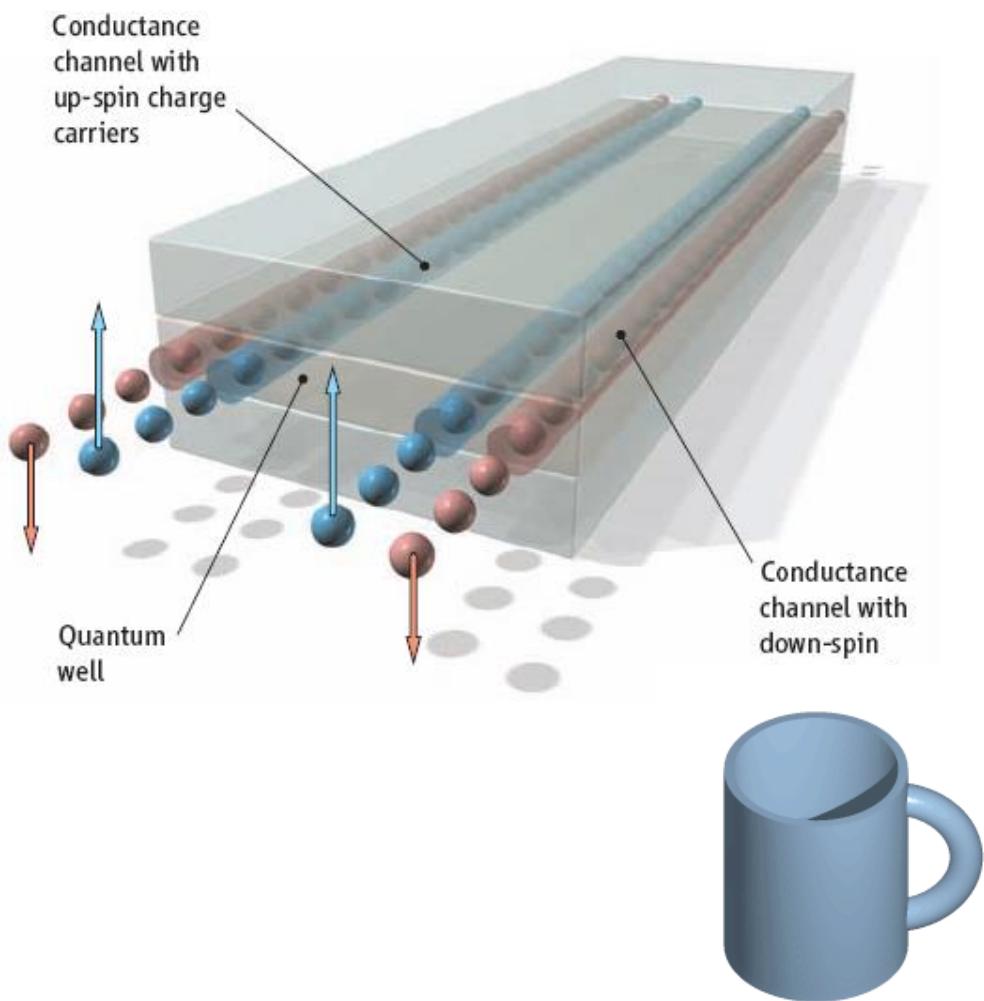


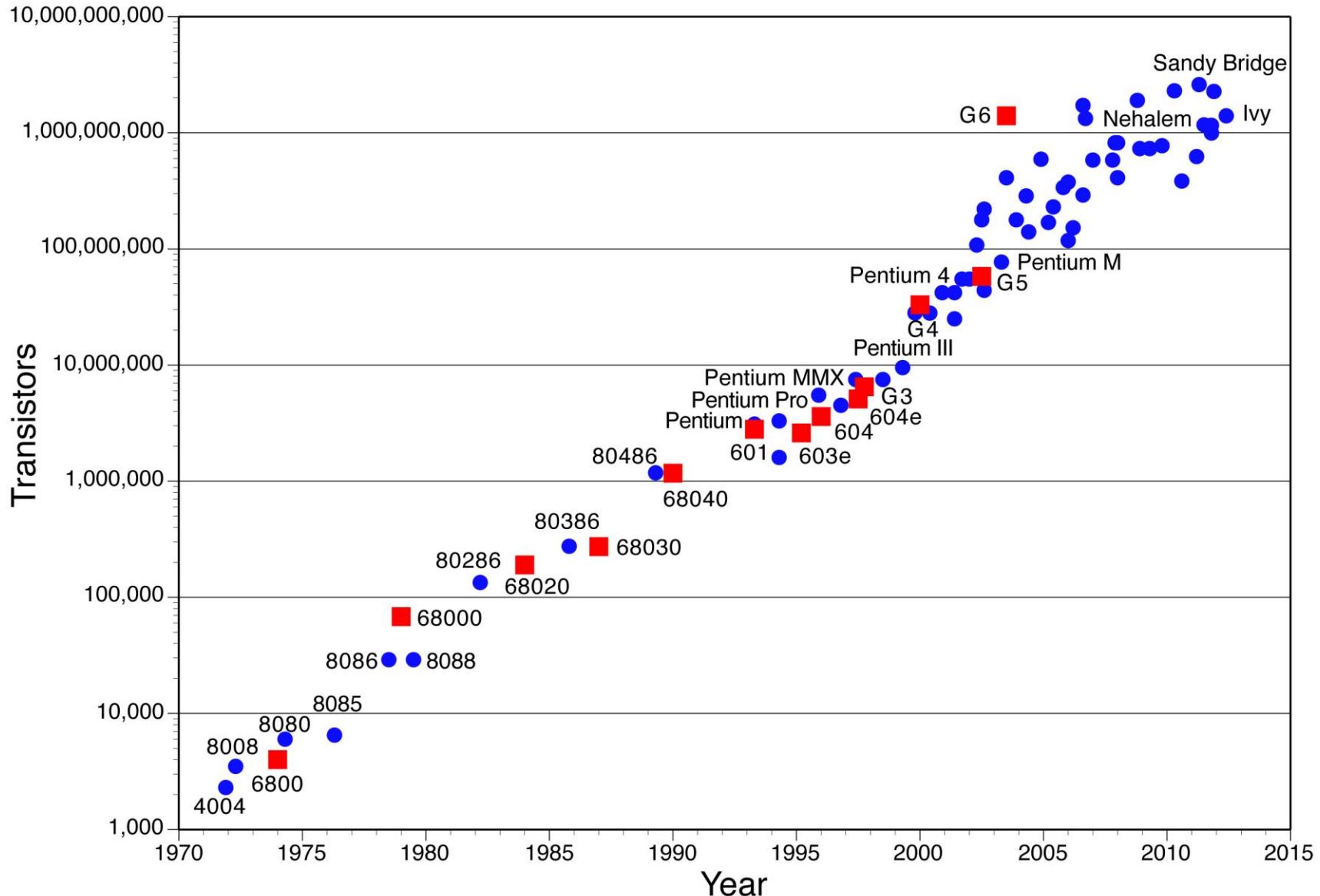
# Science and applications of topological insulators



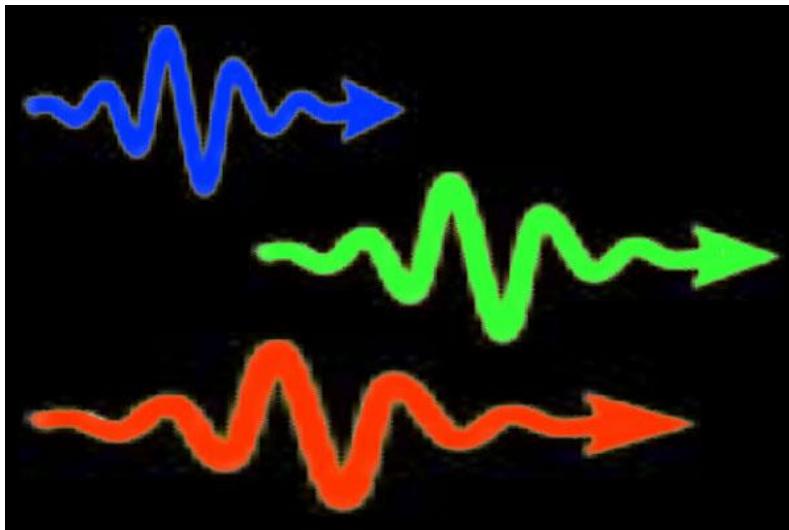
CAS 2014

Shoucheng Zhang, Tsinghua and Stanford University

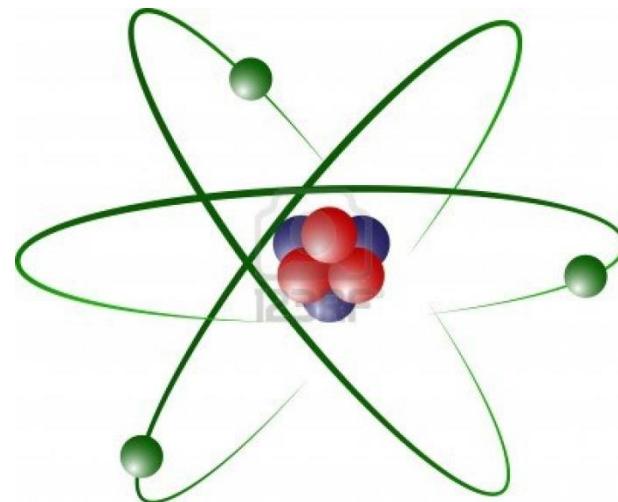
# Moore's law of the information age



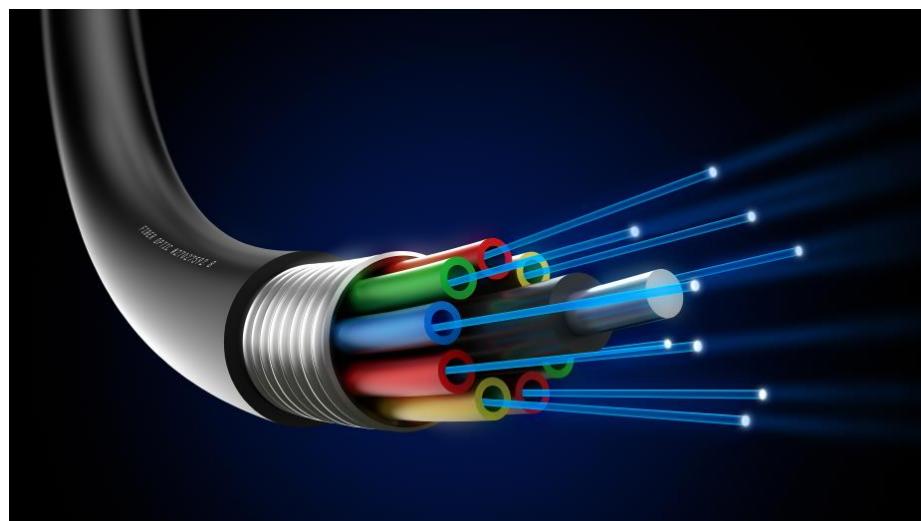
# Two elementary particles of the information age



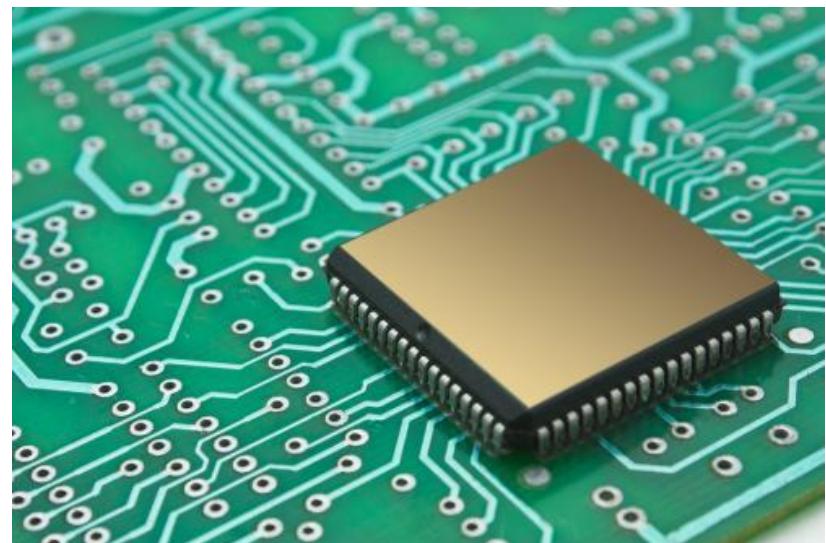
Photon



Electron

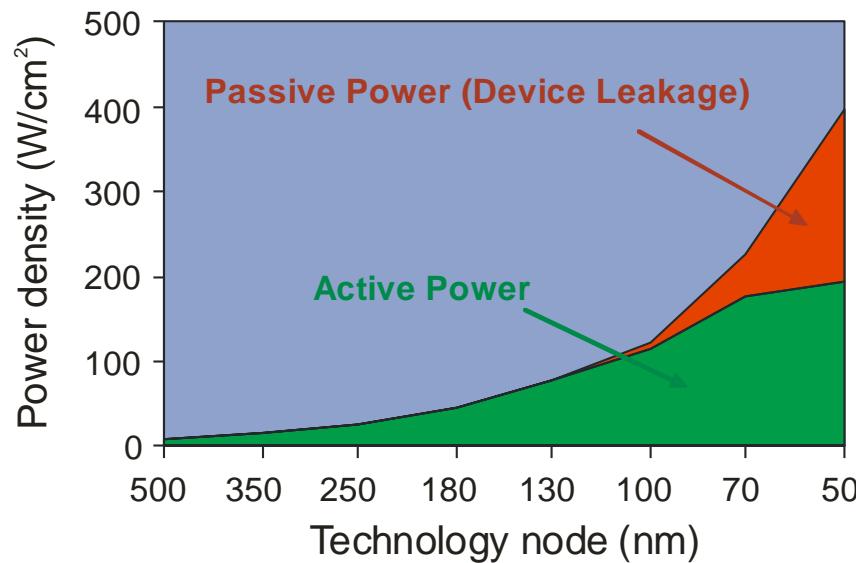


Fiber optical communication



Computing chips

# From traffic jam to info-superhighway

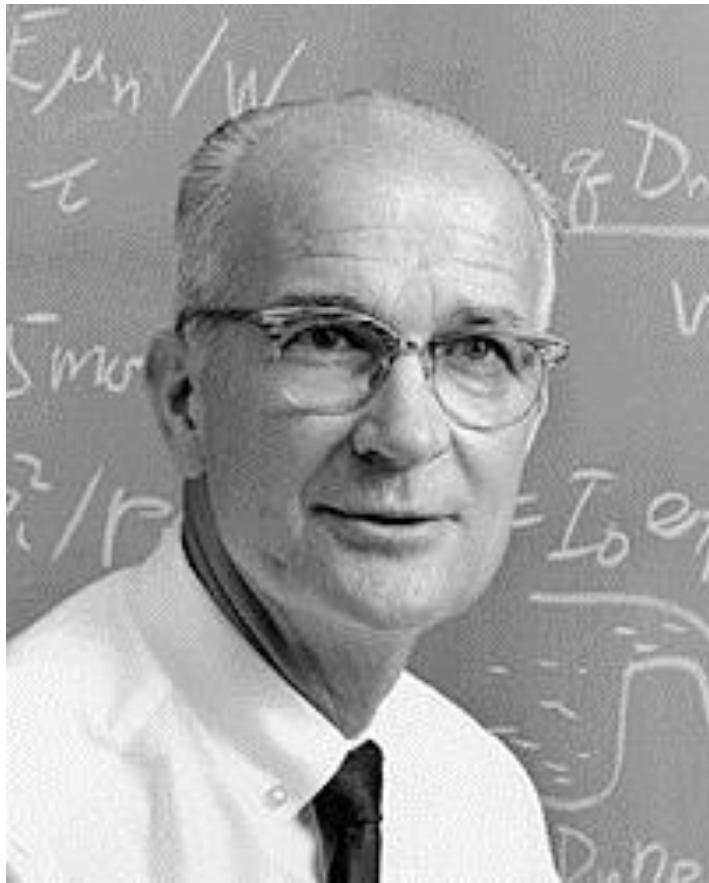


Traffic jam inside chips today

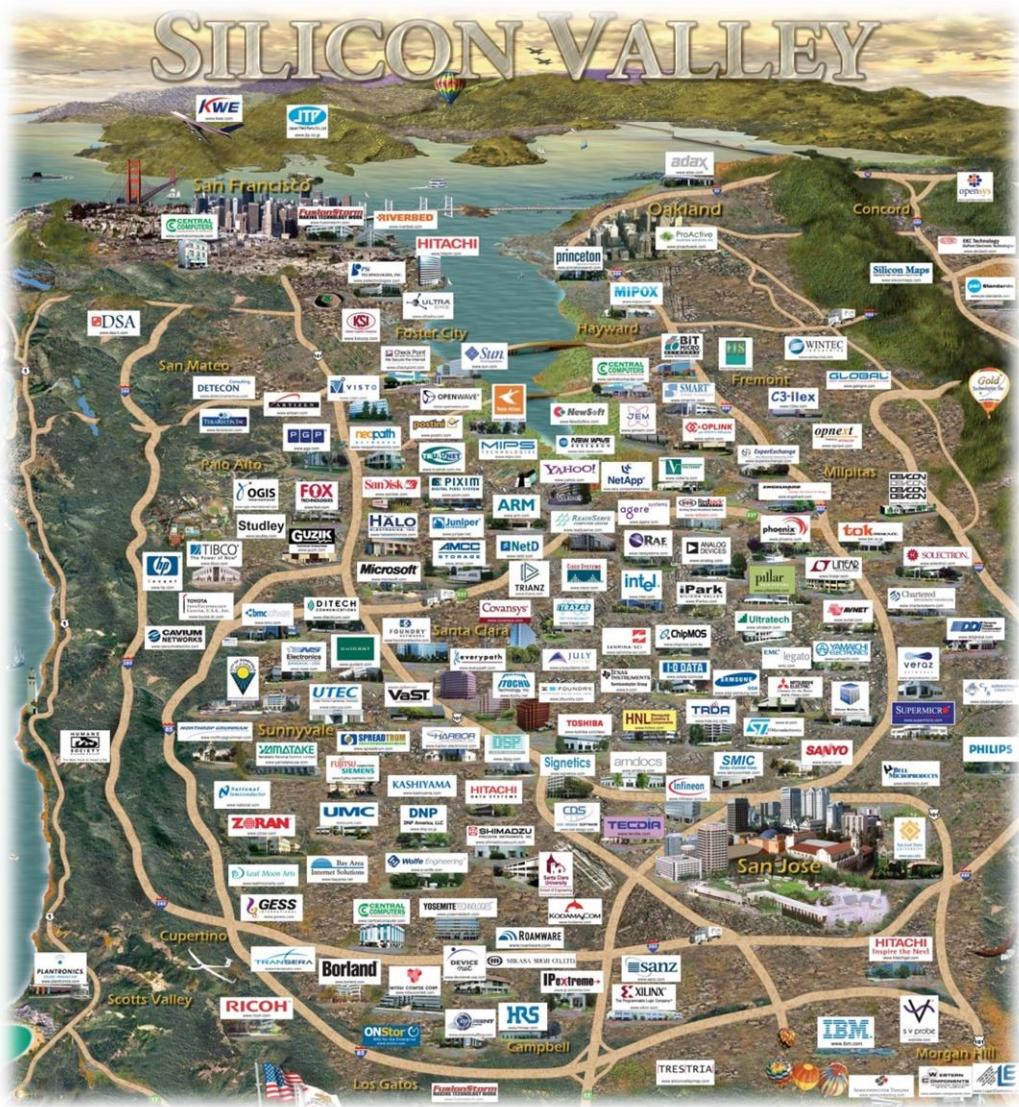


Info highways for the chips in the future

# Crisis and opportunity for basic science and the silicon valley



危 机



# Search for a new electronic states of matter

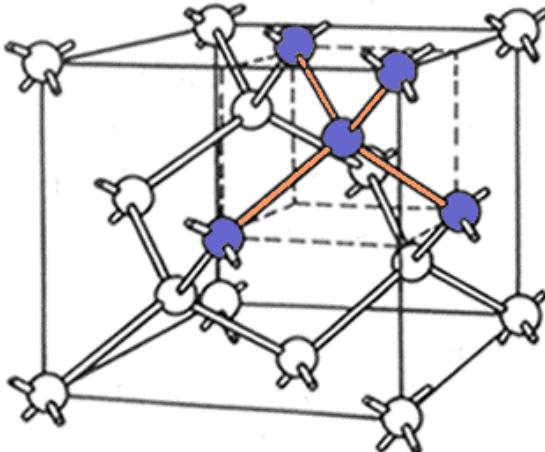
The search for new elements led to a golden age of chemistry.

