

Computability and Complexity

Problem Set 7

Turing machines

Y. Deville

C. Bertrand Van Ouytsel & V. Coppé & A. Gerniers & N. Golenvaux & M. Parmentier

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1. For each of the following functions, build a Turing machine, using $\Sigma = \{0, 1\}$:

(a) $f(n) = n \times 2$

(b) $f(n) = \text{not}(n)$ (f inverts the bits of n)

Check them with a Turing machine simulator and simulate them with different inputs.

2. Suppose we have two Turing machines A and B with $\Sigma = \{a, b\}$. Their starting and halting states are (s_A, h_A) and (s_B, h_B) , respectively. Explain how to combine A and B into a new Turing machine implementing the following pseudo-code :

(a) *if*(a) then $\{A\}$ else $\{B\}$

(b) while *not*(a) do $\{A\}$

In this pseudocode, *if*(a) means “if a is the symbol under the head”, and *not*(a) means “the symbol under the head is not a ”, none of which consumes a symbol.

3. Are those Turing machines equivalent to the standard model of Turing machine? Explain.

(a) “Big jumps” : this Turing machine can move its head of n cells to the left and to the right. The transition function is thus $S \times \Gamma \rightarrow S \times (\{L, R\} \times \mathbb{N}) \times \Gamma$.

(b) “Online” : this Turing machine can only move to the right or stay at the same cell, and cannot move to the left. The transition function is thus $S \times \Gamma \rightarrow S \times \{R, \downarrow\} \times \Gamma$.

4. Is it possible to simulate any deterministic finite automaton with a Turing machine? Explain.

Challenge

What is the computational complexity of the Turing machine described below? Let TM2T be a Turing machine able to use two strips of tapes instead of one. Show that there exists a TM2T that computes the same function as the traditional TM described below, while achieving a quadratic computational speedup.

The following Turing machine has the alphabets $\Sigma = \{a, b\}$ and $\Gamma = \{a, b, x, B, 0, 1\}$.

state	symbol	state	movement	symbol
start	a	seekB	\rightarrow	x
start	b	seekA	\rightarrow	x
start	x	start	\rightarrow	B
start	B	stop	\downarrow	1
seekA	a	restart	\leftarrow	x
seekA	b	seekA	\rightarrow	b
seekA	x	seekA	\rightarrow	x
seekA	B	false	\leftarrow	B
seekB	b	restart	\leftarrow	x
seekB	a	seekB	\rightarrow	a
seekB	x	seekB	\rightarrow	x
seekB	B	false	\leftarrow	B
restart	a	restart	\leftarrow	a
restart	b	restart	\leftarrow	b
restart	x	restart	\leftarrow	x
restart	B	start	\rightarrow	B
false	a	false	\leftarrow	B
false	b	false	\leftarrow	B
false	x	false	\leftarrow	B
false	B	stop	\downarrow	0