Class work 3

Task 1.

(a) Find Bezout's identity for the pair 9876 and 5432.

(b) Find Bezout's identity for the pair 12345 and 6789.

(c) Find Bezout's identity for the pair 8640 and 3780.

(d) Find Bezout's identity for the pair 9991 and 4620.

(e) Find Bezout's identity for the pair 15750 and 9240.

(f) Find Bezout's identity for the pair 2021 and 2897.

(g) Find Bezout's identity for the pair 123456 and 7890.

(h) Find Bezout's identity for the pair 4567 and 3210.

(i) Find Bezout's identity for the pair 31415 and 27182.

(j) Find Bezout's identity for the pair 65536 and 4096.

(k) Find the last digit of decimal form of the number 9^{100}

(l) Find the last digit of decimal form of the number 13^{219}

(m) Find the last digit of decimal form of the number 17^{300}

(n) Find the last digit of decimal form of the number 243^{402}

(o) Find the last digit of decimal form of the number 473^{2026}

(p) Find the last two digits of decimal form of the number 9^{100}

(q) Find the last two digits of decimal form of the number 13^{219}

(r) Find the last two digits of decimal form of the number 17^{300}

(s) Find the last two digits of decimal form of the number 243^{402}

(t) Find the last two digits of decimal form of the number 473^{2026}

(u) Find n if $\varphi(11^n) = 13310$

(v) Find n if $\varphi(7^n) = 705894$

(w) Find all pairs of positive integers (n, m) if $\varphi(2^n 3^m) = 192$.

(x) Find all pairs of positive integers (n, m) if $\varphi(5^n 7^m) = 84000$.

(y) Find all pairs of positive integers (n, m) if $\varphi(3^n 11^m) = 14580$.

(z) Find all pairs of positive integers (n, m) if $\varphi(2^n 5^m) = 4000$.

Task 2. Solve the following equations on integers

(a) $53x - 17y = 25$;

(b) $47x + 105y = 4$;

(c) $18x + 33y = 112$;

(d) $11x + 16y = 156$;

(e) $35x + 16y = 2$;

(f) $64x - 21y = 5$;

(g) $72x + 45y = 9$;

(h) $91x - 35y = 7$;

(i) $84x + 66y = 30$;

(j) $55x - 34y = 1$;

(k) $120x + 77y = 11$;

(l) $143x - 52y = 13$;

(m) $98x + 63y = 14$;

(n) $37x - 29y = 3$;

(o) $81x + 54y = 27$;

(p) $101x - 38y = 19$;

(q) $132x + 48y = 12$;

(r) $75x - 28y = 4$;

(s) $116x + 87y = 29$;

(t) $49x - 18y = 5$;

(u) $90x + 35y = 25$;

(v) $62x - 27y = 8$;

(w) $104x + 39y = 13$;

(x) $73x - 41y = 6$;

(y) $189x + 64y = 17$;

(z) $150x + 84y = 18$.

Task 3. Solve the following systems of congruences with coefficients using the Chinese Remainder Theorem.

(a) $\begin{cases} 3x \equiv 4 \pmod 7, \\ 5x \equiv 2 \pmod{11}, \\ 7x \equiv 3 \pmod{13} \end{cases}$

(b) $\begin{cases} 2x \equiv 3 \pmod 5, \\ 4x \equiv 7 \pmod 9, \\ 6x \equiv 11 \pmod{16} \end{cases}$

(c) $\begin{cases} 5x \equiv 9 \pmod 8, \\ 7x \equiv 4 \pmod{15}, \\ 3x \equiv 6 \pmod{17} \end{cases}$

(d) $\begin{cases} 6x \equiv 1 \pmod 6, \\ 5x \equiv 4 \pmod 7, \\ 8x \equiv 13 \pmod{19} \end{cases}$

(e) $\begin{cases} 7x \equiv 5 \pmod{12}, \\ 9x \equiv 8 \pmod{17}, \\ 11x \equiv 9 \pmod{23} \end{cases}$

(f) $\begin{cases} 3x \equiv 2 \pmod 9, \\ 5x \equiv 7 \pmod{14}, \\ 7x \equiv 11 \pmod{20} \end{cases}$

(g) $\begin{cases} 4x \equiv 3 \pmod 5, \\ 6x \equiv 10 \pmod{18}, \\ 8x \equiv 7 \pmod{11} \end{cases}$