The search for new particles led to the golden age of particle physics.

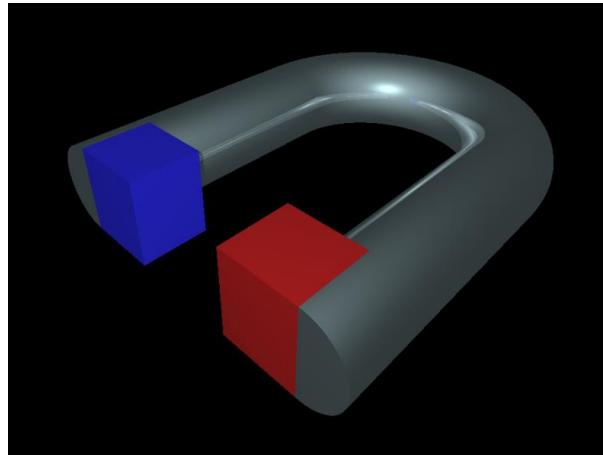
Complex states of matter from the simplicity of the building blocks!

In the classical world we have solid, liquid and gas. The same H<sub>2</sub>O molecules can condense into ice, water or vapor.

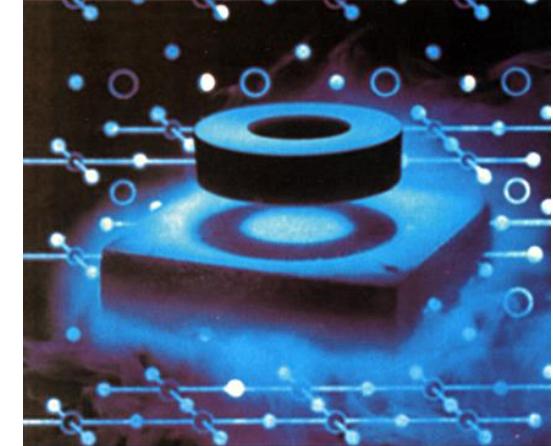
In the quantum world we have metals, insulators, superconductors, magnets etc.



Semiconductors

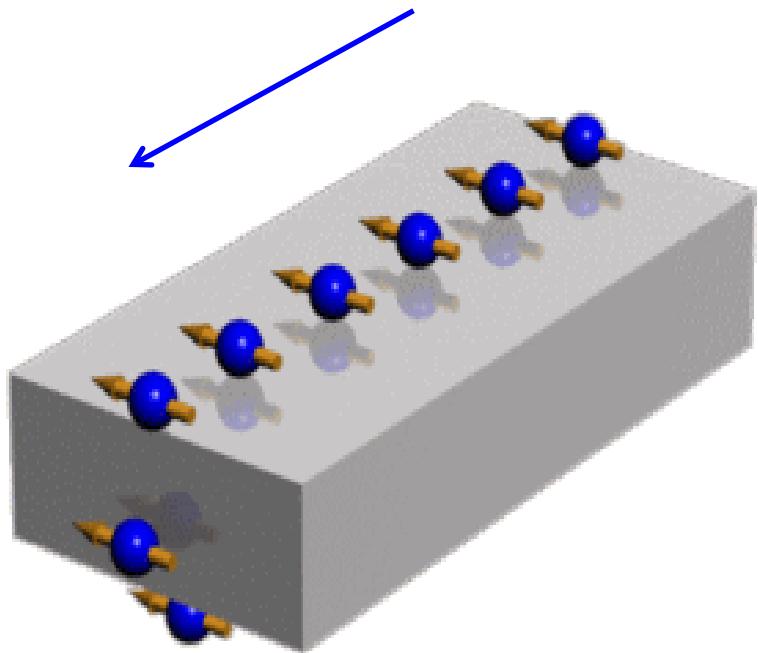
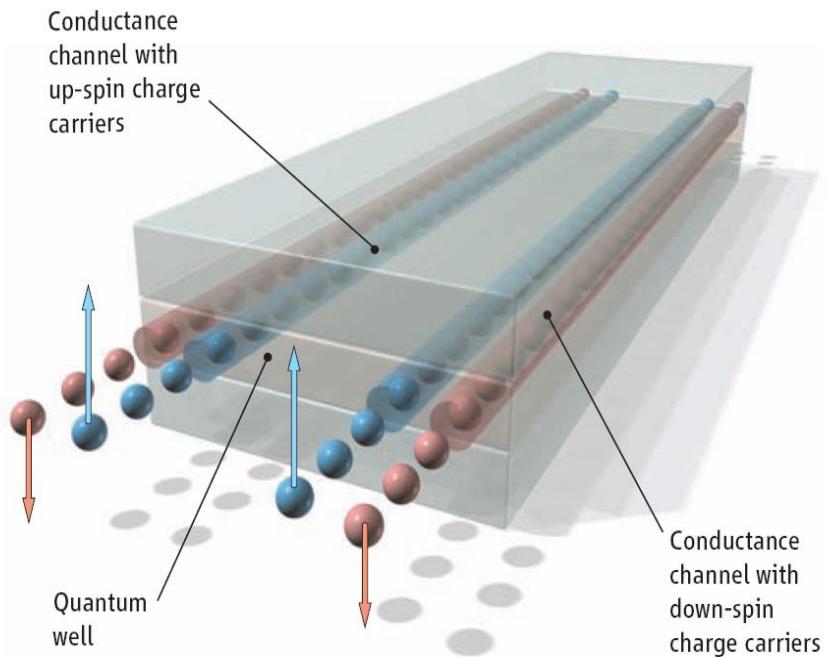


Magnets



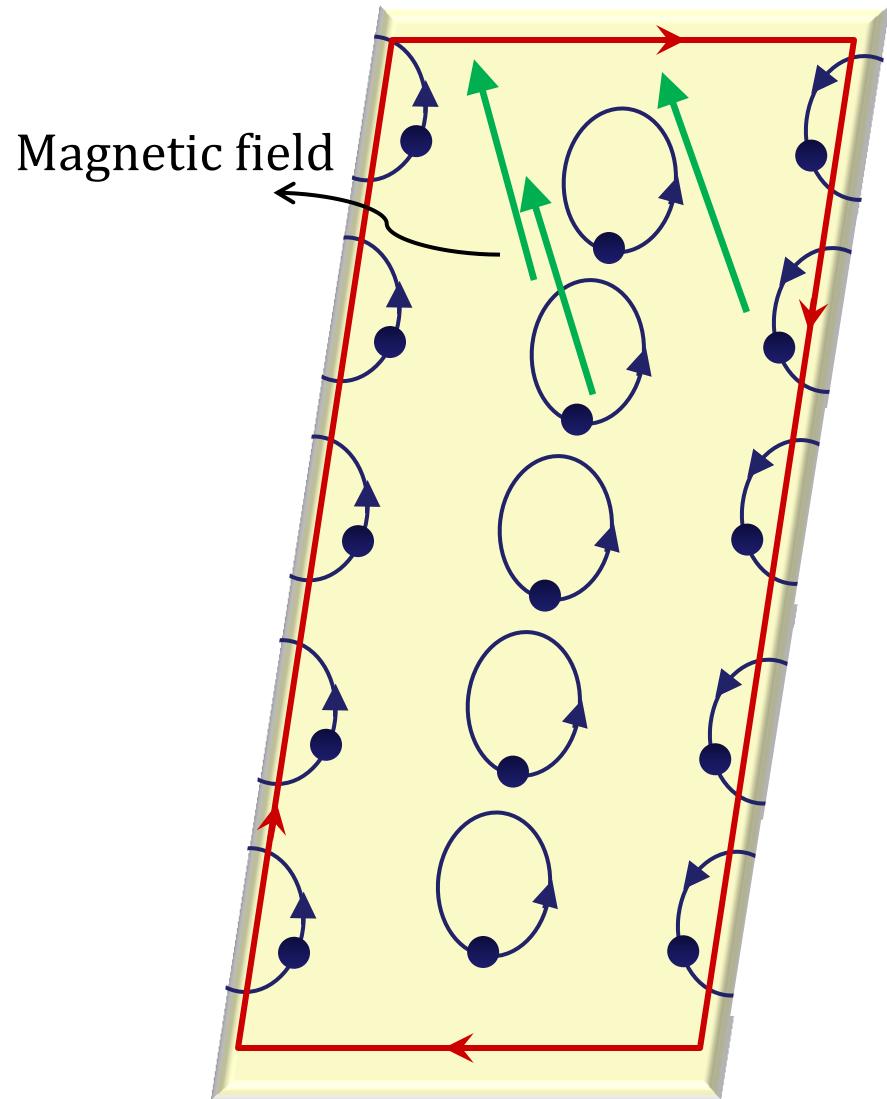
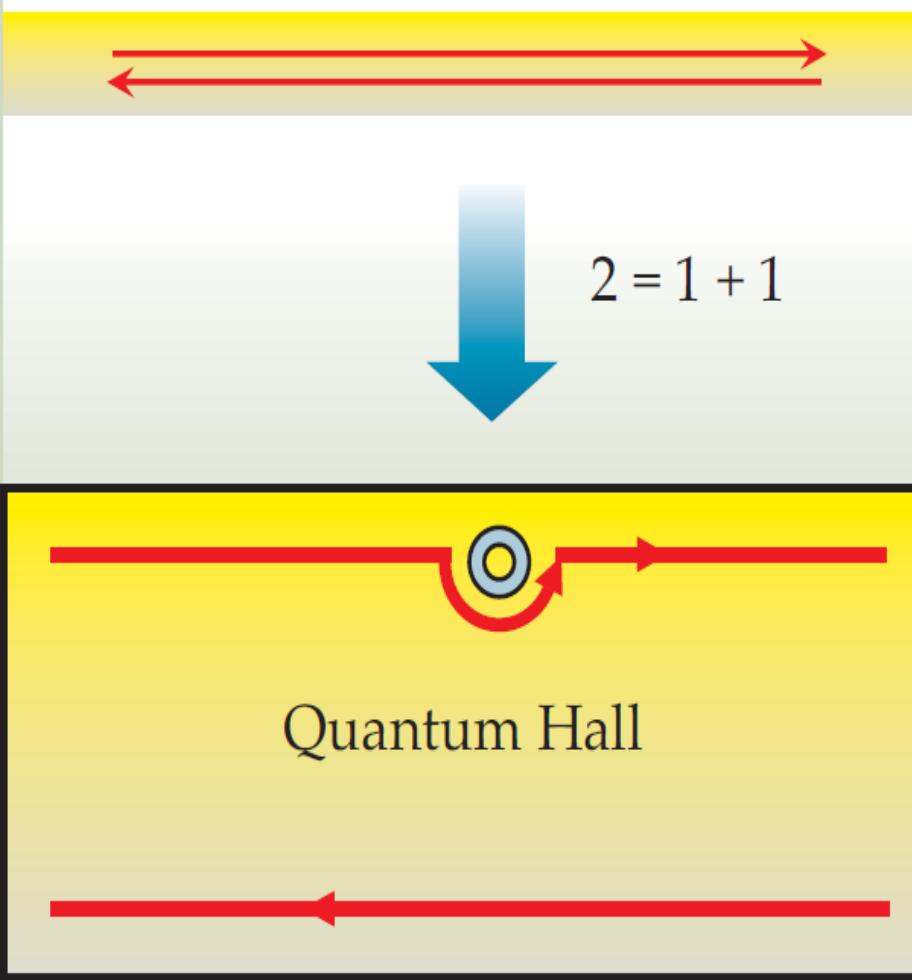
Superconductor

# Discovery of topological insulator materials

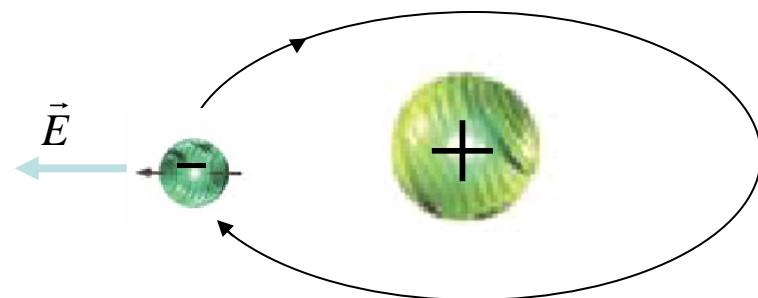
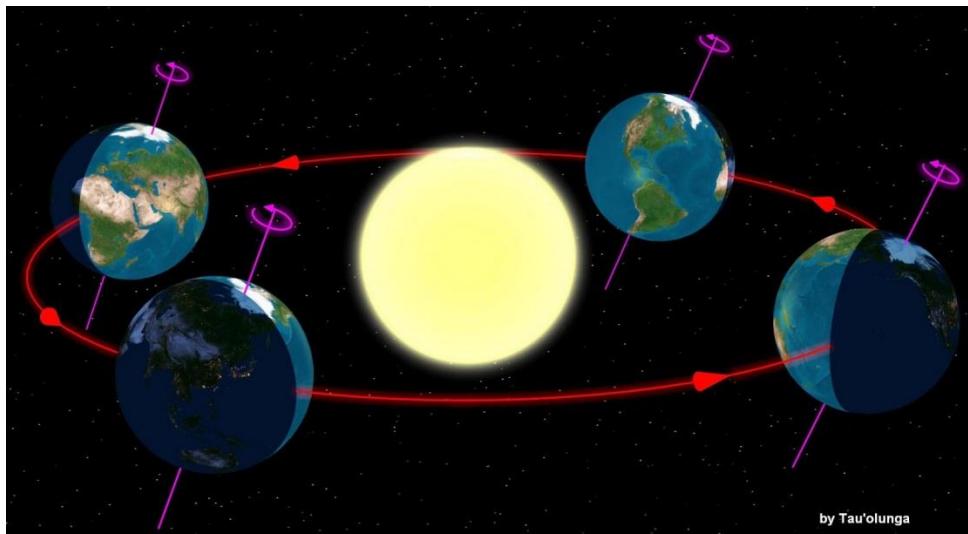
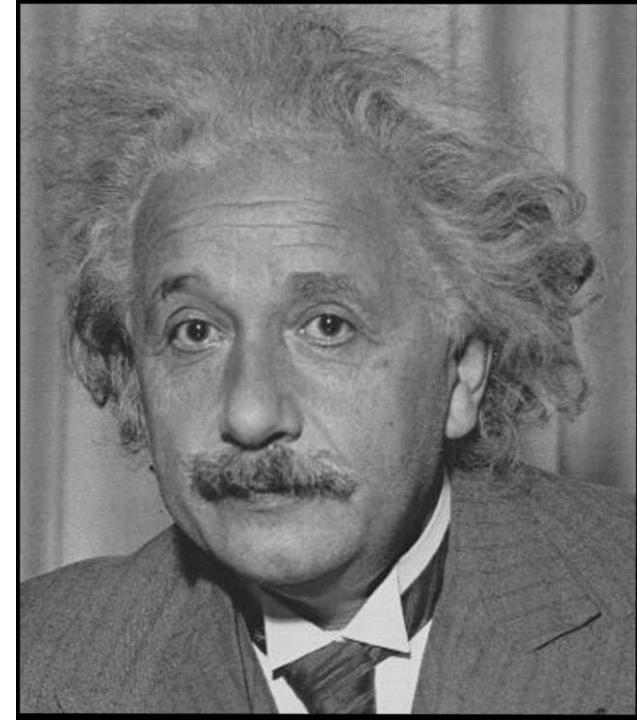
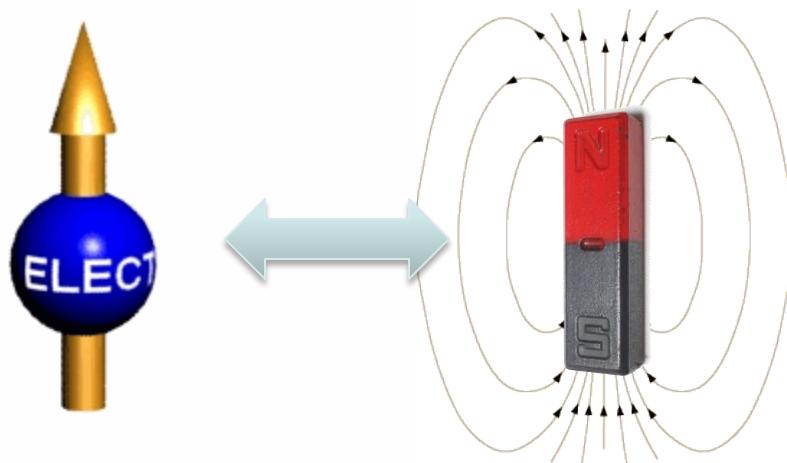


Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

# How to build a highway for the electrons?

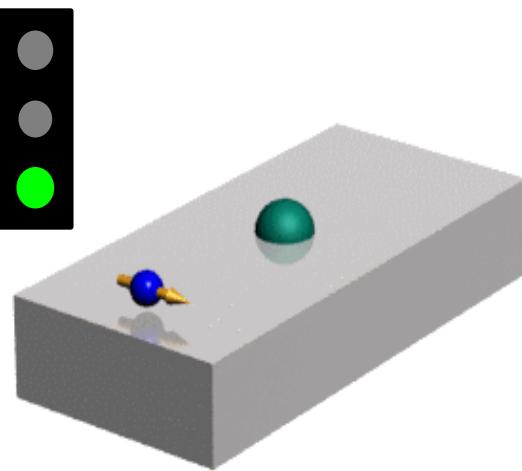
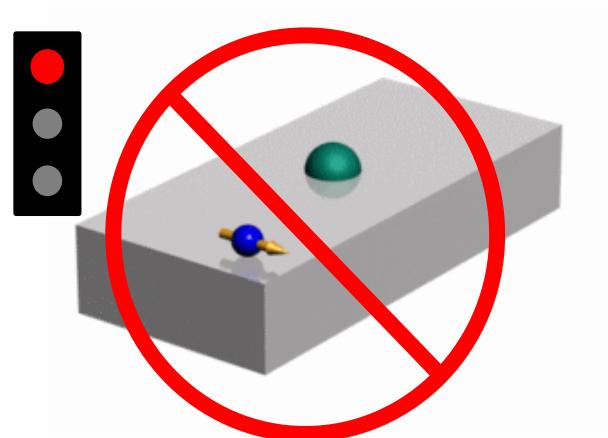
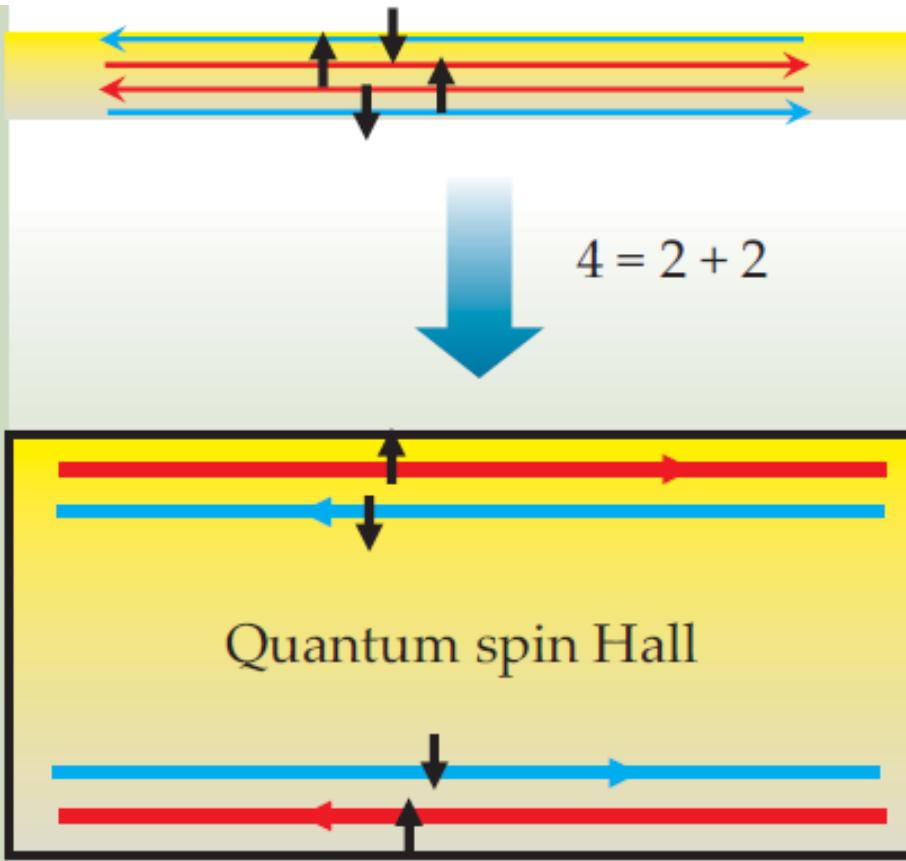


# Electrons have spin, and spin-orbit coupling!



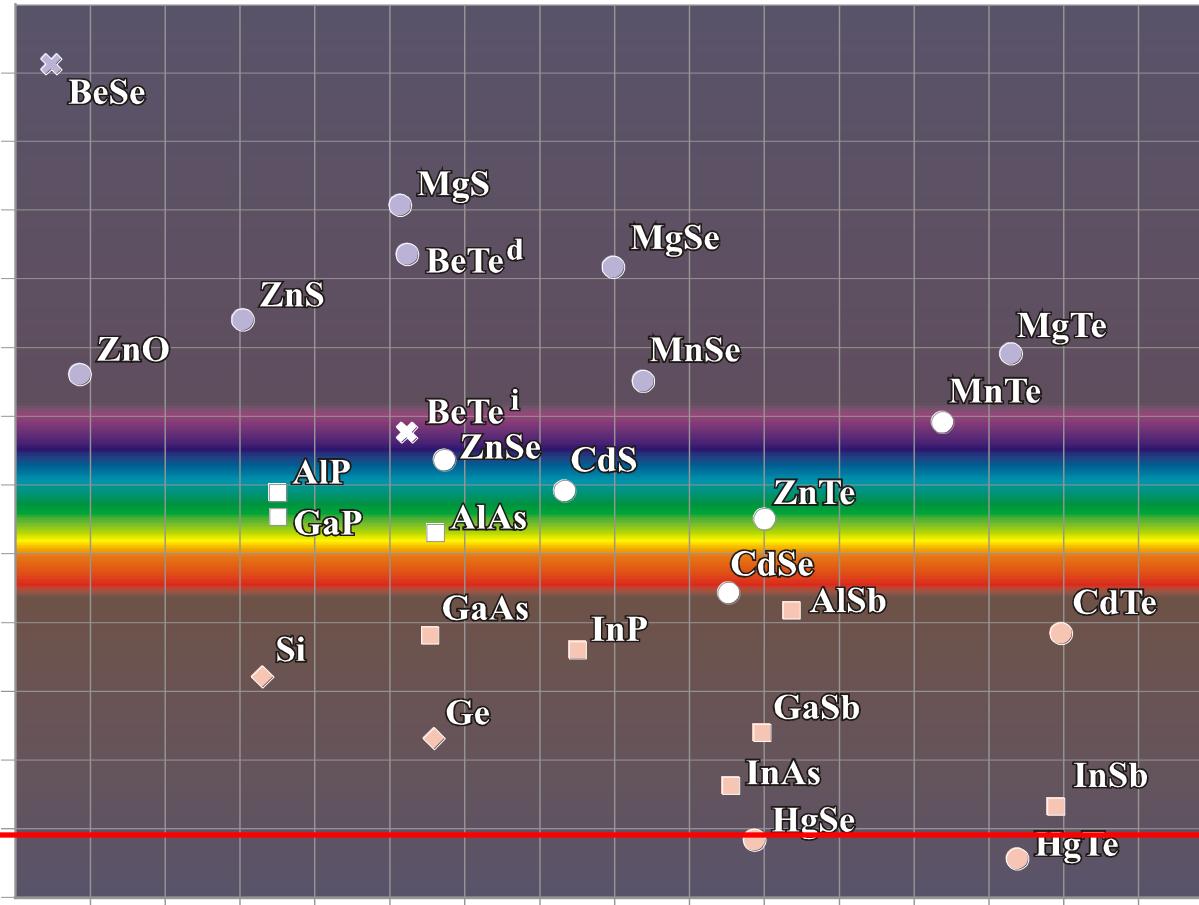
Relativity predicts spin-orbit coupling

# Replace the magnetic field by the spin of electron



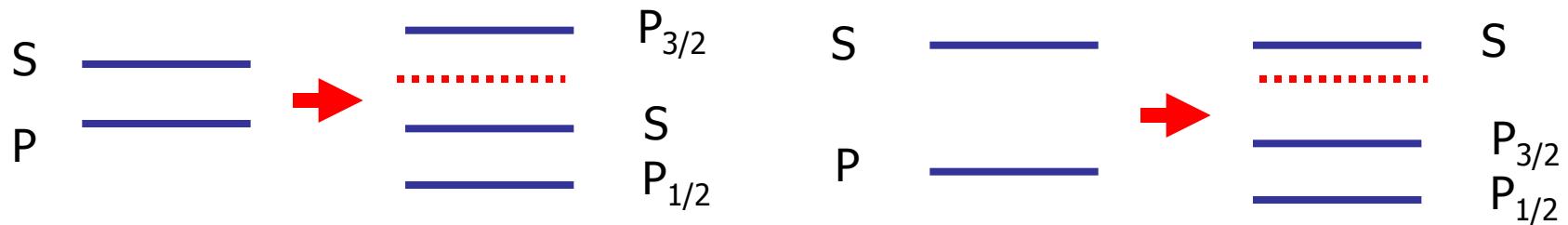
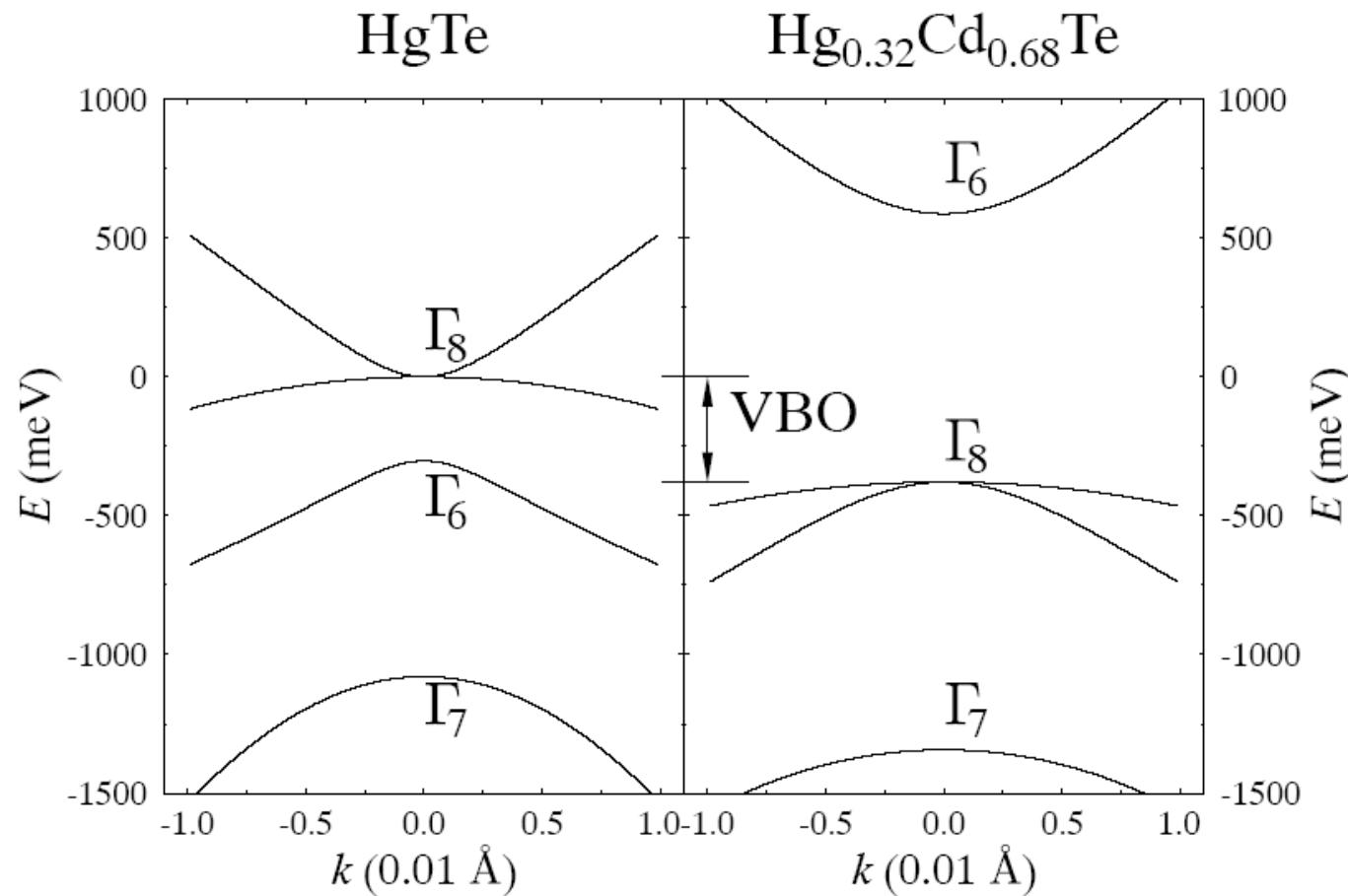
# Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

B. Andrei Bernevig,<sup>1,2</sup> Taylor L. Hughes,<sup>1</sup> Shou-Cheng Zhang<sup>1\*</sup>



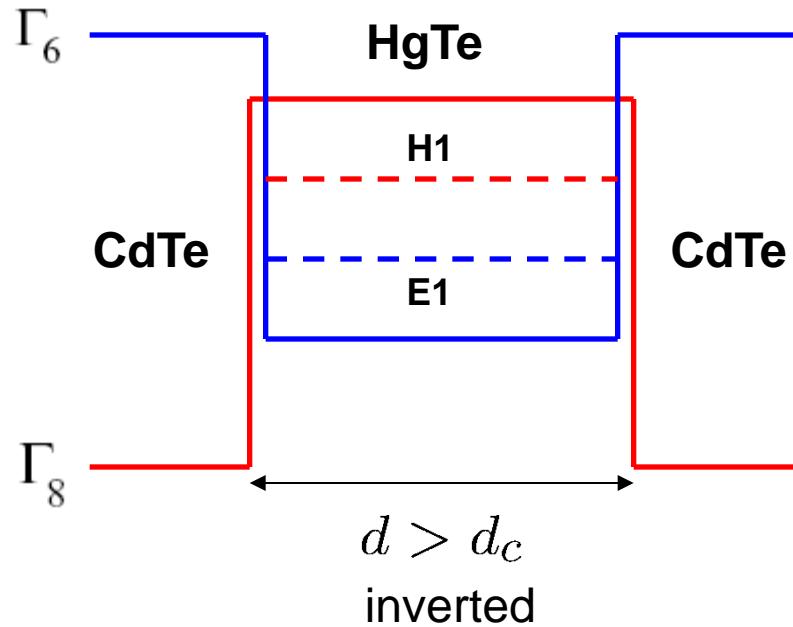
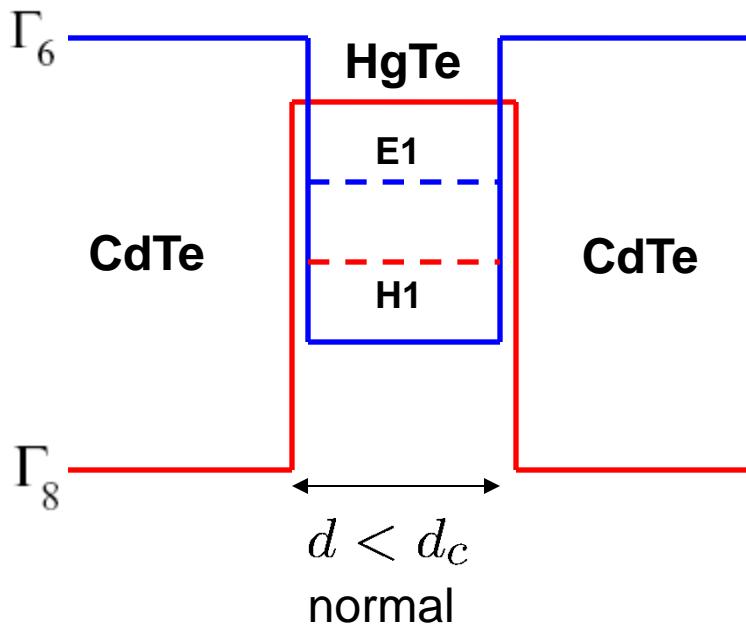
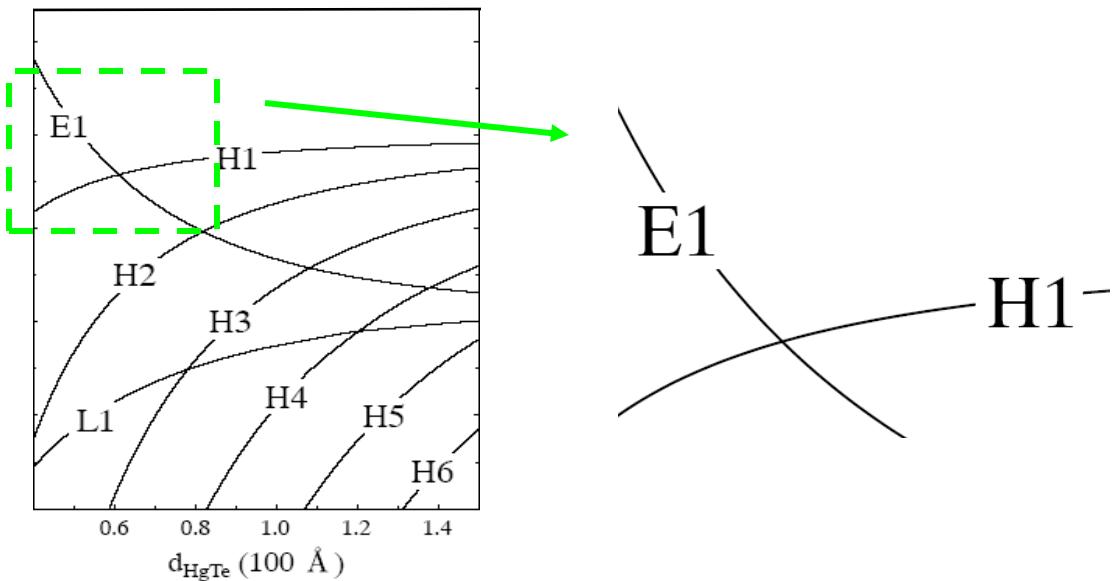
Thanks to  
many  
discussions  
with  
Wurzburg  
group.

# Band Structure of HgTe



# Band inversion in HgTe leads to a topological quantum phase transition

Let us focus on E1, H1 bands close to crossing point



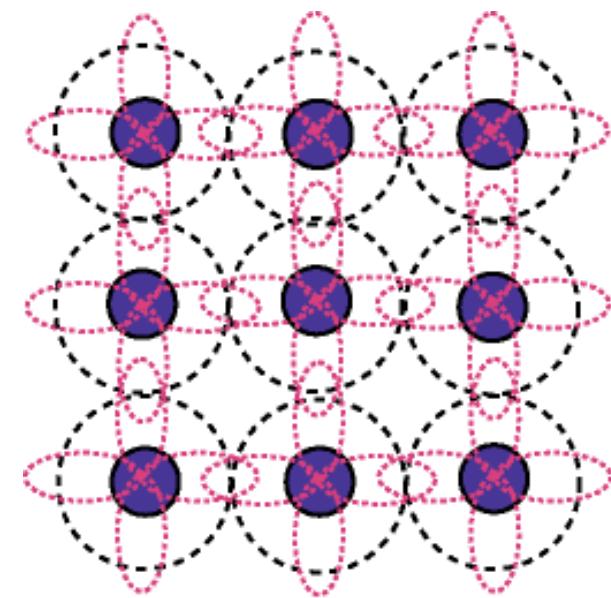
# The model of the 2D topological insulator (BHZ, Science 2006)

Square lattice with 4-orbitals per site:

$$|s, \uparrow\rangle, |s, \downarrow\rangle, |(p_x + ip_y, \uparrow)\rangle, |-(p_x - ip_y), \downarrow\rangle$$

Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in  $k$ .

