

project

April 5, 2021

1 LIF Model

In this notebook, we use the following update rule in order to simulate a single neuron with *LIF* model.

$$u(t + \Delta) = u(t) - \Delta/\tau[(u(t) - u_r) - R.I(t)]$$

```
[1]: import sys
sys.path.append('.')
import numpy as np
from cnsproject.plotting.plotting import time_plot, fi_curve
from cnsproject.utils import run_simulation_with_params
```

```
[2]: # Initializing the simulation variables
iters = 1000
zero_percent = 2
save_monitor_states = True
monitor_vars = ["potential", "s", "u_rest", "threshold", "in_current", "tau"]
```

`neuron_params_1` and `neuron_params_2` are experimenting the effect of `u_rest` and `threshold`

Note: σ_n represents the standard deviation of the added noise.

1.1 Simulation with `neuron_params_1`

$$\tau = 20, |u_r - threshold| = 5mV$$

1.1.1 Without noise:

```
[3]: step_currents = np.linspace(4, 16, 13, dtype=np.float32)
neuron_params_1 = {'threshold': -55, 'u_rest': -60, 'tau': 20}
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_1,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent
```

```

    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])

```

Simulation time: 3.917s

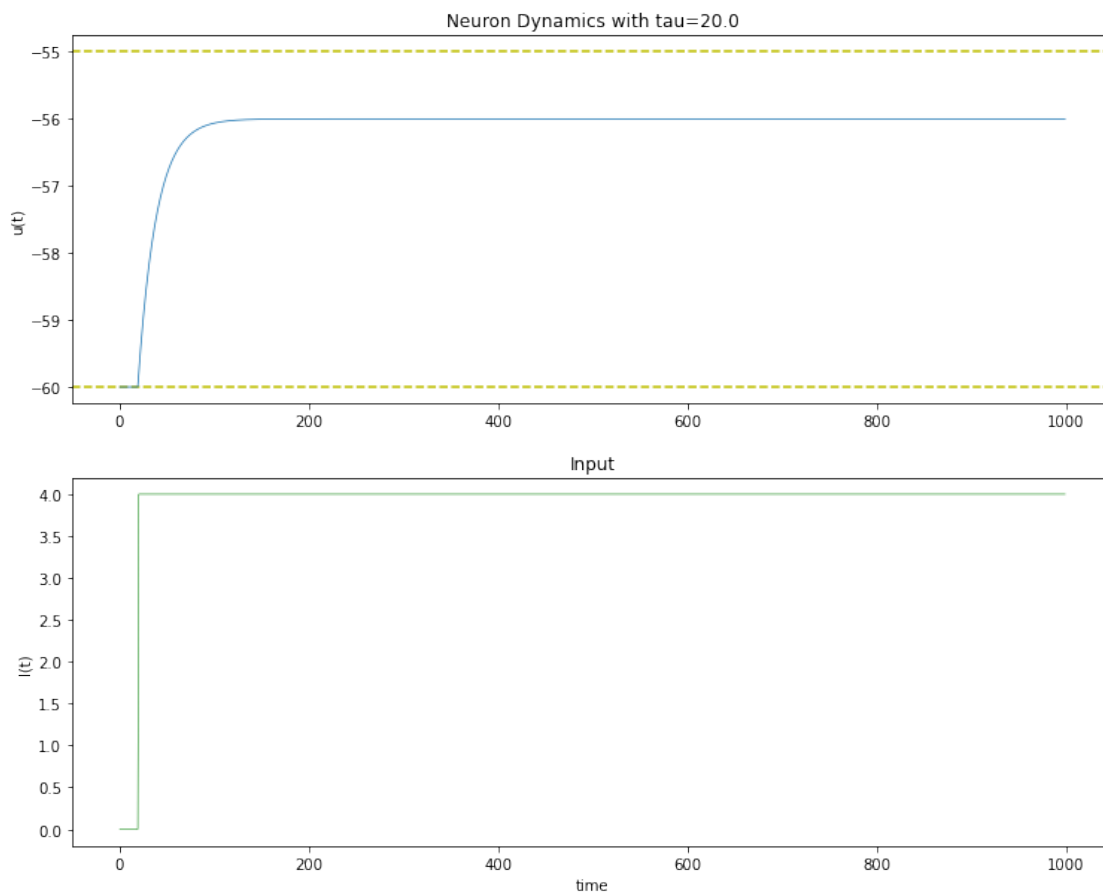
Currents: [4. 6. 8. 10. 12. 14. 16.]

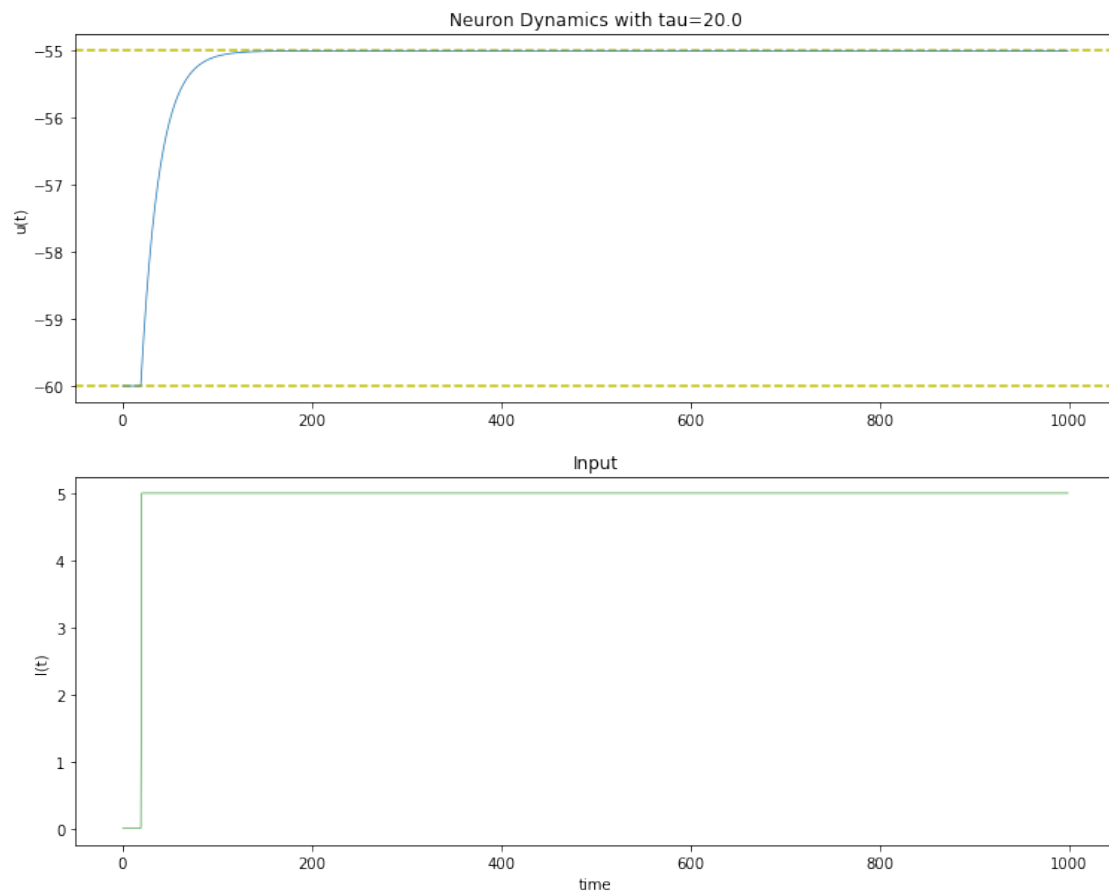
frequencies: [0 29 50 71 91 110 124]

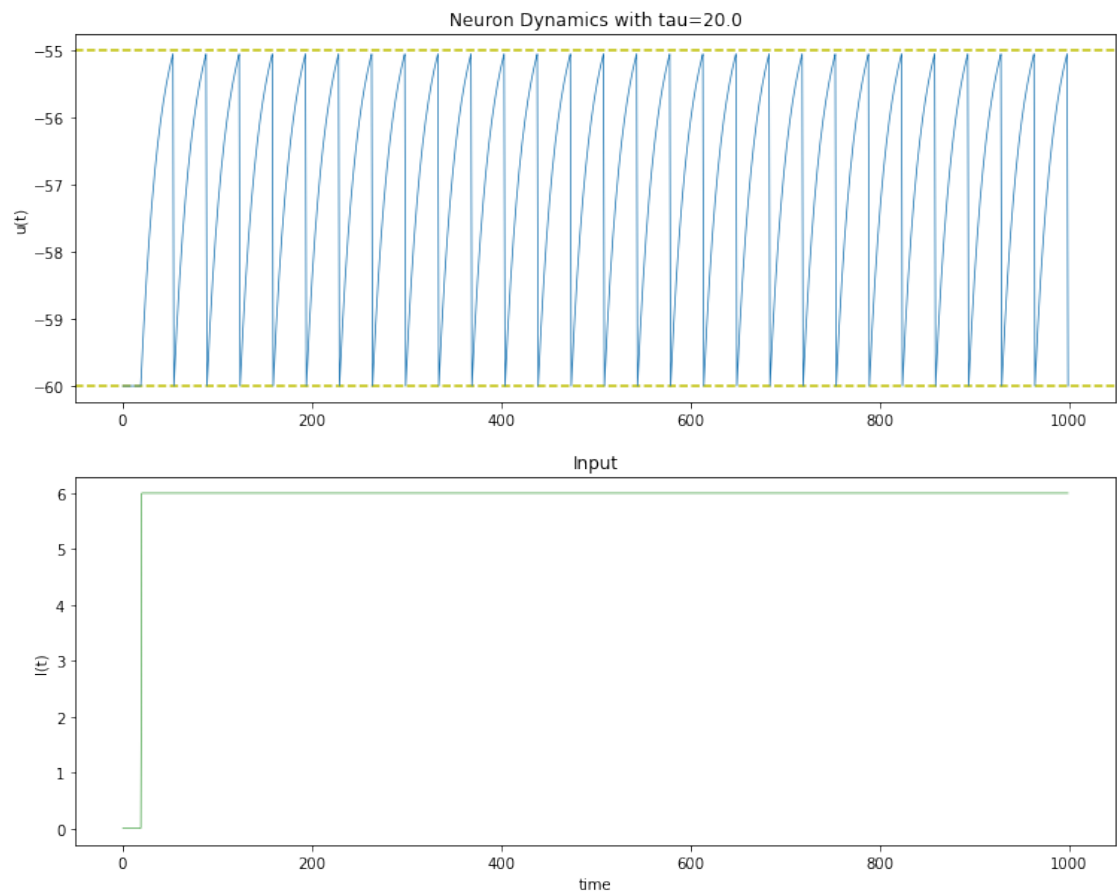
```

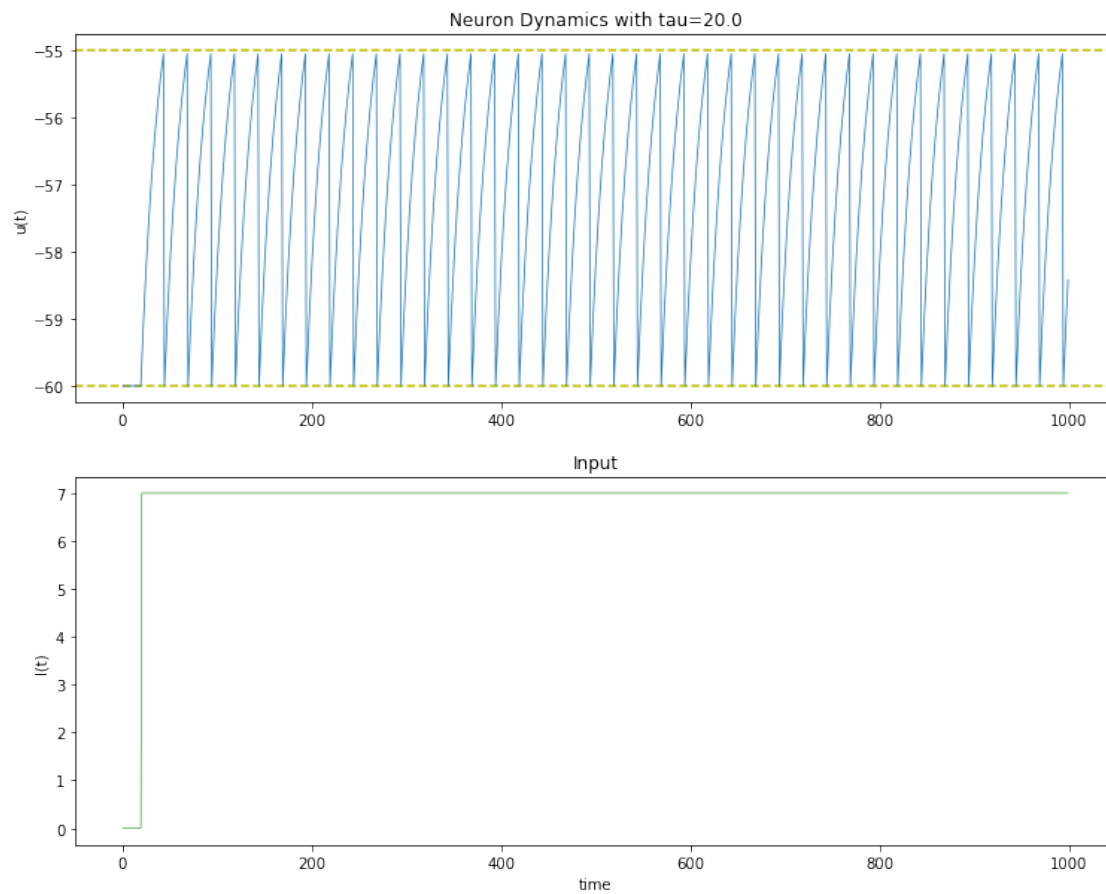
[4]: %matplotlib inline
if save_monitor_states:
    for i in range(4):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)

```

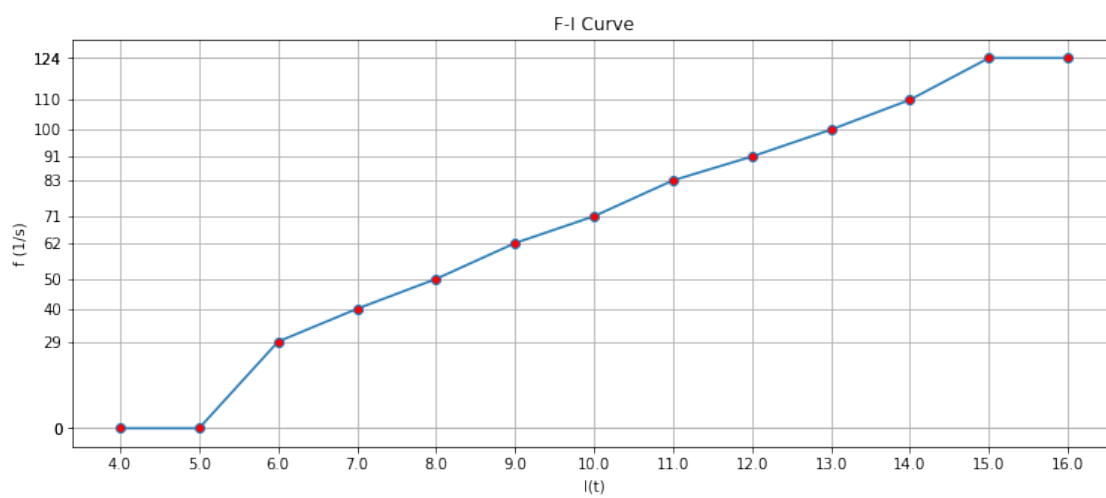








```
[5]: %matplotlib inline
fi_curve(currents, frequencies)
```



According to the update rule: $u(t + \Delta) = u(t) - \Delta/\tau[(u(t) - u_r) - R.I(t)]$ $R.I(t)$ should be greater than $(threshold - u_r)$ before we can see any spike in the neuron's output. This is due to the fact that the output of $[(u(t) - u_r) - R.I(t)]$ should be lower than 0 in order for the $u(t)$ to increase. Now, suppose the opposite; by increasing $u(t)$, $u(t) - u_r$ will be greater than $R.I(t)$ for some $u(t) < threshold$, therefore, $u(t)$ will not increase to reach to the threshold, so we do not get a spike. You can see this effect in the second plot above, where $R.I(t) = |threshold - u_r|$. In this case, $u(t)$ gets very close to $threshold$ but cannot reach it.

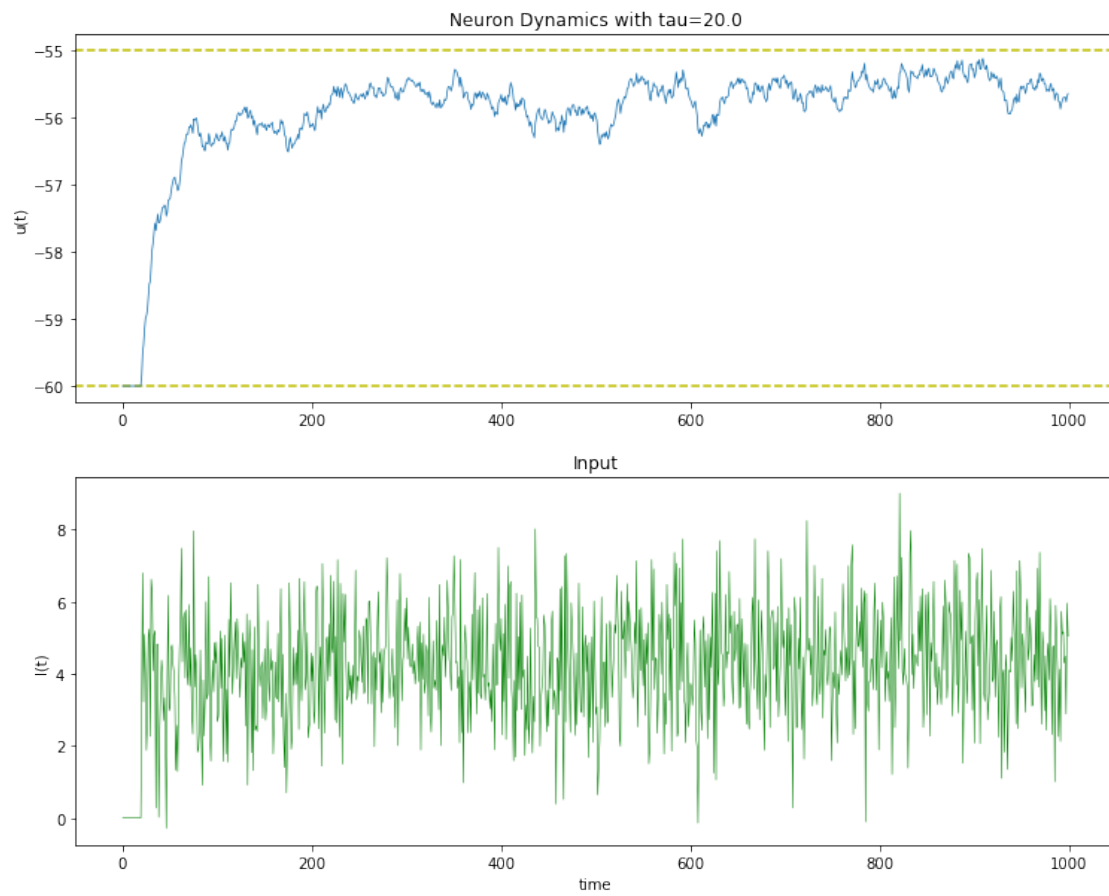
1.1.2 With noise: $\sigma_n = 1.5$

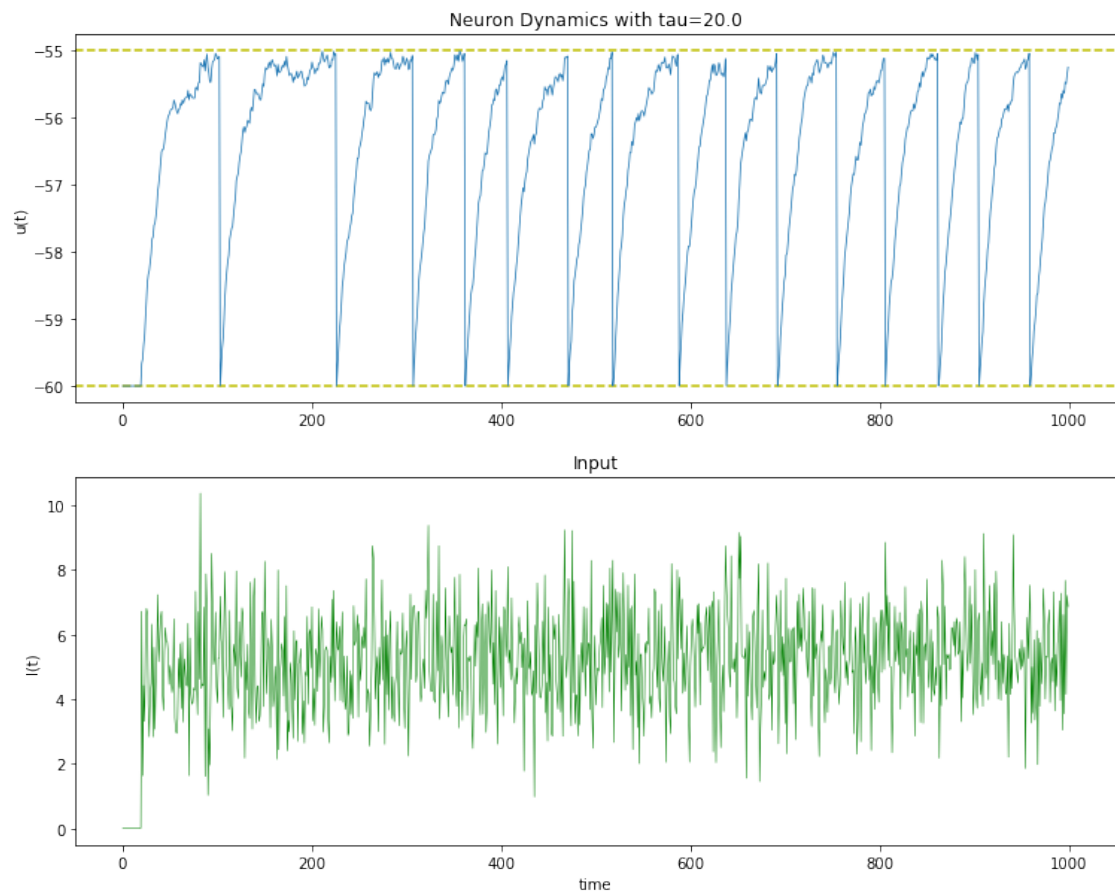
The inputs are linear (after some zero in the beginning), with a *slope* = $5 \cdot 10^{-4}$. Lines with different base values are used for simulation. You can see the implementation at `utils.create_line`.

