SEMINAR LESSON 3

INTRO TO NUMBER THEORY

1) p and q are two different prime numbers. How many divisors has
   a) pq;
   b) p^2q;
   c) p^2q^2;
   d) p^nq^m?
2) Prove that product of any three consecutive natural numbers is divisible by 6.
3) Find the least natural number n such that $n!$ is divisible by 990.
4) Can it happen that $n!$ is end with exactly 5 zeros?
5) How many zeros at the end of $(100!)$?
6) Can it happen that a number which consist of 100 zeros, 100 ones and 100 twosis a full square of some natural number?
7) Find a remainder after dividing
   a) $1989 \cdot 1990 \cdot 1991 + 1992^3$ on 7;
   b) 9^{100} on 8.
8) Prove that $n^3 + 2n$ is divisible by 3 for any natural n.
9) Prove that $n^5 + 4n$ is divisible by 5 for any natural n.
10) Prove that $p^2 - 1$ is divisible by 24 for any prime p, where $p > 3$.
11) x, y, z are natural numbers such that $x^2 + y^2 = z^2$. Prove that at least one ofthese numbers is divisible by 3.
12) Find a last digit of the number 1989^{1989}.
13) Find a last digit of the number 777^{777}.
14) Find a remainder after dividing 2^{300} on 3.
15) Find a remainder after dividing 3^{1989} on 7.
16) Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7.
17) Find a last digit of the number 7^{7^7}.
18) Prove that there exists a natural number n such that the numbers $n + 1, n + 2, \ldots, n + 2018$ are composite.
19) Prove that there exist infinitely many prime numbers.
20) x, y, z are natural numbers such that $x^2 + y^2 = z^2$. Prove that xy is divisibleby 12.

HOME WORK 3

INTRO TO NUMBER THEORY

1) Find a remainder after dividing $2^{70} + 3^{70}$ on 13.
2) Prove that $169 | 3^{3n+3} - 26n - 27$ for any natural n.
3) Prove that $11 \cdot 31 \cdot 61 | 20^{15} - 1$.
4) Find all possible natural n such that $(n+1) | (n^2 + 1)$
5) Prove that there exist infinitely many natural n such that $4n^2 + 1$ is divisible simultaneously on 5 and 13.
6) Find a last digit of the number $1^2 + 2^2 + \cdots + 99^2$.
7) Prove that if $(n-1)! + 1$ is divisible by n then n is a prime number.
8) Find all possible natural solutions of the equation $x^2 - 3y^2 = 8$.
9) Prove that $F_n | 2^{F_n} - 2$ where $F_n = 2^{2^n} + 1$ for any natural n.
10) Prove that $\gcd\left(\frac{a^m - 1}{a-1}, a-1\right) = \gcd(a-1, m)$ for any natural m and $a > 1$.