^2+36)(s+8)} \rightarrow \text{no zeros (0)}$$

$$(s^2+36)(s+8)$$

$$s_1 = -8, s_2 = \pm 6j$$

$$G_4(s) = \frac{s}{(s^2+2s+3)} \rightarrow Z = 0$$

$$(s^2+2s+3)$$

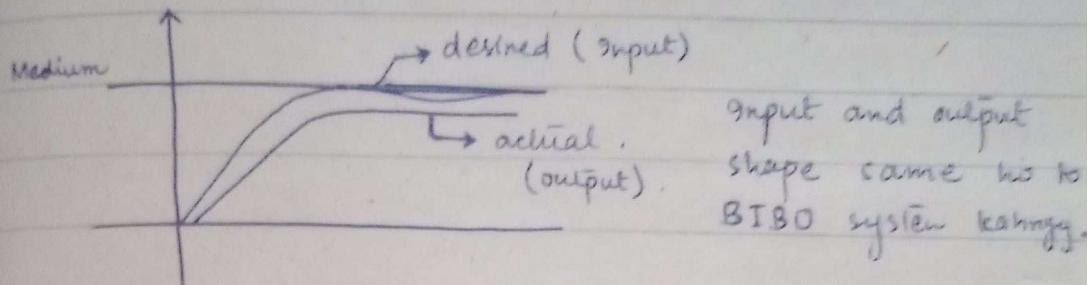
$$s = -1 \pm 1.414j$$

Graphs:-

BIBO system:

stable

Bounded Input bounded output system.



- * A system is said to be stable if and if all the poles of the system are negative i.e lies on the left side of the s-graph.

Dominant poles → (response curve or speed bata hai)

The pole which is near or closest to origin is called dominant pole of the system but if there are complex pole in the system they will always be the dominant pole of the sys.

Ex:

$$G(s) = \frac{s}{(s^2 + 2s + 3)(s + 0.5)}$$

$$\downarrow \quad \downarrow$$

$$\underbrace{s = -1 \pm 1.414j}_{\text{J.}} \quad s_2 = -0.5$$

system mai ➡ complex number
ajay to ab vo O.P. kahlyga.

08/Jan/2023

D.P (dominant pole).

① Speed:

or D.P

The system L which is far from origin is caused and has higher speed although the system L which is near to origin has slowest speed.

Types Of poles:

$$① \text{ Real \& Distinct} \rightarrow G(s) = \frac{1}{(s+5)(s+10)(s+8)(s+2)}$$

→ over damped

$\begin{matrix} & & & \\ \downarrow & \downarrow & \downarrow & \downarrow \\ -5, -10, -8, -2 \end{matrix}$

all are real & distinct.

$$② \text{ Real \& Equal} \rightarrow G(s) = \frac{1}{(s+4)^2(s+9)(s+15)}$$

→ critical damped

$\begin{matrix} & & \\ \downarrow & \downarrow \\ -4, -4, -9, -15 \end{matrix}$

equal and all are real.

$$③ \text{ Complex Conjugate} \rightarrow G(s) = \frac{1}{(s+8)(s^2+2s+3)(s+1)}$$

→ under damped

$\begin{matrix} & & \\ \downarrow & \downarrow & \downarrow \\ -8, -1 \pm 1.41j, -1 \end{matrix}$

overshoot
 zillar range
 time const.

complex

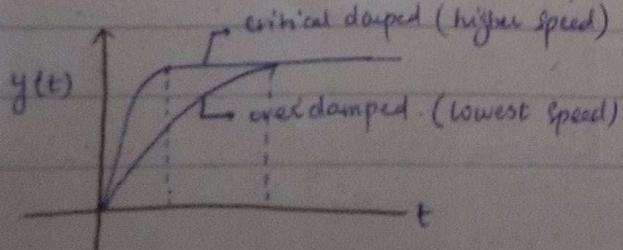
$$④ \text{ Imaginary Poles} \rightarrow G(s) = \frac{1}{(s+8)(s+2)(s^2+144)}$$

→ undamped

$\begin{matrix} & & \\ \downarrow & \downarrow & \downarrow \\ -8, -2, \pm 12j \end{matrix}$

9mag. ↓

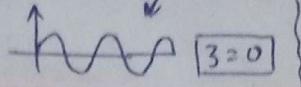
no real part.