(h) $\begin{cases} 5x \equiv 6 \pmod 8, \\ 7x \equiv 13 \pmod{15}, \\ 9x \equiv 4 \pmod{17} \end{cases}$

(i) $\begin{cases} 6x \equiv 9 \pmod{16}, \\ 8x \equiv 5 \pmod{21}, \\ 7x \equiv 7 \pmod{13} \end{cases}$

(j) $\begin{cases} 3x \equiv 2 \pmod 7, \\ 5x \equiv 3 \pmod{11}, \\ 4x \equiv 6 \pmod{19} \end{cases}$

(k) $\begin{cases} 2x \equiv 1 \pmod 5, \\ 3x \equiv 7 \pmod{12}, \\ 4x \equiv 11 \pmod{17}, \\ 5x \equiv 9 \pmod{23} \end{cases}$

(l) $\begin{cases} 5x \equiv 8 \pmod 9, \\ 7x \equiv 14 \pmod{20}, \\ 3x \equiv 5 \pmod{21}, \\ 4x \equiv 12 \pmod{16} \end{cases}$

(m) $\begin{cases} 3x \equiv 3 \pmod 7, \\ 5x \equiv 11 \pmod{13}, \\ 7x \equiv 17 \pmod{19}, \\ 9x \equiv 9 \pmod{23} \end{cases}$

(n) $\begin{cases} 4x \equiv 5 \pmod 8, \\ 6x \equiv 13 \pmod{15}, \\ 8x \equiv 21 \pmod{22}, \\ 10x \equiv 11 \pmod{27} \end{cases}$

(o) $\begin{cases} 3x \equiv 4 \pmod 6, \\ 5x \equiv 7 \pmod{11}, \\ 7x \equiv 19 \pmod{20}, \\ 9x \equiv 13 \pmod{17} \end{cases}$

(p) $\begin{cases} 2x \equiv 6 \pmod 9, \\ 3x \equiv 12 \pmod{14}, \\ 4x \equiv 5 \pmod{15}, \\ 5x \equiv 8 \pmod{11} \end{cases}$

(q) $\begin{cases} 3x \equiv 7 \pmod{10}, \\ 5x \equiv 5 \pmod{21}, \\ 7x \equiv 14 \pmod{22}, \\ 9x \equiv 9 \pmod{13} \end{cases}$

(r) $\begin{cases} 2x \equiv 2 \pmod{5}, \\ 3x \equiv 11 \pmod{16}, \\ 4x \equiv 17 \pmod{19}, \\ 5x \equiv 23 \pmod{29} \end{cases}$

(s) $\begin{cases} 3x \equiv 3 \pmod{7}, \\ 5x \equiv 13 \pmod{18}, \\ 7x \equiv 5 \pmod{11}, \\ 9x \equiv 9 \pmod{20} \end{cases}$

(t) $\begin{cases} 2x \equiv 6 \pmod{9}, \\ 3x \equiv 7 \pmod{10}, \\ 4x \equiv 8 \pmod{11}, \\ 5x \equiv 9 \pmod{12} \end{cases}$

(u) $\begin{cases} 3x \equiv 5 \pmod{8}, \\ 4x \equiv 14 \pmod{15}, \\ 5x \equiv 23 \pmod{26}, \\ 6x \equiv 12 \pmod{17} \end{cases}$

(v) $\begin{cases} 2x \equiv 4 \pmod{9}, \\ 3x \equiv 11 \pmod{16}, \\ 4x \equiv 18 \pmod{21}, \\ 5x \equiv 7 \pmod{13} \end{cases}$

(w) $\begin{cases} 3x \equiv 3 \pmod{5}, \\ 4x \equiv 8 \pmod{12}, \\ 5x \equiv 15 \pmod{17}, \\ 6x \equiv 10 \pmod{19} \end{cases}$

(x) $\begin{cases} 2x \equiv 2 \pmod{6}, \\ 3x \equiv 9 \pmod{14}, \\ 4x \equiv 11 \pmod{15}, \\ 5x \equiv 5 \pmod{16} \end{cases}$

(y) $\begin{cases} 3x \equiv 7 \pmod 8, \\ 4x \equiv 13 \pmod{15}, \\ 5x \equiv 19 \pmod{21}, \\ 6x \equiv 11 \pmod{22} \end{cases}$

(z) $\begin{cases} 2x \equiv 6 \pmod 9, \\ 3x \equiv 14 \pmod{20}, \\ 4x \equiv 5 \pmod{17}, \\ 5x \equiv 8 \pmod{13} \end{cases}$