TP 3
Digital Electronics [ELEC-H-310]
Correction

v1.0.0

Question 1. Find all prime implicant using the Quine-McCluskey method.

N.B.: $\sum_m$ is a sum of minterms, $\sum_d$ is a sum of 'don't care'.

a) $f(a,b,c,d) = \sum_m(0,4,5,8,12,13)$

Answer: We first need to convert decimal numbers into binary.

	a	b	c	d
0	0	0	0	0
4	0	1	0	0
5	0	1	0	1
8	1	0	0	0
12	1	1	0	0
13	1	1	0	1

Then, we can group the elements and merge them. The '✓' states that a line as been merged. The numbers between parenthesis show the values that have been merged.

	a	b	c	d		
G_0	0	0	0	0	0	✓
G_1	0	1	0	0	4	✓
	1	0	0	0	8	✓
G_2	1	1	0	0	12	✓
	0	1	0	1	5	✓
G_3	1	1	0	1	13	✓

	a	b	c	d		
$G_{0'}$	0	–	0	0	(0,4)	✓
	–	0	0	0	(0,8)	✓
$G_{1'}$	–	1	0	0	(4,12)	✓
	0	1	0	–	(4,5)	✓
	1	–	0	0	(8,12)	✓
$G_{2'}$	1	1	0	–	(12,13)	✓
	–	1	0	1	(5,13)	✓

$G_{0''}$	–	–	0	0	(0,4|8,12)	Fusion	IP1
	–	–	0	0	(0,8|4,12)	Fusion	
$G_{1''}$	–	1	0	–	(4,12|5,13)	Fusion	IP2
	–	1	0	–	(4,5|12,13)	Fusion	

The prime implicants hence are $\overline{cd}$ (IP1) and $b\bar{c}$ (IP2).

b) $f(a,b,c,d) = \sum_m(2,3,4,10,12,13) + \sum_d(11,14,15)$

Answer: $f(a,b,c,d) = \bar{b}c + b\bar{c}\bar{d} + ac + ab$

c) $f(a,b,c,d,e,f) = \sum_m(16,28,53,60,63)$

1.b) $\sum m(2,3,4,10,12,13) + \sum d(11,14,15)$

0010 0011 0100 1010 1100 1101 1011 1110 1111

G1	0010	(2)
	0100	(4)
G2	0011	(3)
	1010	(10)
	1100	(12)
G3	1101	(13)
	1011	(11)
	1110	(14)
G4	1111	(15)

G'_1	001-	(2,3) ✓
	-010	(2,10)
	-100	(4,12)
G'_2	-011	(3,11)
	101-	(10,11) ✓
	1-10	(10,14)
	110-	(12,13)
	11-0	(12,14)
G'_3	11-1	(13,15)
	1-11	(11,15)
	111-	(14,15)

G''_1	-01-	(2,3,10,11)
	-01-	(2,10,3,11)
G''_2	1-1-	(10,14,11,15)
	1-1-	
	11--	
	11--	

$$\Rightarrow \quad \begin{array}{cccc} -100 & -01- & 1-1- & 11-- \\ b\bar{c}\bar{d} & \bar{b}c & ac & ab \end{array}$$

$$F = b\bar{c}\bar{d} + \bar{b}c + ac + ab$$

⇒ Vérifier solut° : table de Karnaugh

ab \ cd	00	01	11	10
00	0	1	3	2
01				
11				— 14
10				

on met la valeur 14 de la représentat° décimale

Answer: $f(a,b,c,d,e,f) = abcdef + bcd\overline{ef} + ab\overline{c}d\overline{e}f + \overline{a}b\overline{cdef}$

Question 2. Find the simplified function:

a) $f(a,b,c,d) = \sum_m(2,3,4,10,12,13) + \sum_d(11,14,15)$

Answer: The coverage table with the prime implicants of the previous exercise: $i_1 = b\overline{cd}$, $i_2 = \overline{b}c$, $i_3 = ac$ et $i_4 = ab$.

	i_1	i_2	i_3	i_4
0010		✓		
0011		✓		
0100	✓			
1010		✓	✓	
1100	✓			✓
1101				✓

We can deduce the following coverage function:

$$1 = i_2 \cdot i_1 \cdot (i_2 + i_3) \cdot (i_1 + i_4) \cdot i_4 = i_2 i_1 i_4$$

Using these axioms and theorems:

$$x \cdot x = x$$
$$x \cdot (x + y) = x$$
$$(x + y) \cdot (x + z) = x + y \cdot z$$

We find $f(a,b,c,d) = \overline{b}c + b\overline{cd} + ab$

b) $f(a,b,c,d) = \sum_m(0,2,4,5,10,11,13,15) + \sum_d(6,8)$

Answer: $f(a,b,c,d) = \overline{a}b\overline{c} + \overline{bd} + a\overline{b}c + abd$ ou $f(a,b,c,d) = a\overline{b}c + b\overline{c}d + abd + \overline{ad}$ ou $f(a,b,c,d) = \overline{a}b\overline{c} + a\overline{b}c + abd + \overline{a}\overline{d}$ ou $f(a,b,c,d) = b\overline{c}d + a\overline{b}c + acd + \overline{a}\overline{d}$ ou $f(a,b,c,d) = b\overline{c}d + acd + \overline{a}\overline{d} + \overline{bd}$ ou $f(a,b,c,d) = \overline{a}b\overline{c} + acd + abd + \overline{bd}$ ou $f(a,b,c,d) = \overline{a}b\overline{c} + b\overline{c}d + acd + \overline{bd}$