$\zeta \rightarrow$ damping factor (Dying out of oscillation)

If $\zeta = 0 \rightarrow$ system is sinusoidal

11/Jan/2025

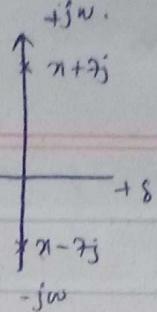


$$G_1(s) = \frac{(s+2)}{(s^2 + 4s + 9)(s+1)}$$

$\boxed{3=0}$

D.P. $\boxed{\pm 7j}, -1$

For $G_1(s)$



Free \rightarrow $0, -0.5 \pm \frac{\sqrt{11}}{2}j$ Underdamped

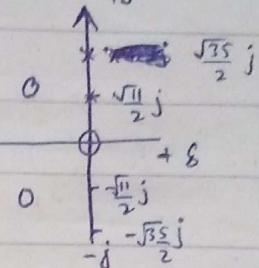
$$G_2(s) = \frac{s(s^2 + s + 3)}{(s^2 + s + 9)(s+6)(s+2)}$$

$\boxed{3 < 1}$

Poles \rightarrow $\boxed{-0.5 \pm \frac{\sqrt{35}}{2}j}, -6, -2$

D.P.

For $G_2(s)$



✓ $G_3(s) = \frac{1}{(s^2 + 8s + 16)(s+6)(s+11)}$ Critical damped.

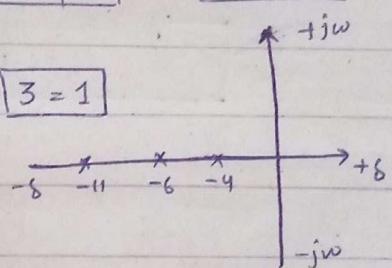
Fastest system

bcs D.P. are far from origin.

$$\boxed{3=1}$$

D.P. $\boxed{-4, -4}, -6, -11$

For $G_3(s)$



✓ $G_4(s) = \frac{s(s+8)}{(s+0.1)(s+2.5)(s+6)}$

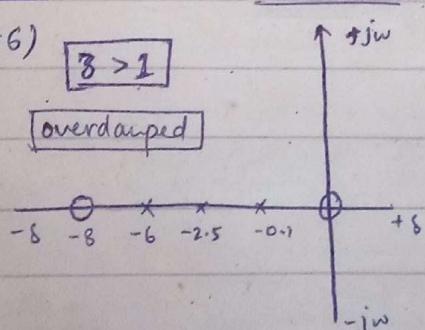
Slowest system

bcs D.P. is very near from origin.

$$\boxed{3 > 1}$$

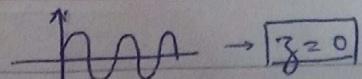
D.P. $\boxed{-0.1}, -2.5, -6$

For $G_4(s)$

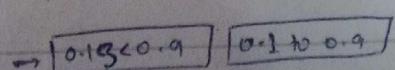


* If system is smooth then the system is critical damped
and it has faster response.

- $\pm 7j \rightarrow$ means undamped \rightarrow



- undamped \rightarrow



* overshoot biada to zeta(ζ) ki value le aao

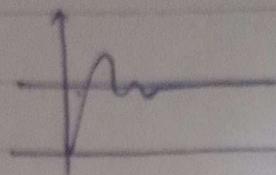
* overshoot kam to zeta(ζ) ki value biada.

Damper \rightarrow Increasing resistance

14/3/2025

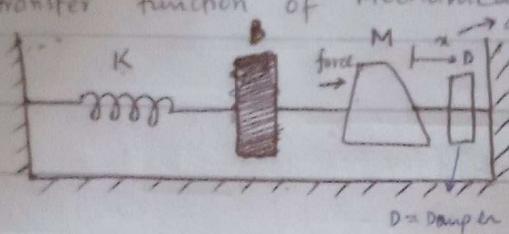
Tuesday

If system has $0.75 < \zeta < 0.8$ then it has less overshoot.



