Exercises for seminar

1) Describe in English the languages expressed by the following regular expressions:
   (a) $(0^*1^*)^*0$
   (b) $(01^*)^*0$
   (c) $\big(00 + 11 + (01 + 10)(00 + 11)^*(01 + 10)\big)$
   (d) $0^* + (0^*1 + 0^*11)(0^+1 + 0^+11)^*0^*$.
2) Simplify the following regular expressions:
   (a) $(00)^*0 + (00)^*$
   (b) $(0 + 1)(\varepsilon + 00)^+ + (0 + 1)$
   (c) $(0 + \varepsilon)0^*1$.
3) Construct regular expressions for the following languages over alphabet $\{0,1\}$:
   (a) The set of all strings whose fifth symbol from right is 0.
   (b) The set of all strings having either 000 or 111 as a substring.
   (c) The set of all strings having neither 000 nor 111 as a substring.
   (d) The set of all strings having no substring 010.
   (e) The set of all strings having an odd number of 0's.
   (f) The set of all strings having an even number of occurrences of substring 011. (Hint: First find the regular expression for the set of binary strings having no substring 011).
4) Show that $(0^2 + 0^3)^* = (0^2 0^*)^*$.
5) Construct a regular expression for the set of all strings over alphabet $\left\{\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$ that represent correct subtractions. For example, $\begin{array}{r} 1\ 0\ 1\ 1 \\ \underline{-\ \ 0\ 1\ 0\ 1} \\ 0\ 1\ 1\ 0 \end{array}$ implies that string $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}$ is in the set.
6) Show that for any regular language L, $L_{odd} = \{x \in L \mid |x| = odd\}$ and $L_{even} = \{x \in L \mid |x| = even\}$ are regular.
7) Show that if L is a regular language, then $L'' = \{u \mid \exists v, uv \in L\}$ is regular.
8) Suppose $h\colon \Sigma^* \to \Gamma^*$ is a mapping satisfying $h(xy) = h(x)h(y)$ for any $x, y \in \Sigma^*$. Show that if A is a regular set over Σ, then $h(A) = \{h(x) \mid x \in A\}$ is a regular set over Γ. Conversely, if B is a regular set over Γ, then $h^{-1}(B) = \{x \in \Sigma^* \mid h(x) \in B\}$ is a regular set over Σ. (*Hint*: Prove by induction).