Question 3. Draw the K-maps of the following function, optimize the functions and find redundant terms to avoid glitches.

a) $f(a,b,c,d) = \sum_m(0,1,2,6,8,9,10,14)$

2. a) $i_1 = b\bar{c}\bar{d}$, $i_2 = \bar{b}c$, $i_3 = ac$, $i_4 = ab$

	i_1	i_2	i_3	i_4
0010		X		
0011		X		
0100	X			
1010		X	X	
1100	X			X
1101				X

3. a)

ab \ cd	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	0	0	0	1
10	1	1	0	1

$\to c\bar{d}$

$\hookrightarrow \bar{b}\bar{c}$

↳ Pour le terme redondant, on choisit le plus grand groupe

$\Rightarrow f = \bar{b}\bar{c} + c\bar{d} + \bar{b}\bar{d}$

Answer:

ab \ cd	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	0	0	0	1
10	1	1	0	1

$F = c\overline{d} + \overline{b}\overline{c} + \overline{b}\overline{d}$

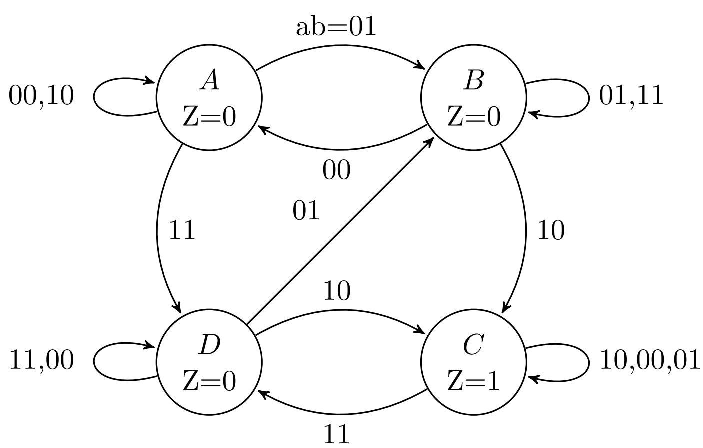
b) $f(a,b,c,d) = \sum_m(1,3,5,7,8,9,12,13)$

Answer:

ab \ cd	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	1	1	0	0
10	1	1	0	0

$F = \overline{a}d + a\overline{c} + \overline{c}d$

Question 4. Build a Huffman table for this graph:



4.

output of system

stable state

ab	00	01	11	10	Z
A	A	B	D	A	0
B	A	B	B	C	0
C	C	C	D	C	1
D	D	B	D	C	0

present state

future state

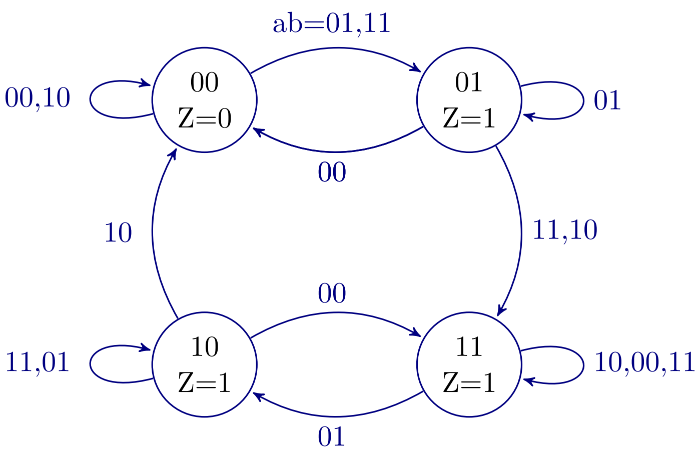
Answer:

	00	01	11	10	ab	Z
A	**A**	B	D	**A**		0
B	A	**B**	**B**	C		0
C	**C**	**C**	D	**C**		1
D	**D**	B	**D**	C		0

Question 5. From this coded Huffman table, find the corresponding state graph and equations.

Y_1 Y_2	00	01	11	10	ab	Z
00	**00**	01	01	**00**		0
01	00	**01**	11	11		1
11	**11**	10	**11**	**11**		1
10	11	**10**	**10**	00		1
y_1 y_2						

Answer:



In order to find the underlying equation, we can deduce Karnaugh tables from the Huffan table.

y_1y_2 \ ab	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	1	0

$$Y_1 = y_2 a + y_1\overline{a} + y_1 b$$

y_1y_2 \ ab	00	01	11	10
00	0	1	1	0
01	0	1	1	1
11	1	0	1	1
10	1	0	0	0

$Y_2 = y_1\overline{ab} + \overline{y_1}b + y_2a$

y_1y_2 \ ab	00	01	11	10
00	0	-	-	0
01	-	1	1	1
11	1	1	1	1
10	1	1	1	-

$Z = y_2 + y_1$

5.

$Y_1\ Y_2$	00	01	11	10	ab	Z
00	**00**	01	01	00		0
01	00	**01**	11	11		1
11	**11**	10	**11**	**11**		1
10	11	**10**	10	00		1
$y_1\ y_2$