

SEMINAR LESSON 10

Advanced Counting Techniques

- 1) Find a recurrence relation for the number of bit strings of length n that contain a pair of consecutive 1s. What are the initial conditions?
- 2) Find a recurrence relation for the number of bit strings of length n that do not contain three consecutive 1s. What are the initial conditions?
- 3) Find a recurrence relation for the number of bit strings of length n that contain the string 10. What are the initial conditions?
- 4) Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive 1s. What are the initial conditions?
- 5) Find a recurrence relation for the number of ways to completely cover a $2 \times n$ checkerboard with 1×2 dominoes.
- 6) A vending machine dispensing books of stamps accepts only \$3 bills and \$5 bills. Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the bills are deposited does not matter.
- 7) Solve these recurrence relations together with the initial conditions given.
 - a) $a_n = a_{n-1} + 12a_{n-2}$ for $n \geq 2$, and $a_0 = 5, a_1 = 7$
 - b) $a_n = 4a_{n-2}$ for $n \geq 2$, and $a_0 = 1, a_1 = -1$
- 8) *Solve the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ with the initial conditions $a_0 = 1$ and $a_1 = 3$.
- 9) *Solve the recurrence relation $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ with the initial conditions $a_0 = 1, a_1 = 2$ and $a_2 = 5$.
- 10) *Solve the recurrence relation $a_n = 2a_{n-1} + 3n$ with the initial condition $a_1 = 5$.