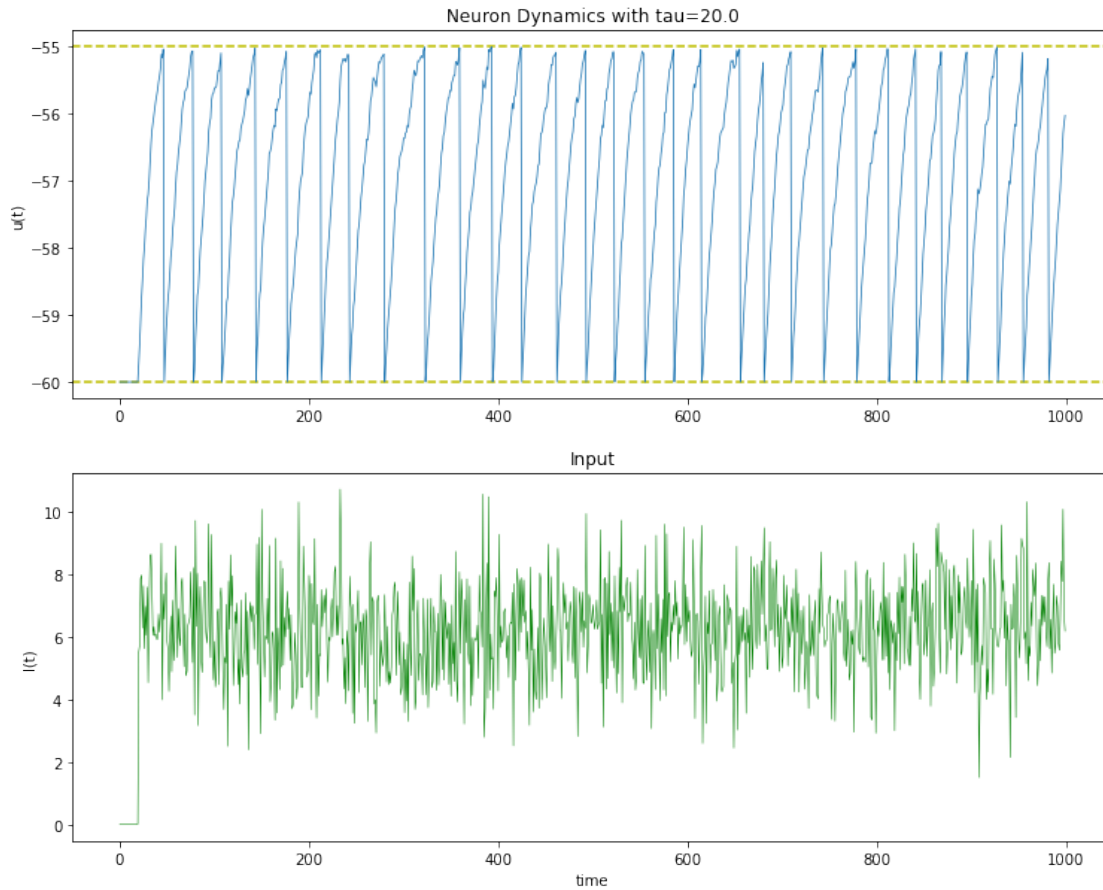
```
[6]: step_currents = np.linspace(4, 8, 5, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_1,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=1.5
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])
```

```
Simulation time: 1.567s
Currents: [4. 6. 8.]
frequencies: [ 0 31 53]
```

```
[7]: %matplotlib inline
if save_monitor_states:
    for i in range(0, len(monitor_states) - 2):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```







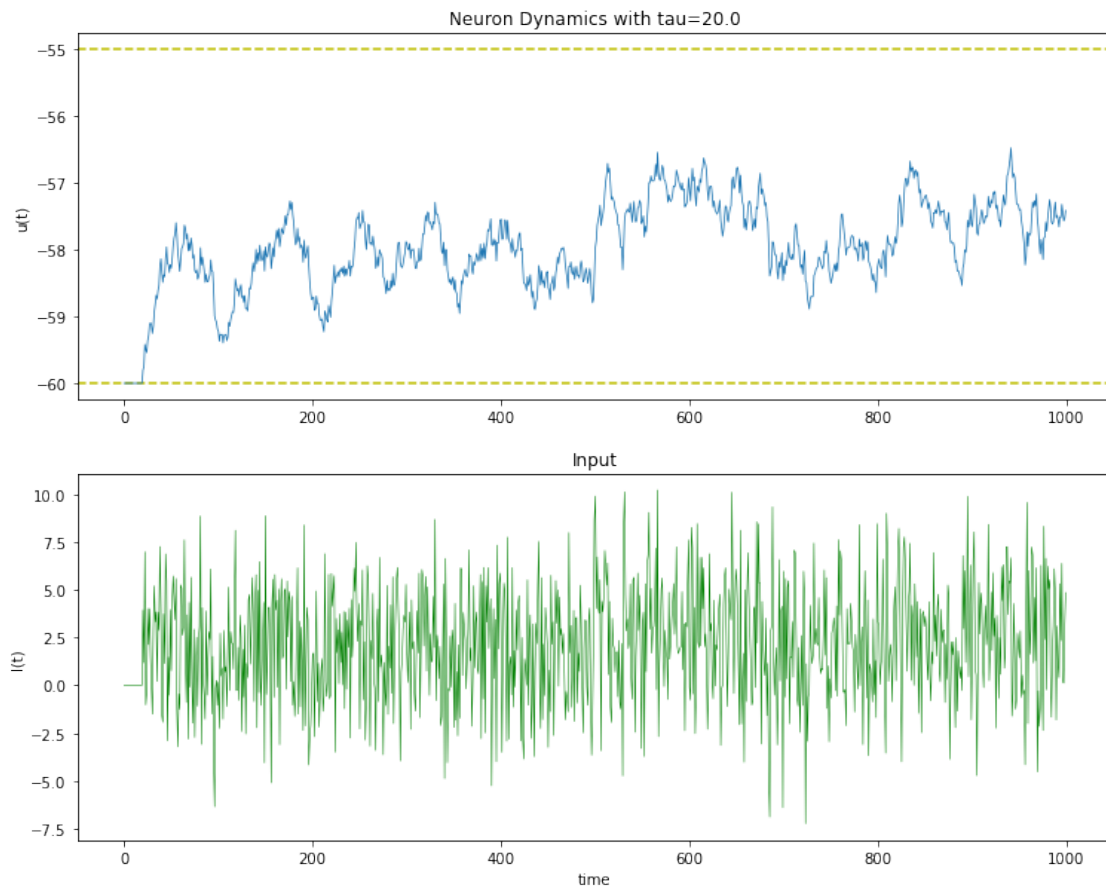
1.1.3 With noise: $\sigma_n = 3$

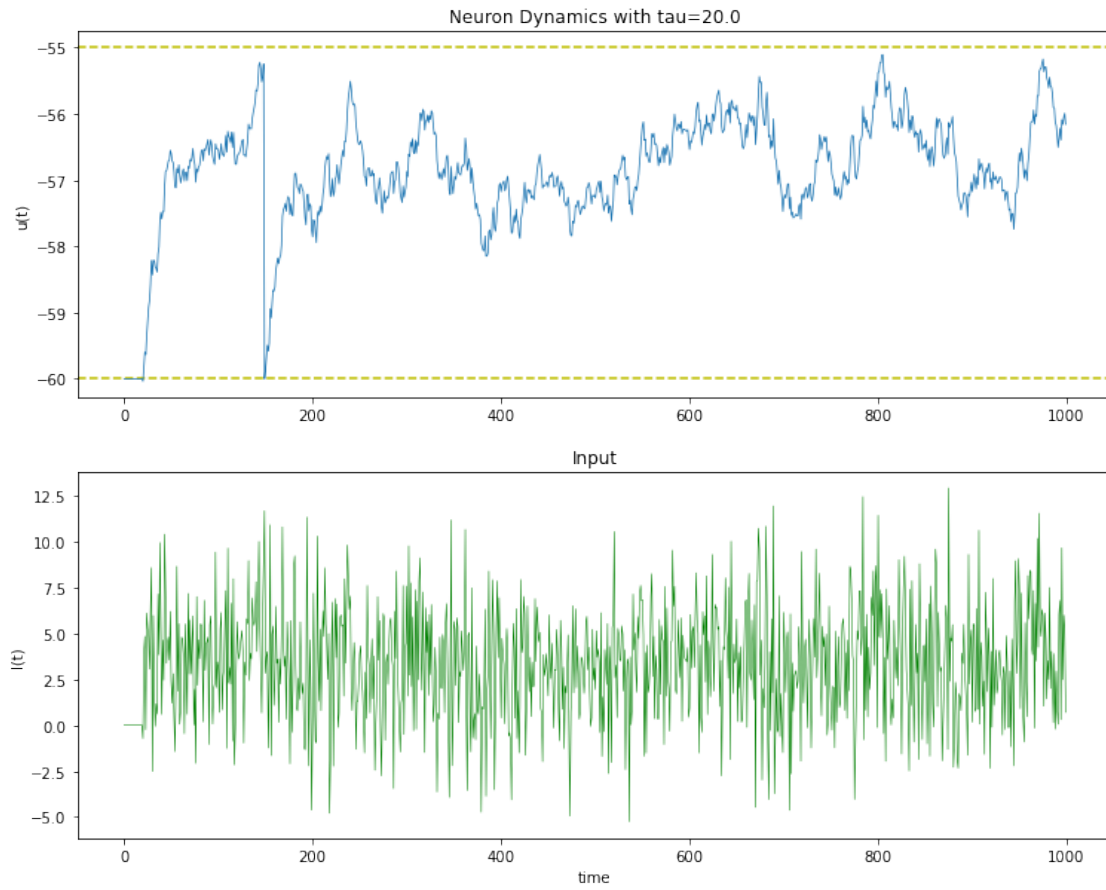
Here, we use inputs with smaller base value to better see the effect of higher noise.

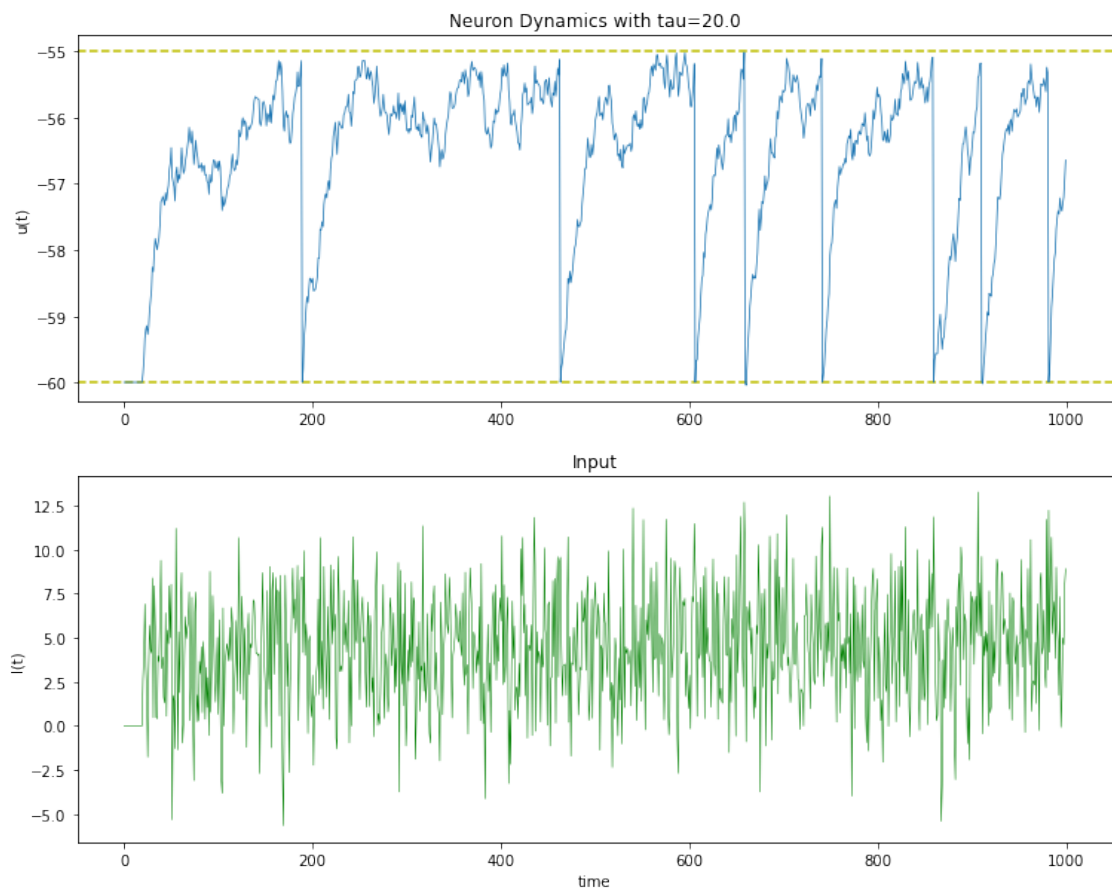
```
[8]: step_currents = np.linspace(2, 6, 5, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_1,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=3
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:2])
print('frequencies:', frequencies[:2])
```

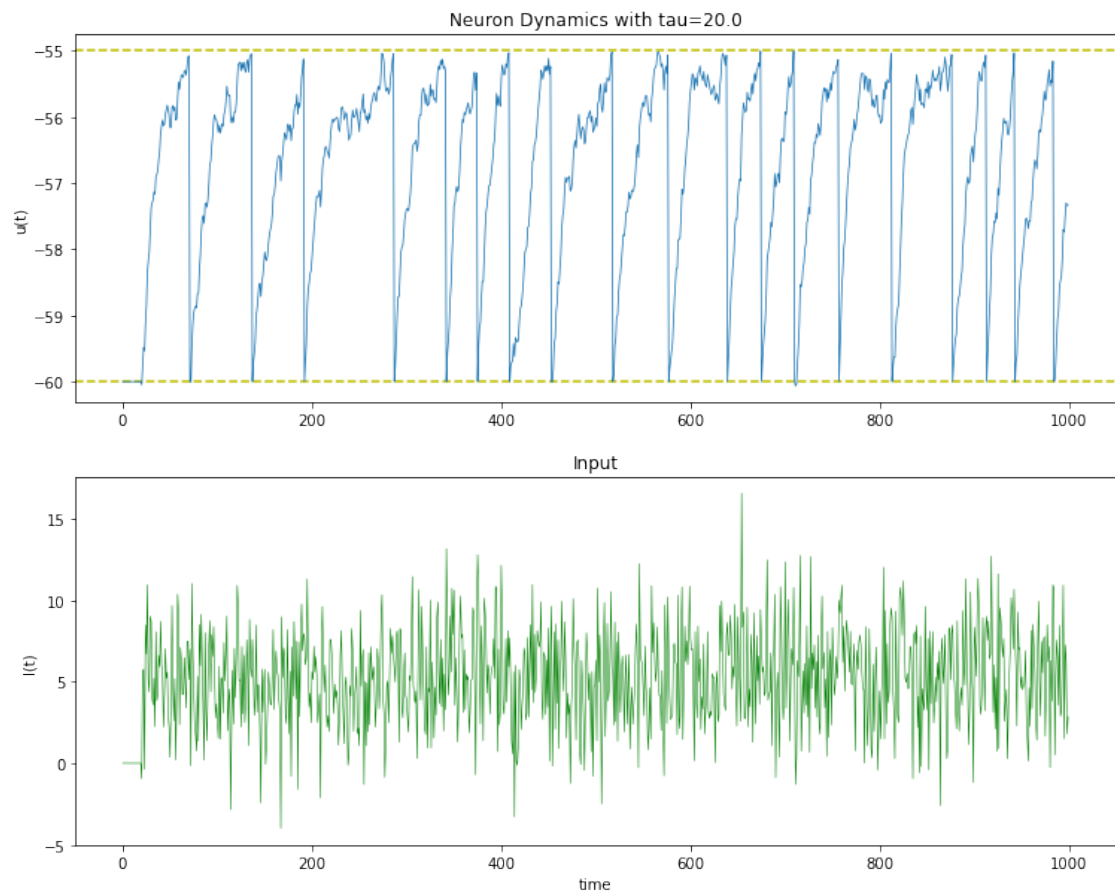
Simulation time: 1.493s
Currents: [2. 4. 6.]
frequencies: [0 8 31]

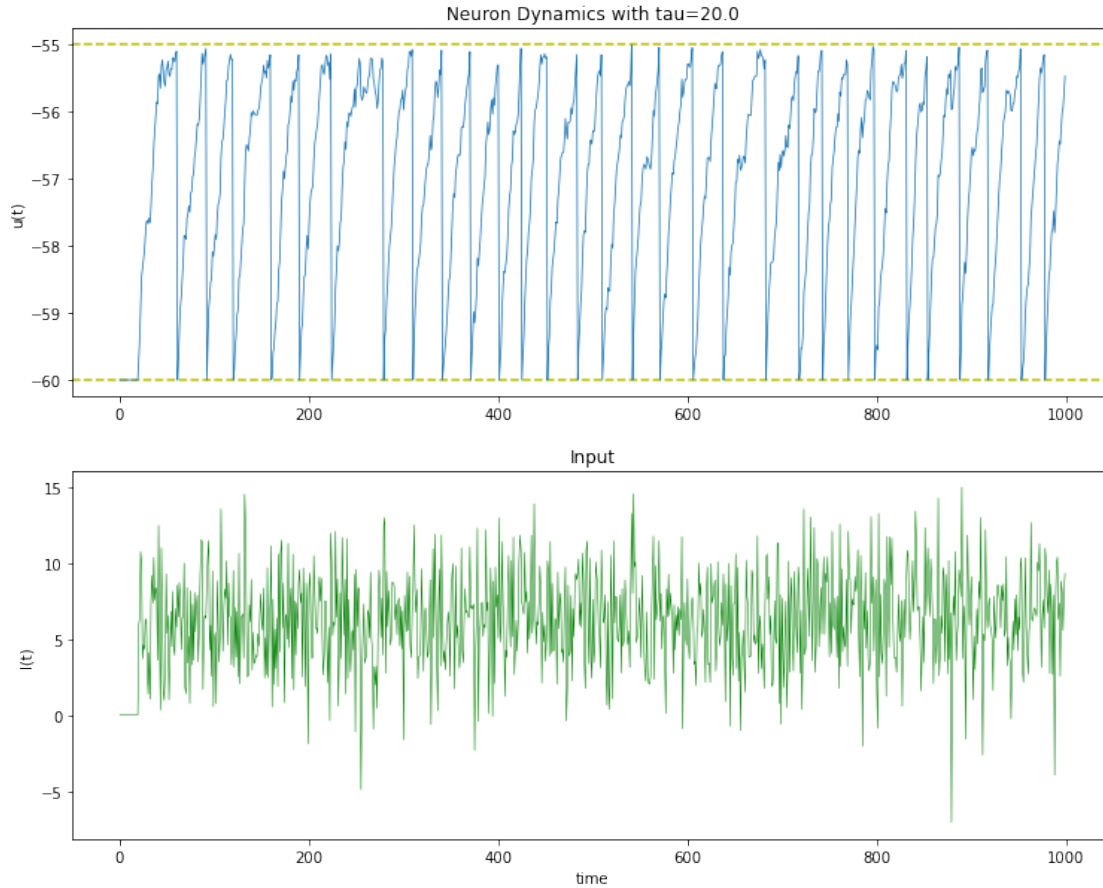
```
[9]: %matplotlib inline
if save_monitor_states:
    for i in range(0, len(monitor_states)):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```











The above simulations with different amounts of noise shows that the chance of getting a spike will increase as the noise increases. With a base value of 4, in the case of $\sigma_n = 1.5$, we got only 1 spike. However, we got 5 spikes with the same base value and $\sigma_n = 3$.

1.2 Simulation with neuron_params_2

$$\tau = 20, |u_r - threshold| = 10mV$$

1.2.1 Without noise

```
[10]: step_currents = np.linspace(8, 32, 13, dtype=np.float32)
neuron_params_2 = {'threshold': -50, 'u_rest': -60, 'tau': 20}
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_2,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent
```

```

    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])

```

Simulation time: 3.982s

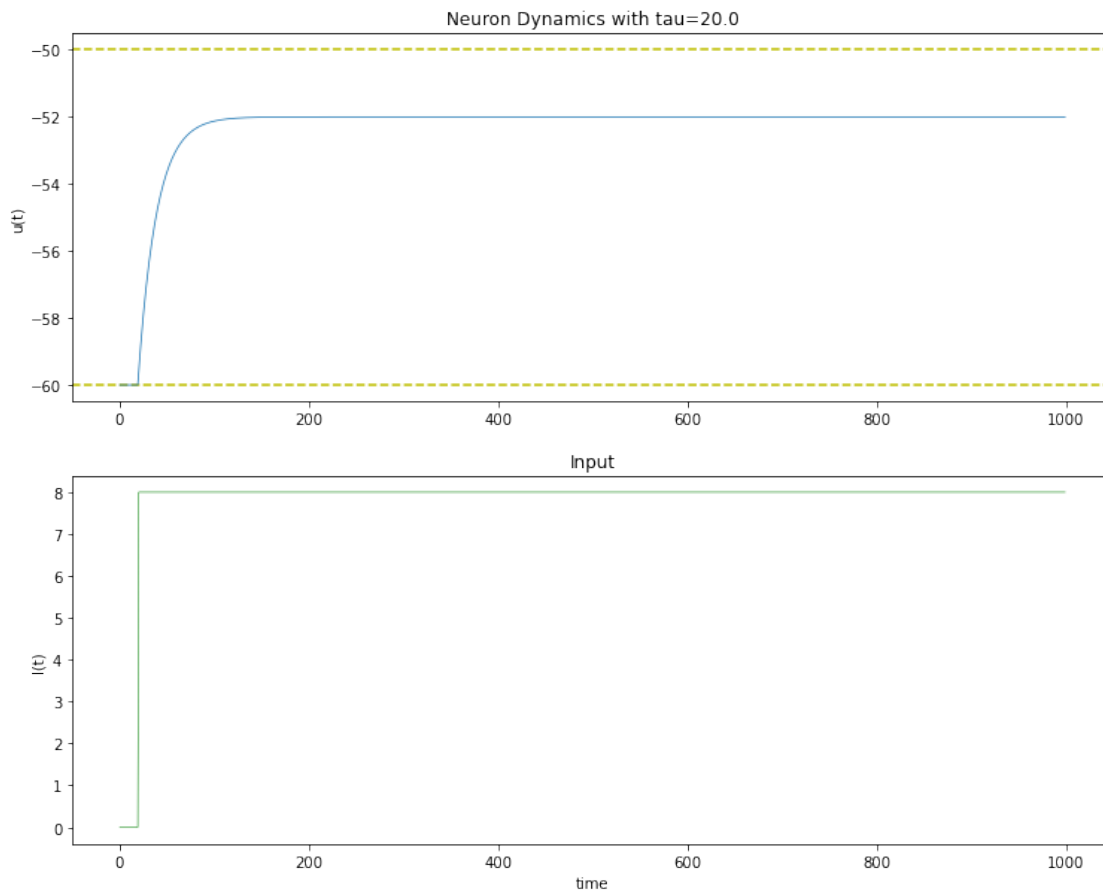
Currents: [8. 12. 16. 20. 24. 28. 32.]