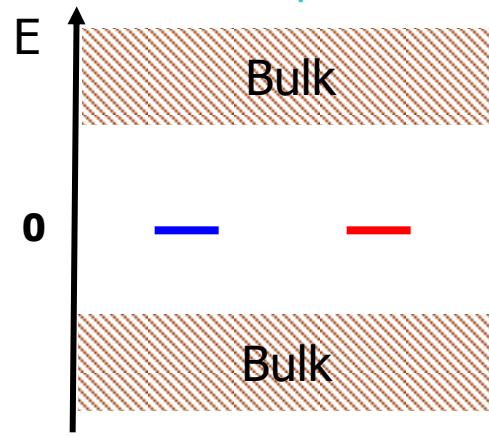
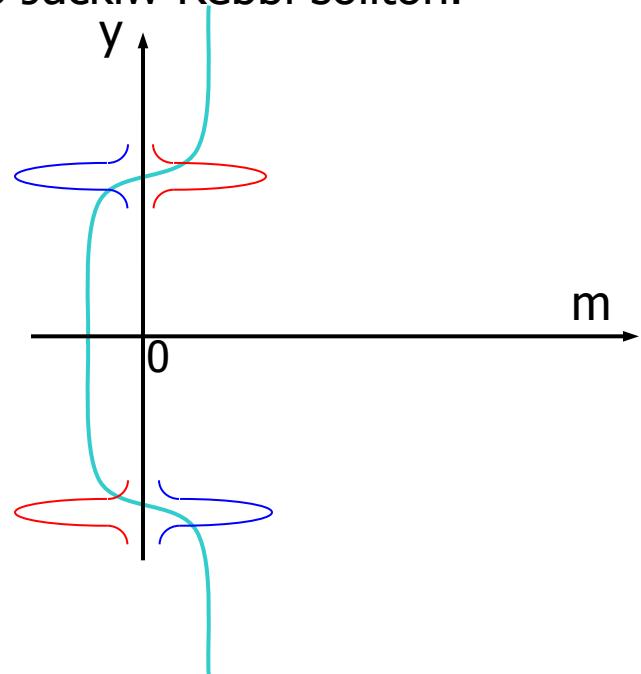
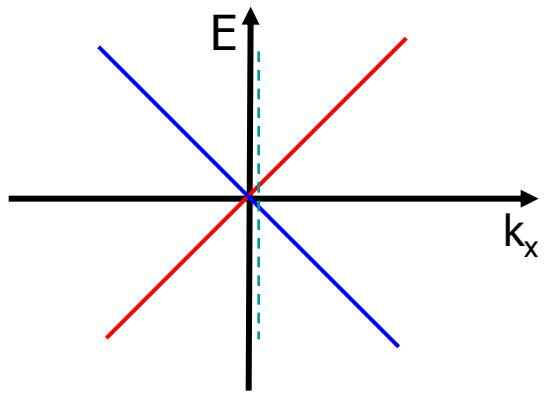
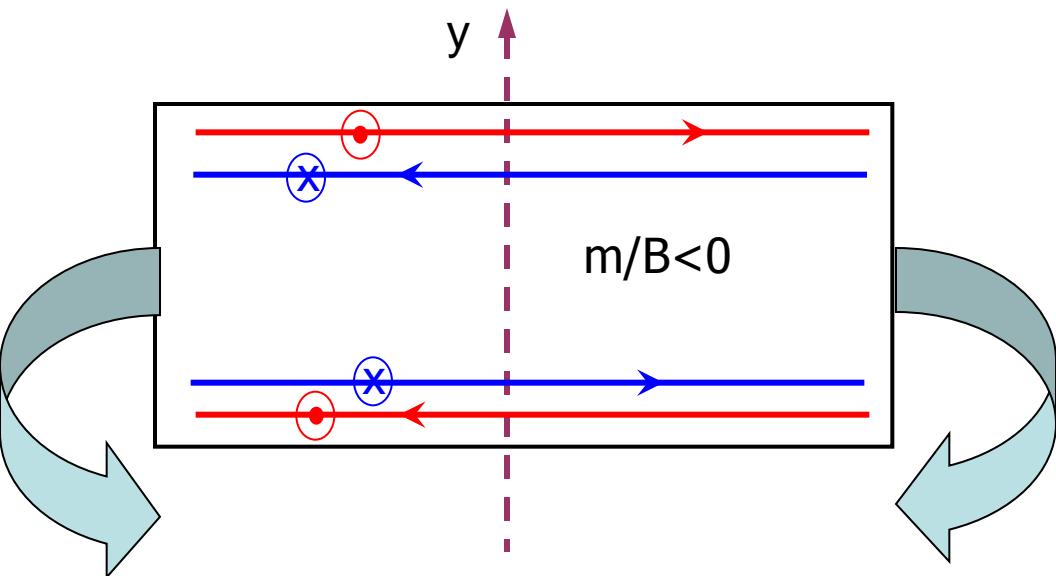
$$\begin{aligned} H_{\text{eff}}(k_x, k_y) &= \begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix} \\ h(k) &= \begin{pmatrix} m(k) & A(\sin k_x - i \sin k_y) \\ A(\sin k_x + i \sin k_y) & -m(k) \end{pmatrix} \equiv d_a(k) \tau^a \\ &\Rightarrow \begin{pmatrix} m + Bk^2 & A(k_x - ik_y) \\ A(k_x + ik_y) & -m - Bk^2 \end{pmatrix} \end{aligned}$$



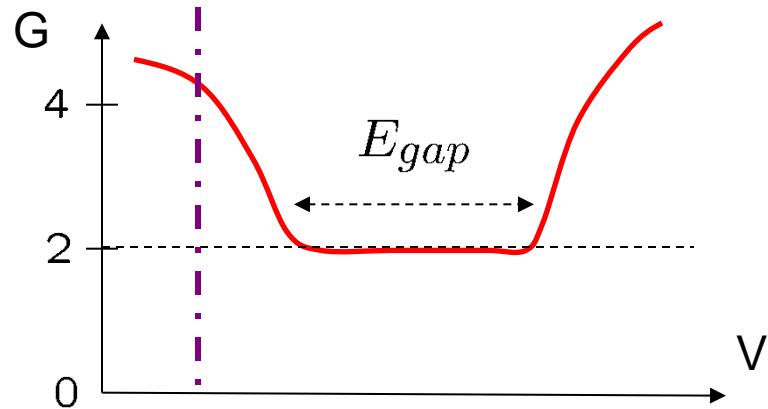
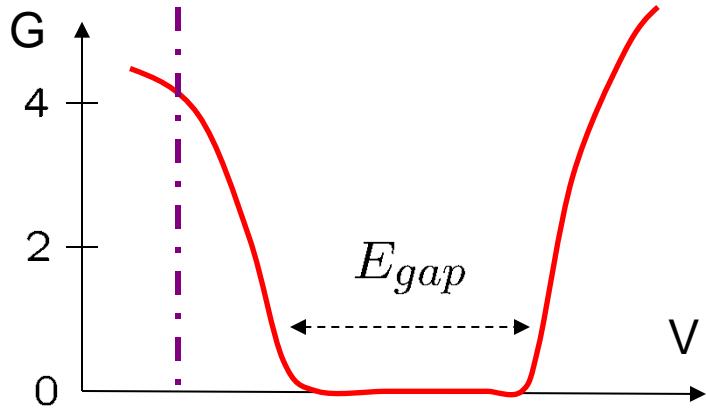
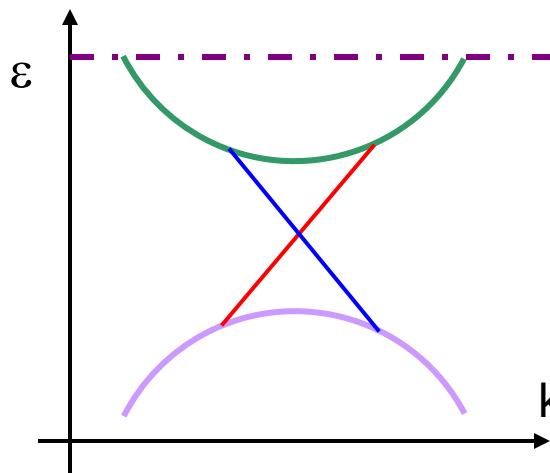
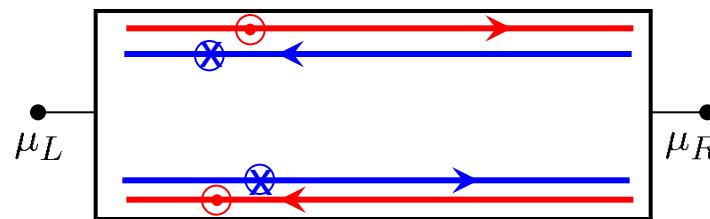
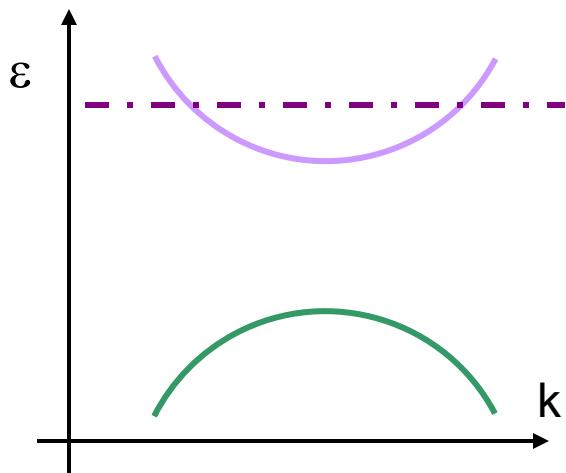
Similar to relativistic Dirac equation in 2+1 dimensions, with a mass term tunable by the sample thickness  $d$ !  $m/B < 0$  for  $d > d_c$ .

# Mass domain wall

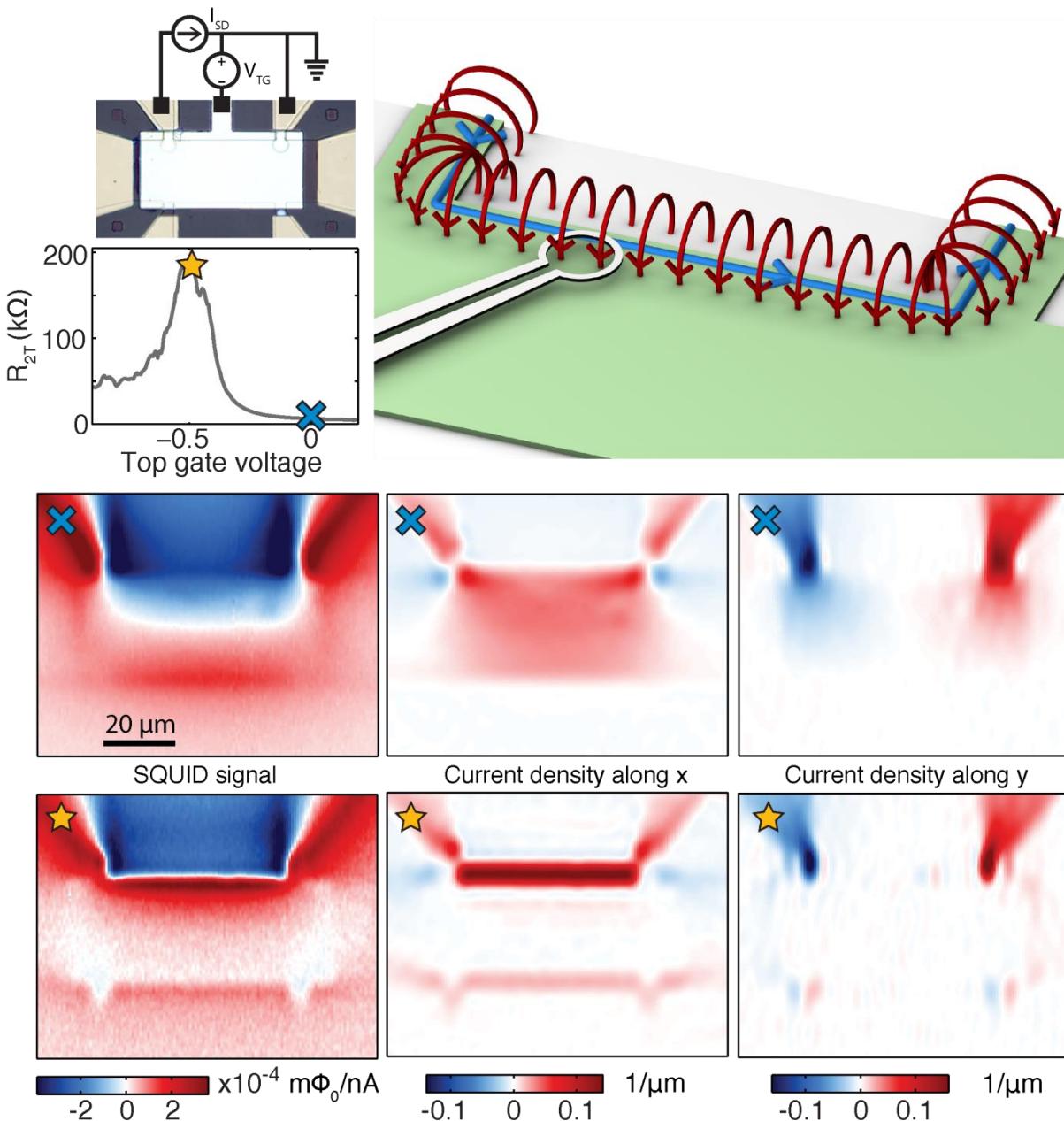
Cutting the Hall bar along the y-direction we see a domain-wall structure in the band structure mass term. This leads to states localized on the domain wall which still disperse along the x-direction, similar to Jackiw-Rebbi soliton.



# Metal, insulator, semiconductor and topological insulators



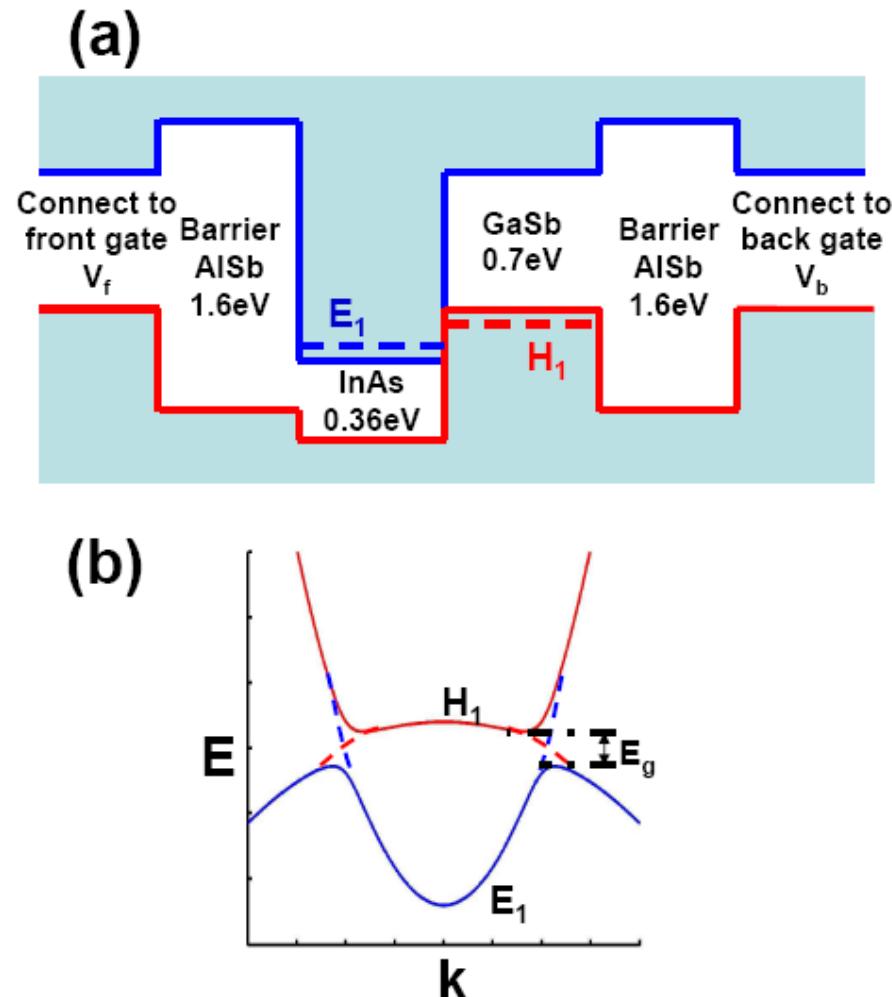
# Measuring the edge current from its magnetic field



# QSH state in InAs/GaSb type II quantum wells

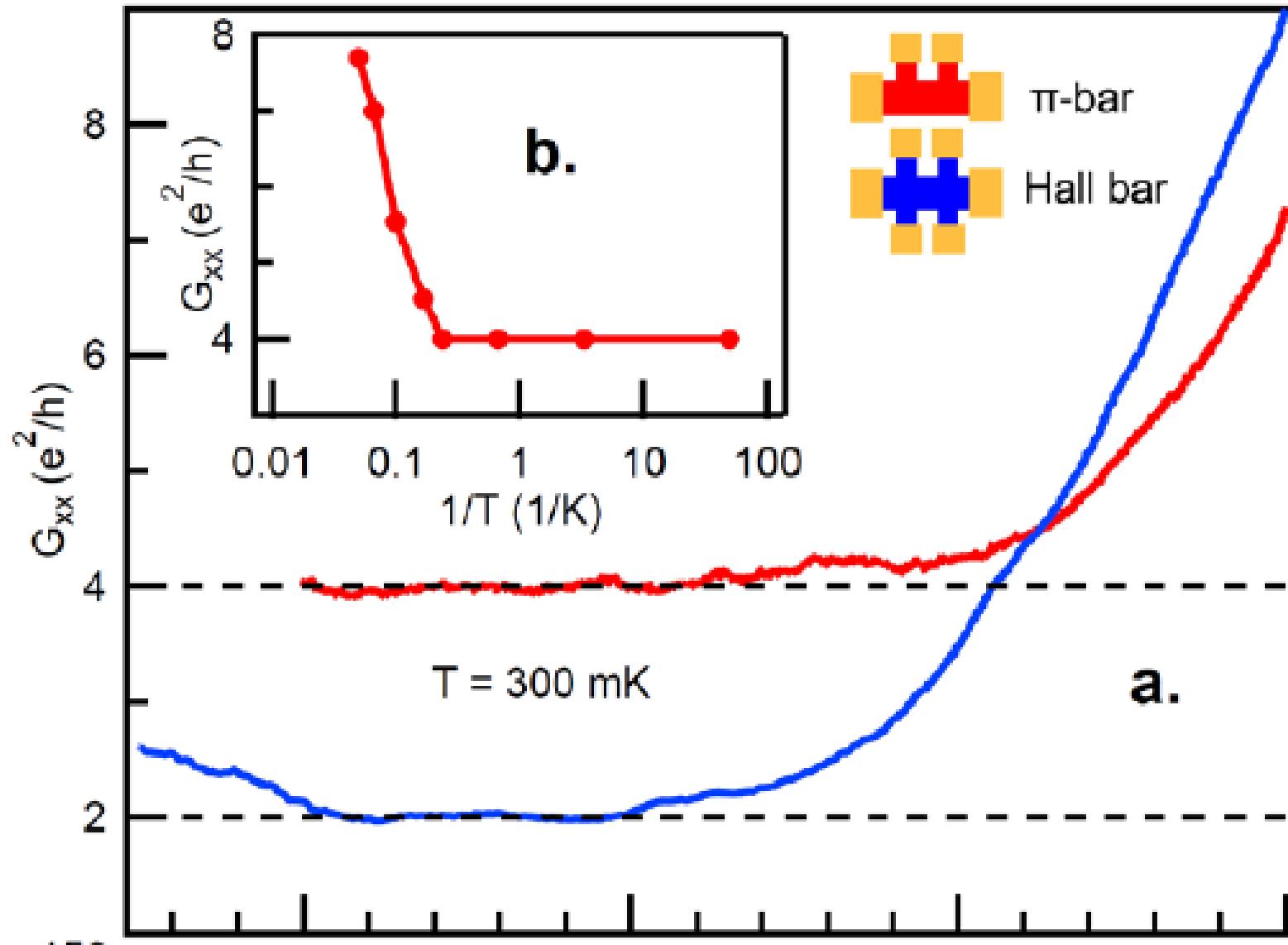
(theoretically predicted by Liu et al PRL 100, 236601 (2008), Zhang group)

- In HgTe, the band inversion occurs intrinsically in the material. However, in InAs/GaSb quantum wells, a similar inversion can occur, since the valence band edge of GaSb lies above the conduction band edge of InAs.
- A small hybridization gap opens up due to tunneling at the interface.
- Theoretical work show that the QSH can occur in InAs/Gab quantum wells. This material can be fabricated commercially in many places around the world.
- InAs can also be used for superconducting proximity effect.