- Damping factor effect speed and overshoot in a system.

* Find out the transfer function of Mechanical system.



$$\int dt \rightarrow \frac{1}{s}$$

$$\frac{d}{dt} \rightarrow s$$

SISO System has
less air force pro
duce outcome settle
Ex: force lag of to
displacement less fly

Spring $\rightarrow F = -Kx$

Damper $\rightarrow F = BV \Rightarrow B \cdot dx/dt$

Mass $\rightarrow F = ma \Rightarrow m \cdot \frac{dv}{dt} = M \frac{d^2x}{dt^2}$

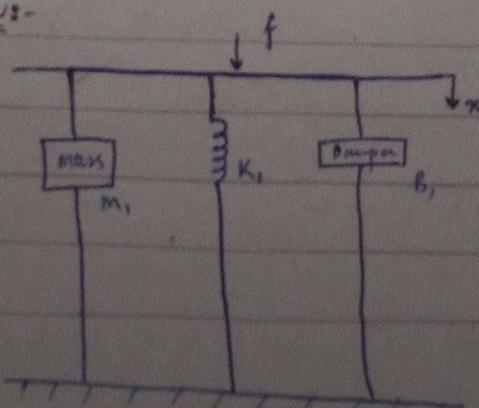
S-Domain Representation:-

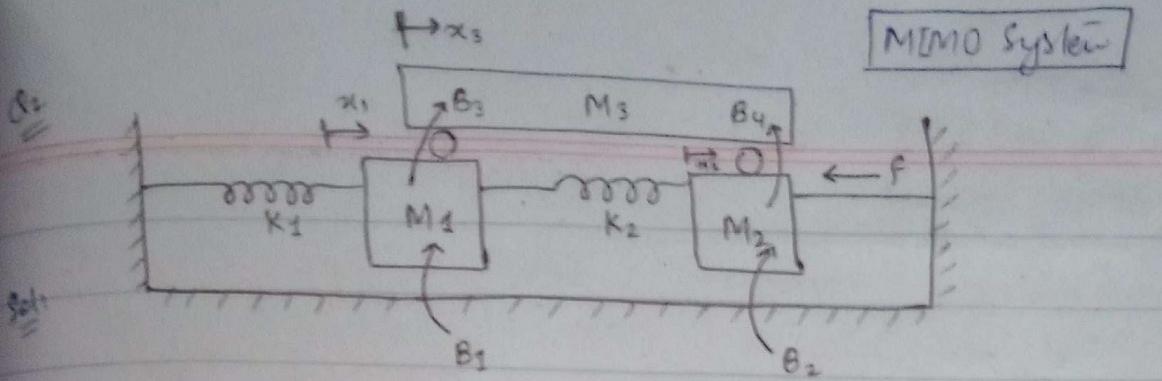
Spring $\rightarrow F(s) = k x(s)$

Damper $\rightarrow F(s) = BV(s) = BS x(s)$

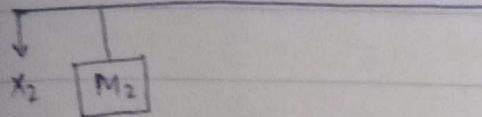
Mass $\rightarrow F(s) = Ma(s) = MsV(s) = Ms^2 x(s)$.

Free Body Diagram:-





Free Body Diagram:



Making Eqn for x_2 :

$$M_2 \frac{d^2}{dt^2} (x_2 - 0) + k_2 (x_2 - 0) + B_3 \frac{d}{dt} (x_2 - 0) + B_2 \frac{d}{dt} (x_2 - x_1) = f_1.$$

$$M_2 \frac{d^2}{dt^2} x_2 + k_2 x_2 + B_3 \frac{d}{dt} x_2 + B_2 \frac{d}{dt} x_2 - B_2 \frac{d}{dt} u_1 = f_1.$$

$$X_2(s) = [M_2 s^2 + (B_2 + B_3)s + k_2] - X_1(s) B_2 s = F_1(s).$$

$$-X_1(s) B_2(s) + [M_2 s^2 + (B_2 + B_3)s + k_2] \cdot X_2(s) = F_1(s). \quad \text{①}$$

Making Eq for x_1 :

15/10/2025.