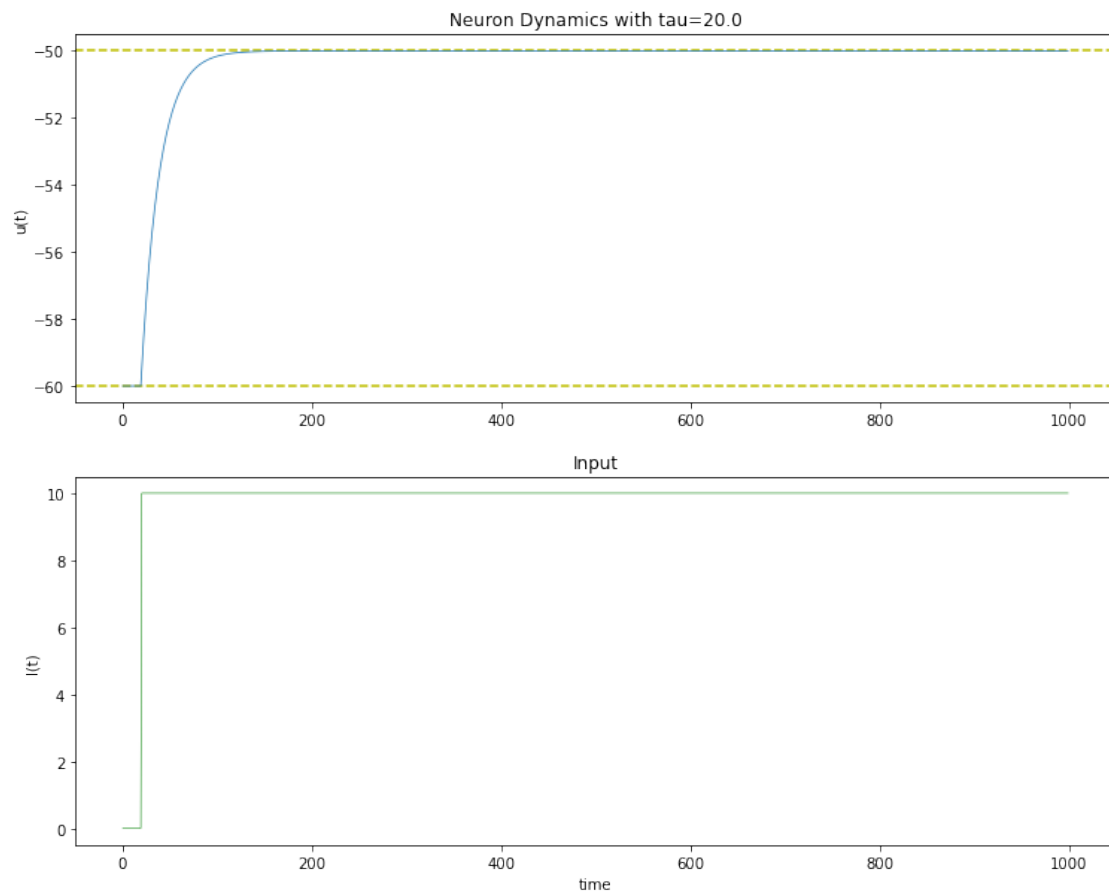
frequencies: [0 29 50 71 91 110 124]

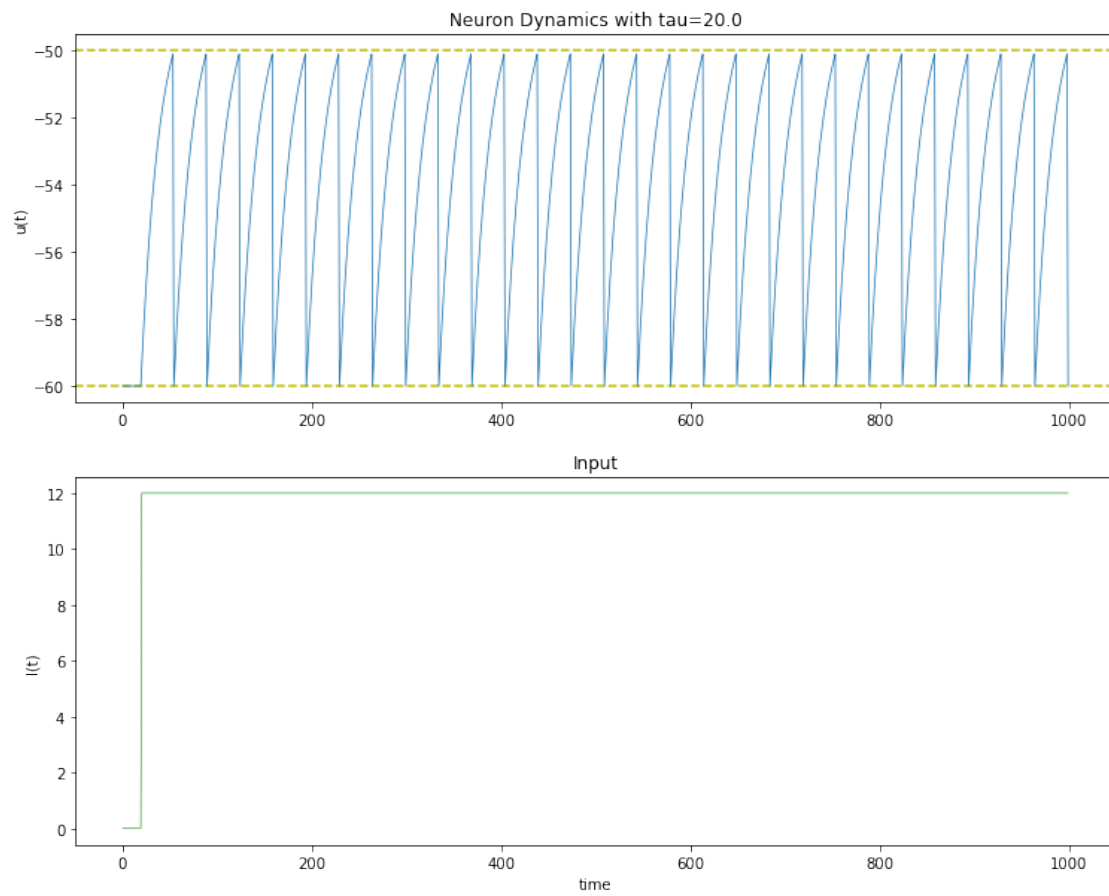
```

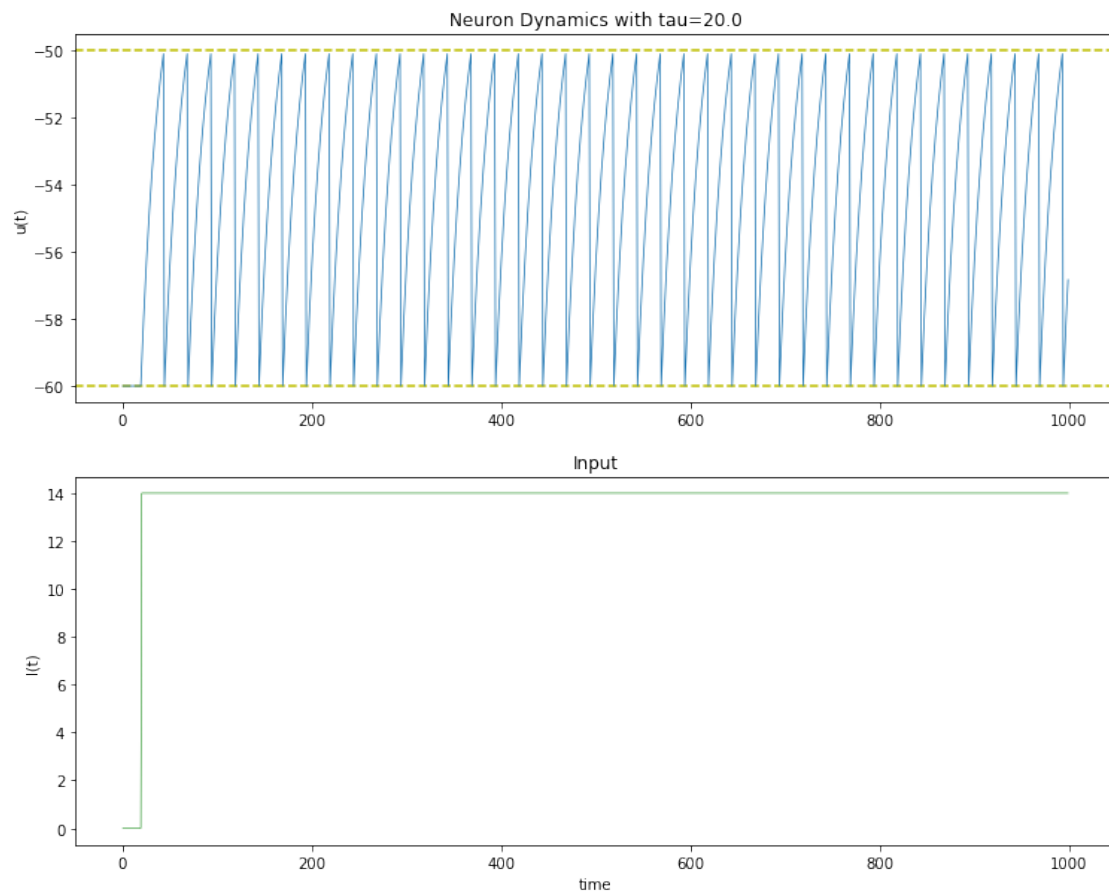
[11]: %matplotlib inline
if save_monitor_states:
    for i in range(4):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)

```

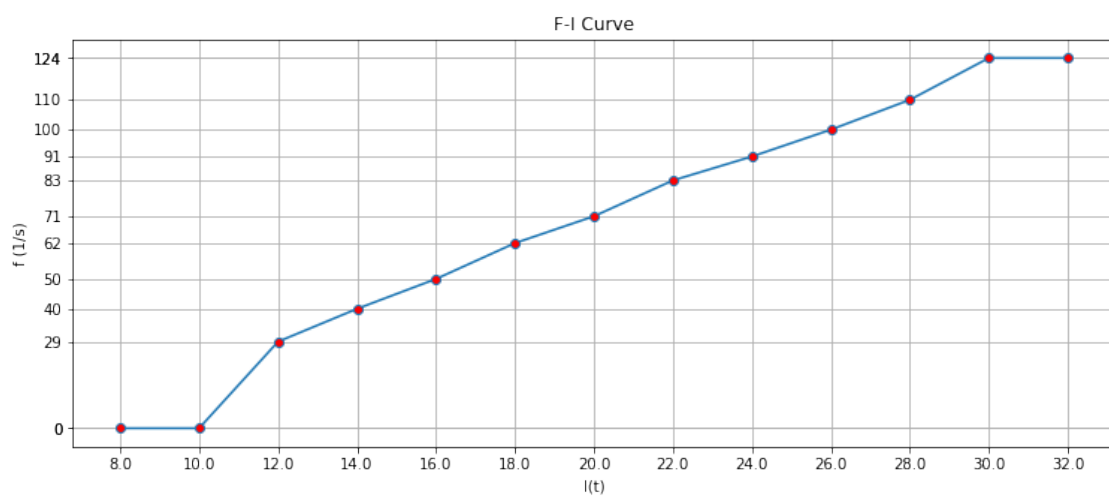








```
[12]: %matplotlib inline
fi_curve(currents, frequencies)
```



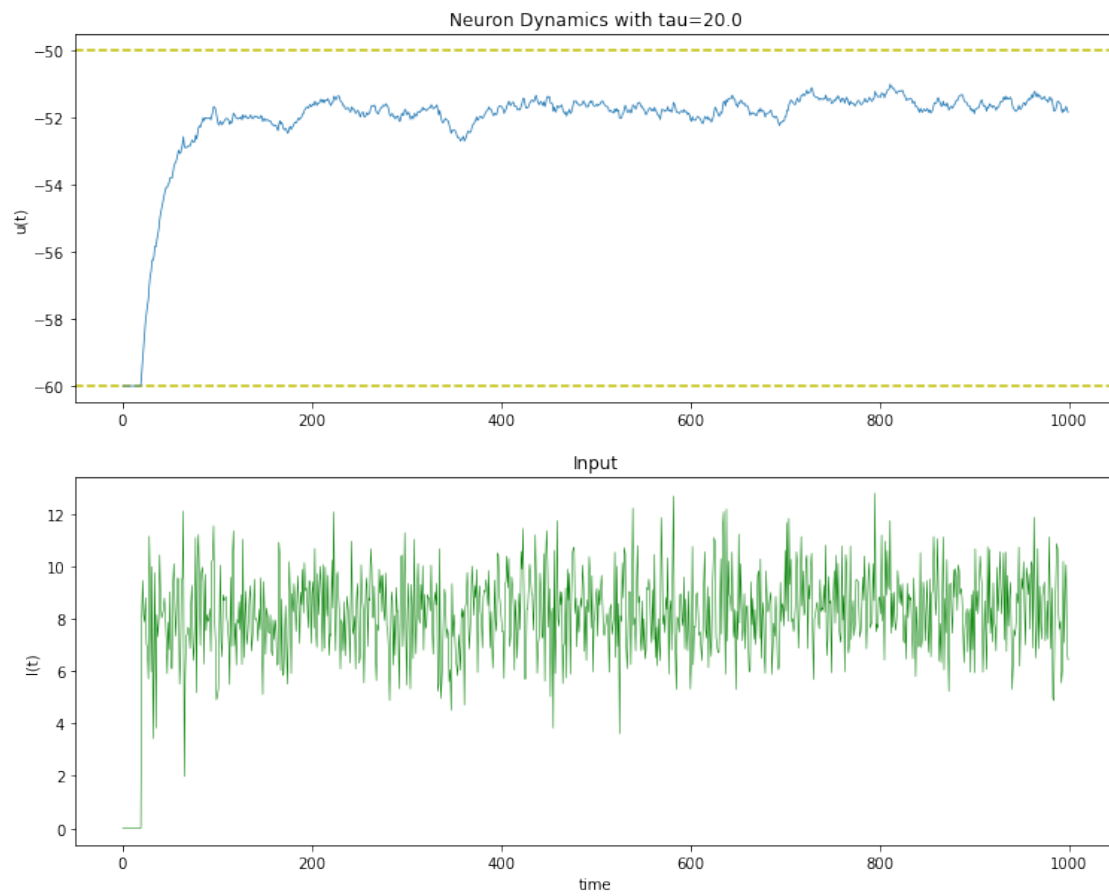
From the above simulations, we conclude that only the difference of u_r and *threshold* is important. If we double their difference (in our examples we doubled the difference from 5 to 10), the input currents should also be doubled to achieve the same result. Therefore, this relationship is linear.

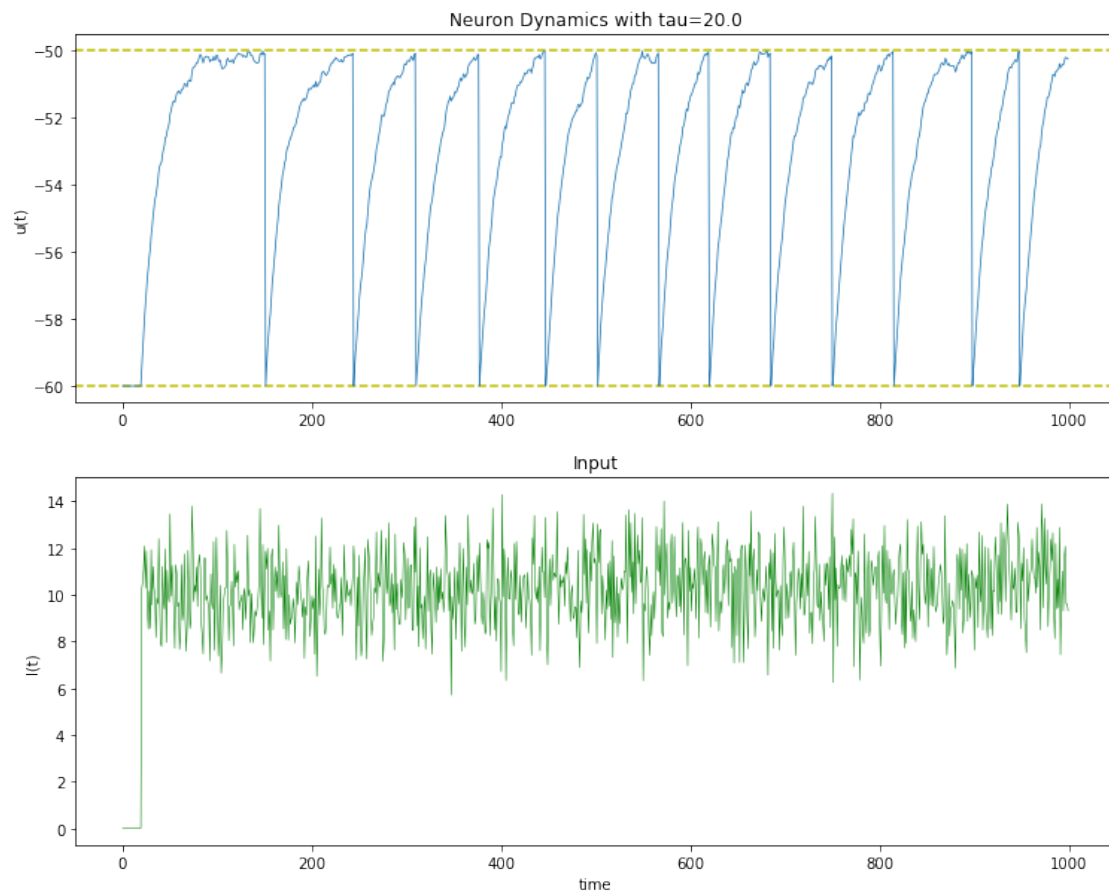
1.2.2 With noise: $\sigma_n = 1.5$

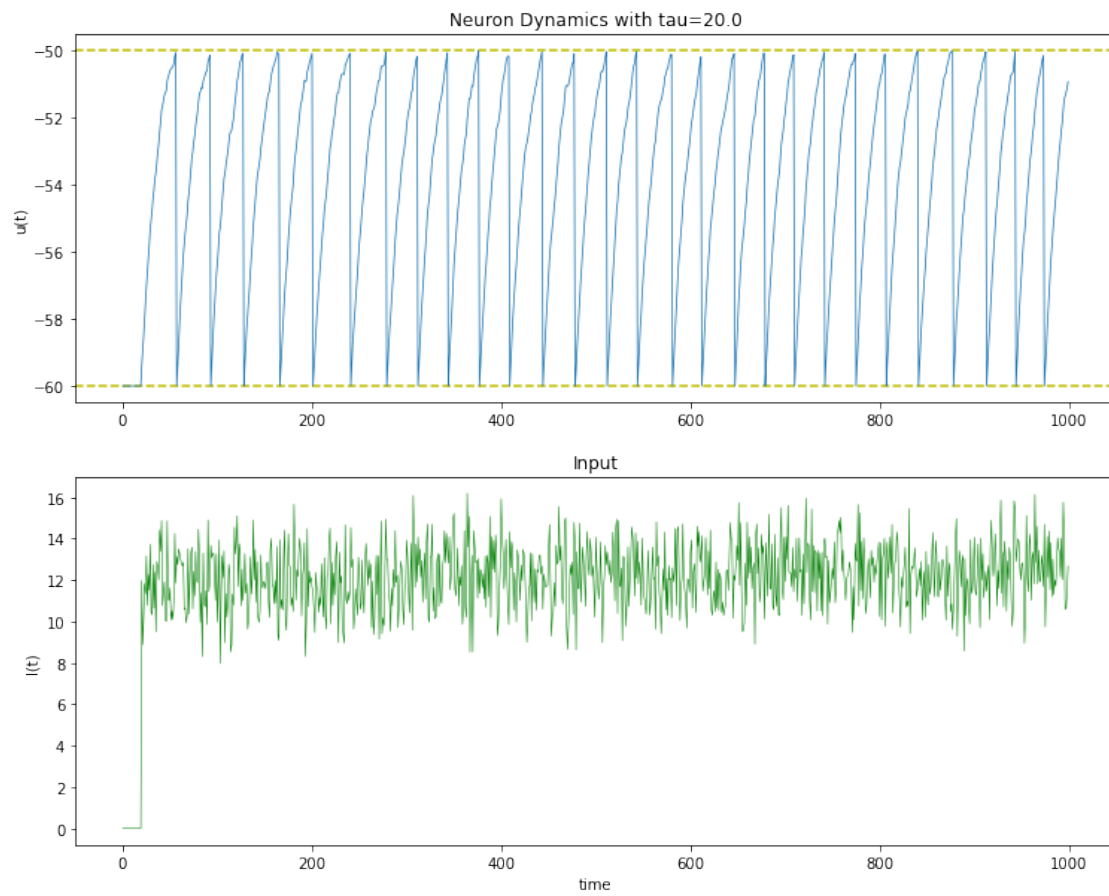
```
[13]: step_currents = np.linspace(8, 16, 5, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_2,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=1.5
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])
```

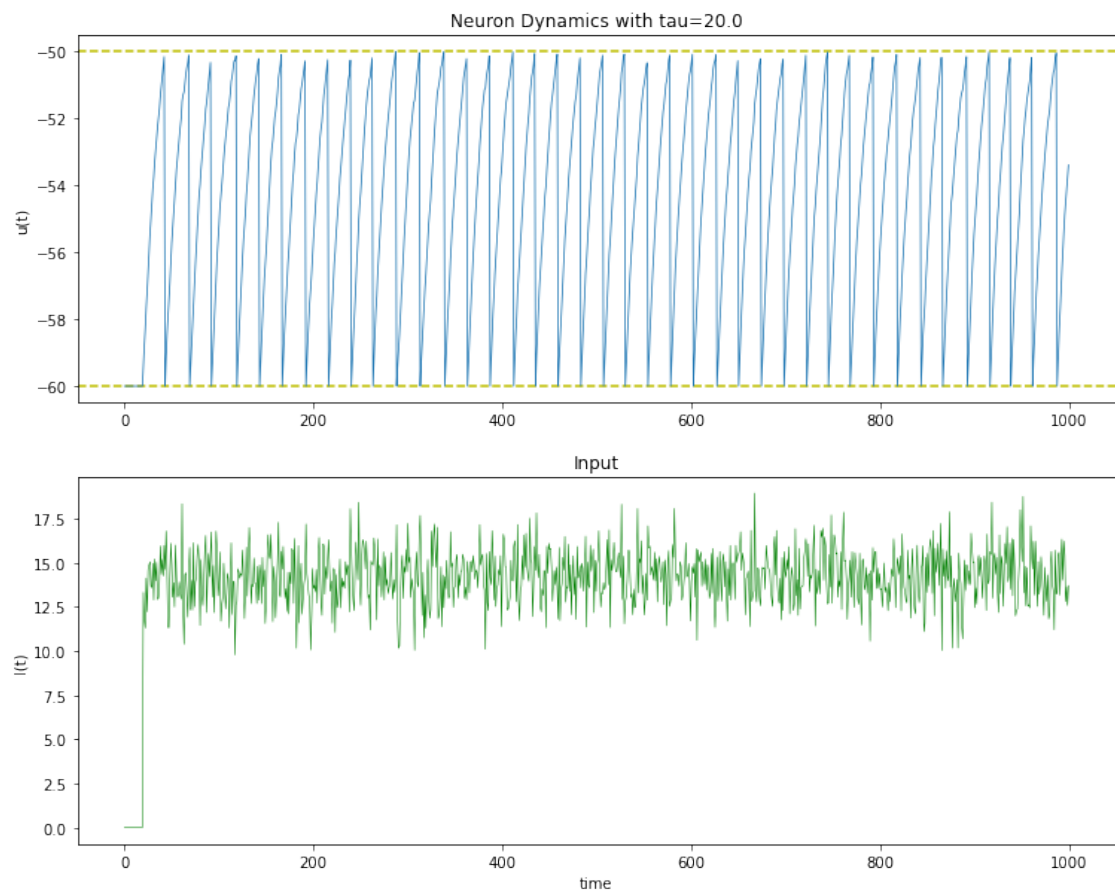
```
Simulation time: 1.620s
Currents: [ 8. 12. 16.]
frequencies: [ 0 29 52]
```

