Computability and Complexity
Problem Set 10
Reductions and complexity classes

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1. Algorithmic versus functional reduction.
   - (a) Prove that for every set A, $A \leq_a \bar{A}$.
   - (b) Prove that for all sets A, B, if $A \leq_f B$ and B is recursively enumerable, then A is recursively enumerable.
   - (c) Show an example for which the relation $A \leq_f \bar{A}$ does not hold.
   - (d) Show an example for which the following relation does not hold : if $A \leq_a B$ and B is recursively enumerable, then A is recursively enumerable.

2. Prove the following relations.
   - (a) Does $A \leq_a B$ imply $A \leq_f B$?
   - (b) Does $A \leq_f B$ imply $A \leq_a B$?

3. Let $re = \{A | A$ is recursively enumerable$\}$.
   - (a) State the conditions to prove that HALT is re-complete wrt $\leq_a$.
   - (b) Prove the conditions.

4. Does the class $DTIME(n^2)$ depend of your calculus model (e.g. Java language)? And what about the class P? Justify your answers.

5. Let A be a recursive set and f a total function from $\mathbb{N}$ to $\mathbb{N}$. Is the following implication true? Explain.

$$A \in NTIME(f) \Rightarrow \exists c > 1 : A \in DTIME(c^f)$$

6. Let A be a recursive set and f a total function from $\mathbb{N}$ to $\mathbb{N}$. Is the following implication true? Explain.

$$A \in DTIME(f) \Rightarrow A \in DSPACE(f)$$