

TP 2
Digital Electronics [ELEC-H-310]
Correction
v1.0.0

⚠ Convention : $\overline{ab} = \overline{a \cdot b}$ PAS $\overline{(ab)}$

N.B.: We use the following convention : $\overline{ab} = \overline{a \cdot b}$ and $\overline{(ab)} = \overline{a \cdot b} = \overline{a} + \overline{b}$

Question 1. Prove this equality by comparing truth tables.

$$\overline{a}c + \overline{a}bc = \overline{a}b + \overline{a}c$$

$$\Rightarrow \overline{a}c + \overline{a}\overline{b}\overline{c} = \overline{a}b + \overline{a}c$$

Answer: We can compare $F1 = F2$

a	b	c	$\overline{a}c$	$\overline{a}bc$	$F1$	$\overline{a}b$	$\overline{a}c$	$F2$
0	0	0	0	1	1	1	0	1
0	0	1	1	0	1	1	1	1
0	1	0	0	0	0	0	0	0
0	1	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0

Question 2. Simplify these expressions using algebraic manipulations.

a) $(a + b) \cdot (a + \overline{b})$

Answer:

$$\begin{aligned}
 (a + b) \cdot (a + \overline{b}) &= aa + a\overline{b} + ab + b\overline{b} \\
 &= a + a\overline{b} + ab + 0 \\
 &= a \cdot (1 + \overline{b} + b) \\
 &= a
 \end{aligned}$$

b) $a + \overline{a}b$

Answer:

$$\begin{aligned} a + \bar{a}b &= (a + \bar{a}) \cdot (a + b) \\ &= 1 \cdot (a + b) \\ &= a + b \end{aligned}$$

c) $\bar{a}bc + \overline{abc} + \bar{a}b\bar{c}$

Answer:

$$\begin{aligned} \bar{a}bc + \overline{abc} + \bar{a}b\bar{c} &= \bar{a}b \cdot (c + \bar{c}) + \bar{a}b\bar{c} \\ &= \bar{a}b + \bar{a}b\bar{c} \\ &= \bar{a} \cdot (\bar{b} + b\bar{c}) \\ &= \bar{a} \cdot ((\bar{b} + b) \cdot (\bar{b} + \bar{c})) \\ &= \bar{a} \cdot (1 \cdot (\bar{b} + \bar{c})) \\ &= \bar{a} \cdot (\bar{b} + \bar{c}) \end{aligned}$$

d) $\overline{((a + b)\bar{c}d + e + \bar{f})}$

Answer:

$$\overline{((a + b) \cdot \bar{c}d + e + \bar{f})} = (\bar{a}\bar{b} + c + d) \cdot \bar{e}f \quad (\text{De Morgan})$$

e) $\bar{a}bc + \bar{a}\bar{b}c + \overline{abc} + \bar{a}\bar{b}c + abc$

Answer:

$$\begin{aligned} \bar{a}bc + \bar{a}\bar{b}c + \overline{abc} + \bar{a}\bar{b}c + abc &= bc \cdot (a + \bar{a}) + \bar{a}\bar{b}c + \bar{b}c \cdot (a + \bar{a}) \\ &= bc + \bar{a}\bar{b}c + \bar{b}c \end{aligned}$$

f) $\overline{(ab + ac)} + \bar{a}\bar{b}c$

Answer:

$$\begin{aligned} \overline{(ab + ac)} + \bar{a}\bar{b}c &= (\bar{a} + \bar{b}) \cdot (\bar{a} + \bar{c}) + \bar{a}\bar{b}c \\ &= \bar{a} + \bar{a}\bar{c} + \bar{a}\bar{b} + \bar{b}\bar{c} + \bar{a}\bar{b}c \\ &= \bar{a} \cdot (1 + \bar{c} + \bar{b} + \bar{b}c) + \bar{b}\bar{c} \\ &= \bar{a} + \bar{b}\bar{c} \end{aligned}$$

g) $\overline{(a + b)} \cdot \overline{(a + b)}$

Answer:

$$\begin{aligned}\overline{(a+b)} \cdot \overline{(\bar{a}+b)} &= (\bar{a}\bar{b}) \cdot (a\bar{b}) \\ &= \bar{a} \cdot a \cdot \bar{b} \\ &= 0\end{aligned}$$

h) $a + \bar{a}b + \bar{a}\bar{b}$

Answer:

$$\begin{aligned}a + \bar{a}b + \bar{a}\bar{b} &= a + \bar{a} \cdot (b + \bar{b}) \\ &= a + \bar{a} \\ &= 1\end{aligned}$$

Question 3. Write these expressions as minterms (disjunctive normal form).

a) $F(a, b, c, d) = \bar{a}\bar{b}c + \bar{a}\bar{b} + ab\bar{c}d$ → minterm

Answer:

on fait apparaître 0

$$\bar{a}b\bar{c}d + \bar{a}\bar{b}\bar{c}d + \bar{a}\bar{b}cd + \bar{a}b\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + \bar{a}b\bar{c}d + \bar{a}\bar{b}cd + \bar{a}b\bar{c}d$$

b) $F(a, b, c, d) = ab + \bar{b}c + cd$

Answer:

$$abcd + ab\bar{c}d + ab\bar{c}\bar{d} + ab\bar{c}d + \bar{a}bcd + \bar{a}b\bar{c}d + \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd$$

c) $F(a, b, c, d) = a + d$

Answer:

$$abcd + ab\bar{c}d + ab\bar{c}\bar{d} + ab\bar{c}d + \bar{a}bcd + \bar{a}b\bar{c}d + \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + \bar{a}b\bar{c}d + \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d$$

