SEMINAR LESSON 1

Operations on sets

1) Prove that if A is the set of roots of the equation $x^2 - 7x + 6 = 0$ and $B = \{1; 6\}$, then $A = B$.

2) Prove that $\emptyset \neq \{\emptyset\}$.

3) Prove that $\{\{1; 2\}; \{2; 3\}\} \neq \{1; 2; 3\}$.

4) Are there sets A, B, C such that $A \cap B \neq \emptyset, A \cap C = \emptyset, (A \cap B)\backslash C = \emptyset$?

5) Prove that the set of all roots of the polynomial $\alpha(x) = \beta(x) \cdot \gamma(x)$ with real coefficients is the union of the sets of roots of the polynomials $\beta(x)$ and $\gamma(x)$.

6) Prove the following identities:
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;
   (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

7) Prove the following identities:
   (a) $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$;
   (b) $A\backslash(A\backslash B) = A \cap B$;

8) Prove the following identities:
   (a) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$;
   (b) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$;

9) Prove that $A \cap B = \emptyset \Leftrightarrow A \cup B = A \oplus B$.

10) Express the operations $\cup, \cap, \backslash$ in terms of:
    (a) $\oplus, \cap$;
    (b) $\oplus, \cup$.

11) Prove that: $\backslash$ is non-expressible in terms of $\cap$ and $\cup$.

12) Find all subsets of the sets $\emptyset, \{\emptyset\}, \{x\}, \{1; 2\}$.

13) (a) Prove that a set of n elements has 2^n subsets;
    (b) How many k-element subsets does a set of n elements contain $(k \leq n)$?

14) Prove that $P(A \cap B) = P(A) \cap P(B)$.

15) Which of the following statements are true for all A, B and C:
    (a) if $A \in B$ and $B \in C$, then $A \in C$?
    (b) if $A \subseteq B$ and $B \in C$, then $A \in C$?
    (c) if $A \cap B \subseteq \bar{C}$ and $A \cup C \subseteq B$, then $A \cap C = \emptyset$?

16) Solve the system of equations $\begin{cases} A \cap X = B, \\ A \cup X = C, \end{cases}$ given sets A, B and C such that $B \subseteq A \subseteq C$.

17) Solve the system of equations $\begin{cases} A\backslash X = B, \\ X\backslash A = C, \end{cases}$ given sets A, B and C such that $B \subseteq A, A \cap C = \emptyset$.

HOME WORK 1

Operations on sets

1) Prove that the intersection of the sets of real roots of polynomials $\alpha(x)$ and $\beta(x)$ with real coefficients coincides with the set of all real roots of the polynomial $\gamma(x) = \alpha^2(x) + \beta^2(x)$.

2) Prove the following identities:
   (a) $A \backslash B = A \backslash (A \cap B)$;
   (b) $A \cap (B \backslash C) = (A \cap B) \backslash (A \cap C) = (A \cap B) \backslash C$.

3) Prove the following identities:
   (a) $A \oplus (A \oplus B) = B$;
   (b) $A \cup B = (A \oplus B) \oplus (A \cap B)$.

4) Express the operations $\cup$, $\cap$, $\backslash$ in terms of: $\backslash$, $\oplus$.

5) Prove that: $\cup$ is non-expressible in terms of $\cap$ and $\backslash$.

6) Which of the following statements are true for all A, B and C:
   (a) if $A \neq B$ and $B \neq C$, then $A \neq C$?
   (b) if $A \subseteq \overline{B \cup C}$ and $B \subseteq \overline{A \cup C}$, then $B = \emptyset$?

7) Solve the system of equations $\begin{cases} A \backslash X = B, \\ A \cup X = C, \end{cases}$ given sets A, B and C such that $B \subseteq A \subseteq C$.