Solving Q # 01

$$F = ma$$

$$F = m \frac{dv}{dt}$$

$$F = M \frac{d^2x}{dt^2}$$

$$G(s) = \frac{x(s)}{F(s)} = ?$$

$$M_1 \frac{d^2}{dt^2}(x-0) + K_1(x-0) + B_1 \frac{dx}{dt}(x-0) = f.$$

$$M_1 \frac{d^2}{dt^2}(x) + K_1(x) + B_1 \frac{dx}{dt}x = f$$

Convert into time domain:-

$$M_1 \frac{d^2}{dt^2}x(t) + K_1 x(t) + B_1 \frac{dx}{dt}x(t) = f(t).$$

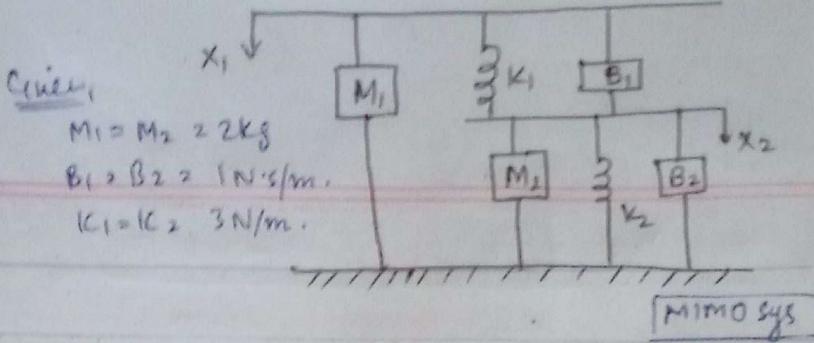
$$M_1 s^2 x(s) + K_1 x(s) + B_1 s x(s) = f(s).$$

$$x(s) [M_1 s^2 + K_1 + B_1 s] = f(s)$$

$$\frac{x(s)}{F(s)} = \frac{1}{M_1 s^2 + B_1 s + K_1}$$

$$\boxed{\frac{x(s)}{F(s)} = \frac{1/M_1}{s^2 + \frac{B_1 s}{M_1} + \frac{K_1}{M_1}}}$$

$$\text{Mass} = M s^2(x)$$



$$M_1 s^2 + (B_1 + B_2) s + K_1$$

$$\textcircled{1} \quad G_1(s) = x_1(s)/F(s) = ?$$

$$\textcircled{2} \quad G_2(s) = x_2(s)/F(s) = ?$$

For x_1 :

$$M_1 s^2 (x_1 - 0) + K_1 (x_1 - x_2) + B_1 s (x_1 - x_2) = F(s).$$

$$M_1 s^2 x_1(s) + K_1 x_1(s) - K_1 x_2(s) + B_1 s x_1(s) - B_1 s x_2(s) = F(s)$$

$$x_1(s) [M_1 s^2 + B_1 s + K_1] - [B_1 s + K_1] x_2(s) = F(s). \rightarrow \textcircled{A}$$

For x_2 :

$$K_2 (x_2 - x_1) = 0$$

$$M_2 s^2 (x_2 - 0) + K_2 (x_2 - 0) + B_2 s (x - 0) + B_1 s (x_2 - x_1) + \\ - (B_1 s + K_1) x_1(s) + [M_2 s^2 + (B_1 + B_2) s + (K_1 + K_2)] x_2(s) = 0 \rightarrow \textcircled{B}$$

Putting values of $M_1, M_2, B_1, B_2, K_1, K_2$ in eq \textcircled{A} & eq \textcircled{B},

$$\text{eq } \textcircled{1} \Rightarrow x_1(s) [2s^2 + s + 3] - [s + 3] x_2(s) = F(s). \rightarrow \textcircled{1}$$

$$\text{eq } \textcircled{2} \Rightarrow - (s + 3) x_1(s) + [2s^2 + 2s + 6] x_2(s) = 0 \rightarrow \textcircled{2}$$

Applying Cramer's Rule,

$$\begin{bmatrix} 2s^2 + s + 3 & -(s+3) \\ -(s+3) & 2s^2 + 2s + 6 \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}.$$

$$\therefore G_1(s) = \frac{x_1(s)}{F(s)} = ?$$

$x_1, x_2 \rightarrow 2(x)$ hence
2nd ~~order~~
degree of freedom.