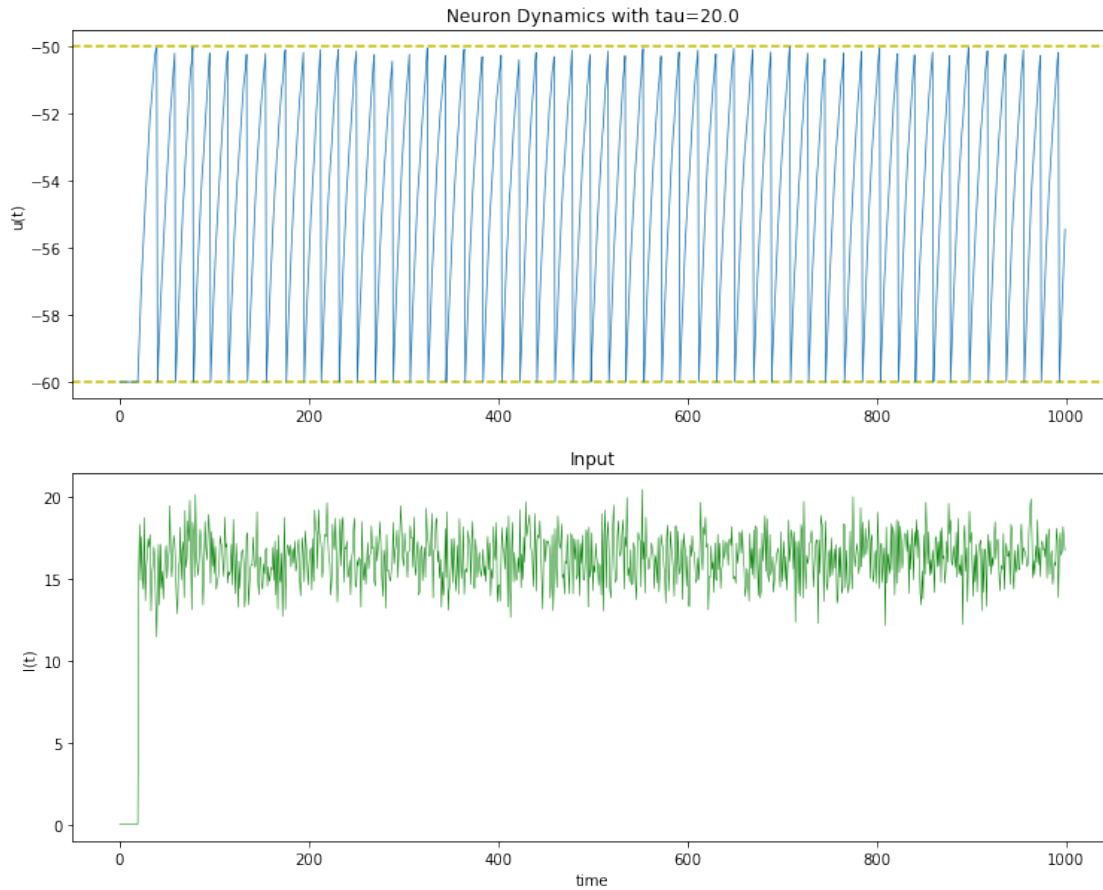
```
[14]: %matplotlib inline
if save_monitor_states:
    for i in range(0, len(monitor_states)):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```











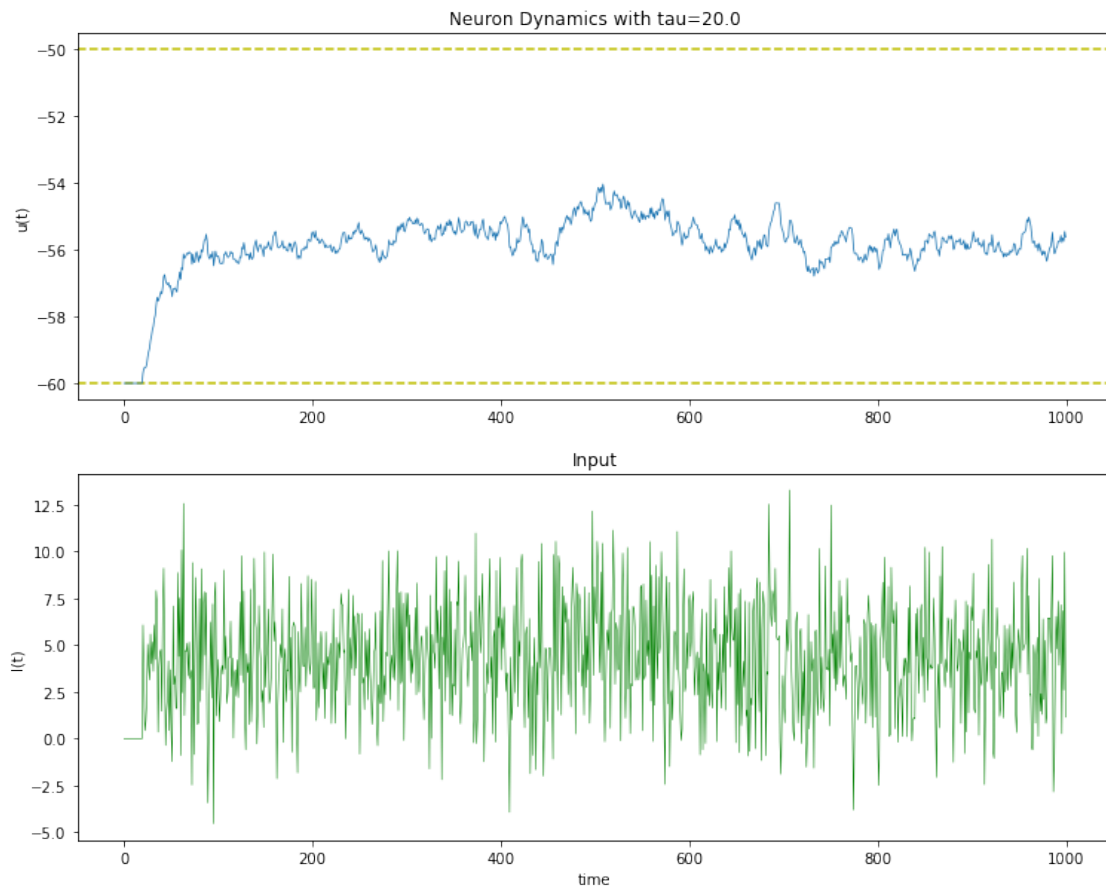
1.2.3 With noise: $\sigma_n = 3$

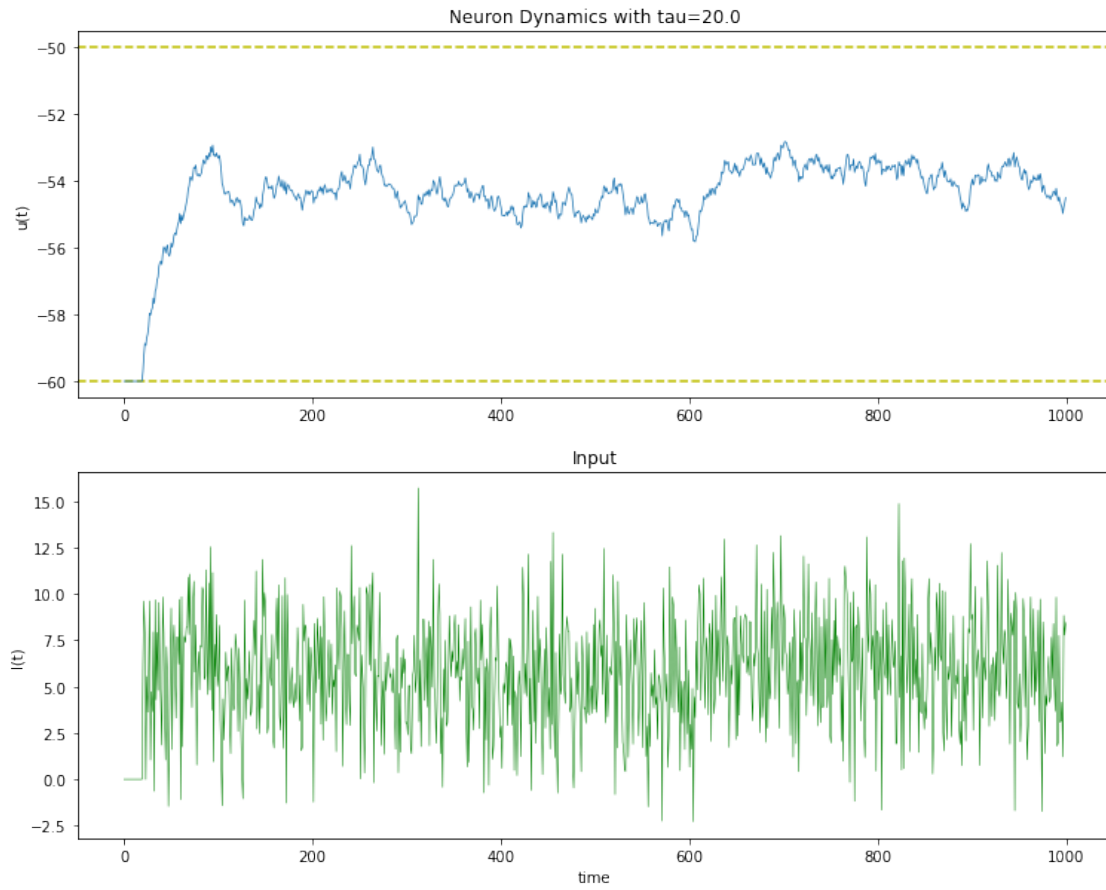
```
[15]: step_currents = np.linspace(4, 10, 5, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_2,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=3
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:2])
print('frequencies:', frequencies[:2])
```

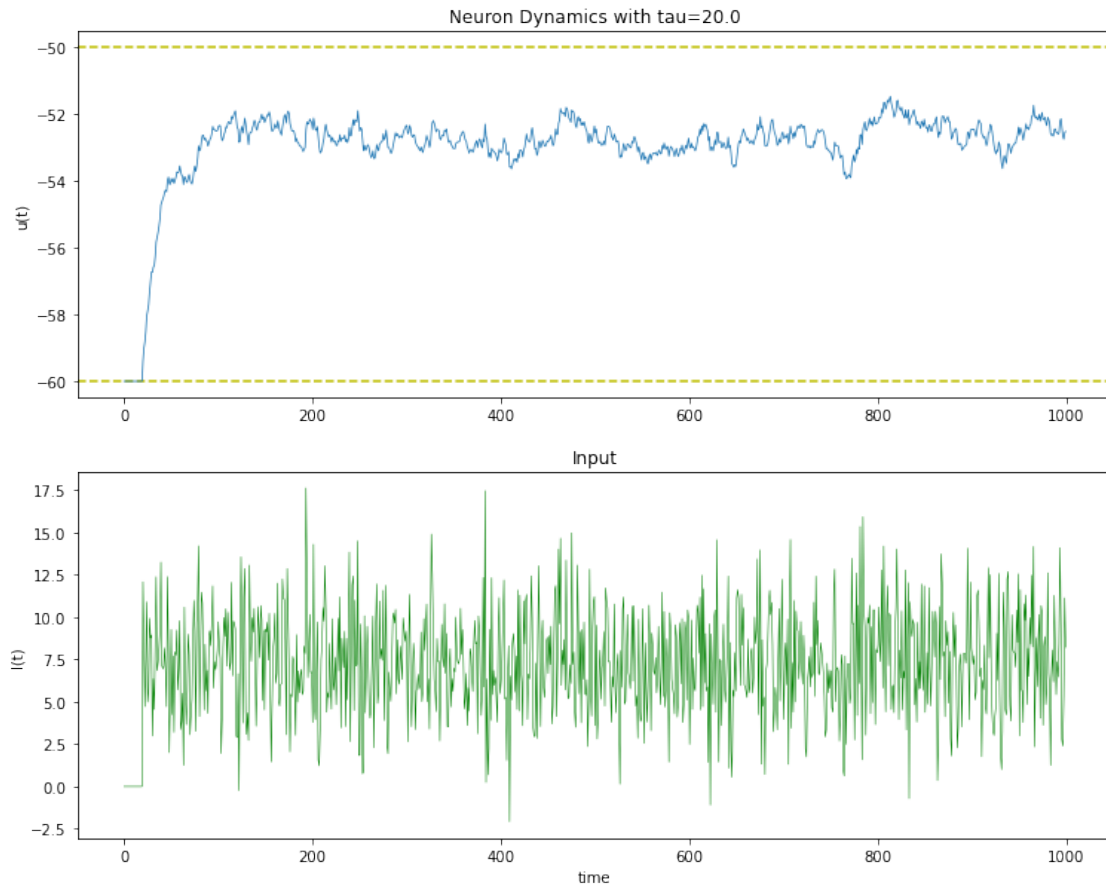
Simulation time: 1.572s

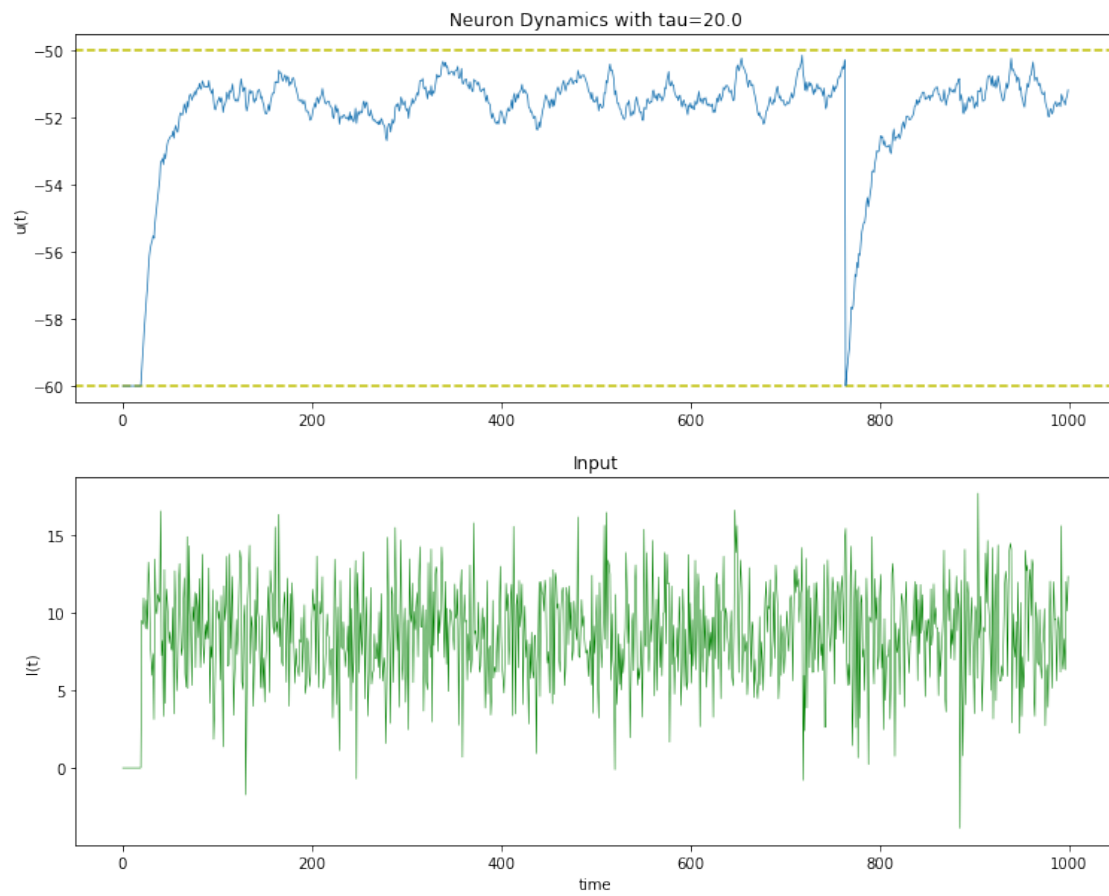
Currents: [4. 7. 10.]
frequencies: [0 0 15]

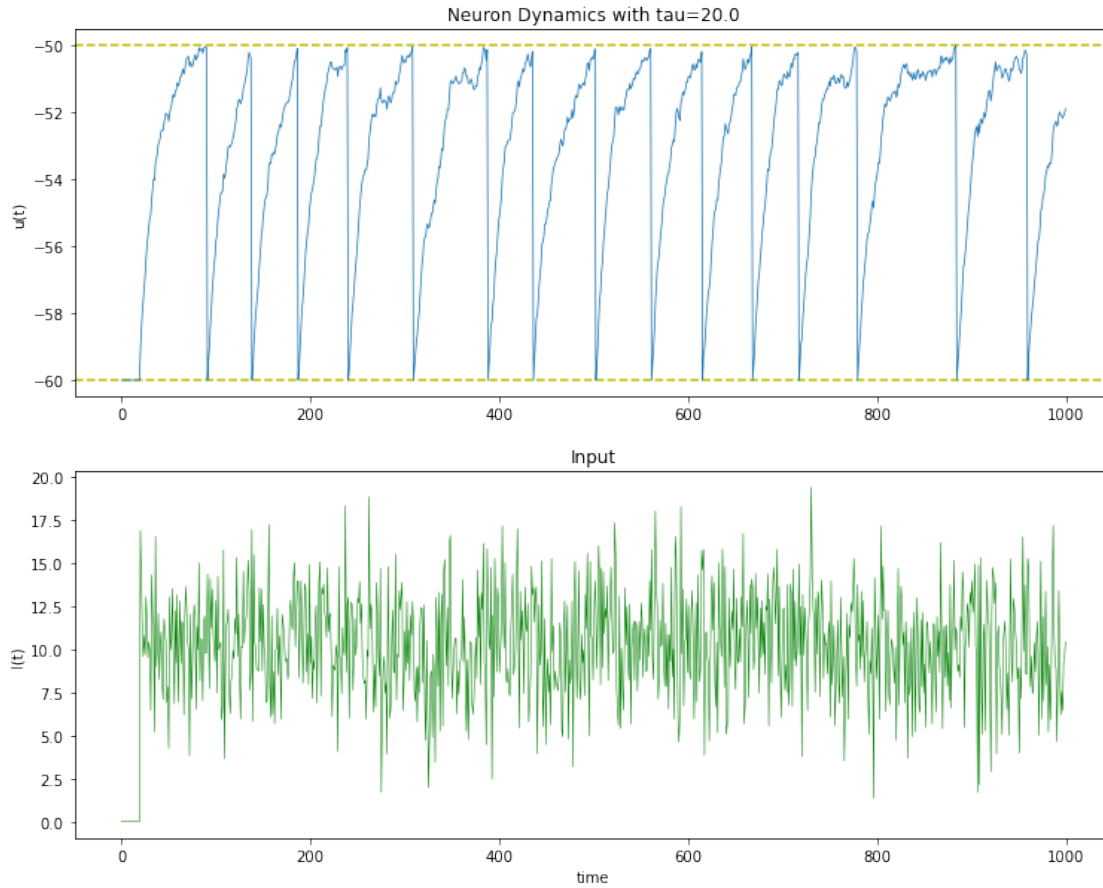
```
[16]: %matplotlib inline
if save_monitor_states:
    for i in range(0, len(monitor_states)):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```











By comparing the `params_1` and `params_2` output with noisy inputs, we can see that the second parameter set is more robust to noisy input.

Explanation: Since the noise has a normal distribution, its effect will be lowered if it takes longer for the neuron to reach its threshold.

`neuron_params_3` and `neuron_params_4` are experimenting the effect of τ

1.3 Simulation with `neuron_params_3`

$$\tau = 40, |u_r - threshold| = 5mV$$

1.3.1 without noise

```
[17]: step_currents = np.linspace(4, 16, 13, dtype=np.float32)
neuron_params_3 = {'threshold': -55, 'u_rest': -60, 'tau': 40}
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_3,
        monitor_vars=monitor_vars,
        currents=step_currents,
```

```

        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])

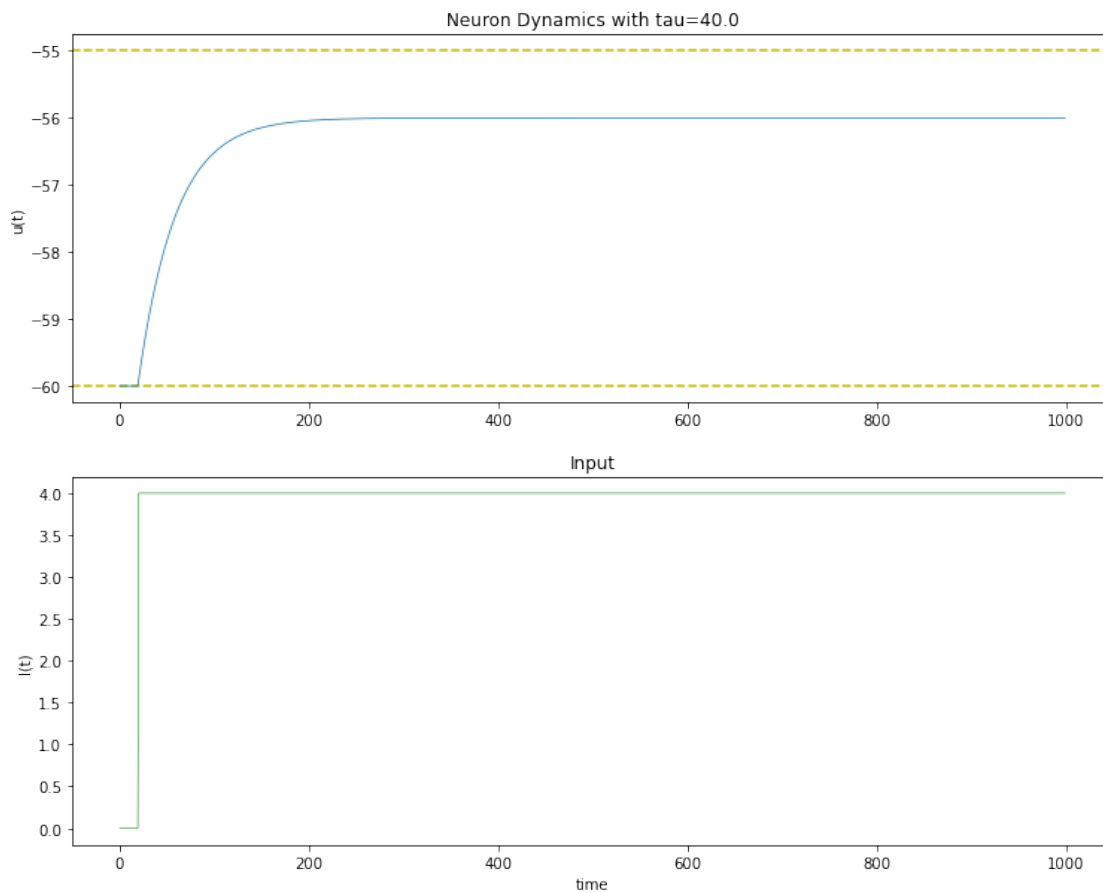
```

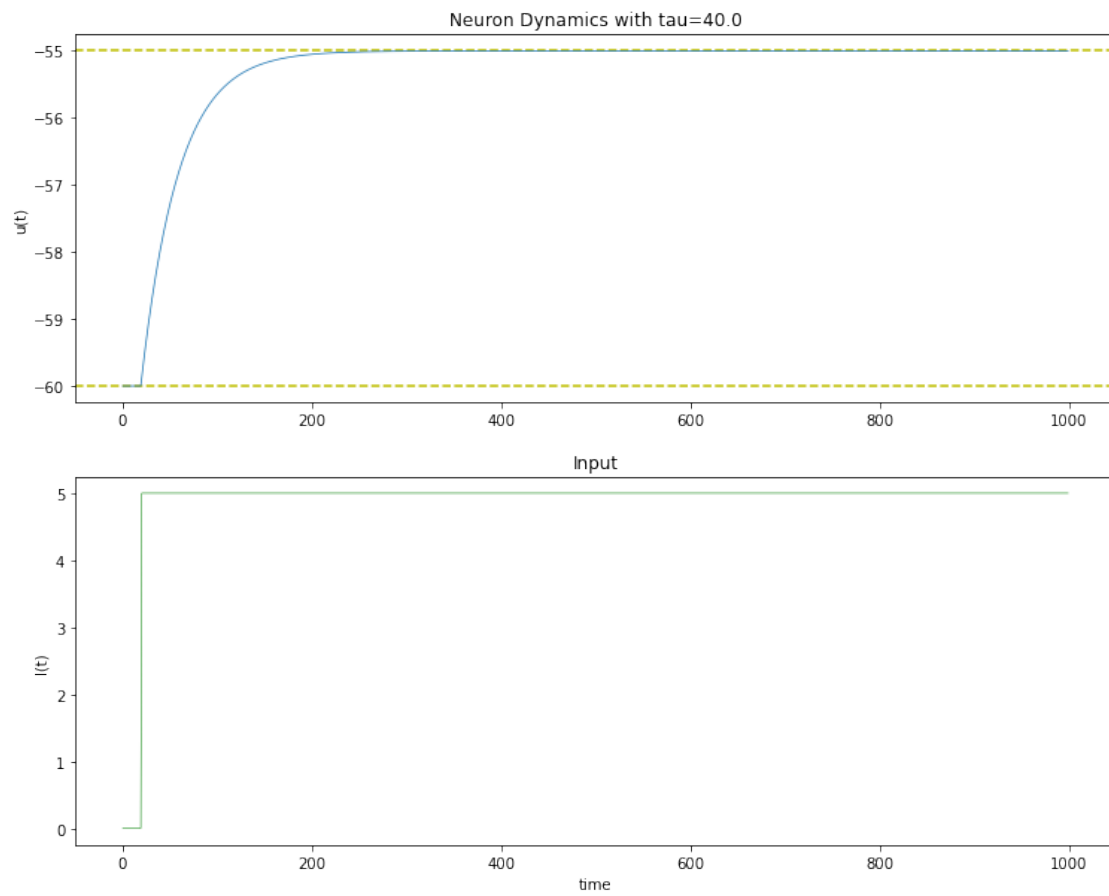
Simulation time: 4.065s
Currents: [4. 6. 8. 10. 12. 14. 16.]
frequencies: [0 13 26 36 45 55 66]

