Class work 1

Task 1. Prove the following identities: (By $A \triangle B := (A \setminus B) \cup (B \setminus A)$ we denote symmetric difference)

(a) $(A \cup B)^c = A^c \cap B^c$;
(b) $(A \cap B)^c = A^c \cup B^c$;
(c) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$;
(d) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$;
(e) $A \setminus (A \setminus B) = A \cap B$;
(f) $A \setminus B = A \setminus (A \cap B)$;
(g) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$;
(h) $A \setminus (B \setminus C) = (A \setminus C) \setminus (B \setminus C)$;
(i) $A \cup B = A \cup (B \setminus A)$;
(j) $(A \cap B) \cup (C \cap D) = (A \cup C) \cap (B \cup C) \cap (A \cup D) \cap (B \cup D)$;
(k) $(A^c \cup B) \cap A = A \cap B$;
(l) $A \cap (B \setminus A) = \emptyset$;
(m) $(A \cap B) \cup (A \cap B^c) = A$;
(n) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$;
(o) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$;
(p) $A \setminus (B \cup C) = (A \setminus B) \setminus C$;
(q) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$;
(r) $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$;
(s) $A \triangle (A \triangle B) = B$;
(t) $A \cup B = (A \triangle B) \triangle (A \cup B)$;
(u) $A \setminus B = A \triangle (A \cap B)$;
(v) $A \cup B = (A \triangle B) \cup (A \cap B)$;
(w) $(A \cup B) \times C = (A \times C) \cup (B \times C)$;
(x) $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$;
(y) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$;
(z) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times C) \cup (A \times D) \cup (B \times D)$.

Task 2. Find $\mathrm{Dom}(R), \mathrm{Ran}(R)$, $R \circ R$, $R \circ R^{-1}$, and $R^{-1} \circ R$

(a) $R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$;
(b) $R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,3),(3,4),(1,3),(2,4)\}$;
(c) $R = \{(1,2),(2,3),(3,4),(4,1),(1,3),(2,4)\}$;
(d) $R = \{(a,a),(b,b),(c,c),(a,b),(b,c),(a,c),(c,a)\}$;
(e) $R = \{(x,y) \mid x,y \in \{1,2,3\},\ x \neq y\}$;
(f) $R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,3),(3,4),(1,3),(2,4)\}$;
(g) $R = \{(1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(1,3),(2,4)\}$;
(h) $R = \{(a,b),(b,c),(c,d),(d,a),(a,c),(b,d)\}$;
(i) $R = \{(1,1),(2,2),(3,3),(1,2),(2,3),(3,1)\}$;
(j) $R = \{(x,y) \mid x,y \in \{1,2,3,4\},\ x \leq y\}$;
(k) $R = \{(x,y) \mid x,y \in \{1,2,3\},\ x - y \text{ is even}\}$;
(l) $R = \{(a,a),(b,b),(c,c),(d,d),(a,b),(b,c),(c,d),(a,c),(b,d)\}$;
(m) $R = \{(1,2),(2,3),(3,1),(1,3),(2,1),(3,2)\}$;
(n) $R = \{(x,y) \mid x,y \in \{1,2,3\},\ x + y \leq 4\}$;
(o) $R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(3,4),(4,3)\}$;

(p) $R = \{(a, b), (b, c), (c, b), (b, a), (a, c), (c, a)\}$;
(q) $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (1, 3), (2, 4), (3, 5)\}$;
(r) $R = \{(x, y) \mid x, y \in \{1, 2, 3, 4\},\ x \neq y\}$;
(s) $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$;
(t) $R = \{(a, b), (b, c), (c, d), (d, e), (a, c), (b, d), (c, e)\}$;
(u) $R = \{(x, y) \mid x, y \in \{1, 2, 3\},\ x = y \text{ or } x + y = 4\}$;
(v) $R = \{(1, 2), (2, 3), (3, 4), (4, 1), (1, 4), (2, 1), (3, 2), (4, 3)\}$;
(w) $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c, b)\}$;
(x) $R = \{(x, y) \mid x, y \in \{1, 2, 3, 4\},\ |x - y| \leq 1\}$;
(y) $R = \{(1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1)\}$;
(z) $R = \{(x, y) \mid x, y \in \{1, 2, 3\},\ x \leq y\}$.

Task 3. Prove the following statements:
(a) R_1 and R_2 are reflexive $\Rightarrow R_1 \cup R_2$ is reflexive;
(b) R_1 and R_2 are reflexive $\Rightarrow R_1 \cap R_2$ is reflexive;
(c) R_1 and R_2 are reflexive $\Rightarrow R_1 \circ R_2$ is reflexive;
(d) R_1 and R_2 are irreflexive $\Rightarrow R_1 \cup R_2$ is irreflexive;
(e) R_1 and R_2 are irreflexive $\Rightarrow R_1 \cap R_2$ is irreflexive;
(f) R_1 and R_2 are symmetric $\Rightarrow R_1 \cup R_2$ is symmetric;
(g) R_1 and R_2 are symmetric $\Rightarrow R_1 \cap R_2$ is symmetric;
(h) R is symmetric $\Rightarrow R^{-1}$ is symmetric;
(i) $R_1 \circ R_2$ is symmetric $\Rightarrow R_1 \circ R_2 = R_2 \circ R_1$;
(j) $R_1 \circ R_2 = R_2 \circ R_1 \Rightarrow R_1 \circ R_2$ is symmetric;
(k) R_1 and R_2 are antisymmetric $\Rightarrow R_1 \cap R_2$ is antisymmetric;
(l) $R_1 \cup R_2$ is antisymmetric $\Rightarrow R_1 \circ R_2^{-1} \subseteq \{(x, x) : x \in A\}$;
(m) $R_1 \circ R_2^{-1} \subseteq \{(x, x) : x \in A\} \Rightarrow R_1 \cup R_2$ is antisymmetric;
(n) R is symmetric and antisymmetric $\Rightarrow R$ is transitive;
(o) R_1 and R_2 are reflexive $\Rightarrow R_1 \cup R_2$ is reflexive;
(p) R_1 and R_2 are reflexive $\Rightarrow R_1 \cap R_2$ is reflexive;
(q) R_1 and R_2 are reflexive $\Rightarrow R_1 \circ R_2$ is reflexive;
(r) R_1 and R_2 are irreflexive $\Rightarrow R_1 \cup R_2$ is irreflexive;
(s) R_1 and R_2 are irreflexive $\Rightarrow R_1 \cap R_2$ is irreflexive;
(t) R_1 and R_2 are symmetric $\Rightarrow R_1 \cup R_2$ is symmetric;
(u) R_1 and R_2 are symmetric $\Rightarrow R_1 \cap R_2$ is symmetric;
(v) R is symmetric $\Rightarrow R^{-1}$ is symmetric;
(w) $R_1 \circ R_2$ is symmetric $\Rightarrow R_1 \circ R_2 = R_2 \circ R_1$;
(x) $R_1 \circ R_2 = R_2 \circ R_1 \Rightarrow R_1 \circ R_2$ is symmetric;
(y) R_1 and R_2 are antisymmetric $\Rightarrow R_1 \cap R_2$ is antisymmetric;
(z) $R_1 \cup R_2$ is antisymmetric $\Rightarrow R_1 \circ R_2^{-1} \subseteq \{(x, x) : x \in A\}$;

Task 4. Assume A and B are finite sets of n and m elements, respectively.
(1) How many relations between the elements of A and B are there?
(2) How many functions from A to B are there?
(3) How many injective and surjective function from A to B are there?