TP 2
Digital Electronics [ELEC-H-310]
Correction

v1.0.0

⚠ Convention : $\overline{a}\,\overline{b} = \overline{a} \cdot \overline{b}$ PAS $\overline{(ab)}$

N.B.: We use the following convention : $\overline{ab} = \overline{a} \cdot \overline{b}$ and $\overline{(ab)} = \overline{a \cdot b} = \overline{a} + \overline{b}$

Question 1. Proove this equality by comparing truth tables.

$$\overline{a}c + \overline{abc} = \overline{ab} + \overline{a}c$$

$\Rightarrow \overline{a}c + \overline{a}\,\overline{b}\,\overline{c} = \overline{a}\,\overline{b} + \overline{a}c$

Answer: We can compare $F1 = F2$

a	b	c	$\overline{a}c$	$\overline{abc}$	$F1$	$\overline{ab}$	$\overline{a}c$	$F2$
0	0	0	0	1	1	1	0	1
0	0	1	1	0	1	1	1	1
0	1	0	0	0	0	0	0	0
0	1	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0

Question 2. Simplify these expressions using algebraic manipulations.

a) $(a + b) \cdot (a + \overline{b})$

Answer:

$$\begin{aligned}(a + b) \cdot (a + \overline{b}) &= aa + a\overline{b} + ab + b\overline{b} \\ &= a + a\overline{b} + ab + 0 \\ &= a \cdot (1 + \overline{b} + b) \\ &= a\end{aligned}$$

b) $a + \overline{a}b$

Answer:

$$
\begin{aligned}
a + \overline{a}b &= (a + \overline{a}) \cdot (a + b) \\
&= 1 \cdot (a + b) \\
&= a + b
\end{aligned}
$$

c) $\overline{ab}c + \overline{abc} + \overline{a}b\overline{c}$

Answer:

$$
\begin{aligned}
\overline{ab}c + \overline{abc} + \overline{a}b\overline{c} &= \overline{ab} \cdot (c + \overline{c}) + \overline{a}b\overline{c} \\
&= \overline{ab} + \overline{a}b\overline{c} \\
&= \overline{a} \cdot (\overline{b} + b\overline{c}) \\
&= \overline{a} \cdot ((\overline{b} + b) \cdot (\overline{b} + \overline{c})) \\
&= \overline{a} \cdot (1 \cdot (\overline{b} + \overline{c})) \\
&= \overline{a} \cdot (\overline{b} + \overline{c})
\end{aligned}
$$

d) $\overline{((a+b)\overline{cd} + e + \overline{f})}$

Answer:

$$
\overline{((a+b) \cdot \overline{cd} + e + \overline{f})} = (\overline{ab} + c + d) \cdot \overline{e}f \quad \text{(De Morgan)}
$$

e) $\overline{a}bc + a\overline{bc} + \overline{abc} + a\overline{b}c + abc$

Answer:

$$
\begin{aligned}
\overline{a}bc + a\overline{bc} + \overline{abc} + a\overline{b}c + abc &= bc \cdot (a + \overline{a}) + a\overline{b}c + \overline{bc} \cdot (a + \overline{a}) \\
&= bc + a\overline{b}c + \overline{bc}
\end{aligned}
$$

f) $\overline{(ab + ac)} + \overline{ab}c$

Answer:

$$
\begin{aligned}
\overline{(ab + ac)} + \overline{ab}c &= (\overline{a} + \overline{b}) \cdot (\overline{a} + \overline{c}) + \overline{ab}c \\
&= \overline{a} + \overline{ac} + \overline{ab} + \overline{bc} + \overline{ab}c \\
&= \overline{a} \cdot (1 + \overline{c} + \overline{b} + \overline{b}c) + \overline{bc} \\
&= \overline{a} + \overline{bc}
\end{aligned}
$$

g) $\overline{(a+b)}\ \overline{(\overline{a}+b)}$

Answer:

$$\begin{aligned}\overline{(a+b)} \cdot \overline{(\overline{a}+b)} &= (\overline{a}\overline{b}) \cdot (a\overline{b}) \\ &= \overline{a} \cdot a \cdot \overline{b} \\ &= 0\end{aligned}$$

h) $a + \overline{a}b + \overline{ab}$

Answer:

$$\begin{aligned}a + \overline{a}b + \overline{ab} &= a + \overline{a} \cdot (b + \overline{b}) \\ &= a + \overline{a} \\ &= 1\end{aligned}$$

Question 3. Write these expressions as minterms (disjunctive normal form).