Order \rightarrow no. of mass
given mass = 2

X_1 is the remainder
in 1st column & put
it replace by zero.

$$X_1(s) = \begin{bmatrix} F(s) & -(s+3) \\ 0 & 2s^2 + 2s + 6 \end{bmatrix}$$

$$\begin{bmatrix} 2s^2 + 2s + 3 & -(s+3) \\ -(s+3) & 2s^2 + 2s + 6 \end{bmatrix}$$

$$X_1(s) = \frac{F(s) [2s^2 + 2s + 6]}{(2s^2 + s + 3)(2s^2 + 2s + 6) - (s+3)^2}$$

$$\boxed{\frac{X_1(s)}{F(s)} = \frac{2s^2 + 2s + 6}{4s^2}}$$

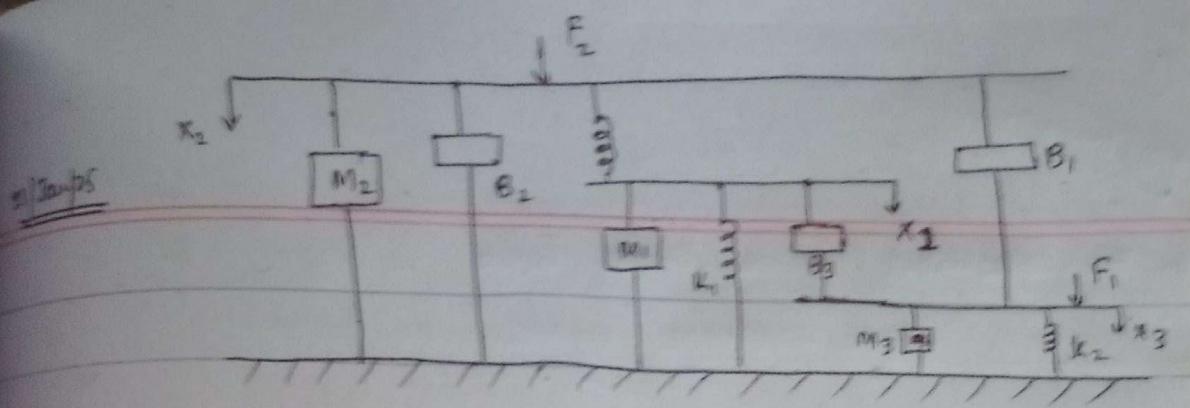
For $G_2(s) = \frac{x_2(s)}{F(s)}$

$$X_2(s) = \frac{\begin{bmatrix} 2s^2 + s + 3 & F(s) \\ -(s+3) & 0 \end{bmatrix}}{\begin{bmatrix} 2s^2 + 2s + 3 & -(s+3) \\ -(s+3) & 2s^2 + 2s + 6 \end{bmatrix}}$$

$$X_2(s) = \frac{F(s) [-(s+3)]}{(2s^2 + s + 3)(2s^2 + 2s + 6) - (s+3)^2}$$

middle term minus
of the cross terms

$$X_2(s) =$$



For x_1

$$[M_1 s^2 + B_3 s + (k_1 + k_3)] x_1(s) - k_3 x_2(s) - B_3 s x_3(s) = 0 \quad (1)$$

For x_2

$$-k_3 x_1(s) + [M_2 s^2 + (B_1 + B_2)s + k_3] x_2(s) - B_1 s x_3(s) = F_2(s).$$

For x_3

$$-B_3 s x_1(s) - B_1 s x_2(s) + [M_3 s^2 + (B_1 + B_2)s + k_2] x_3(s) = F_1(s)$$

