

## SEMINAR LESSON 10

### Advanced Counting Techniques

- 1) Find a recurrence relation for the number of bit strings of length  $n$  that contain a pair of consecutive 1s. What are the initial conditions?
- 2) Find a recurrence relation for the number of bit strings of length  $n$  that do not contain three consecutive 1s. What are the initial conditions?
- 3) Find a recurrence relation for the number of bit strings of length  $n$  that contain the string 10. What are the initial conditions?
- 4) Find a recurrence relation for the number of ternary strings of length  $n$  that do not contain two consecutive 1s. What are the initial conditions?
- 5) Find a recurrence relation for the number of ways to completely cover a  $2 \times n$  checkerboard with  $1 \times 2$  dominoes.
- 6) A vending machine dispensing books of stamps accepts only \$3 bills and \$5 bills. Find a recurrence relation for the number of ways to deposit  $n$  dollars in the vending machine, where the order in which the bills are deposited does not matter.
- 7) Solve these recurrence relations together with the initial conditions given.
  - a)  $a_n = a_{n-1} + 12a_{n-2}$  for  $n \geq 2$ , and  $a_0 = 5, a_1 = 7$
  - b)  $a_n = 4a_{n-2}$  for  $n \geq 2$ , and  $a_0 = 1, a_1 = -1$
- 8) \*Solve the recurrence relation  $a_n = 8a_{n-1} - 16a_{n-2}$  with the initial conditions  $a_0 = 1$  and  $a_1 = 3$ .
- 9) \*Solve the recurrence relation  $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$  with the initial conditions  $a_0 = 1, a_1 = 2$  and  $a_2 = 5$ .
- 10) \*Solve the recurrence relation  $a_n = 2a_{n-1} + 3n$  with the initial condition  $a_1 = 5$ .