

Graphs

1) Show that every simple graph has two vertices of the same degree.

2) Let $G = \{V, E\}$ be a graph.

Draw a graph with the following specified properties, or explain why no such graph exists.

A graph having $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$,

where $e_1 = (v_1, v_3)$, $e_2 = (v_2, v_4)$, $e_3 = (v_1, v_4)$, $e_4 = (v_3, v_3)$, $e_5 = (v_3, v_5)$.

How many components does G have?

3) Is there a graph on 10 nodes that contains an Eulerian circuit and the sum of the degrees is 34?

4) Each of the 17 scientists exchanges emails with the rest of the colleagues.

Every two scientists exchange emails in one of the three languages: Kazakh, English, or German. Prove that at least three scientists email with each other in the same language. (Generalization of Ramsey's Problem)

5) In a group everyone knows 4 other people. (We assume that acquaintance is mutual.) Show that they can sit down around some round tables in a way that everyone knows his/her two neighbors.

6) Construct a niche overlap graph for six species of birds, where the hermit thrush competes with the robin and with the blue jay, the robin also competes with the mockingbird, the mockingbird also competes with the blue jay, and the nuthatch competes with the hairy woodpecker.

7) Describe a graph model that represents whether each person at a party knows the name of each other person at the party. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

8) If the simple graph G has v vertices and e edges, how many edges does G have?