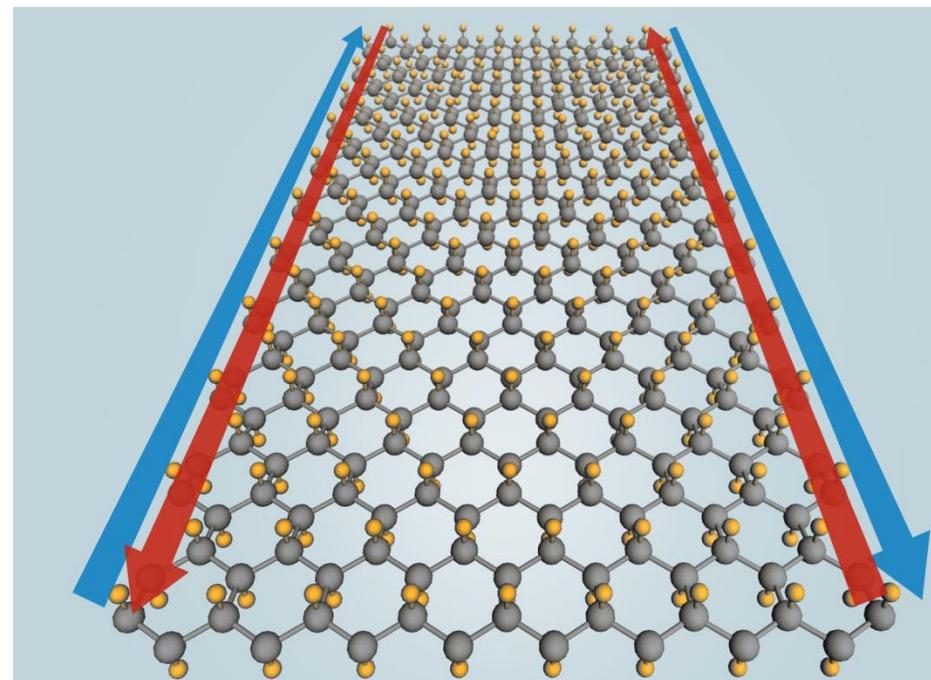
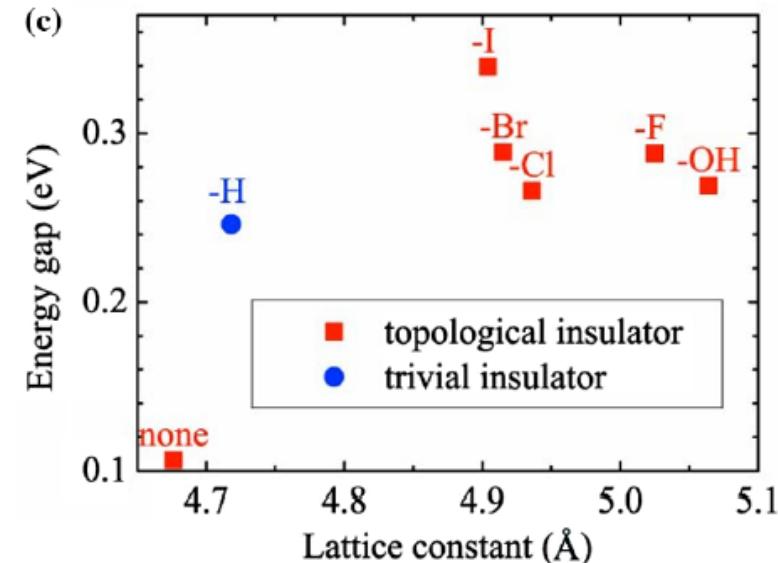
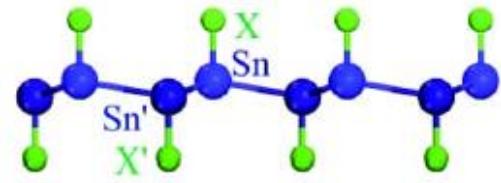
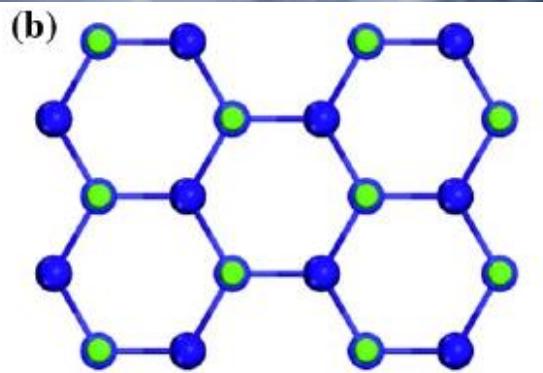
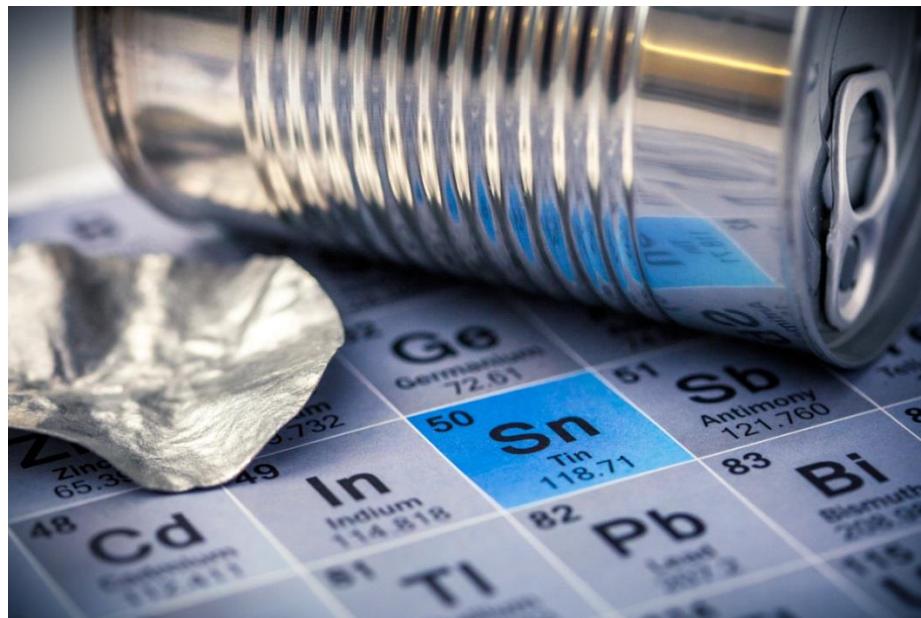


# QSH state in InAs/GaSb type II quantum wells

(experimentally observed in Ruirui Du group)



# Stanene: new materials for 2D topological insulators



# 3D topological insulators

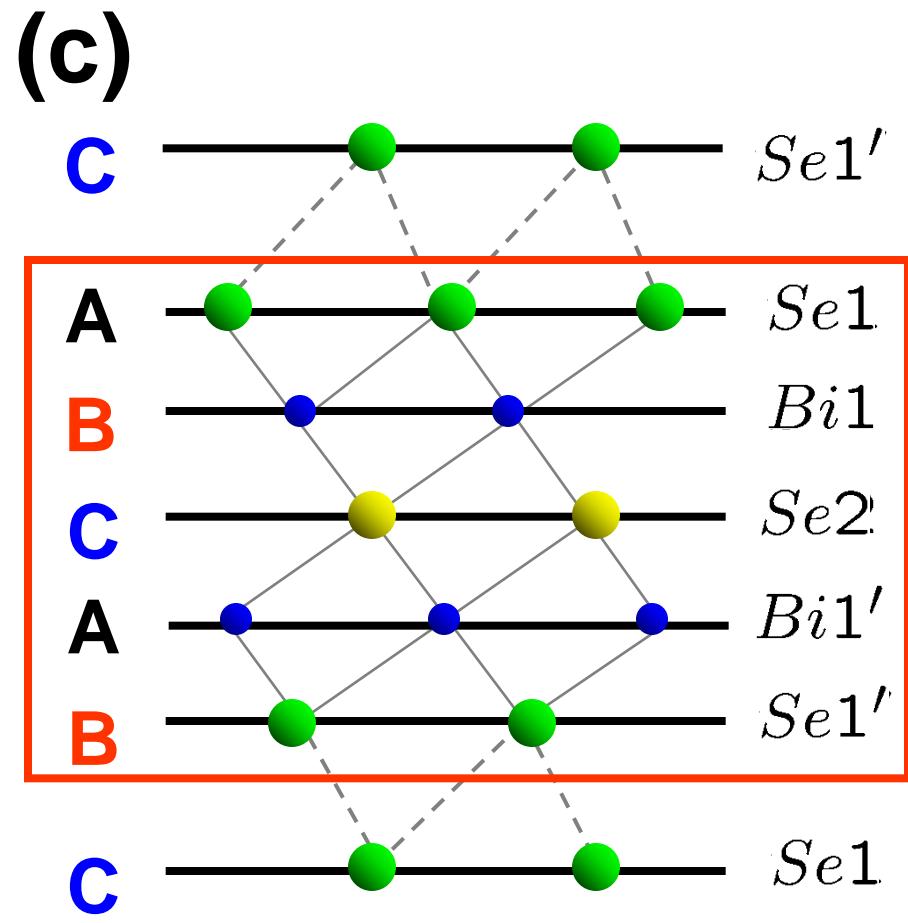
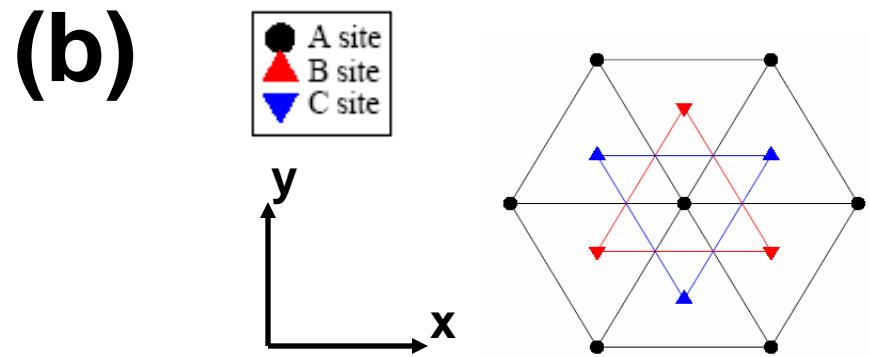
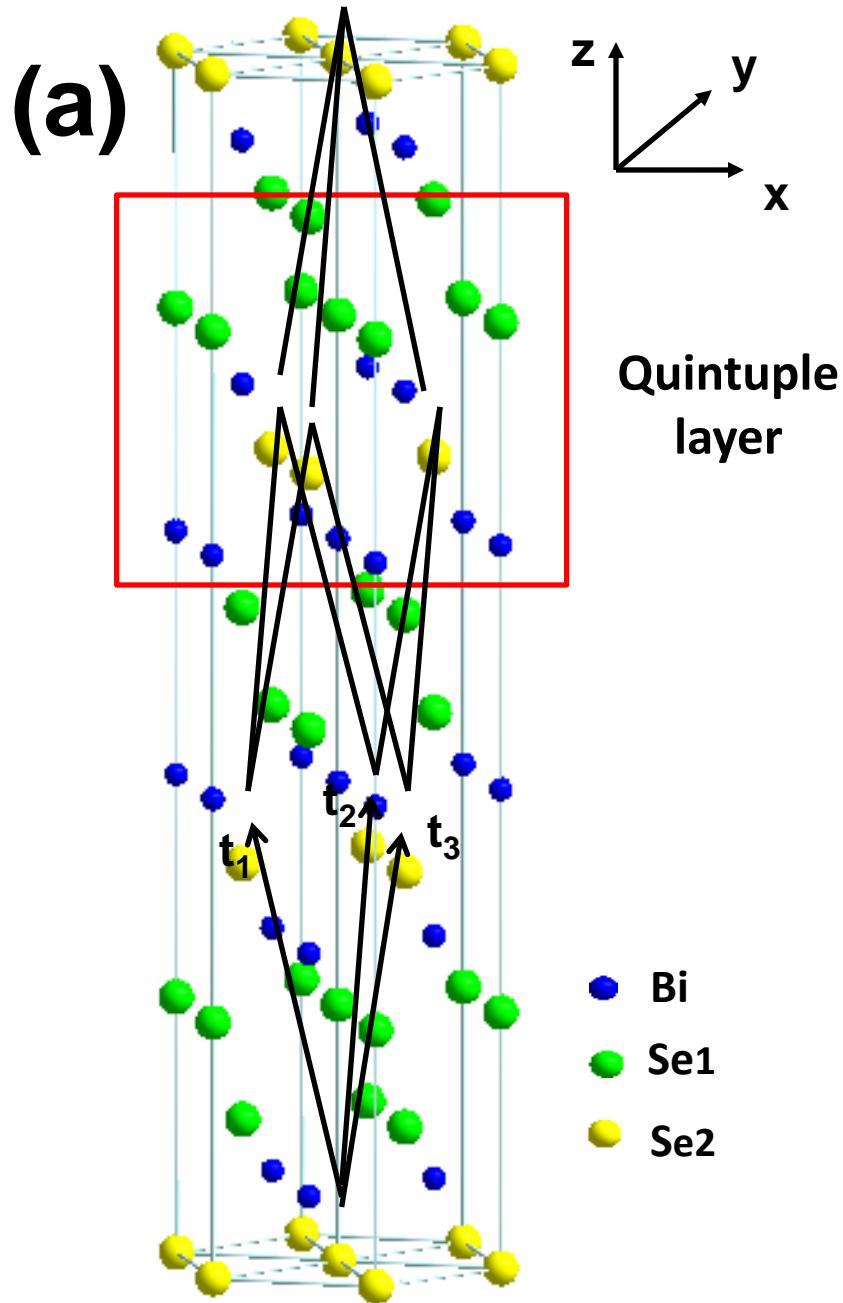
## Periodic Table of the Elements

	IA	Periodic Table of the Elements																		O			
1	H	IIA																		He			
2	Li	Be																		Ne			
3	Na	Mg	IIIB	IVB	VB	VIB	VIIIB	VII				IB	IIB										
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn								Kr			
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	53		Xe				
6	Cs	Ba	*La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	85		Rn				
7	Fr	Ra	+Ac	Rf	Ha	Sg	Ns	Hs	Mt	110	111	112	113										
* Lanthanide Series		58	59	60	61	62	63	64	65	66	67	68	69	70	71								
+ Actinide Series		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu								
		90	91	92	93	94	95	96	97	98	99	100	101	102	103								
		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr								

\* Lanthanide Series

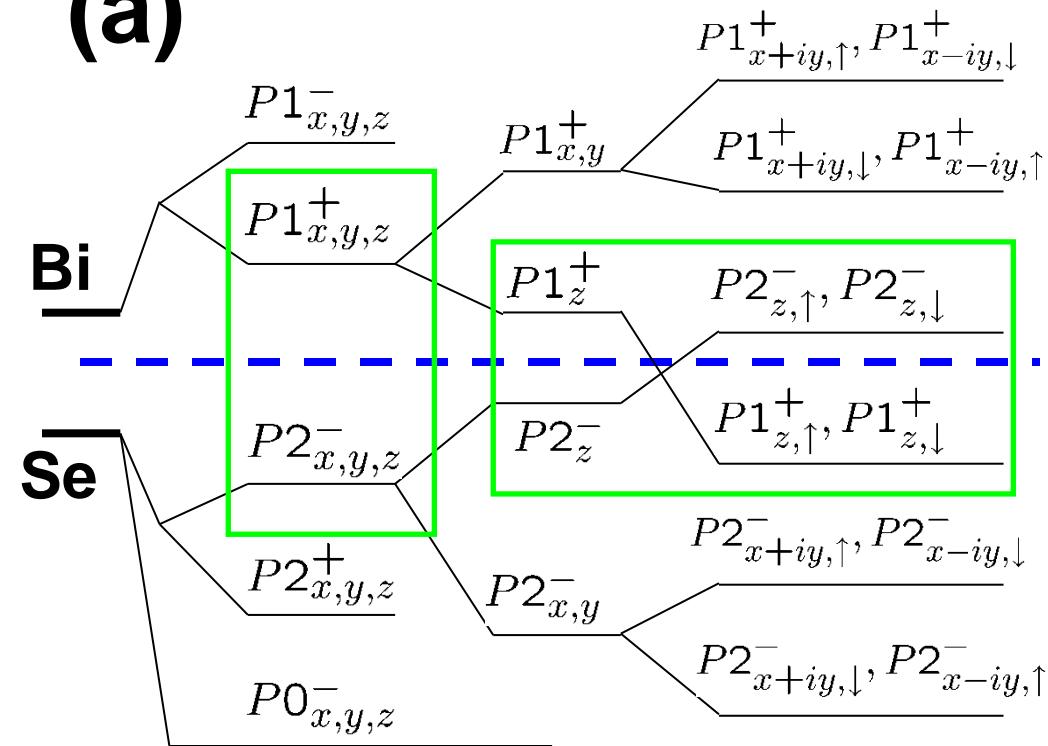
+ Actinide Series

# 3D insulators with a single Dirac cone on the surface

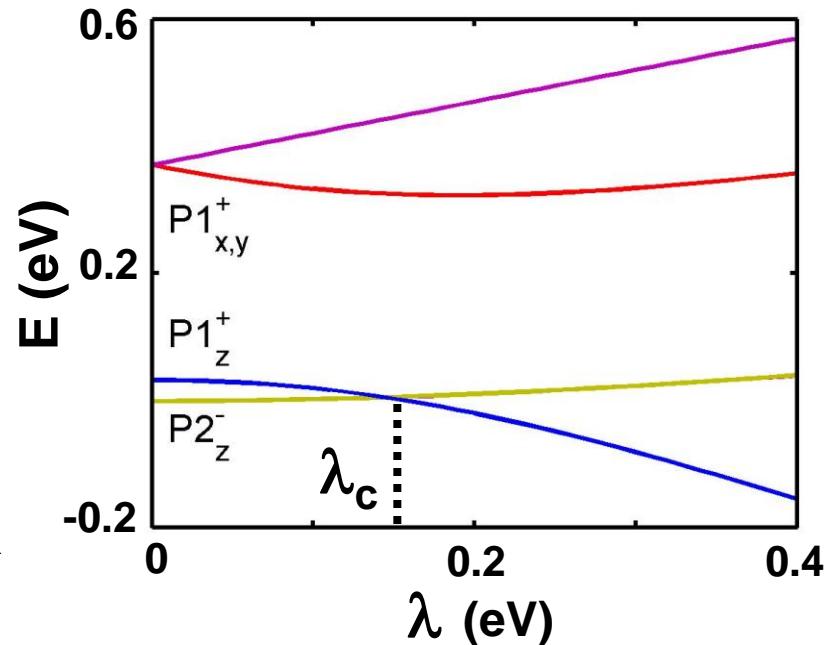


# Relevant orbitals of Bi<sub>2</sub>Se<sub>3</sub> and the band inversion

(a)



(b)



