

## Graphs

- 1) Sports competition is held on a circular system. This means that each pair of players meets each other exactly once. Prove that at any time there will be at least two players who have held the same number of meetings (*The Concept of a Graph*)
- 2) In the competition on a circular system with twelve participants held all the meetings. How many meetings were played? (*Complete Graph*)
- 3) In the football tournament, 20 teams played 8 rounds: each team played with 8 different teams. Prove that there are three teams that have not played each other yet.
- 4) In some country the airline system is arranged so that any city is connected by airlines to no more than three others and from any city in any other one you can fly making no more than one transfer. What is the largest possible number of cities in this country? (*Petersen's Graph*)
- 5) In the tennis tournament, each player of the "KBTU Rockets" team meets with each player of the "KBTU Sharks" team. The number of players in teams is the same and not more than eight. "KBTU Rockets" won four times more meetings than the "KBTU Sharks ". How many people in each team? (*Bipartite Graph*)
- 6) Each student of the specialty "MCM" is a friend with three students of the specialty "IS", and each student of the specialty "IS" is a friend with three students of the specialty "MCM." Prove that the numbers of students in these specialties are the same. (*Regular Graph*)
- 7) \*In each of the three universities 300 students study. Any student has in total of 301 acquaintances from two other universities. Prove that you can select one student from each university so that the selected three students are familiar with each other.
- 8) A summer Arman rested at the youth camp "Little Britain", where he was accompanied by another 52 students. After the rest, some pairs of students exchanged phone numbers, and each of the students had at least 26 numbers. After a while, Arman needed the number of Aizhan, with whom he did not exchange the numbers. Prove that Arman can find the number of Aizhan, i.e. there is a chain of students that begins with Arman and ends with Aizhan in which each neighbor pair exchanged numbers. (*Connected Graph*)
- 9) Aizhan rested at the youth camp "Little Britain", where together with her there were 45 students. After the rest, 950 pairs of students exchanged phone numbers. After a while, Aizhan needed Arman's number, with whom she did

- not exchange numbers. Prove that with the help of students rested in the camp, Aizhan can find Arman's number (*see the previous problem*).
- 10) Prove that in a group of six people there are always either three people who are familiar with each other or three people who are not familiar with each other. (*Ramsey's Problem*)
  - 11) Each of the 17 scientists exchanges emails with the rest of the colleagues. Every two scientists exchange emails in one of the three languages: Kazakh, English, or German. Prove that at least three scientists email with each other in the same language. (*Generalization of Ramsey's Problem*)
  - 12) \*At a company any two acquaintances do not have common acquaintances, and any two non-acquaintances have exactly two common acquaintances. Prove that in this company all persons have the same number of acquaintances.
  - 13) \*The lecture group has 50 students. It is known that among any four students there is at least one student familiar with three others. Prove that there is a student familiar with all other students.
  - 14) \*11 people participate in a chess tournament. Currently, among any three participants at least two did not play with each other. Prove that no more than 30 games have been played.
  - 15) Can a country, where exactly three roads out of each city, have exactly 100 roads between cities? (*Handshaking Lemma*)
  - 16) Cities of a country are connected by airlines. 21 airlines are started from Capital City, one starts from Miracle City, and twelve airlines start from each of the other cities. Prove that you can arrive at Miracle City from Capital City, possibly with transfers. (*Connected Components*)