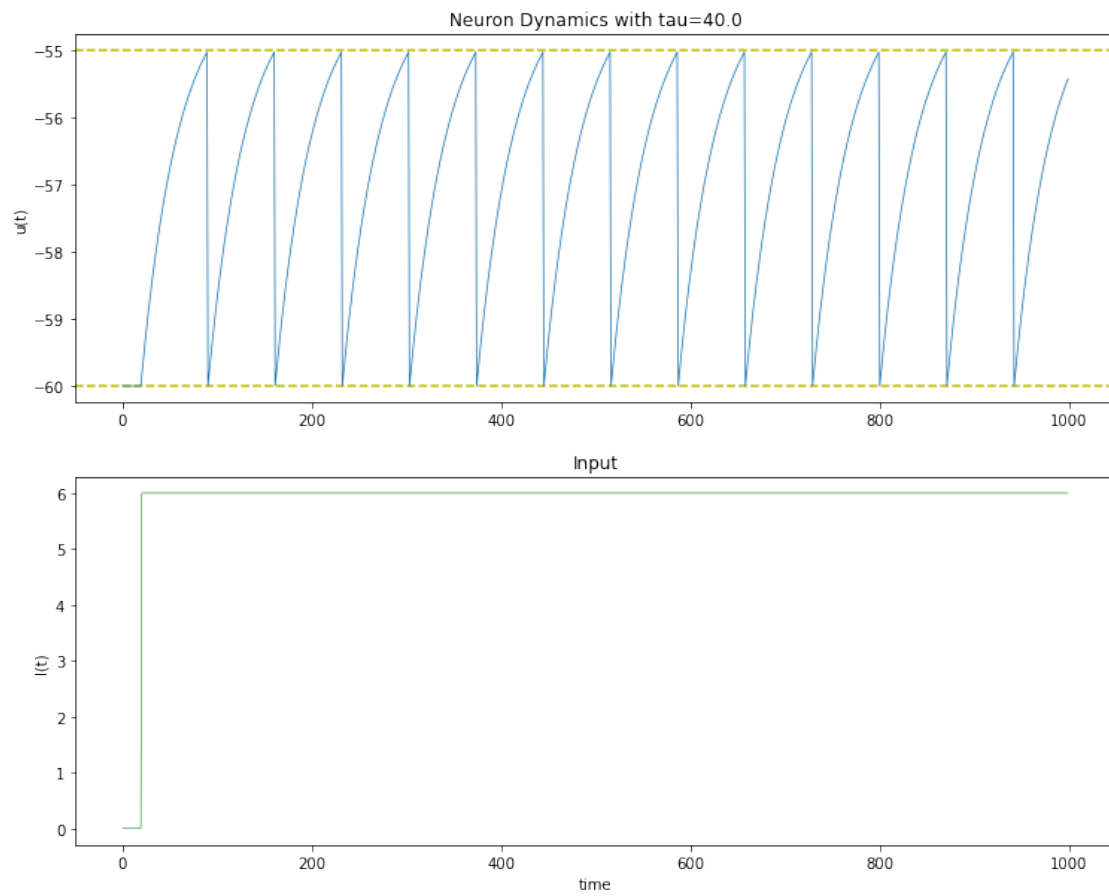
```

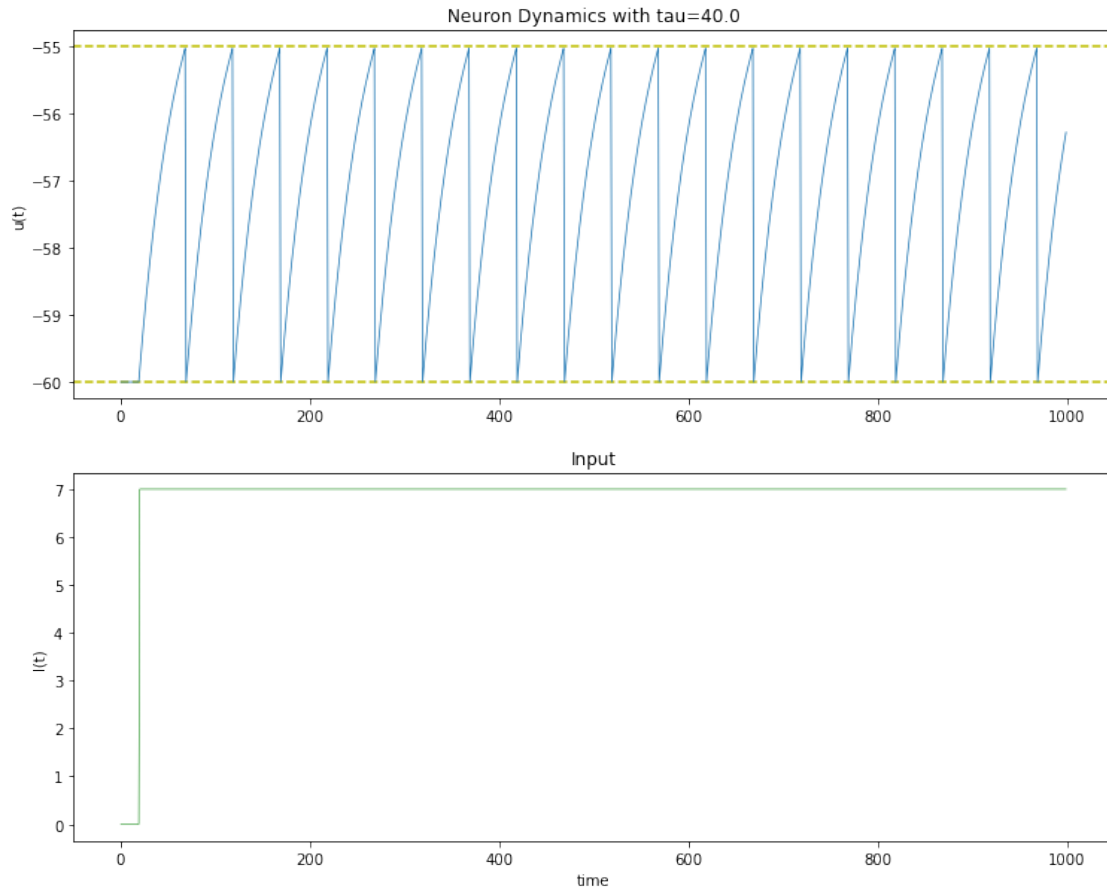
[18]: %matplotlib inline
if save_monitor_states:
    for i in range(4):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)

```

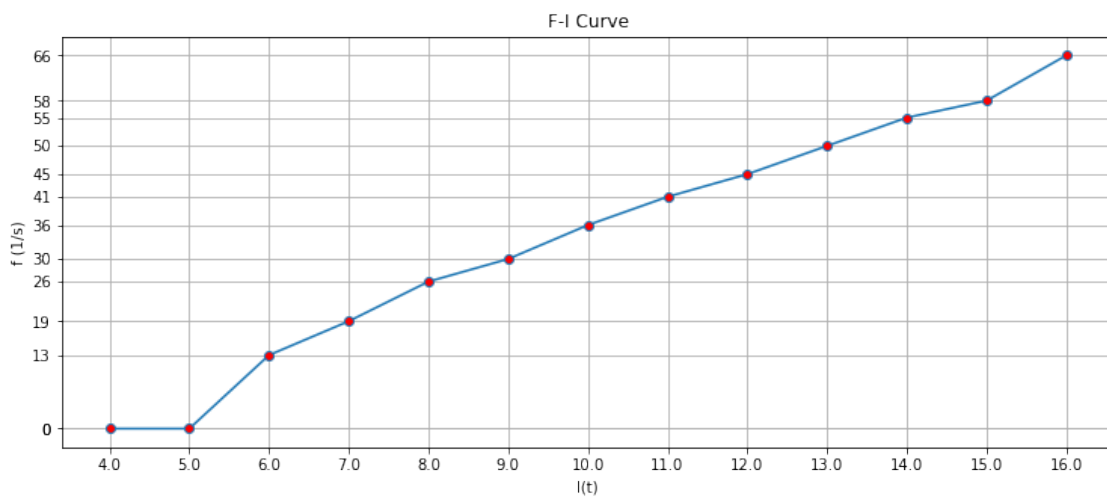








```
[19]: %matplotlib inline
fi_curve(currents, frequencies)
```



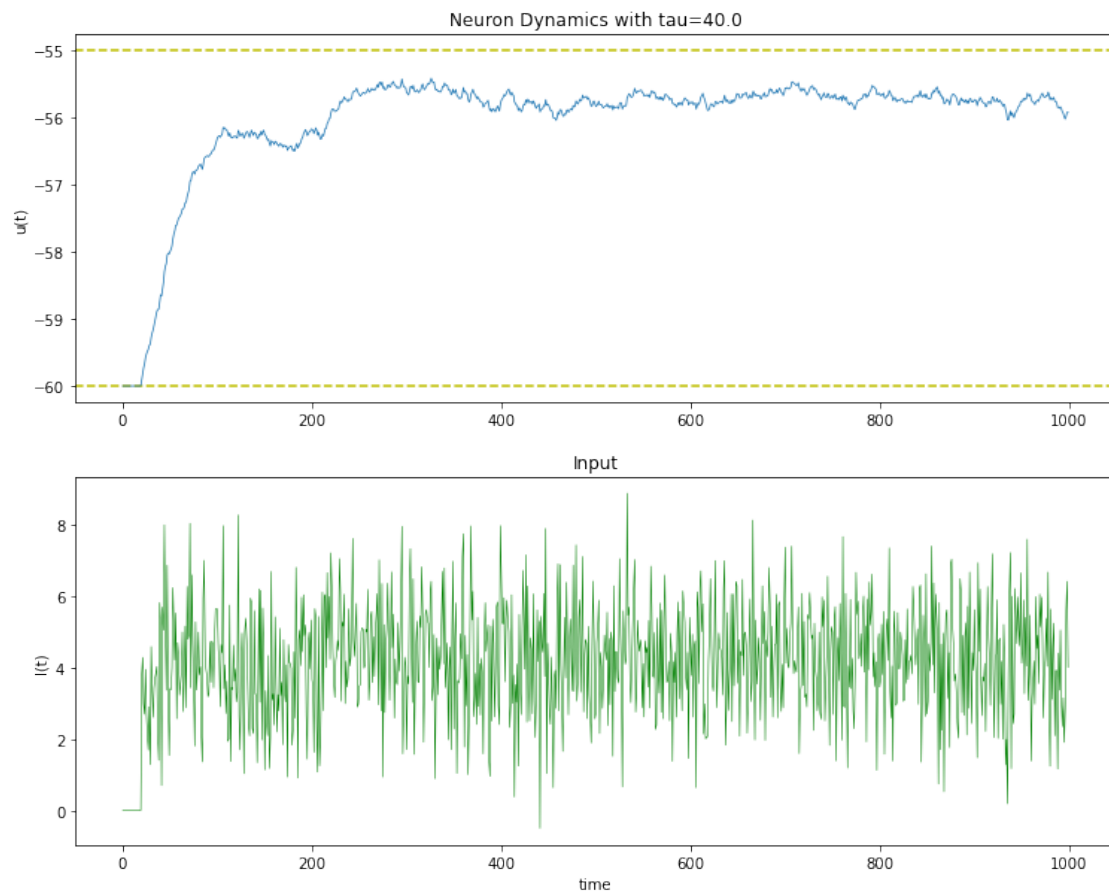
By comparing the **F_I curve** of this simulation and the first simulation, we observe that the frequency of the spikes almost (not exactly in some cases) will be halved if we double the tau. So, spike-frequency and the tau are inversely proportional. Considering the update rule $u(t + \Delta) = u(t) - \Delta/\tau[(u(t) - u_r) - R.I(t)]$ it is obvious why this is the case. If we increase τ , the update term will be decreased by a factor of $1/\tau$. In other words, if we double τ , the time we update the $u(t)$ should almost (because $u(t)$ itself appears in the update term) be doubled if we want to achieve the same potential.