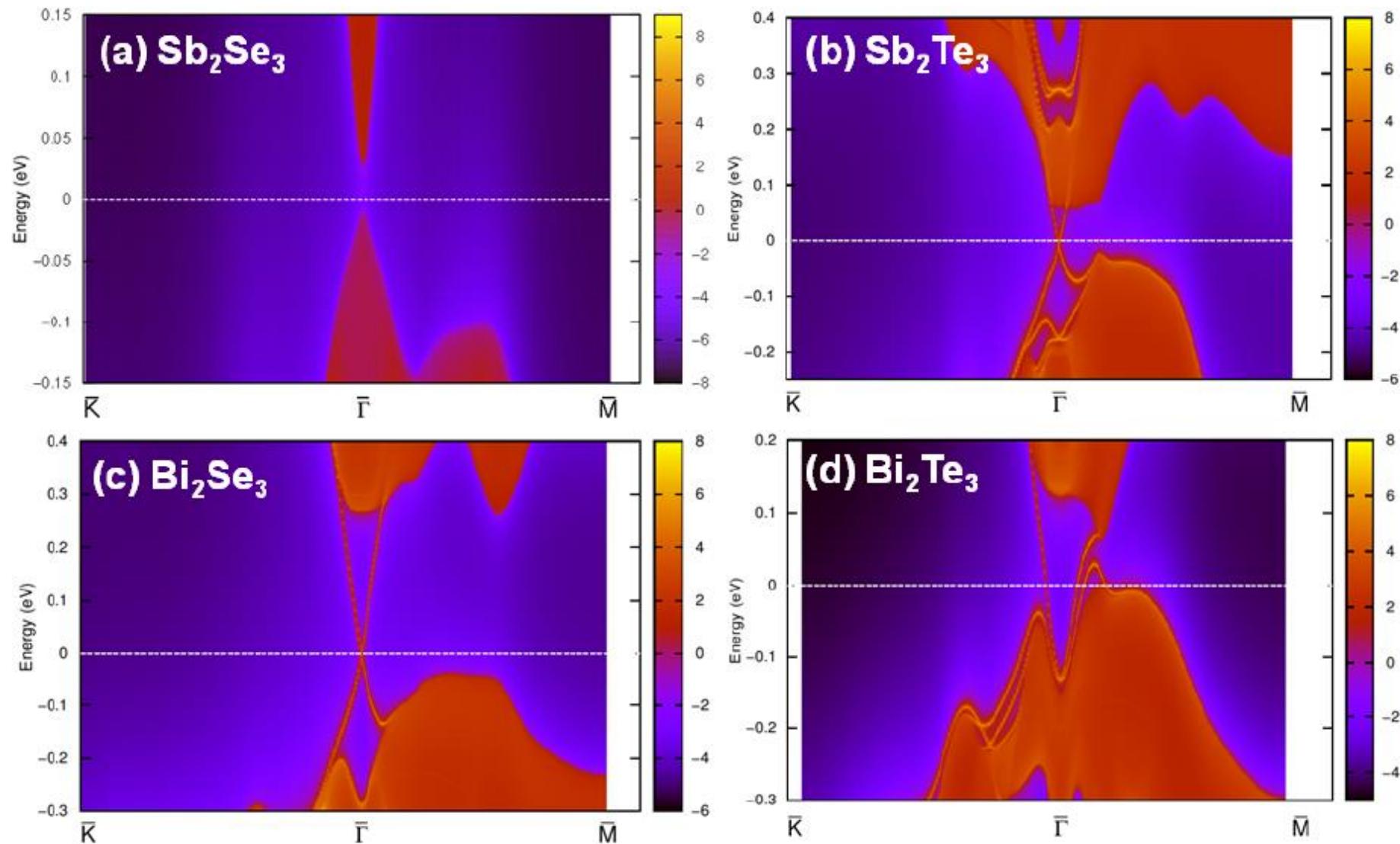
(I)

(II)

(III)

# Topological insulators in $\text{Bi}_2\text{Se}_3$ , $\text{Bi}_2\text{Te}_3$ and $\text{Sb}_2\text{Te}_3$ with a single Dirac cone on the surface

Haijun Zhang<sup>1</sup>, Chao-Xing Liu<sup>2</sup>, Xiao-Liang Qi<sup>3</sup>, Xi Dai<sup>1</sup>, Zhong Fang<sup>1</sup> and Shou-Cheng Zhang<sup>3\*</sup>



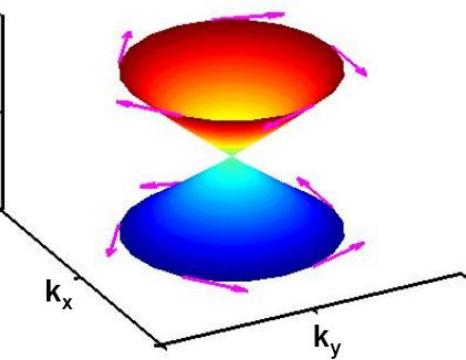
# Model for topological insulator Bi<sub>2</sub>Te<sub>3</sub>, (Zhang et al, 2009)

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k})\mathbf{I}_{4 \times 4} + \begin{pmatrix} \mathcal{M}(\mathbf{k}) & A_1 k_z & 0 & A_2 k_- \\ A_1 k_z & -\mathcal{M}(\mathbf{k}) & A_2 k_- & 0 \\ 0 & A_2 k_+ & \mathcal{M}(\mathbf{k}) & -A_1 k_z \\ A_2 k_+ & 0 & -A_1 k_z & -\mathcal{M}(\mathbf{k}) \end{pmatrix} + o(\mathbf{k}^2)$$

Pz+, up, Pz-, up, Pz+, down, Pz-, down

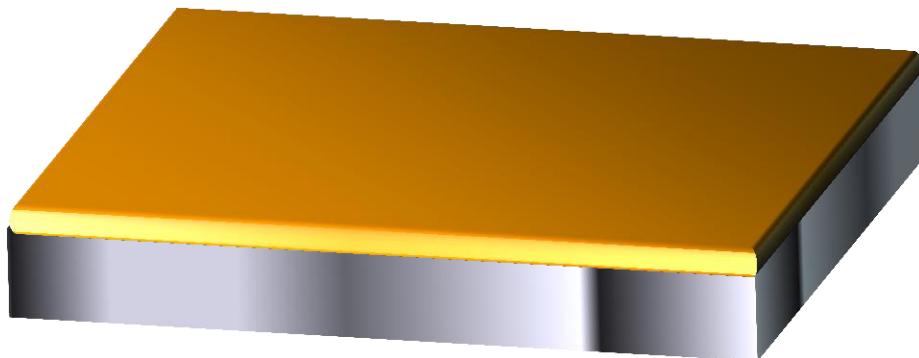
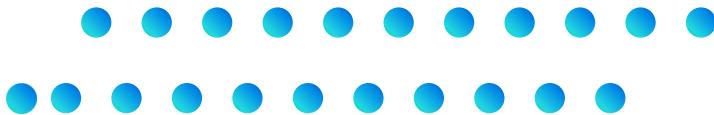
## Single Dirac cone on the surface of Bi<sub>2</sub>Te<sub>3</sub>

$$H = \int d^2\mathbf{x} \psi^\dagger(\mathbf{x}) [\hbar v_f (\hat{\mathbf{z}} \times (-i\nabla)) \cdot \boldsymbol{\sigma} - \mu] \psi(\mathbf{x}),$$

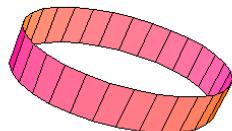


Surface of Bi<sub>2</sub>Te<sub>3</sub> = 1/4 Graphene !

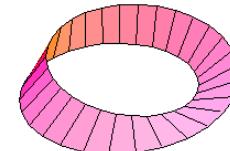
# Magic of the topological surface state



**Coating insulator with a metallic surface**

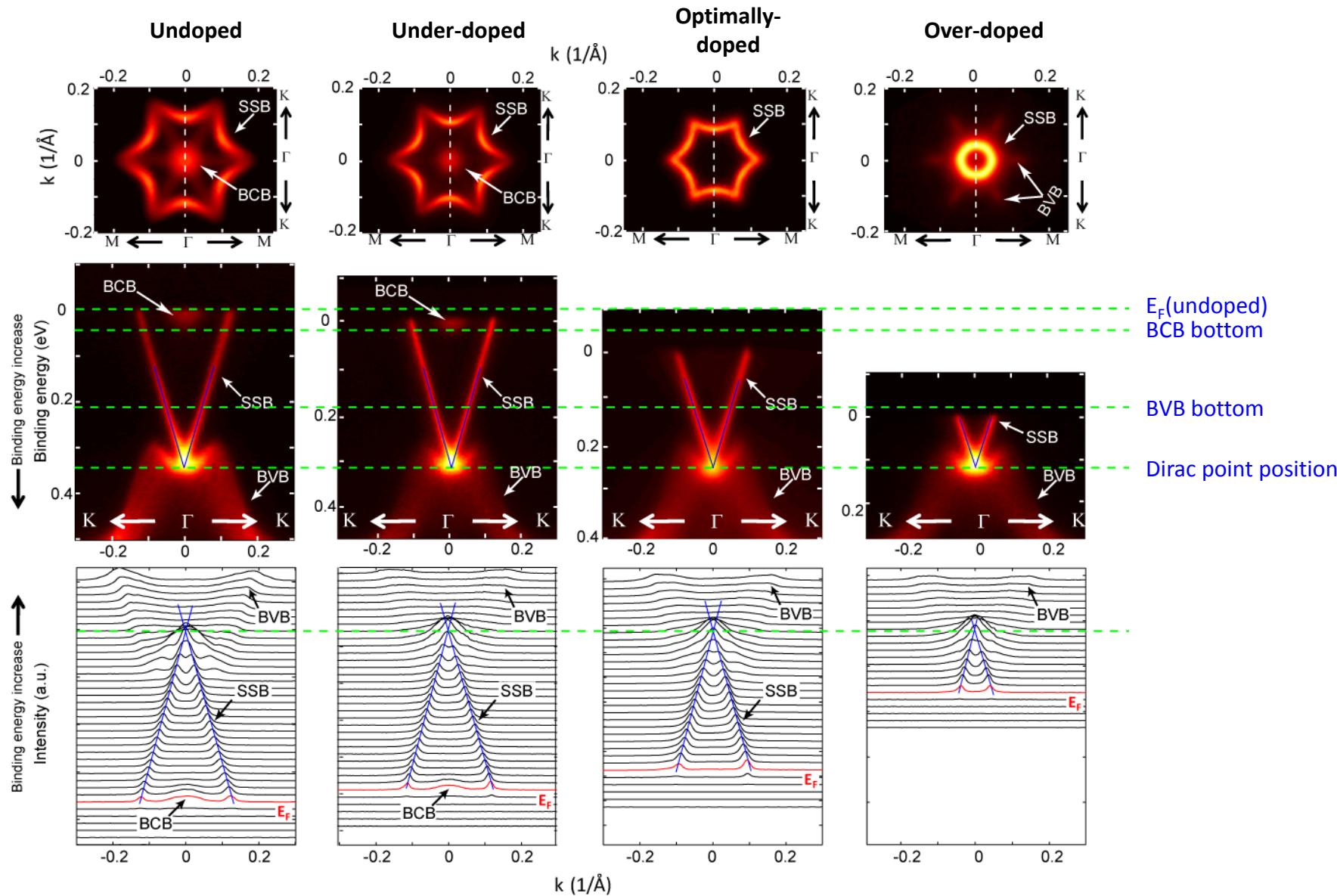


**Topological surface state**

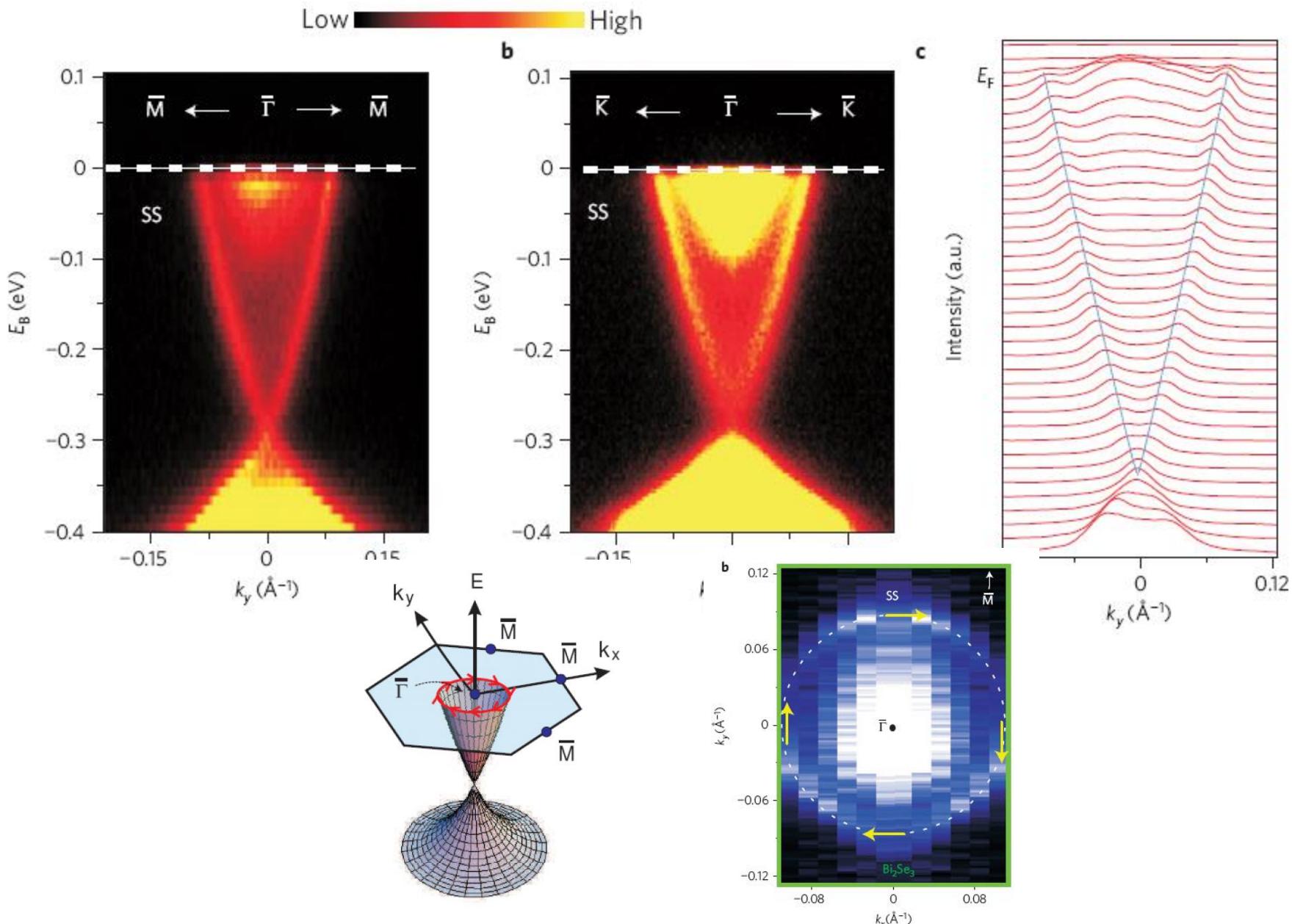


# Arpes experiment on Bi<sub>2</sub>Te<sub>3</sub> surface states, Shen group

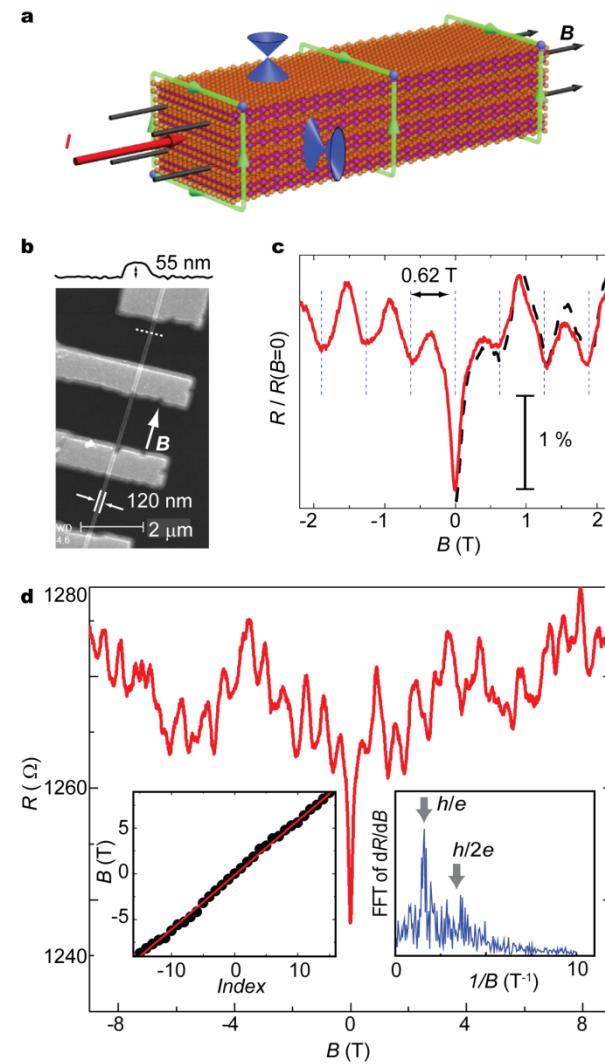
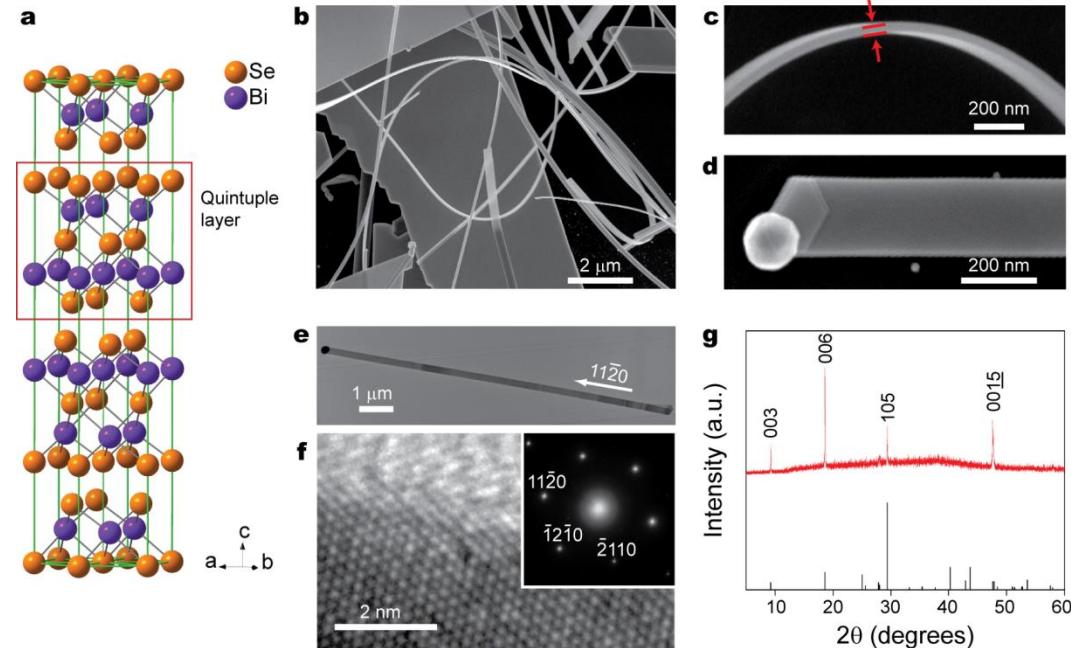
## Doping evolution of the FS and band structure