a) $F(a,b,c,d) = a\overline{b}c + \overline{ab} + ab\overline{c}d$ → minterm

on fait apparaître d

Answer:

$$a\overline{b}cd \quad + \quad a\overline{b}c\overline{d} \quad + \quad \overline{ab}cd \quad + \quad \overline{ab}c\overline{d} \quad + \quad \overline{abc}d \quad + \quad \overline{abcd} \quad + \quad ab\overline{c}d$$

b) $F(a,b,c,d) = ab + \overline{b}c + cd$

Answer:

$$abcd + abc\overline{d} + ab\overline{c}d + ab\overline{cd} + a\overline{b}cd + a\overline{b}c\overline{d} + \overline{ab}cd + \overline{ab}c\overline{d} + \overline{a}bcd$$

c) $F(a,b,c,d) = a + d$

Answer:

$$abcd+abc\overline{d}+ab\overline{c}d+ab\overline{cd}+a\overline{b}cd+a\overline{b}c\overline{d}+a\overline{bc}d+a\overline{bcd}+\overline{a}bcd+\overline{a}b\overline{c}d+\overline{ab}cd+\overline{abc}d$$

Question 4. Simplify $F(a,b)$ using K-maps. Reminder: to fill the Karnaugh table, you can develop the function into one of its canonic forms.

a) $F(a,b) = a + \bar{a}b + \overline{ab}$

Answer:

b \ a	0	1
0	1	1
1	1	1

$F = 1$

b) $F(a,b) = (a+b)\cdot(a+\bar{b})$

Answer:

b \ a	0	1
0	0	1
1	0	1

$F = a$

c) $F(a,b) = a + \bar{a}b$

Answer:

b \ a	0	1
0	0	1
1	1	1

$F = a + b$

résultats à partir de table vérité

a	b	F
0	0	0
0	1	1
1	0	1
1	1	1

→ a est constant

↳ b est constant

il faut entourer 2^n 1 où la distance de Hamming vaut 1

Question 5. Simplify $F(a,b,c)$ using K-maps.

a) $F(a,b,c) = \bar{a}c + \overline{abc}$

Answer:

$\overline{a}\,\overline{b}$ est constant

a \ bc	00	01	11	10
0	1	1	1	0
1	0	0	0	0

$F = \overline{a}\overline{b} + \overline{a}c$

$\overline{a}c$ est constant

b) $F(a,b,c) = a\overline{b}c + \overline{a}b\overline{c} + \overline{a}bc + \overline{ab}c$

Answer:

a \ bc	00	01	11	10
0	0	1	1	1
1	0	1	0	0

$F = \overline{a}b + \overline{b}c$

c) $F(a,b,c) = ab\overline{c} + \overline{abc} + \overline{a}b\overline{c} + a\overline{bc}$

Answer:

a \ bc	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$F = \overline{c}$

Question 6. Simplify $F(a,b,c,d)$ using K-maps.

a) $F(a,b,c,d) = abd + acd + bcd + ab + \overline{a}cd + \overline{ab}cd$

Answer:

$F = ab + cd$

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	1	1	1	1
10	0	0	1	0

b) $F(a,b,c,d) = \overline{abcd} + \overline{ac}d + \overline{a}b\overline{c} + abc + a\overline{b}c + abcd$

Answer:

$F = ac + \overline{ac}$

ab \ cd	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	0	1	1
10	0	0	1	1

c) $F(a,b,c,d) = \overline{bcd} + \overline{a}c\overline{d} + \overline{ac}d + a\overline{d} + \overline{a}b\overline{d}$

Answer:

$F = \overline{d} + \overline{ac}$

ab \ cd	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	1	0	0	1
10	1	0	0	1

on entoure ici les 2 extrémités car distance Hamming=1

Question 7. Simplify $F(a,b,c,d,e)$ using K-maps.

a) $F(a,b,c,d,e) = a\overline{e} + b\overline{e} + a\overline{b}ce + a\overline{b}cde + a\overline{b}c\overline{e} + \overline{acde} + \overline{abe} + \overline{abce}$

Answer:

de \ abc	000	001	011	010	100	101	111	110
00	1	1	1	1	1	1	1	1
01	0	1	0	0	0	1	0	0
11	0	1	0	0	0	1	0	0
10	1	1	1	1	1	1	1	1

$F = \overline{e} + \overline{b}c$

Développem- : - on construit la table
- " commence avec les plus petits termes

de \ abc	000	001	011	010	100	101	111	110
00	1	1	1	1	1	1	1	1
01	0	1	0	0	0	1	0	0
11	0	1	0	0	0	1	0	0
10	1	1	1	1	1	1	1	1

→ $\overline{e}$

$\overline{b}c$

$\Rightarrow F = \overline{b}c + \overline{e}$

$\Rightarrow$ Pour les avoir adjacents en 2D

000	010	011	001	101	111	110	100