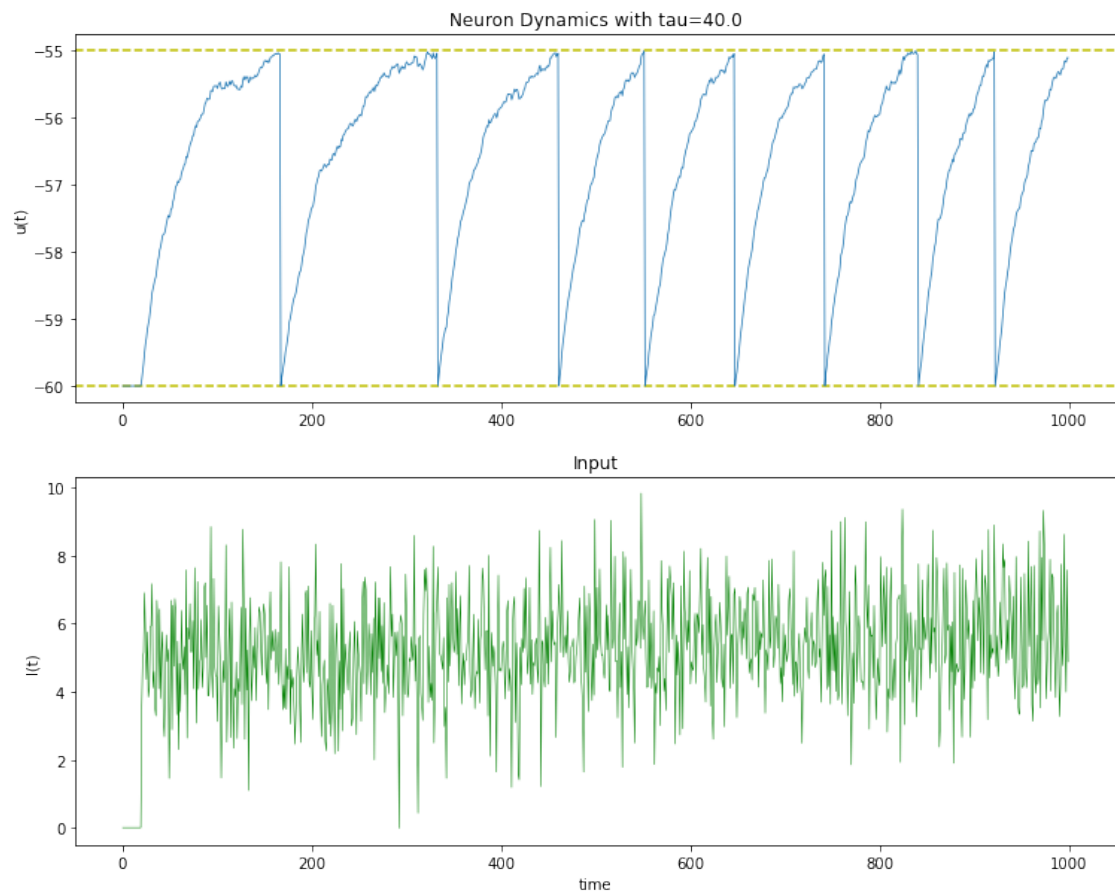
1.3.2 With noise: $\sigma_n = 1.5$

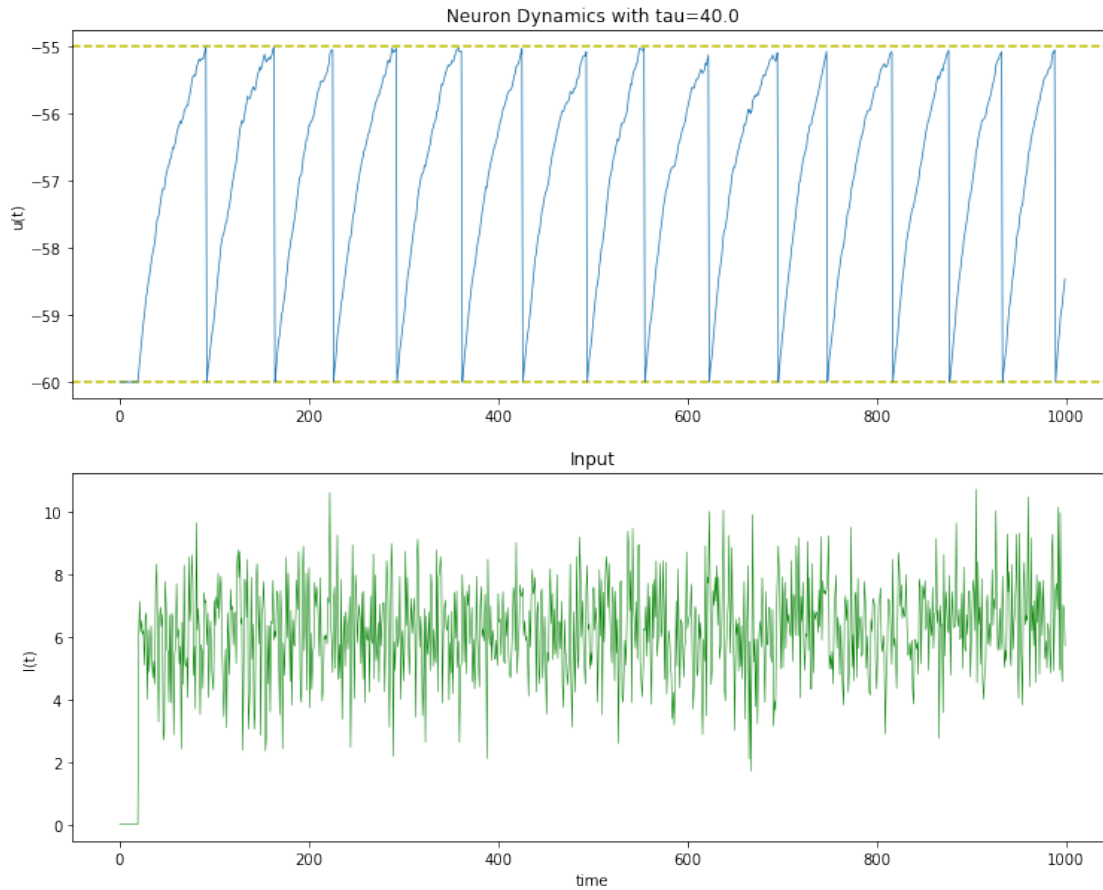
```
[20]: step_currents = np.linspace(4, 8, 5, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_3,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=1.5
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])
```

```
Simulation time: 1.581s
Currents: [4. 6. 8.]
frequencies: [ 0 15 27]
```

```
[21]: %matplotlib inline
if save_monitor_states:
    for i in range(len(monitor_states) - 2):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```





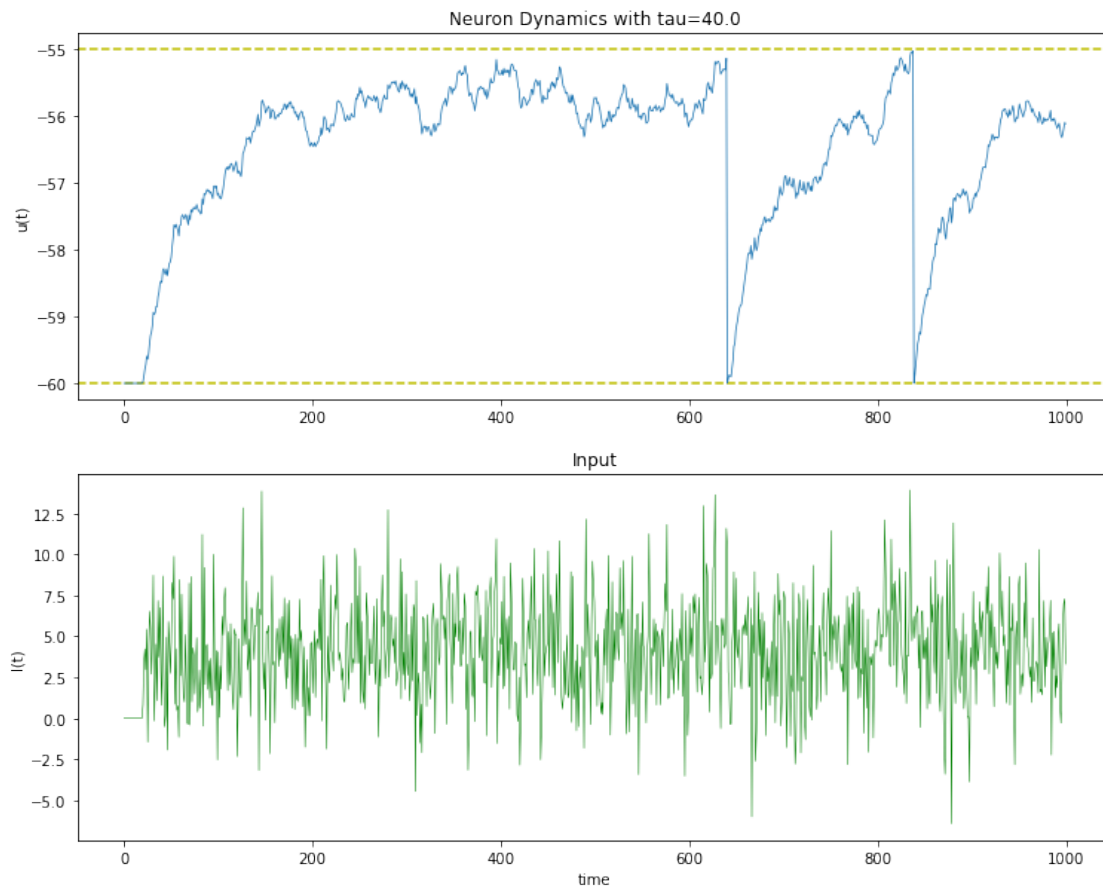


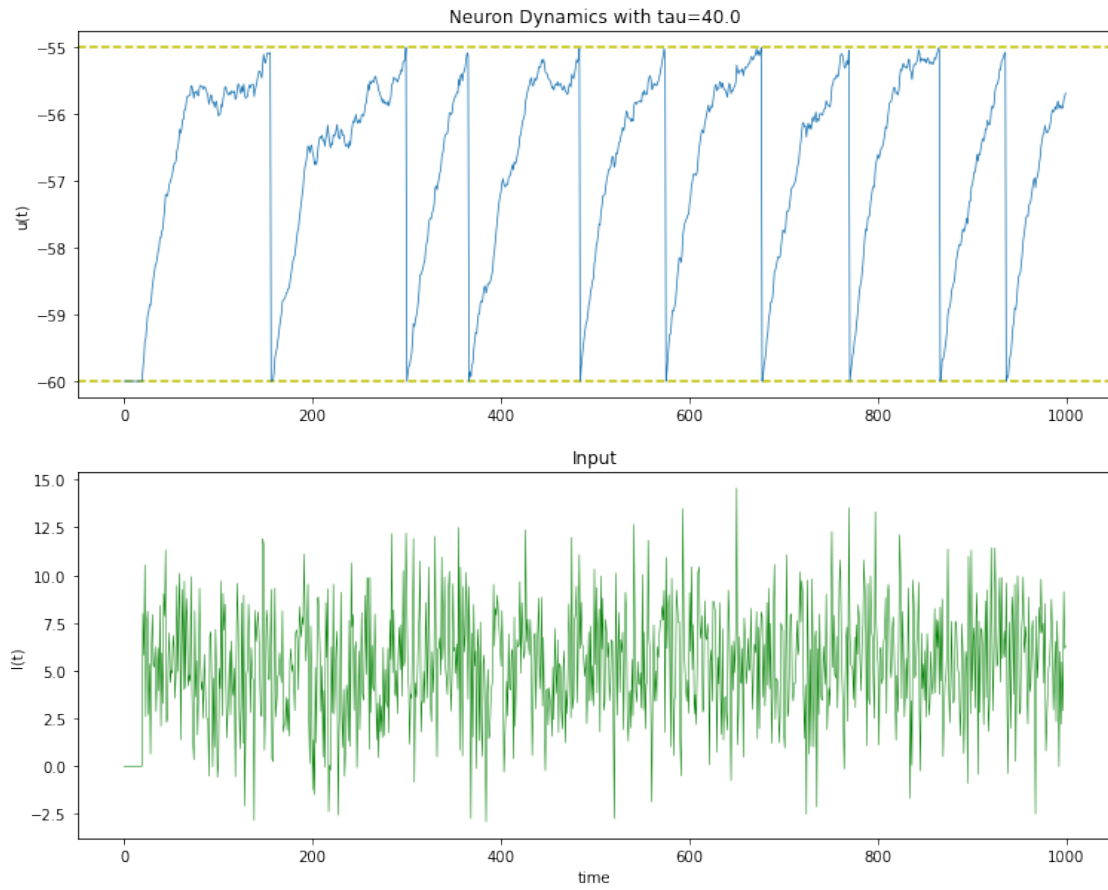
1.3.3 With noise: $\sigma_n = 3$

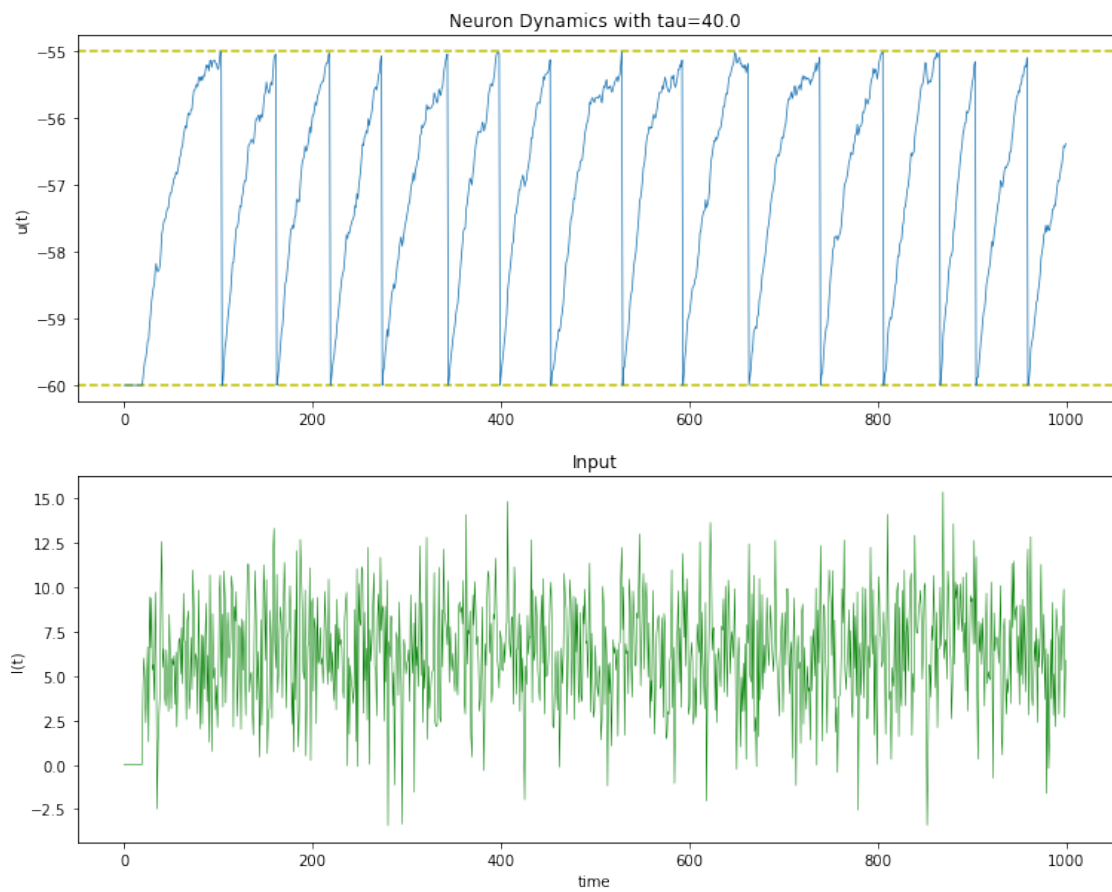
```
[22]: step_currents = np.linspace(4, 8, 5, dtype=np.float32)
neuron_params_3 = {'threshold': -55, 'u_rest': -60, 'tau': 40}
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_3,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=3
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])
```

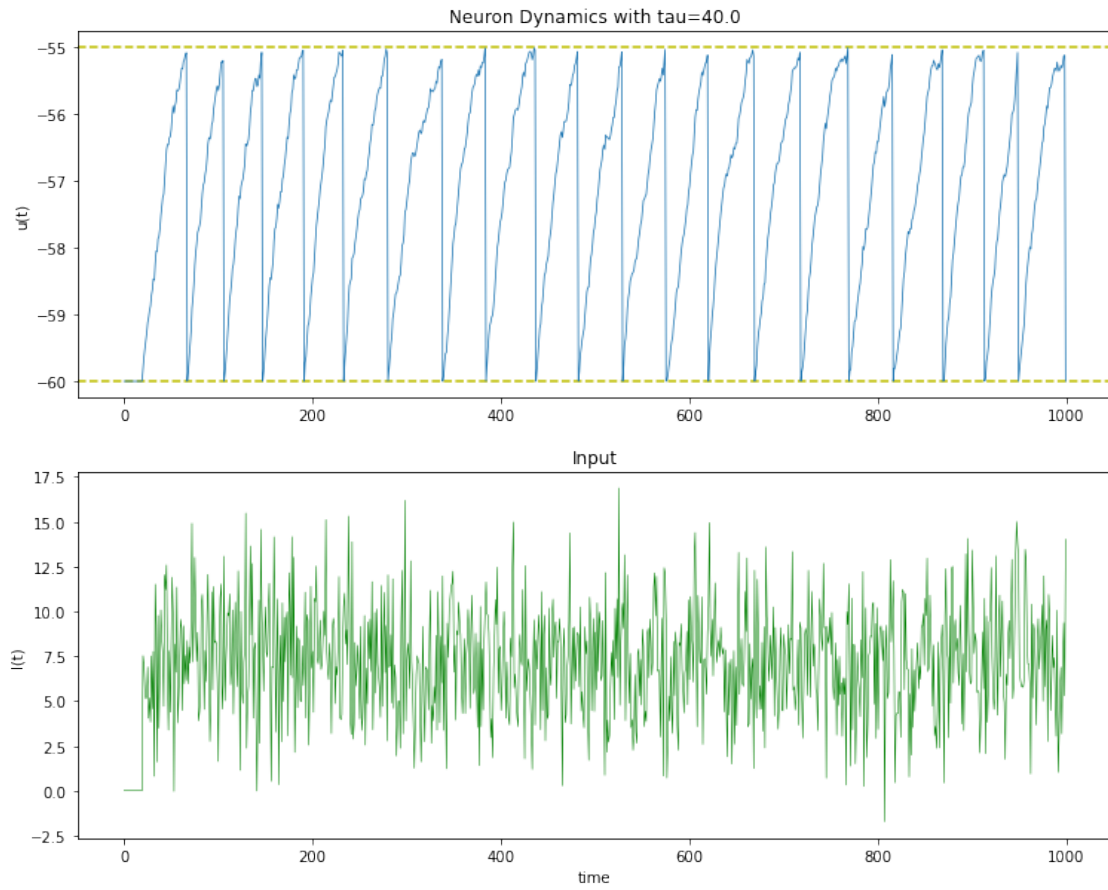
Simulation time: 1.605s
Currents: [4. 6. 8.]
frequencies: [2 15 26]

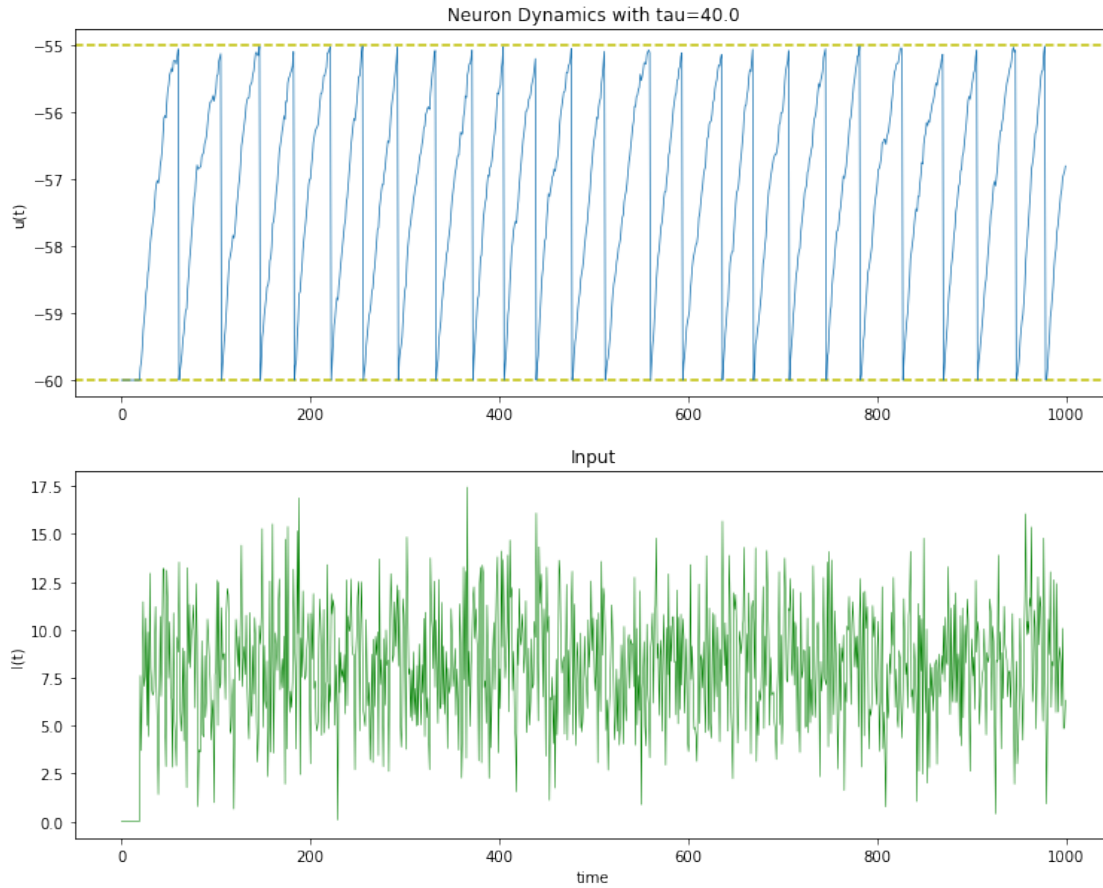
```
[23]: %matplotlib inline
if save_monitor_states:
    for i in range(len(monitor_states)):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```











1.4 Simulation with neuron_params_4

$\tau = 10$, $|u_r - threshold| = 5mV$

1.4.1 without noise

```
[24]: neuron_params_4 = {'threshold': -55, 'u_rest': -60, 'tau': 10}
step_currents = np.linspace(4, 16, 13, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_4,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[::2])
```

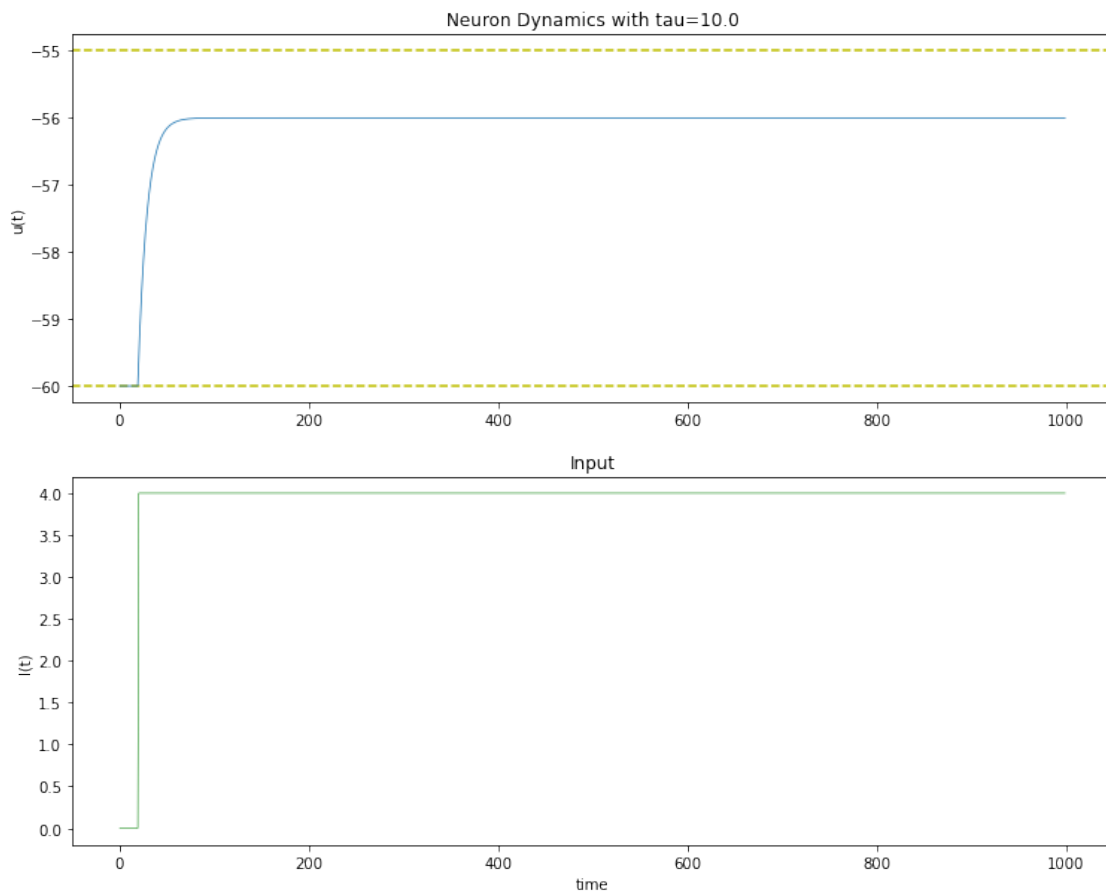
```
print('frequencies:', frequencies[:,2])
```

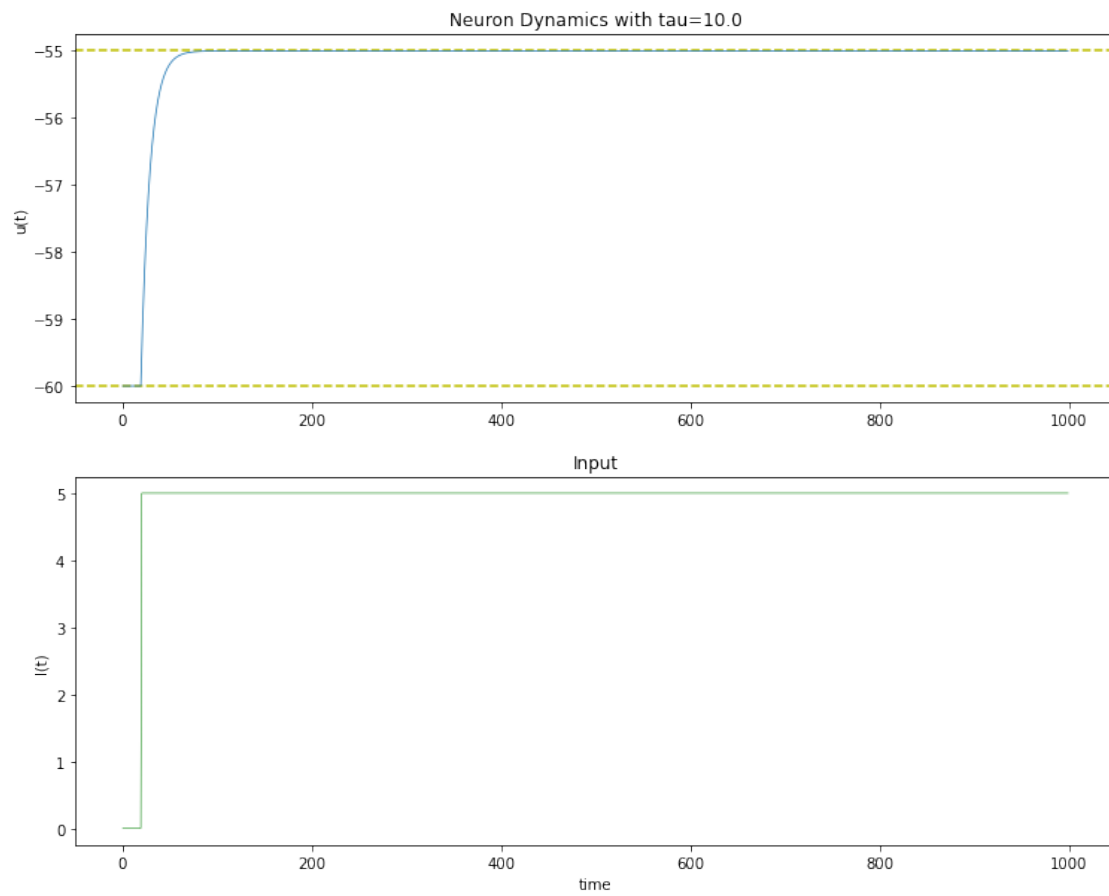
Simulation time: 4.039s

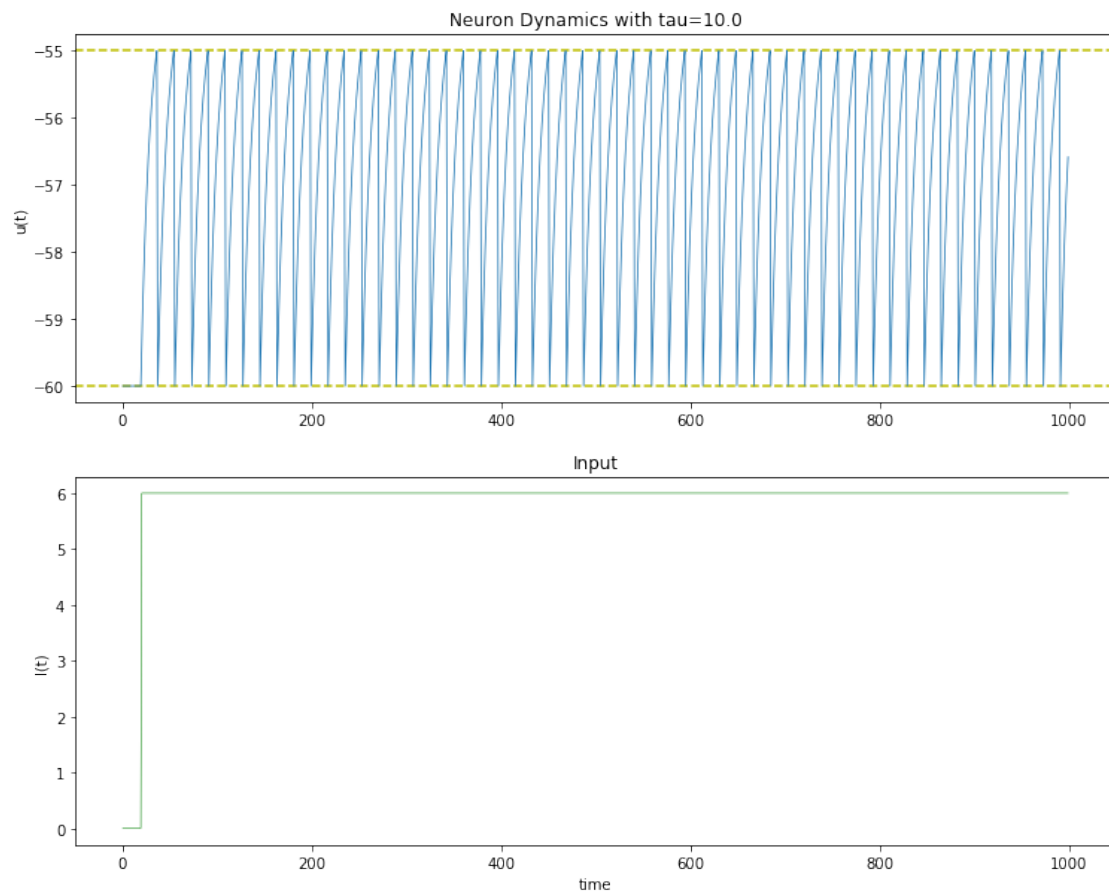
Currents: [4. 6. 8. 10. 12. 14. 16.]