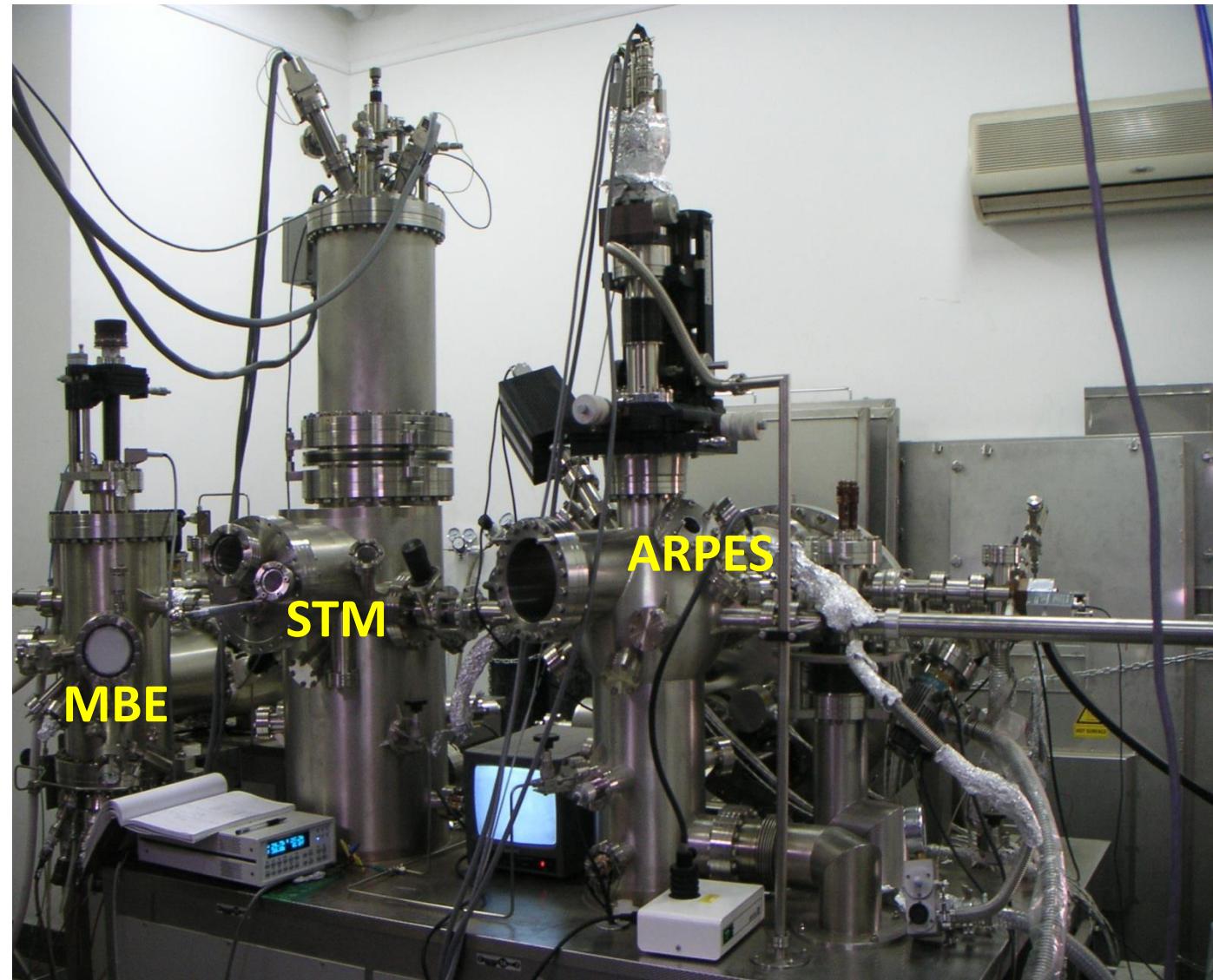
# Arpes experiment&theory on Bi<sub>2</sub>Se<sub>3</sub> surface states, Hasan+Cava group



# AB oscillations in nano-ribbon of Bi<sub>2</sub>Se<sub>3</sub>, Cui group



# MBE-STM-ARPES System at Tsinghua University



**Results in the TI field:**

**First MBE growth.**

**QAH effect.**

**Landau levels for the Dirac surface state.**

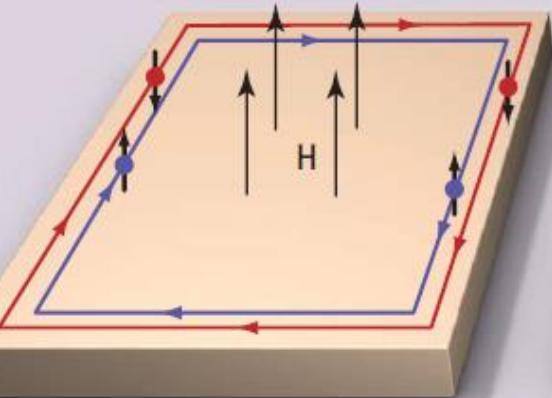
# The quantum Hall trio

Hall  
(1879)

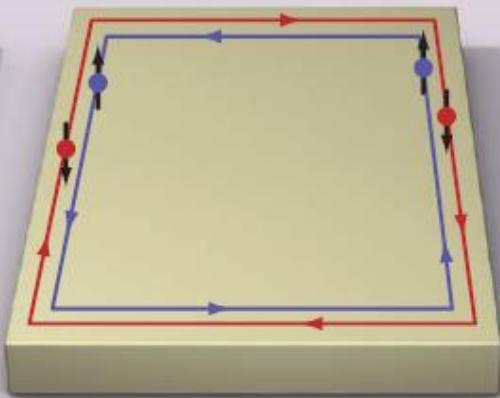
Quantum Hall  
(1980)

Spin Hall  
(2004)

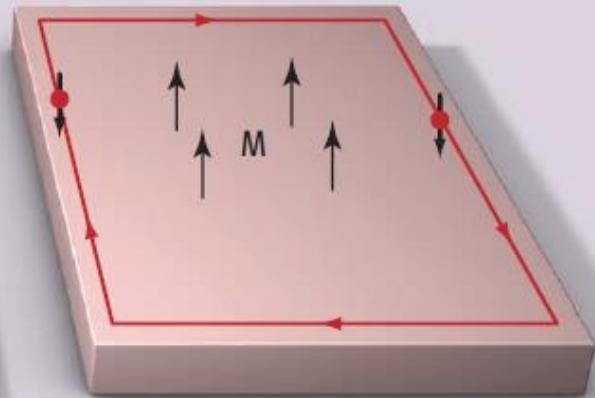
Anomalous Hall  
(1881)



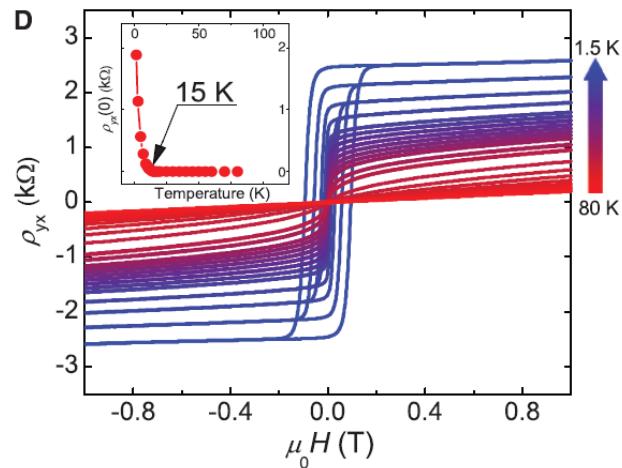
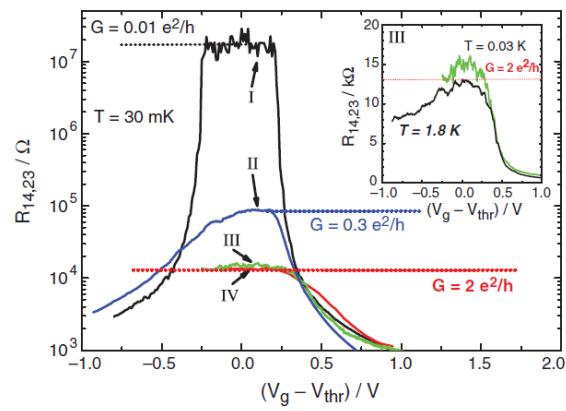
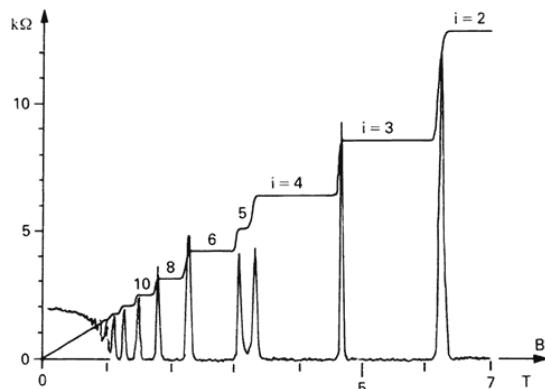
Quantum Hall



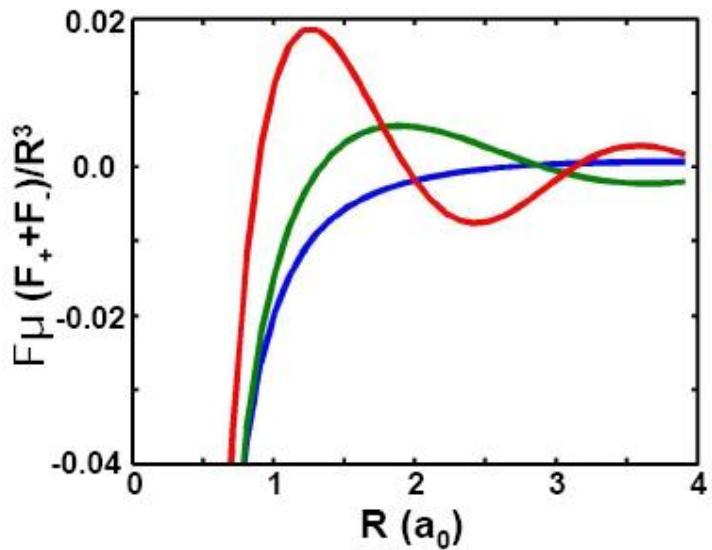
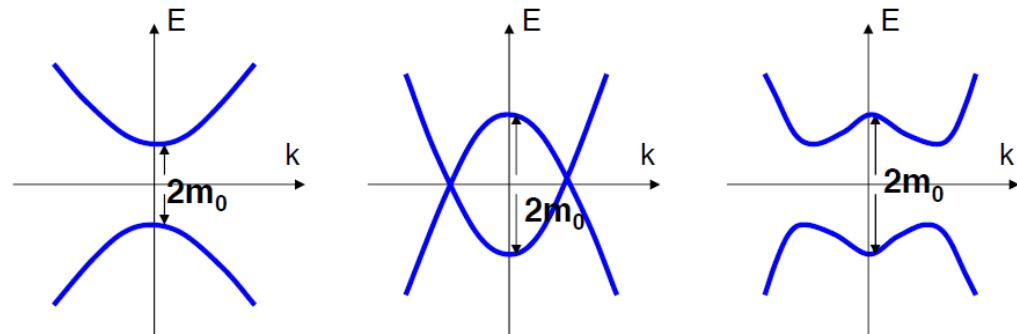
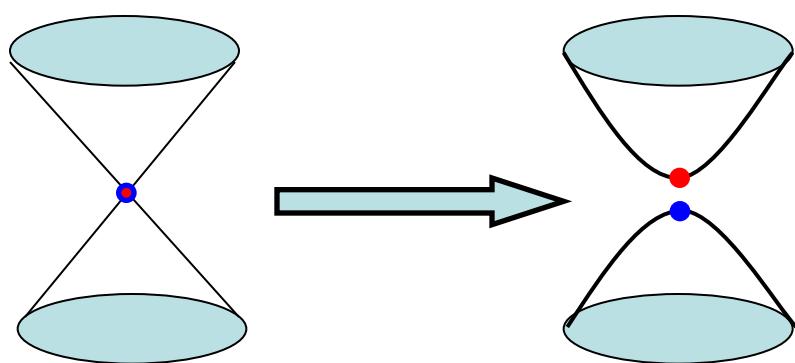
Quantum spin Hall



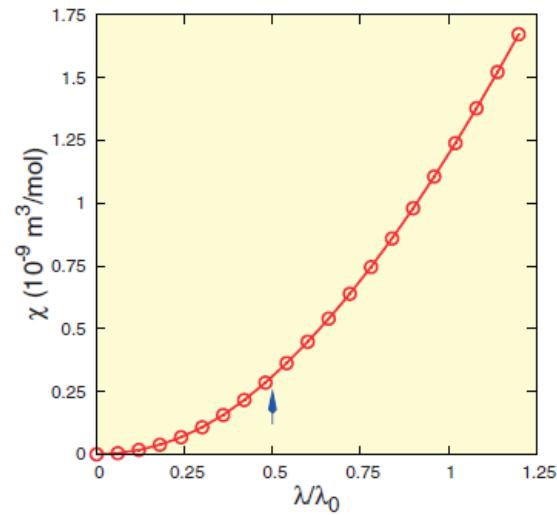
Quantum anomalous Hall



# Mechanism of magnetic TI



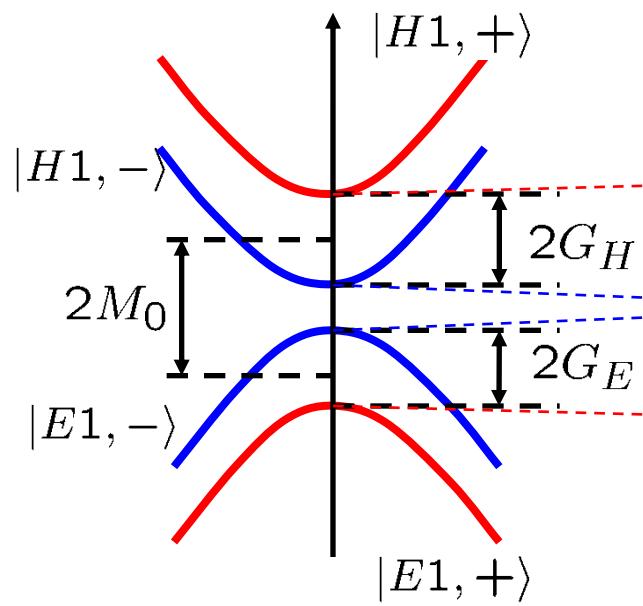
Ferromagnetic coupling through the RKKY interaction on the TI surface. (Zhang group, PRL2009).



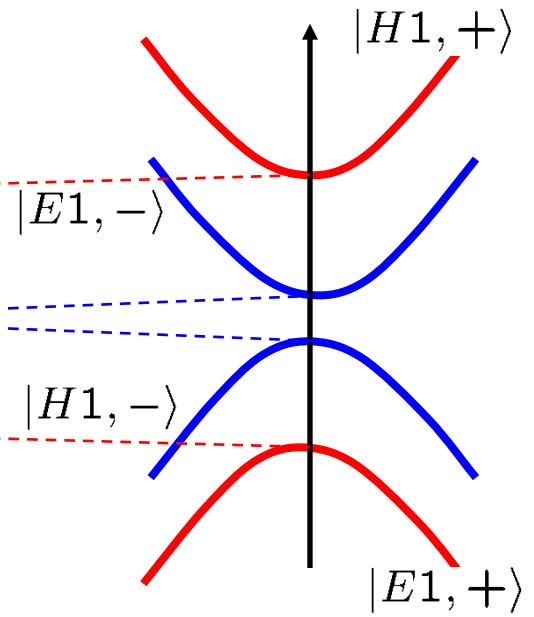
Spin-orbit coupling and band inversion increases magnetic susceptibility, leading to magnetic order in the TI state. (IOP+Zhang group, Science 2010).

# Understanding from edge states: QAH

Small spin splitting,  $\sigma_H=0$

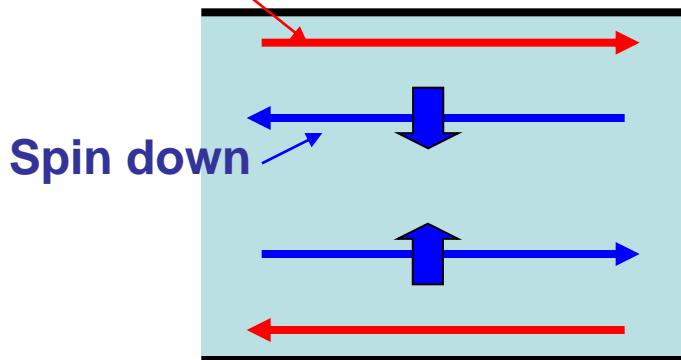


Large splitting,  $\sigma_H=e^2/h$

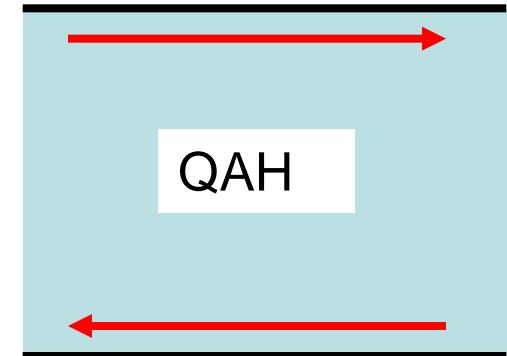
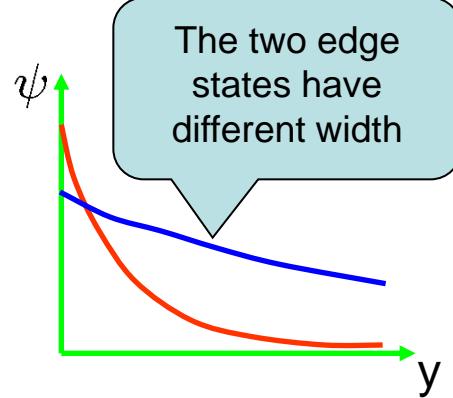


