```
import numpy as np
from matplotlib import pyplot as plt
from sklearn.datasets import fetch_openml
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels \
    import RBF, WhiteKernel, RationalQuadratic, ExpSineSquared
print(__doc__)
def load_mauna_loa_atmospheric_co2():
    ml_data = fetch_openml(data_id=41187, as_frame=False)
    months = []
    ppmv_sums = []
    counts = []
    y = ml_data.data[:, 0]
    m = ml_data.data[:, 1]
    month_float = y + (m - 1) / 12
    ppmvs = ml_data.target
    for month, ppmv in zip(month_float, ppmvs):
        if not months or month != months[-1]:
            months.append(month)
            ppmv_sums.append(ppmv)
            counts.append(1)
        else:
            # aggregate monthly sum to produce average
            ppmv_sums[-1] += ppmv
            counts[-1] += 1
    months = np.asarray(months).reshape(-1, 1)
    avg_ppmvs = np.asarray(ppmv_sums) / counts
    return months, avg_ppmvs
X, y = load_mauna_loa_atmospheric_co2()
# Kernel with parameters given in GPML book
k1 = 66.0**2 * RBF(length_scale=67.0) # long term smooth rising trend
k2 = 2.4**2 * RBF(length_scale=90.0) \
    * ExpSineSquared(length_scale=1.3, periodicity=1.0) # seasonal component
# medium term irregularity
k3 = 0.66**2 \
    * RationalQuadratic(length_scale=1.2, alpha=0.78)
k4 = 0.18**2 * RBF(length scale=0.134) \
    + WhiteKernel(noise_level=0.19**2) # noise terms
kernel_gpml = k1 + k2 + k3 + k4
```

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                                              20191ISE0017.jpynb - Colaboratory
   gp = GaussianProcessRegressor(kernel=kernel gpml, alpha=0,
                                  optimizer=None, normalize y=True)
   gp.fit(X, y)
   print("GPML kernel: %s" % gp.kernel_)
   print("Log-marginal-likelihood: %.3f"
         % gp.log_marginal_likelihood(gp.kernel_.theta))
   # Kernel with optimized parameters
   k1 = 50.0**2 * RBF(length_scale=50.0) # long term smooth rising trend
   k2 = 2.0**2 * RBF(length_scale=100.0) \
       * ExpSineSquared(length_scale=1.0, periodicity=1.0,
                        periodicity_bounds="fixed") # seasonal component
   # medium term irregularities
   k3 = 0.5**2 * RationalQuadratic(length_scale=1.0, alpha=1.0)
   k4 = 0.1**2 * RBF(length_scale=0.1) \
       + WhiteKernel(noise_level=0.1**2,
                     noise_level_bounds=(1e-5, np.inf)) # noise terms
   kernel = k1 + k2 + k3 + k4
   gp = GaussianProcessRegressor(kernel=kernel, alpha=0,
                                 normalize_y=True)
   gp.fit(X, y)
   print("\nLearned kernel: %s" % gp.kernel_)
   print("Log-marginal-likelihood: %.3f"
         % gp.log_marginal_likelihood(gp.kernel_.theta))
   X_= np.linspace(X.min(), X.max() + 30, 1000)[:, np.newaxis]
   y_pred, y_std = gp.predict(X_, return_std=True)
   # Illustration
   plt.scatter(X, y, c='k')
   plt.plot(X_, y_pred)
   plt.fill_between(X_[:, 0], y_pred - y_std, y_pred + y_std,
                    alpha=0.5, color='k')
   plt.xlim(X_.min(), X_.max())
   plt.xlabel("Year")
   plt.ylabel(r"CO$_2$ in ppm")
   plt.title(r"Atmospheric CO$_2$ concentration at Mauna Loa")
   plt.tight_layout()
   plt.show()
    D.
```

Automatically created module for IPython interactive environment GPML kernel: 66**2 * RBF(length_scale=67) + 2.4**2 * RBF(length_scale=90) * ExpSineSquar Log-marginal-likelihood: -117.023

Learned kernel: $44.8**2 * RBF(length_scale=51.6) + 2.64**2 * RBF(length_scale=91.5) * Example Log-marginal-likelihood: -115.050$

