HW2: Attribute Selection with Information Gain

$Info(D) = I(9,5) = \frac{9}{14}\log_2(\frac{9}{14}) - \frac{5}{14}\log_2(\frac{5}{14}) = 0.940$ - Expected information (entropy) ความต้องการ

$Info_{age}(D) = \frac{5}{14}I(2,3) + \frac{4}{14}I(4,0) + \frac{5}{14}I(3,2) = 0.694$ - Expected information ความต้องการ

$Info_{income}(D) = \frac{4}{14}I(3,2) + \frac{6}{14}I(4,2) + \frac{4}{14}I(6,2)$

$= \frac{4}{14}\left(-\frac{3}{4}\log_2(\frac{3}{4}) - \frac{1}{4}\log_2(\frac{1}{4})\right) + \frac{6}{14}\left(-\frac{4}{6}\log_2(\frac{4}{6}) - \frac{2}{6}\log_2(\frac{2}{6})\right) + \frac{4}{14}\left(-\frac{3}{4}\log_2(\frac{3}{4}) - \frac{1}{4}\log_2(\frac{1}{4})\right)$

$= 0.911$

$Info_{student}(D) = \frac{7}{14}I(4,3) + \frac{7}{14}I(6,1)$

$= \frac{7}{14}\left(-\frac{4}{7}\log_2(\frac{4}{7}) - \frac{3}{7}\log_2(\frac{3}{7})\right) + \frac{7}{14}\left(-\frac{6}{7}\log_2(\frac{6}{7}) - \frac{1}{7}\log_2(\frac{1}{7})\right)$

$= 0.788$

$Info_{credit\text{-}rating}(D) = \frac{8}{14}\left(-\frac{6}{8}\log_2(\frac{6}{8}) - \frac{2}{8}\log_2(\frac{2}{8})\right) + \frac{6}{14}\left(-\frac{3}{6}\log_2(\frac{3}{6}) - \frac{3}{6}\log_2(\frac{3}{6})\right)$

$= \frac{8}{14}I(6,2) + \frac{6}{14}I(3,3)$

$= 0.892$

$Gain(age) = Info(D) - Info_{age}(D) = 0.940 - 0.694 = 0.246$

$Gain(income) = Info(D) - Info_{income}(D) = 0.940 - 0.911 = 0.029$

$Gain(student) = Info(D) - Info_{student}(D) = 0.940 - 0.788 = 0.152$

$Gain(Credit\text{-}rating) = Info(D) - Info_{credit\text{-}rating}(D) = 0.940 - 0.892 = 0.048$

ดังนั้นจาก Root node คือ ค่า Gain ที่มากที่สุดคือ age ได้ค่า Gain = 0.246

ที่ age: <=30

$Info_{age<=30}(D) = I(2,3) = -\frac{2}{5}\log_2(\frac{2}{5}) - \frac{3}{5}\log_2(\frac{3}{5}) = 0.971$

$Info_{income}(D) = \frac{1}{5}I(1,0) + \frac{2}{5}I(1,1) + \frac{2}{5}I(0,2)$

$= \frac{1}{5}(-\frac{1}{1}\log_2(\frac{1}{1}) - 0) + \frac{2}{5}(-\frac{1}{2}\log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2})) + \frac{2}{5}(-\frac{2}{2}\log_2(\frac{2}{2}) - 0)$

$Info_{student}(D) = \frac{3}{5}(0,3) + \frac{2}{5}(2,0) = \frac{2}{5}(-\frac{2}{2}\log_2(\frac{2}{2})) + \frac{3}{5}(-\frac{3}{3}\log_2(\frac{3}{3}))$

$= 0$

$Info_{credit\_rating}(D) = \frac{3}{5} I(1,2) + \frac{2}{5}(1,1)$ (fair, excellent)

$= \frac{3}{5}\left(-\frac{1}{3}\log_2\left(\frac{1}{3}\right) - \frac{2}{3}\log_2\left(\frac{2}{3}\right)\right) + \frac{2}{3}\left(-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log\left(\frac{1}{2}\right)\right)$

$= 0.951$

$Gain_{(income)} = Info_{age:<=30}(D) - Info_{income}(D) = 0.971 - 0.400 = 0.571$

$Gain(student) = Info_{age:<=30}(D) - Info_{student}(D) = 0.971 - 0 = 0.971$

$Gain(credit\_rating) = Info_{age:<=30}(D) - Info_{credit\_rating}(D) = 0.971 - 0.951 = 0.020$

ดังนั้น decision node แรก คือ student เพราะ มีค่า Gain สูงสุด

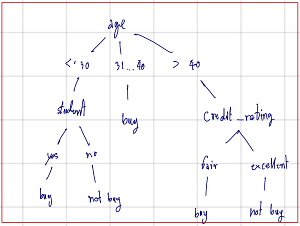
$Info_{age:31...40}(D) = I(4,0)$

$= -\frac{4}{4}\log_2\left(\frac{4}{4}\right) - \frac{0}{4}\log_2\left(\frac{0}{4}\right)$

ดังนั้น 31...40 จึงไม่มี decision node หลังจากสมการ

I(4,0) ทำให้พบว่า age : 31...40 จะซื้อทั้งหมด

$Info_{age:>40}(D) = I(3,2) = -\frac{3}{5}\log_2\left(\frac{3}{5}\right) - \frac{2}{5}\log_2\left(\frac{2}{5}\right) = 0.971$



$Info_{income}(D) = \frac{2}{5}(I(1,1)) + \frac{3}{5} I(2,1)$

$= \frac{2}{5}\left(-\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right) + \frac{3}{5}\left(-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \left(\frac{1}{3}\right)\log_2\left(\frac{1}{3}\right)\right)$

$= 0.951$

$Info_{(student)}(D) = \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1) = \frac{3}{5}\left(-\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)\right) + \frac{2}{5}\left(-\frac{1}{2}\log_2\left(\frac{1}{1}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right)$

$= 0.951$

$Info_{(credit\_rating)}(D) = \frac{3}{5} I(3,0) + \frac{2}{5} I(0,2)$

$= \frac{3}{5}\left(-\frac{3}{3}\log_2\left(\frac{3}{3}\right)\right) + \frac{2}{5}\left(-\frac{2}{2}\log_2\left(\frac{2}{2}\right)\right) = 0$

$Gain(income) = Info_{age:>40}(D) - Info_{income}(D) = 0.971 - 0.951 = 0.02$

$Gain(student) = Info_{age:>40}(D) - Info_{student}(D) = 0.971 - 0.951 = 0.02$

$Gain(credit\_rating) = Info_{age:>40}(D) - Info_{cr}(D) = 0.971 - 0 = 0.971$

สรุปดังนั้น credit_rating เป็น decision node สอง เพราะ มีค่า Gain สูงสุด และ Info (D)

= 0 ทำให้สามารถ แบ่งค่าซื้อไม่ซื้อได้เลย