Question 4. Simplify $F(a, b)$ using K-maps. Reminder: to fill the Karnaugh table, you can develop the function into one of its canonic forms.

a) $F(a, b) = a + \bar{a}b + \bar{a}\bar{b}$

Answer:

		a	
		0	1
b	0	1	1
	1	1	1

$F = 1$

b) $F(a, b) = (a + b) \cdot (a + \bar{b})$

Answer:

		a	
		0	1
b	0	0	1
	1	0	1

$F = a$

c) $F(a, b) = a + \bar{a}b$

Answer:

		a	
		0	1
b	0	0	1
	1	1	1

$F = a + b$

*résultats à partir
de table vérité*

a	b	F
0	0	0
0	1	1
1	0	1
1	1	1

a est constant

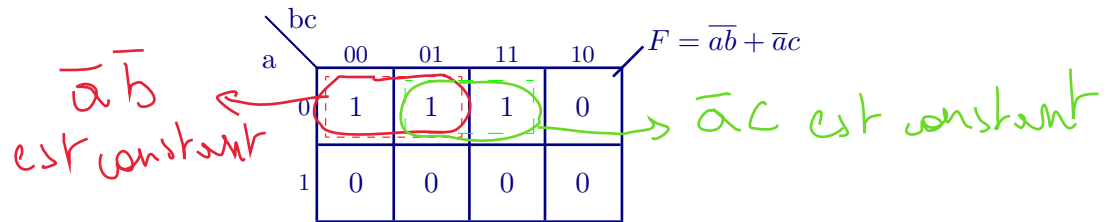
b est constant

il faut entourer 2^m 1 où la distance de Hamming vaut 1

Question 5. Simplify $F(a, b, c)$ using K-maps.

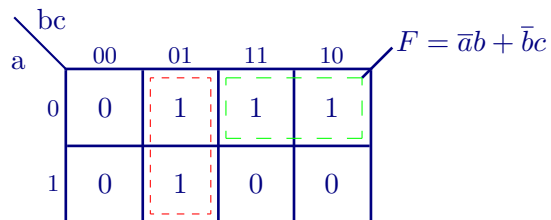
a) $F(a, b, c) = \bar{a}c + \bar{a}bc$

Answer:



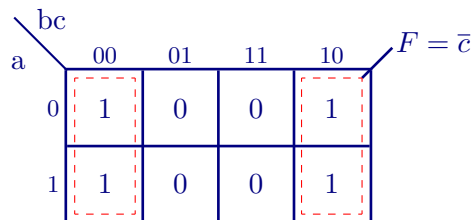
b) $F(a,b,c) = \overline{a}bc + \overline{a}b\overline{c} + \overline{a}bc + \overline{a}bc$

Answer:



c) $F(a,b,c) = ab\overline{c} + \overline{a}b\overline{c} + \overline{a}b\overline{c} + \overline{a}b\overline{c}$

Answer:



Question 6. Simplify $F(a,b,c,d)$ using K-maps.

a) $F(a,b,c,d) = abd + acd + bcd + ab + \overline{a}cd + \overline{a}bcd$

Answer:

		cd				
ab		00	01	11	10	$F = ab + cd$
	00	0	0	1	0	
	01	0	0	1	0	
	11	1	1	1	1	
	10	0	0	1	0	

b) $F(a, b, c, d) = \overline{abcd} + \overline{acd} + \overline{abc} + abc + \overline{abc} + abcd$

Answer:

		cd				
ab		00	01	11	10	$F = ac + \overline{ac}$
	00	1	1	0	0	
	01	1	1	0	0	
	11	0	0	1	1	
	10	0	0	1	1	

c) $F(a, b, c, d) = \overline{bcd} + \overline{acd} + \overline{acd} + \overline{ad} + \overline{abd}$

Answer:

		cd				
ab		00	01	11	10	$F = \overline{d} + \overline{ac}$
	00	1	1	0	1	
	01	1	1	0	1	
	11	1	0	0	1	
	10	1	0	0	1	

on encadre ici les
2 extrémités car
distance Hamming = 1

Question 7. Simplify $F(a, b, c, d, e)$ using K-maps.

a) $F(a, b, c, d, e) = a\overline{e} + b\overline{e} + \overline{a}bce + \overline{a}bcd\overline{e} + \overline{a}b\overline{c}\overline{e} + \overline{a}cde + \overline{a}b\overline{e} + \overline{a}bce$

Answer:

abc de	000	001	011	010	100	101	111	110
00	1	1	1	1	1	1	1	1
01	0	1	0	0	0	1	0	0
11	0	1	0	0	0	1	0	0
10	1	1	1	1	1	1	1	1

$F = \bar{e} + \bar{b}c$

Développement : - on construit la table
- on commence avec les plus petits termes

abc de	000	001	011	010	100	101	111	110
00	1	1	1	1	1	1	1	1
01	0	1	0	0	0	1	0	0
11	0	1	0	0	0	1	0	0
10	1	1	1	1	1	1	1	1

Handwritten annotations: A red line groups the first row (00) and the last row (10), labeled \bar{e} . A green line groups the second and third columns (001, 011), labeled $\bar{b}c$. A blue vertical line is drawn between columns 010 and 100.

$$\Rightarrow F = \bar{b}c + \bar{e}$$

\Rightarrow Pour les avoir adjacents en 2D

000 | 010 | 011 | 001 | 101 | 111 | 110 | 100