For $G_1(s) = \frac{x_3(s)}{F_2(s)}$

$x_1(s)$

$x_2(s)$

$x_3(s)$

x_3 max $F_1(s)$ arriva
now at home $F_1(s)$
in a real max

For x_3

$$x_3(s) = \begin{bmatrix} M_1 s^2 + B_3 s + (k_1 + k_3) & -k_3 & 0 \\ -k_3 & M_2 s^2 + (B_1 + B_2)s + k_3 & 0 \\ -B_3 s & -B_1 s & F_1(s) \end{bmatrix}$$

$$\begin{bmatrix} M_1 s^2 + B_3 s + (k_1 + k_3) & -k_3 & -B_3 s \\ -k_3 & M_2 s^2 + (B_1 + B_2)s + k_3 & -B_1 s \\ -B_3 s & -B_1 s & M_3 s^2 + (B_1 + B_2)s + k_2 \end{bmatrix}$$

When system is 3rd degree of freedom, then we will leave the solution here.

Ans

21 Jan 2025

Thursday

* In electrical system, we have 2 input system called force - voltage system or force - current system.

Voltage changed into → force.
Current changed into → force

when electrical sys.
changed by mech sys.

Force-Velocity System 1-

- *- $F = ma \Rightarrow F = m dv/dt \rightarrow$ Mass
- *- $F = Bv^2 \Rightarrow F = Bv \rightarrow$ Dumper.
- *- $F = kx \Rightarrow F = k \int v dt \rightarrow$ Spring.

\rightarrow Force - voltage
F - V

\rightarrow Force - current
F - I

V → input

V → output

I → output

I → input

$$V = IR \rightarrow \text{Resistor}$$

$$I = V/R$$

$$V_C = \frac{1}{C} \int i_C dt \rightarrow \text{capacitor}$$

$$i_C = \frac{dV_C}{dt}$$

$$V_L = \frac{L di_L}{dt} \rightarrow \text{inductor}$$

$$i_L = \frac{1}{L} \int V_L dt$$

(F - V Analogous)

(F - I Analogous)

F - V

F - I

$v - I$

$v - V$

M - L

M - C

B - R

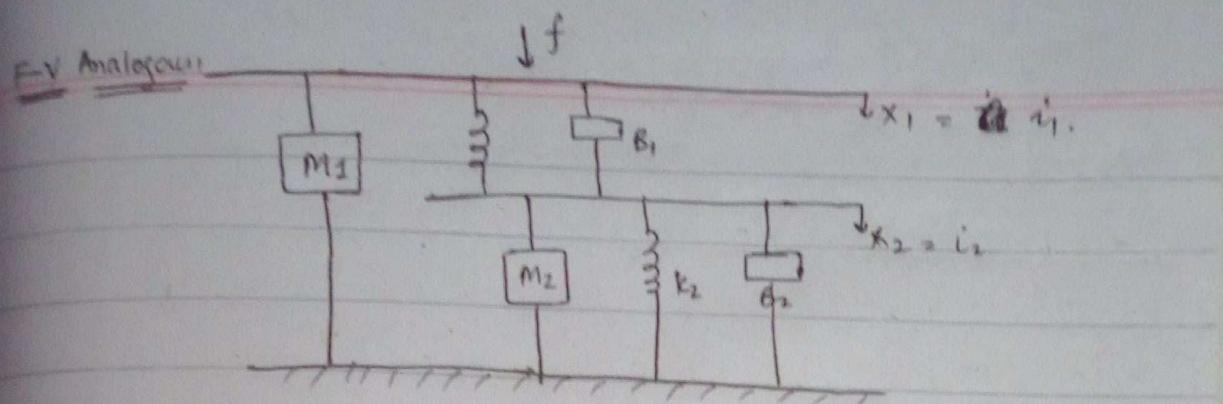
B - $1/R$

$K - \frac{1}{C}$

$K - 1/L$

electric
→
mechanical
charge

Parallel \rightarrow voltage same.
Series \rightarrow current same.



electrical circuit,