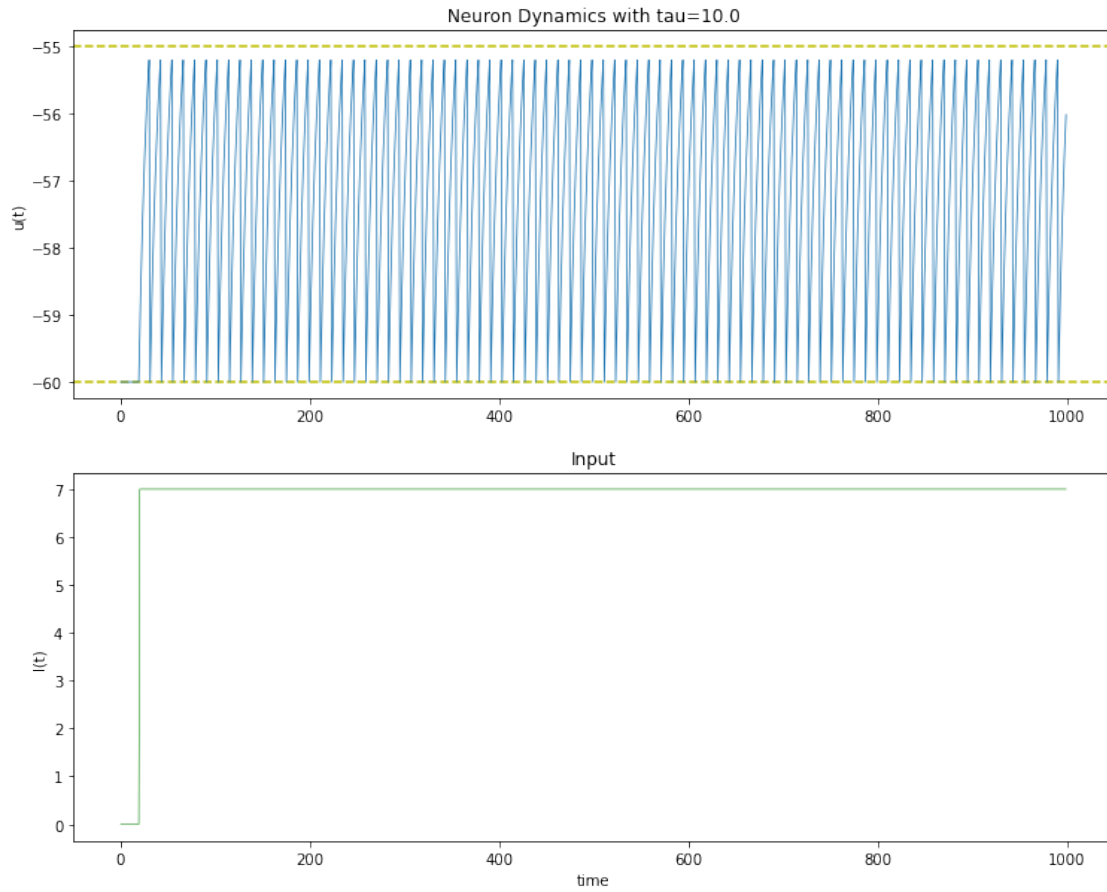
frequencies: [0 55 100 143 166 200 250]

```
[25]: %matplotlib inline
if save_monitor_states:
    for i in range(4):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```

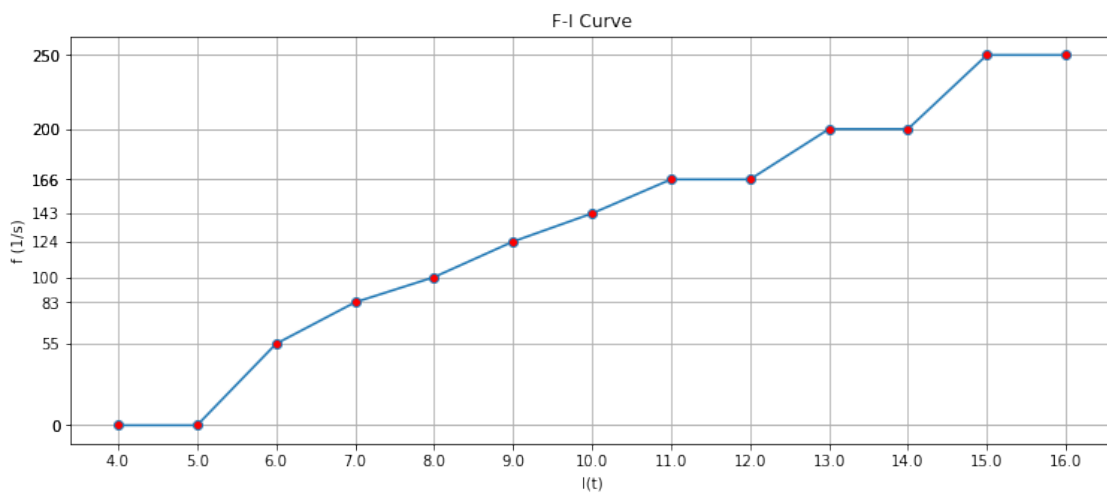








```
[26]: %matplotlib inline
fi_curve(currents, frequencies)
```



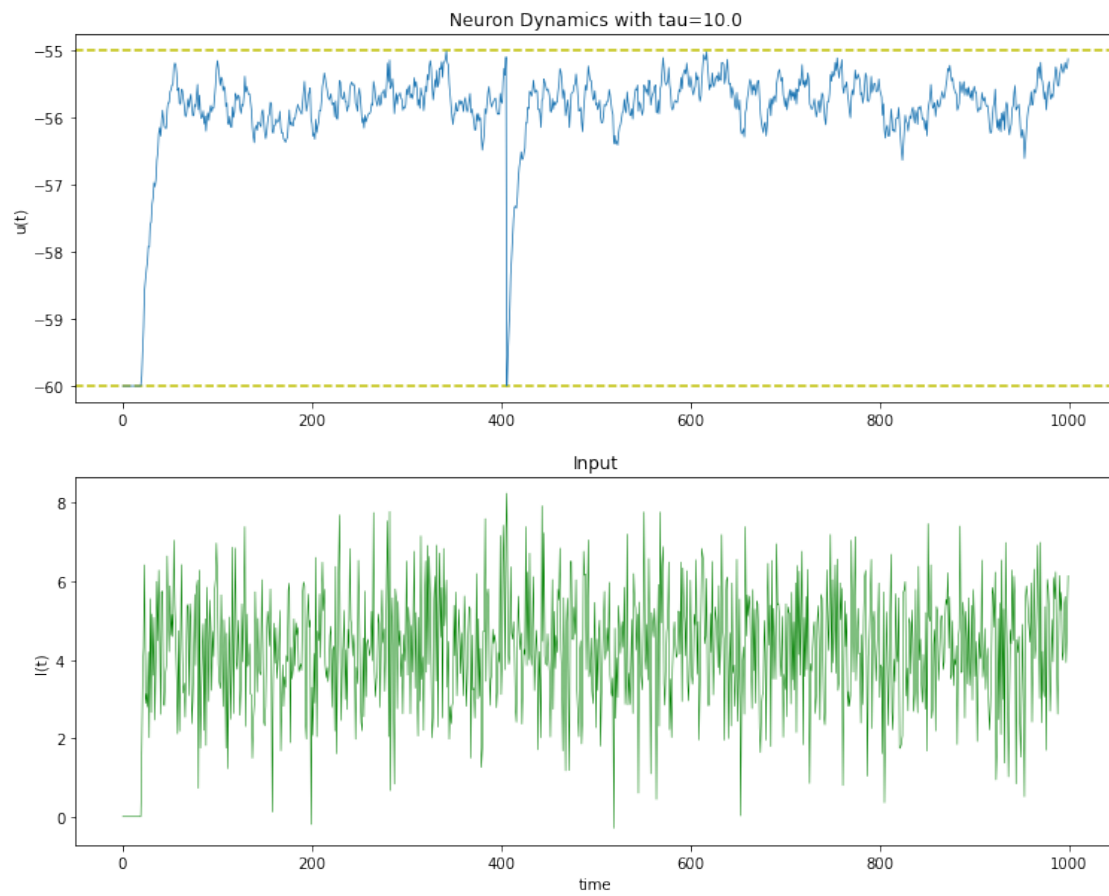
Similarly, If we halve the value of τ , the output frequency will be almost doubled. In other words, if we decrease the value of τ , the neuron reaches the threshold faster.

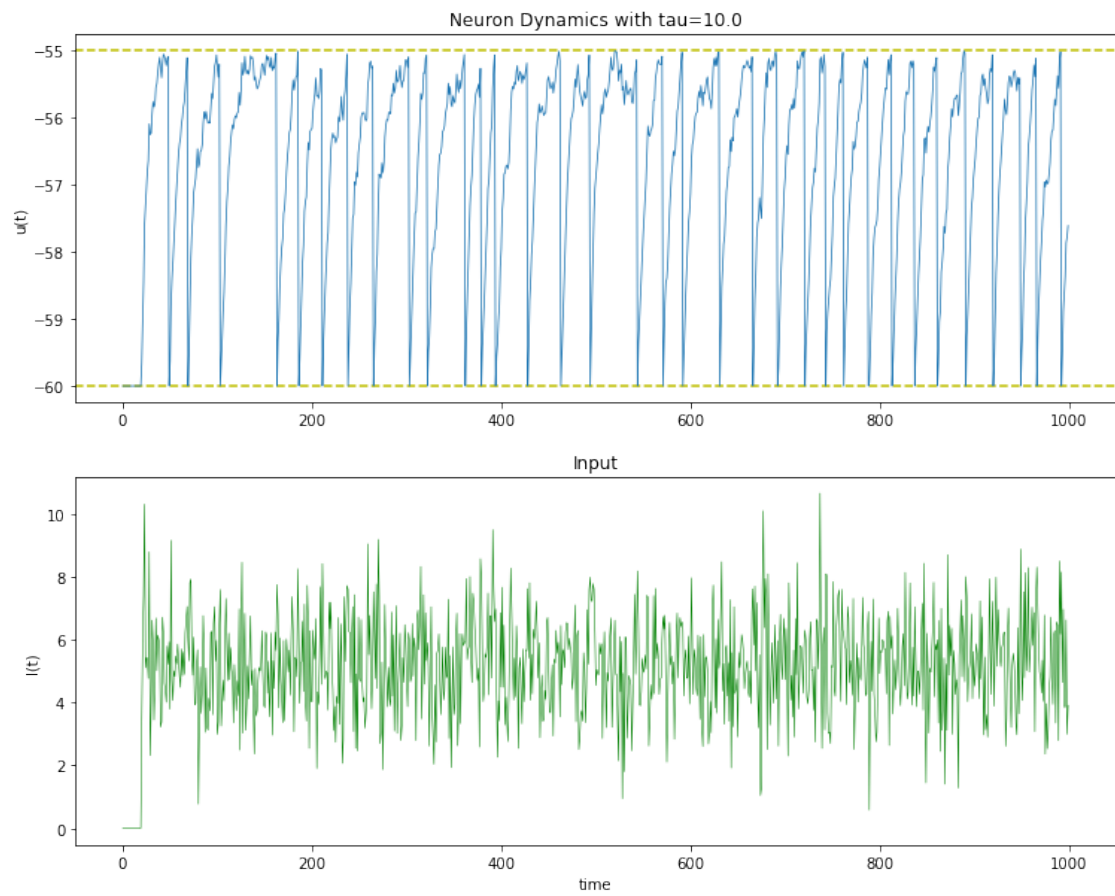
1.4.2 With noise: $\sigma_n = 1.5$

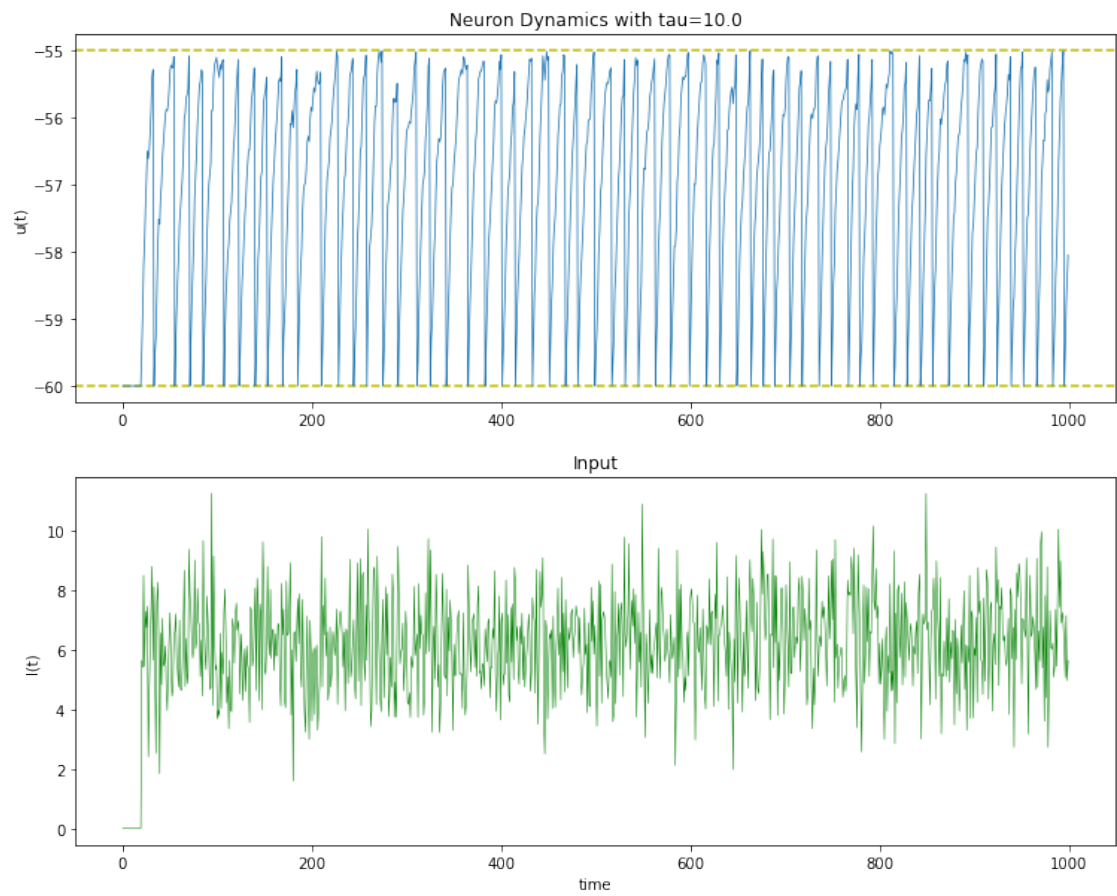
```
[27]: step_currents = np.linspace(4, 8, 5, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_4,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=1.5
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])
```

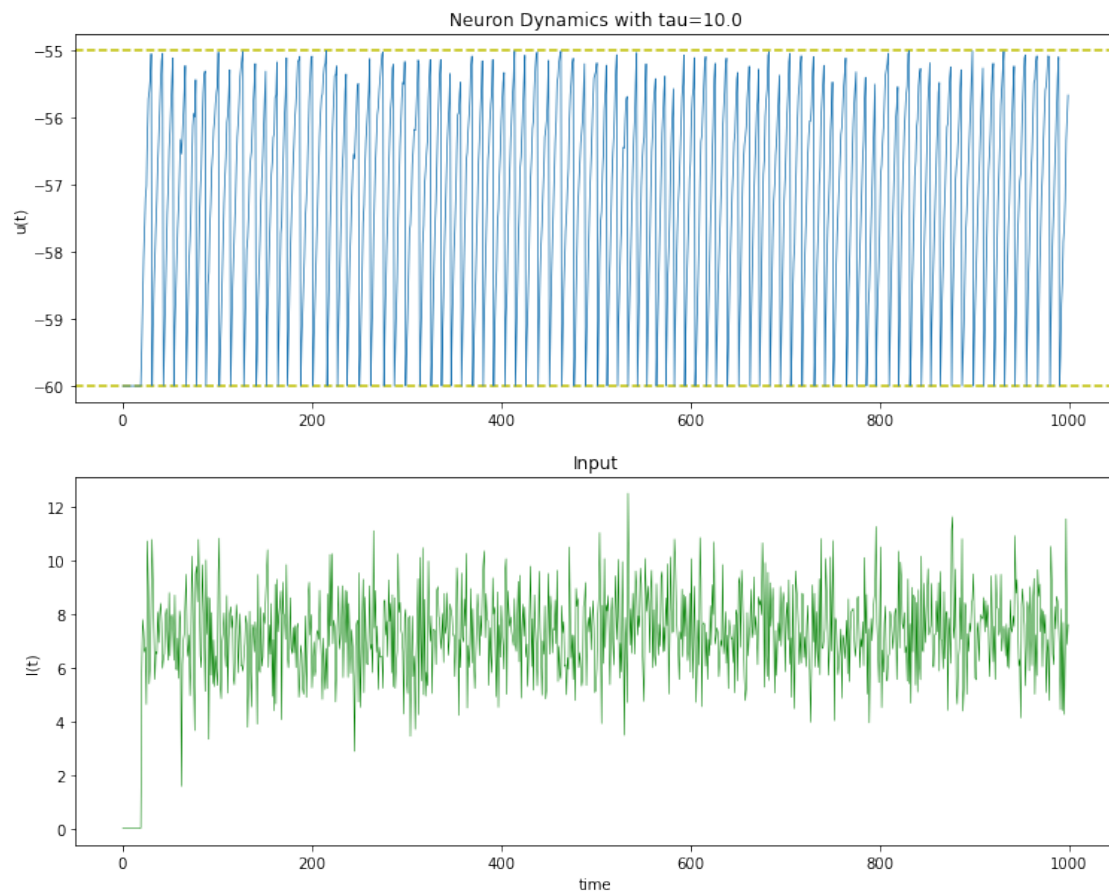
```
Simulation time: 1.589s
Currents: [4. 6. 8.]
frequencies: [ 1 61 107]
```

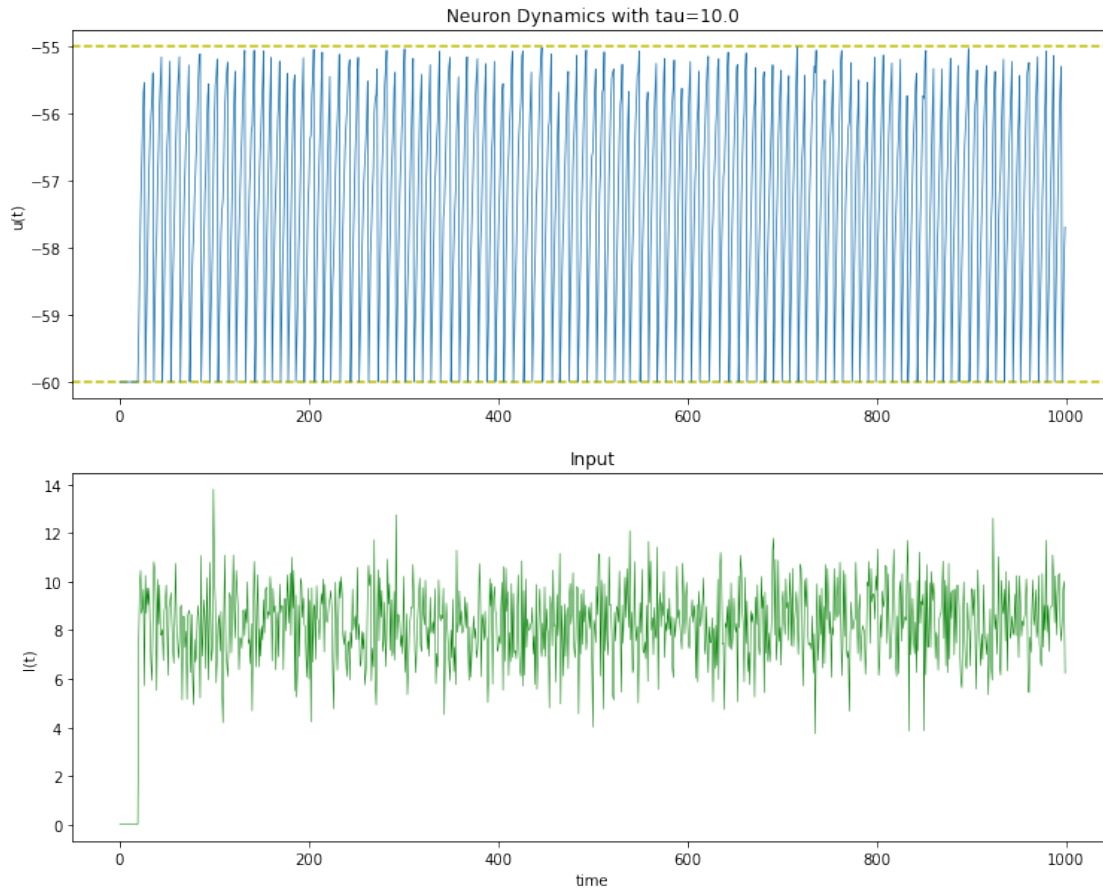
```
[28]: %matplotlib inline
if save_monitor_states:
    for i in range(len(monitor_states)):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```











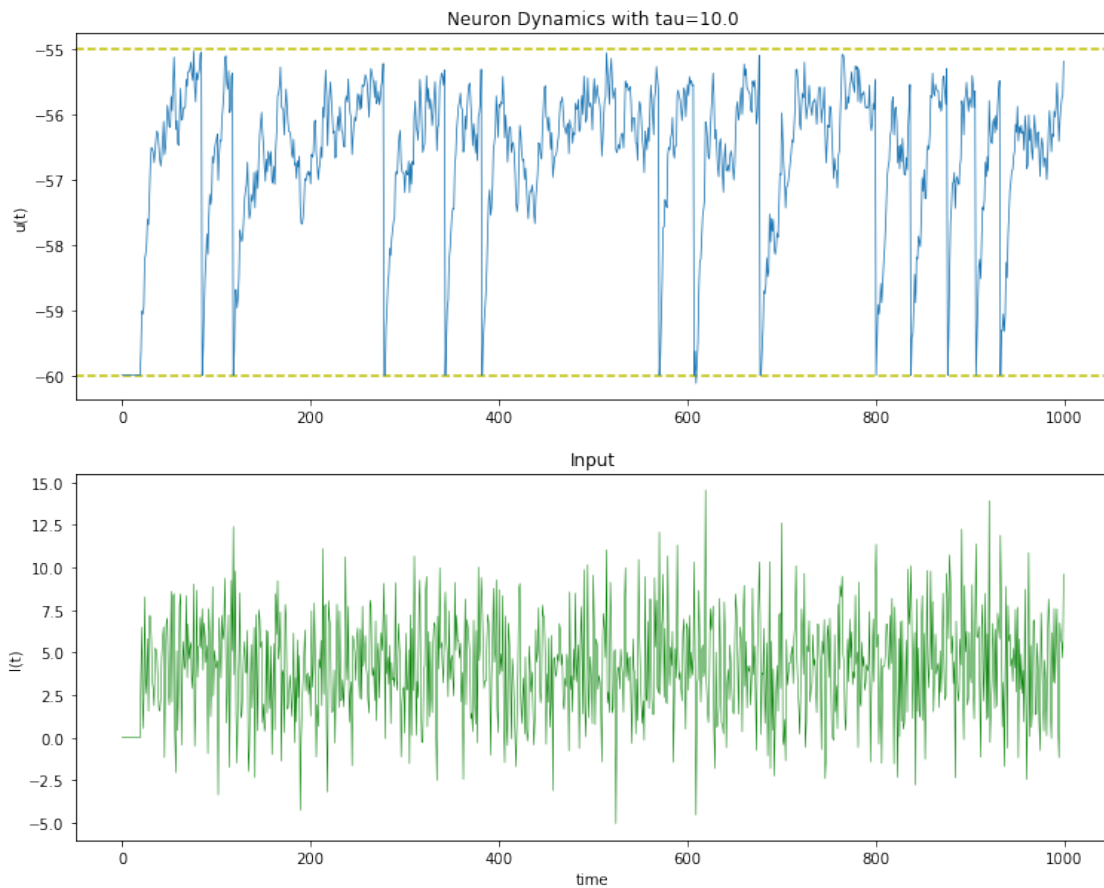
1.4.3 With noise: $\sigma_n = 3$

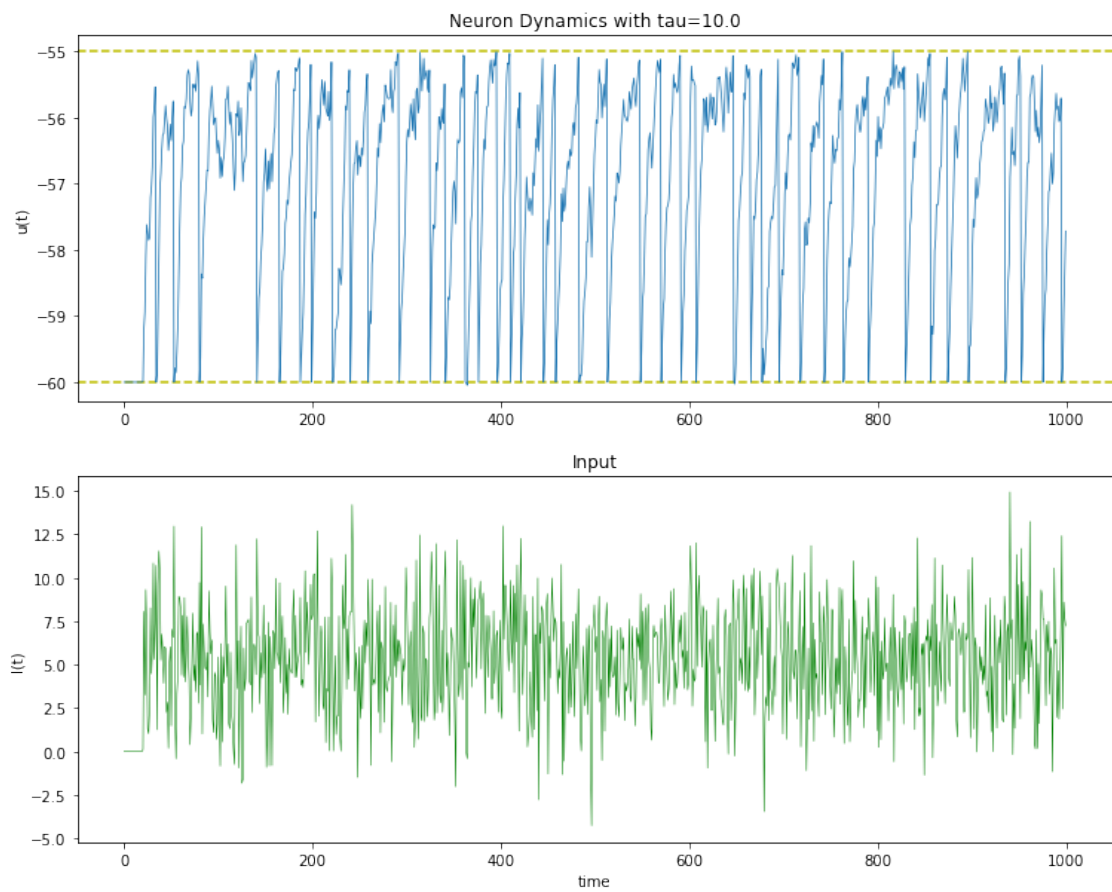
```
[29]: step_currents = np.linspace(4, 8, 5, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
        neuron_params=neuron_params_4,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent,
        line_slop=5e-4,
        noise_std=3
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:2])
print('frequencies:', frequencies[:2])
```

