Exercises for seminar

1) Let $A = \{grand, \varepsilon\}$ and $B = \{mother, father\}$. What are AB and A^*B?
2) Let A be a language over $\{a, b\}$ and $x \in \{a, b\}^*$. Find necessary and sufficient conditions in terms of x and A for the equation $A^* - \{x\} = A^+$.
3) For each of the following equations, determine whether it is true for all languages A, B or not. Present a proof or a counterexample.
   (a) $(A^R)^* = (A^*)^R$.
   (b) $(A^+)^* = A^*$.
   (c) $(A \cup A^R)^* = A^* \cup (A^*)^R$.
   (d) $A^2 \cup B^2 = (A \cup B)^2$.
   (e) $A^* \cap B^* = (A \cap B)^*$.
4) (a) Show that, for $k \geq 1$, $\cup_{i=0}^{k} A^i = (\{\varepsilon\} \cup A)^k$.
   (b) Show that , for $n \geq 1, (A^*)^n = A^*$.
   (c) Assume that $\varepsilon \notin A$. Show that, for $\geq 1, (A^+)^n = A^n A^*$.
5) Prove the following identities on languages A, B, C, D:
   (a) $A(BA)^* = (AB)^* A$.
   (b) $(A \cup B)^* = (A^* B^*)^*$.
   (c) $A(B \cup C) = AB \cup AC$.
   (d) $(A \cup B)C = AC \cup BC$.
   (e) $A^* B(DA^* B \cup C)^* = (A \cup BC^* D)^* BC^*$.
6) Find the shortest string over alphabet $\{0\}$ which is not in $\{\varepsilon, 0, 0^2, 0^5\}^3$.
7) *Find the general solutions for the equation $xy = yx$ for $x, y \in \{0,1\}^*$.
8) Solve the following language equations for languages $A, B, C \subseteq \{a, b\}^*$:
   $A = \{a\}C \cup \{b\}B$,
   $B = \{\varepsilon\} \cup \{b\}A \cup \{a\}C$,
   $C = \{\varepsilon\} \cup \{a\}A$.