SEMINAR LESSON 6

Elements of Combinatorics

1) The bag contains balls of two different colors: red and blue. What is the smallest number of balls you need to blindly remove from the bag so that there are obviously two balls of the same color among them?
2) Given 12 integers. Prove that you can choose two of them, the difference of which is divisible by 11.
3) 10 students solved 35 problems at the Olympiad, and it is known that among them there are a student who solved exactly one problem, a student who solved exactly two problems and a student who solved exactly three problems. Prove that there is a student who has solved at least five problems.
4) 51 points were thrown into a square with a side of 1 meter. Prove that some three of them can be covered with a square which side is 20 cm.
5) Prove that among the numbers written only by ones, there is a number that is divisible by 2019.
6) 15 students collected 100 mushrooms. Prove that any two of them have collected the same number of mushrooms.
7) *Prove that among any 10 integers there are several (or maybe one), the sum of which is divisible by 10.

8) Asan has 8 books on mathematics and Aizhan has 6 books on computer science. In how many ways they can exchange three books on mathematics to the three books on computer science?
9) There are 31 people in the class in which Kairat and Timur are studying. How many ways can you choose a football team (11 people) from the class so that Kairat and Timur are not part of the team at the same time?
10) How many ways can you choose 10 cards from a full deck (52 cards) so that there is exactly one ace among them?
11) 30 people vote on 4 proposals. In how many ways can votes be distributed if everyone votes for only one proposal and only the number of votes for each proposal is taken into account?
12) There are 12 books on the shelf. How many ways can you choose 5 books from them, no two of which stand side by side?