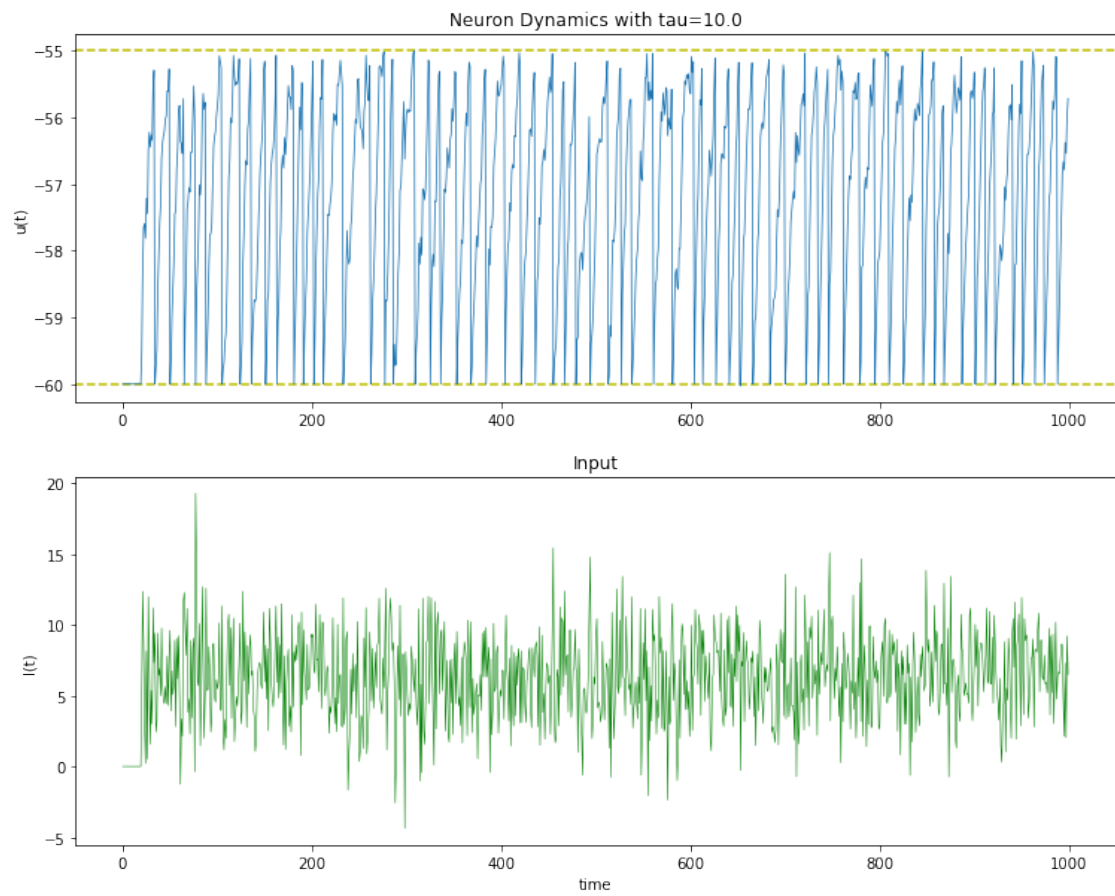
Simulation time: 1.600s

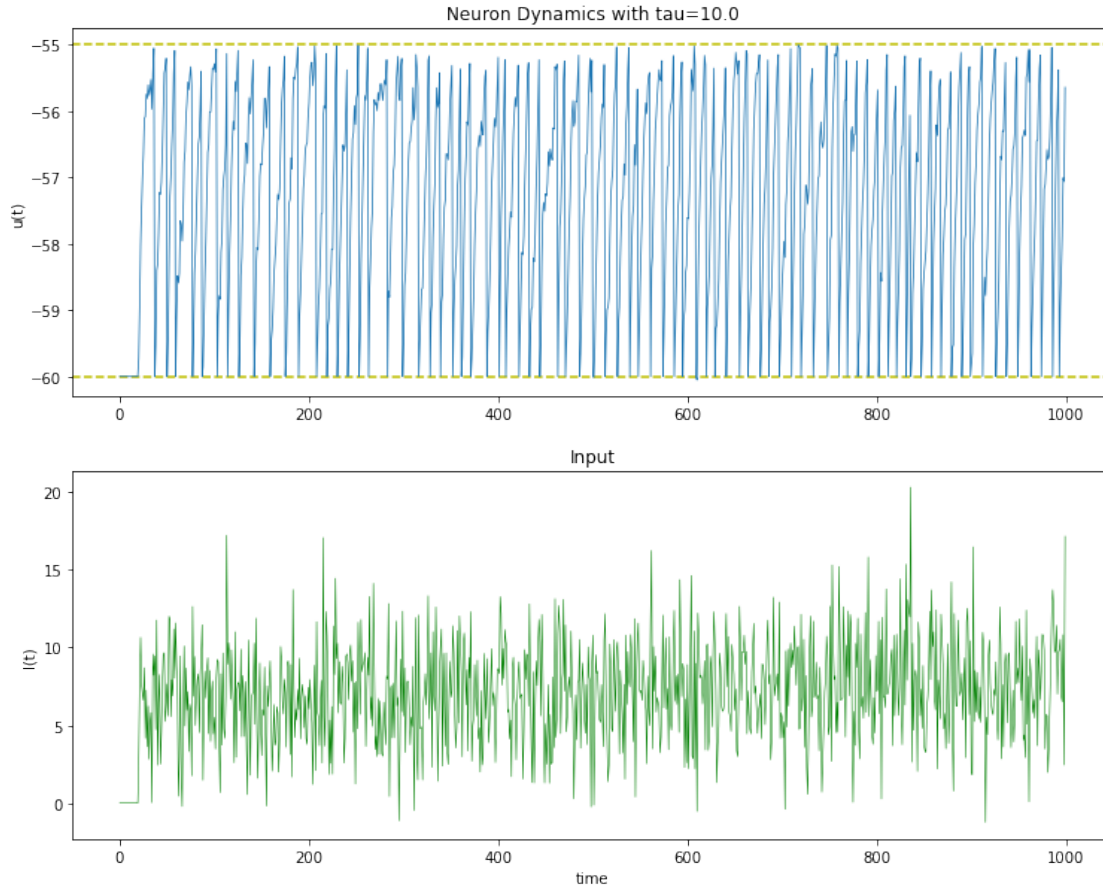
Currents: [4. 6. 8.]
frequencies: [13 65 106]

```
[30]: %matplotlib inline
if save_monitor_states:
    for i in range(4):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)
```









By comparing the results from the simulations with $\tau = 10$ and $\tau = 40$, we can clearly see that higher values of τ will cause the noise to be dampened to some extent. With lower values of τ , the effect of noise is more visible in the output. This could be explained by the **update rule**. By increasing the value of τ we actually are lowering the effect of noise in the output, because we are dividing the update terms by τ .

1.5 Simulation with neuron_params_5

$\tau = 20$, $|u_r - threshold| = 5mV$, $r = 2$

1.5.1 without noise

If we consider the update rule, it is clear that the effect of increasing the value of r is just like increasing the input value. So, in the below simulation, we halve the input values and set $r = 2$ to achieve the same result as with **neuron_params_5**.

```
[31]: neuron_params_5 = {'threshold': -55, 'u_rest': -60, 'tau': 20, 'r': 2}
step_currents = np.linspace(2, 8, 13, dtype=np.float32)
currents, frequencies, monitor, monitor_states, sim_time = \
    run_simulation_with_params(
```



```

        neuron_params=neuron_params_5,
        monitor_vars=monitor_vars,
        currents=step_currents,
        iters=iters,
        save_monitor_states=save_monitor_states,
        zero_percent=zero_percent
    )
print('Simulation time: {:.3f}s'.format(sim_time))
print('Currents:', currents[:,2])
print('frequencies:', frequencies[:,2])

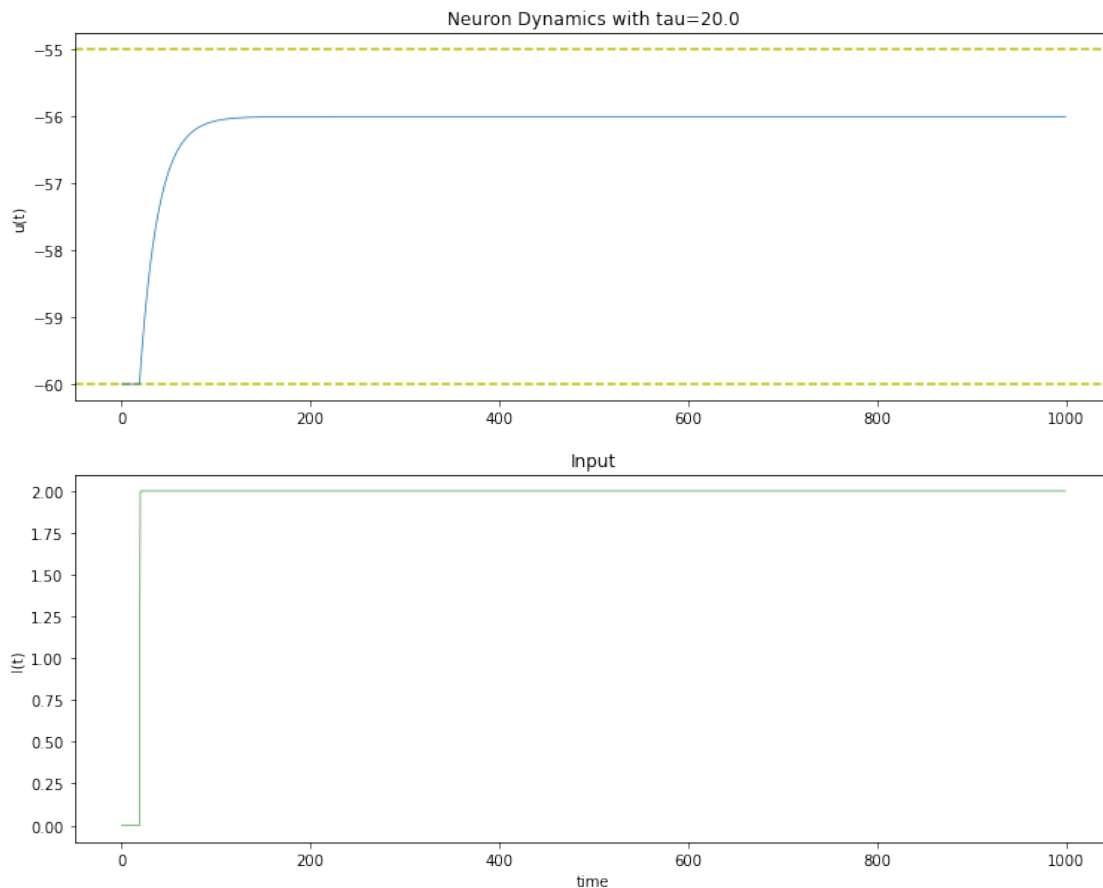
```

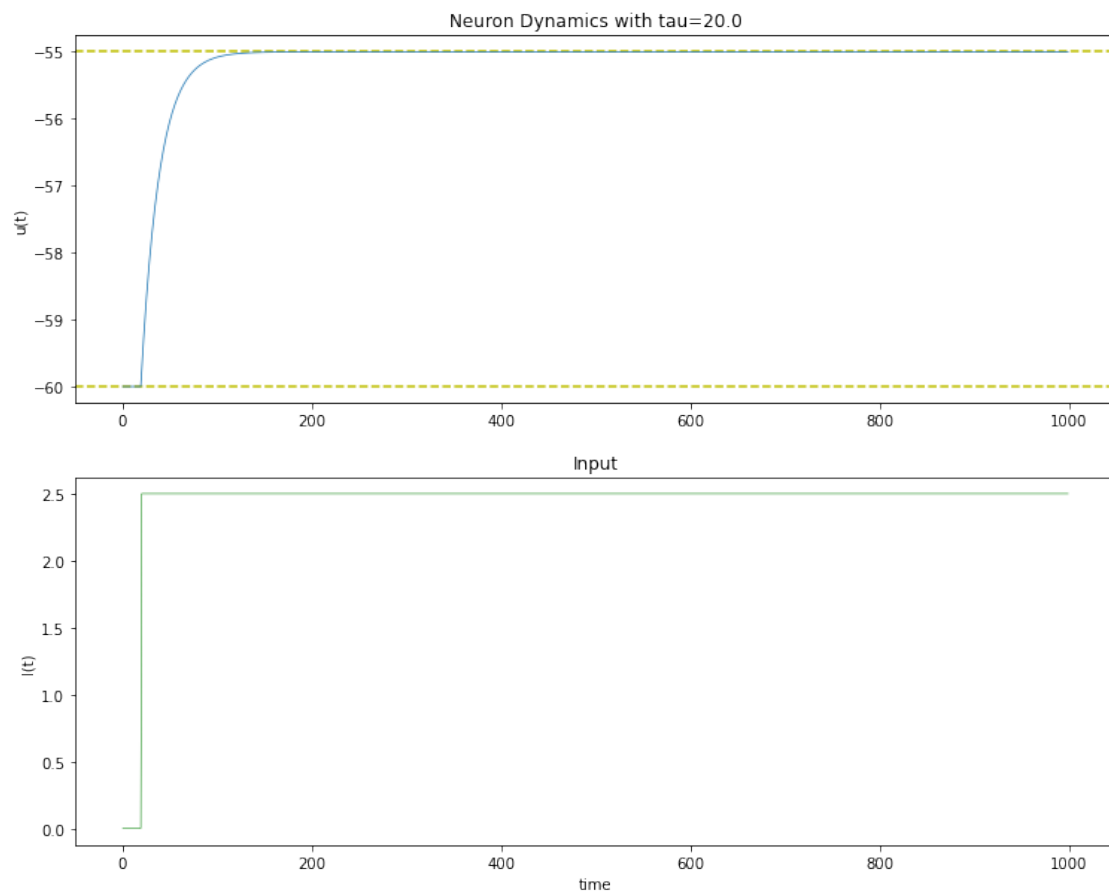
Simulation time: 4.062s
 Currents: [2. 3. 4. 5. 6. 7. 8.]
 frequencies: [0 29 50 71 91 110 124]

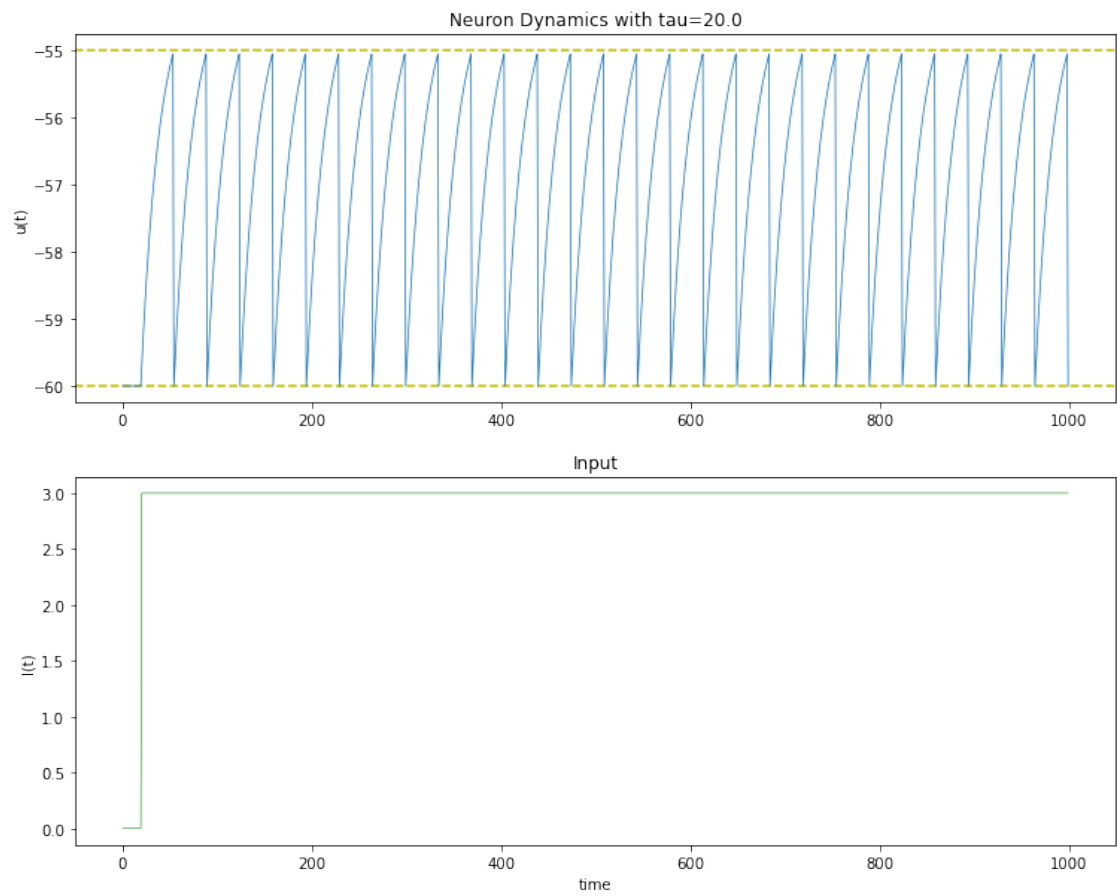
```

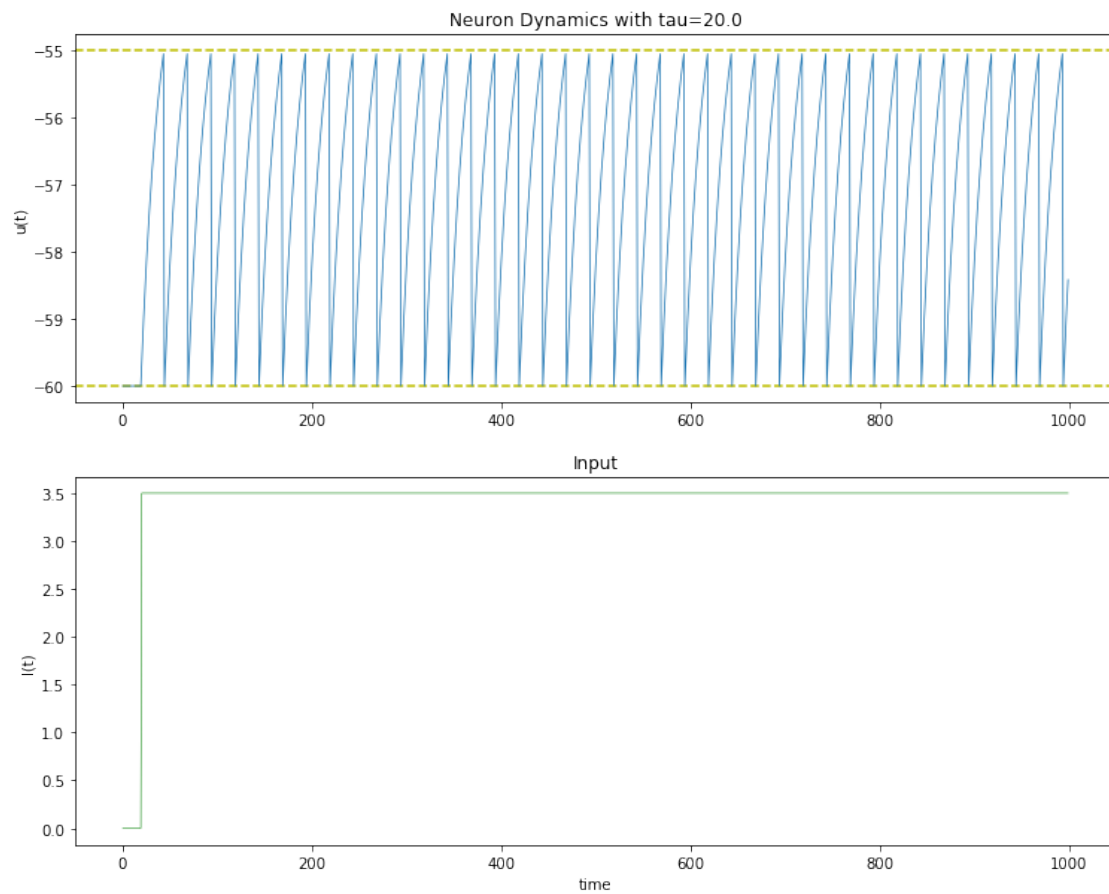
[32]: %matplotlib inline
if save_monitor_states:
    for i in range(4):
        time_plot(monitor_states[i], plot_spikes=False, figsize=(12.5, 10))
else:
    time_plot(monitor)

```

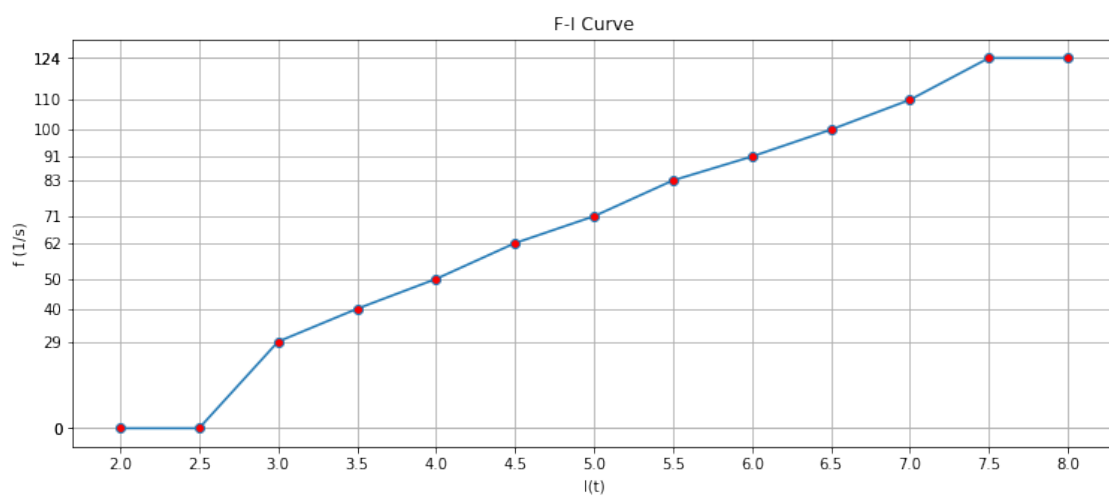








```
[33]: %matplotlib inline
fi_curve(currents, frequencies)
```



Since the effect of changing r is linear to the output, we do not run the simulations with noisy inputs. It is clear that by increasing r , we should have lower amounts of σ_n to get the same result.

1.6 Summary of observations:

1. Higher values of input increases the spikes' frequency.
2. The slope of FI-Curve is high at the beginning, then it tends to be lower. Also, for some different (but close) values of input, the frequency do not change. Maybe if we increase the simulation time, the difference would show up.
3. By increasing $|u_r - threshold|$ the input should also be increased, if we want to get the same frequency (relation between $|u_r - threshold|$, and the input intensity is **linear** in this case).
4. The higher amounts of noise tend to increase the spikes' frequency in some cases. Also, the noise in the potential curve will be increases as well.
5. Higher values of τ will dampen the effect of noise. Therefore, if the input's noise is large, the model with a larger value of τ tend to perform better. In other words, it is more robust to noise.
6. Increasing r will increase the effect of noise in the output as it will increase the σ_n felt by the neuron.
7. Also, increasing $|u_r - threshold|$ will dampen the effect of noise. It is because neuron takes a longer time to reach its threshold, therefore; the noise effect would be lower (assuming the noise distribution is normal).
8. τ and the spikes' frequency are almost inversely proportional.