Note that  $G_E$  and  $G_H$  have opposite sign

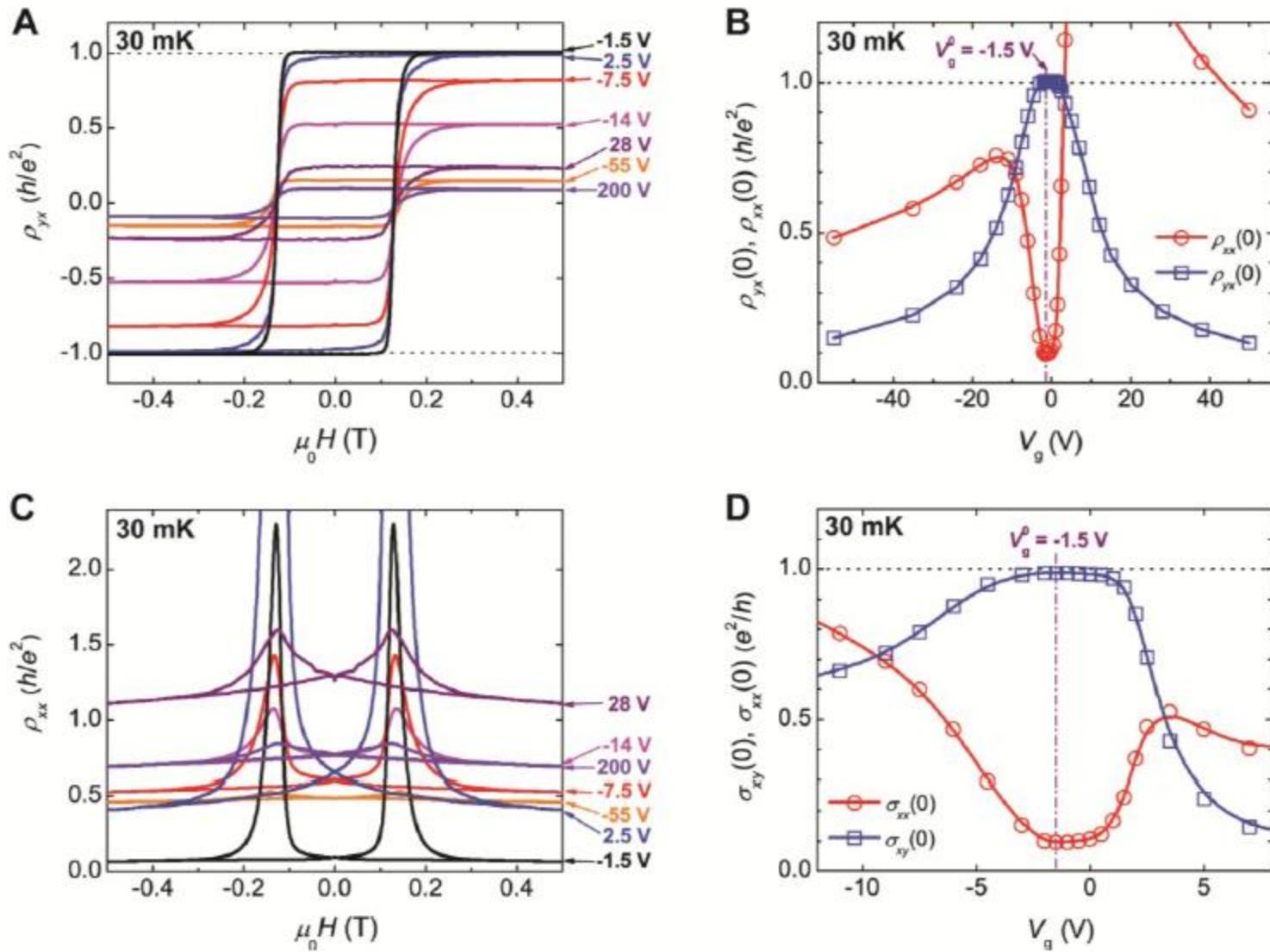
Spin up



Spin down

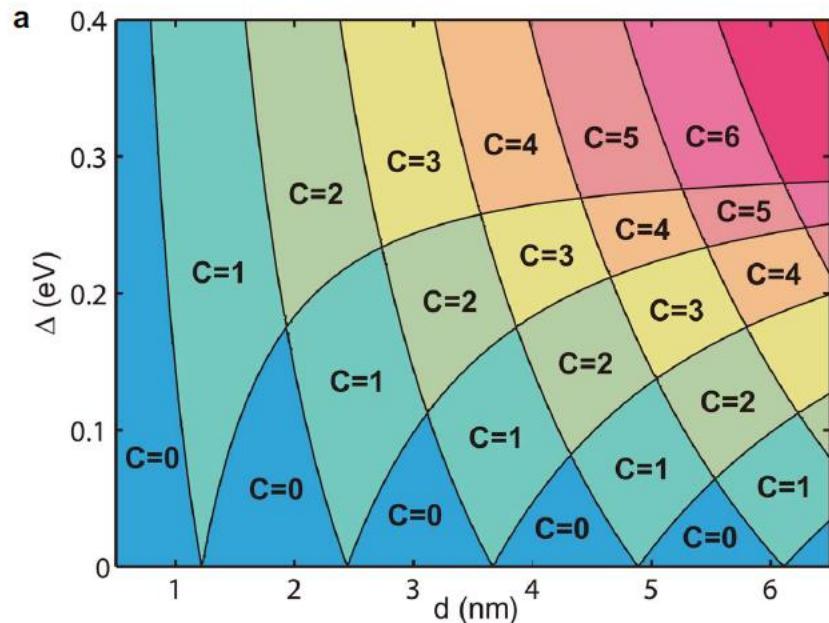


# Discovery of the QAH (Tsinghua University, China)

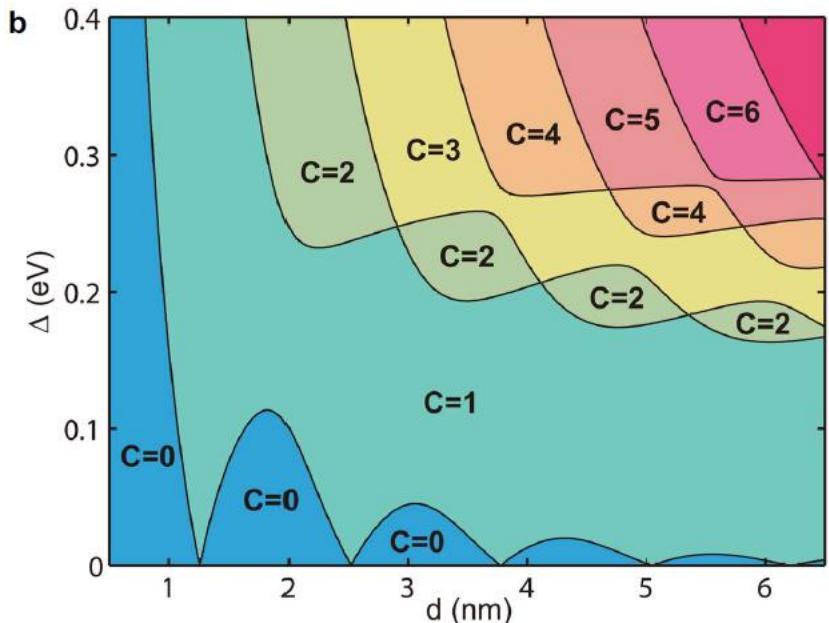


# Quantum Anomalous Hall with Higher plateaus

Theoretical phase diagram of QAH ( thickness  $d$  , exchange field  $\Delta$ )

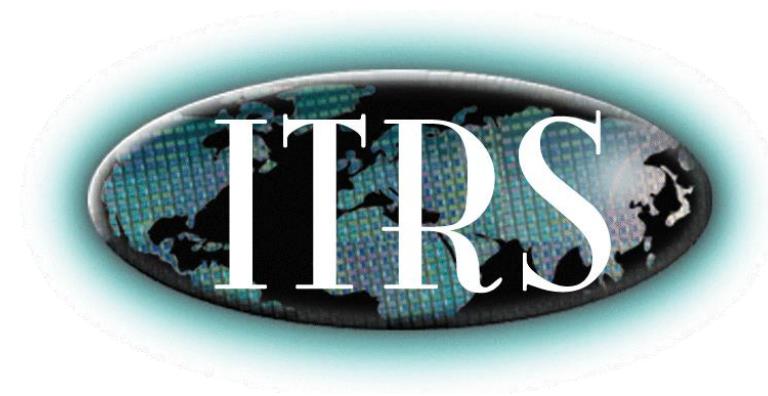
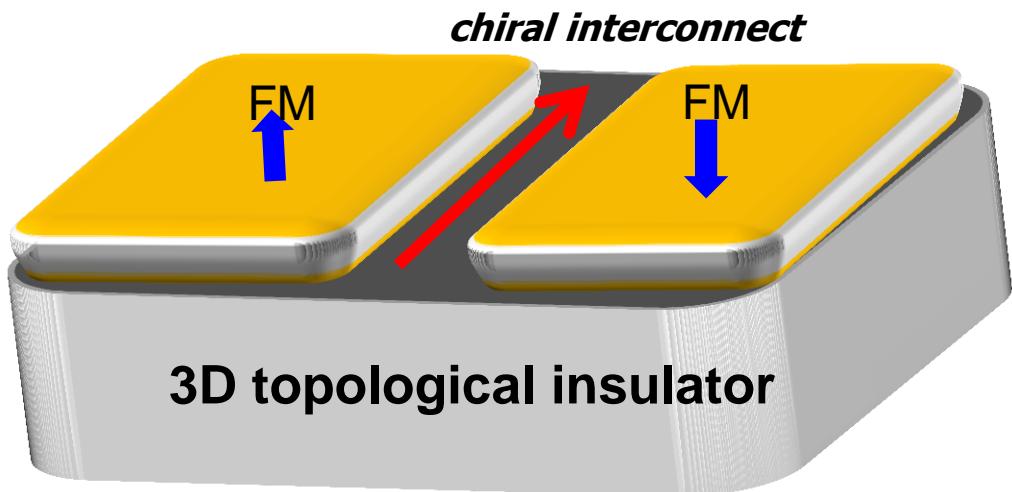
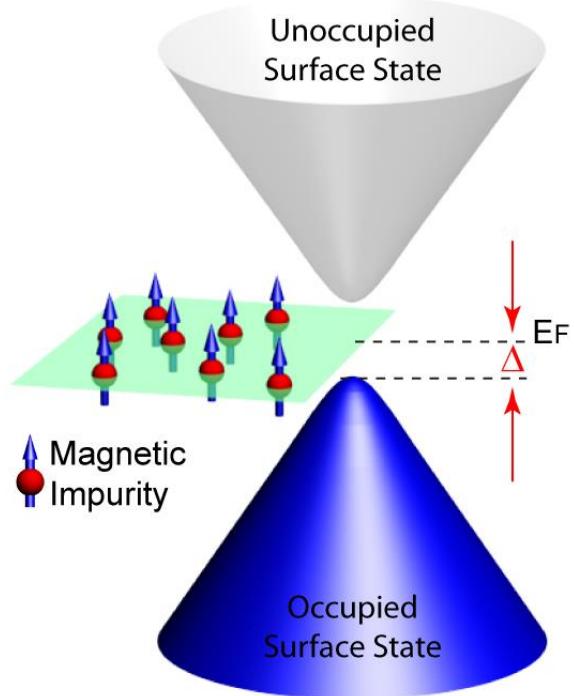


Without  $z$  direction SOC

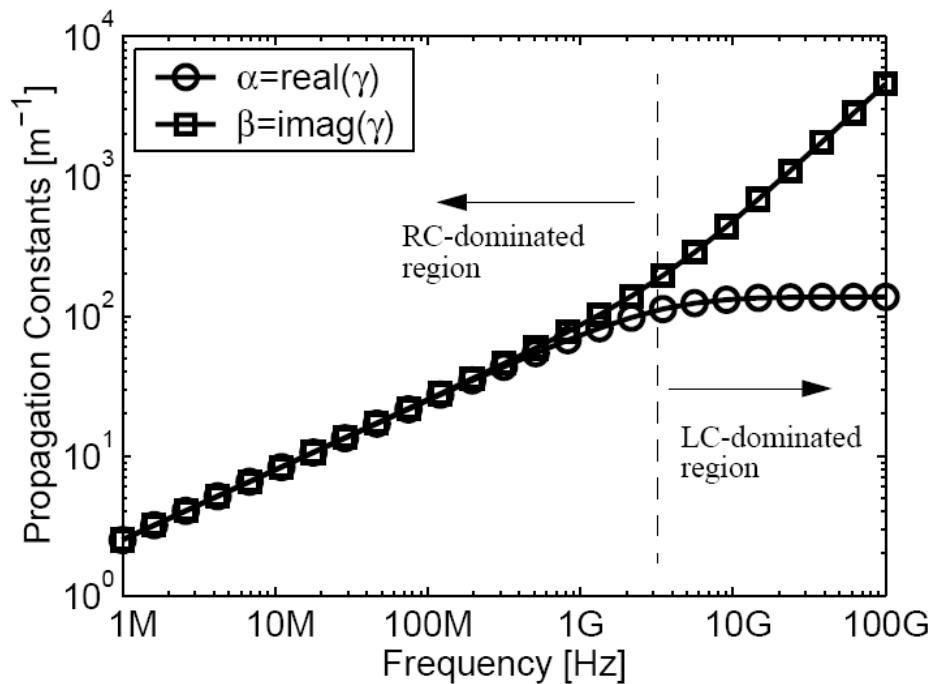


With  $z$  direction SOC

# Applications: chiral interconnects



## Bottleneck of the current copper interconnect: high dissipation

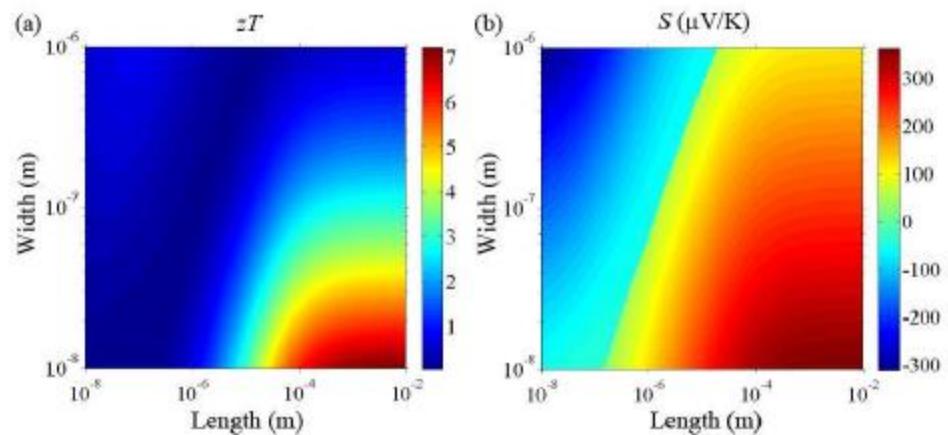
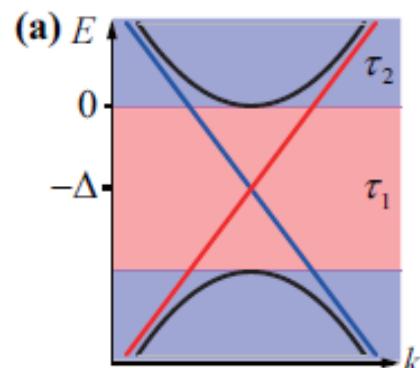
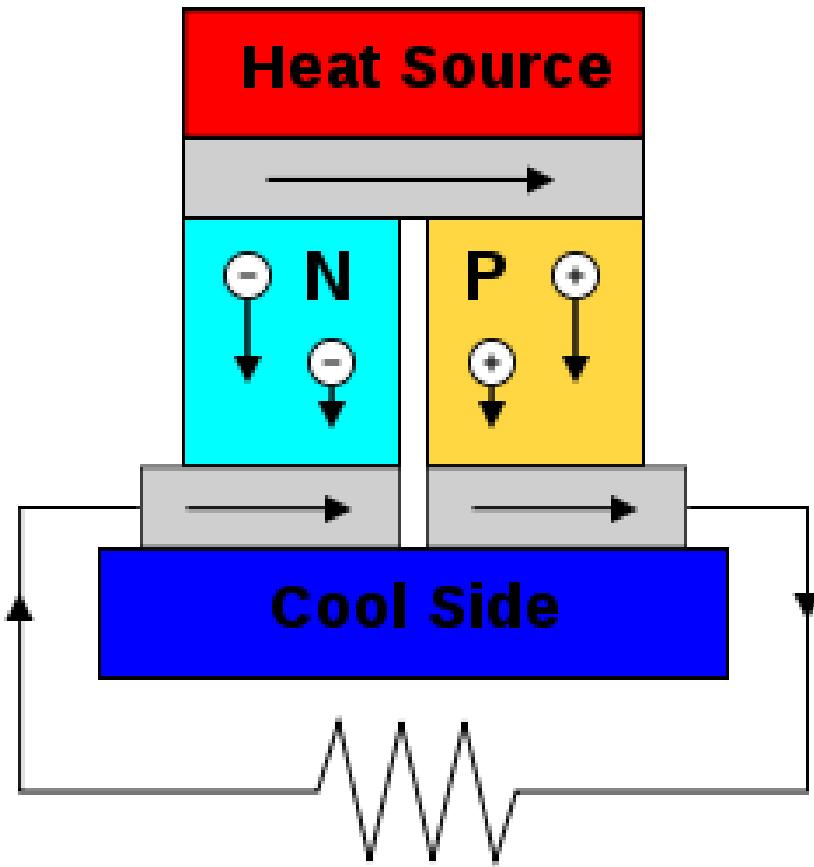


Around 3GHz, the current interconnect works in the RC dominated region; delay time 158 ns/mm; signal distortion due to  $\omega = iD k^2$  dispersion law.

Currently the industry is considering optical interconnect solution inside the chip!

# Energy applications: thermal electrics

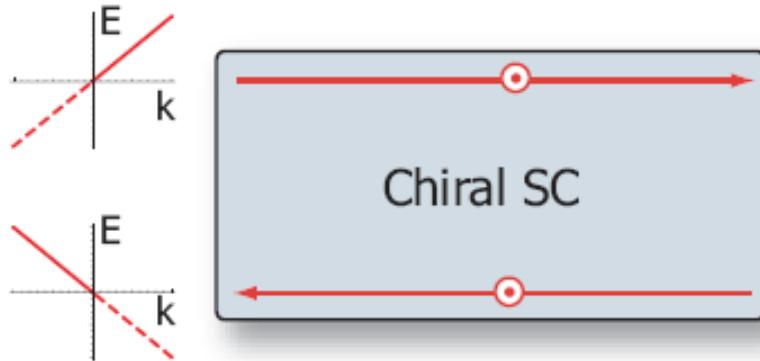
$$zT = \frac{\sigma S^2 T}{\kappa} \quad zT = \frac{G S^2 T}{K}$$



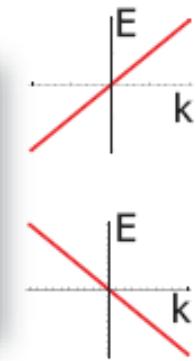
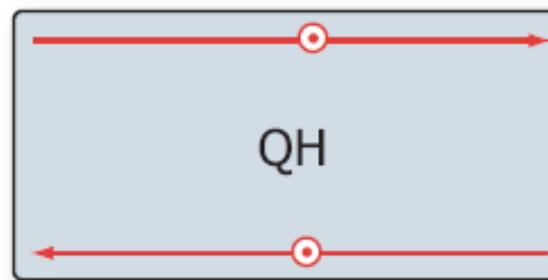
# Topological insulators and superconductors

Full pairing gap in the bulk, gapless Majorana edge and surface states

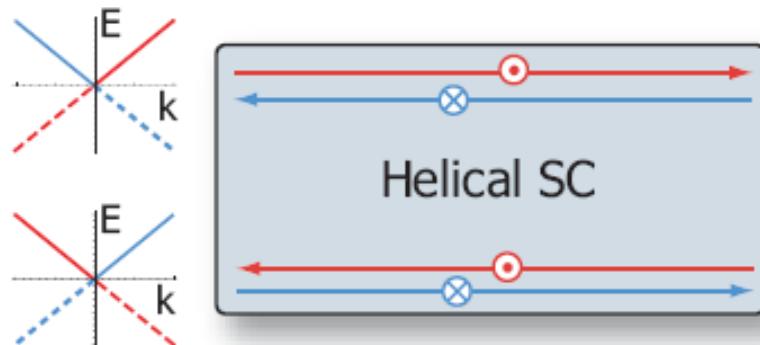
Chiral Majorana fermions



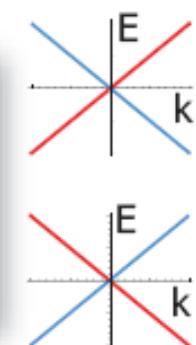
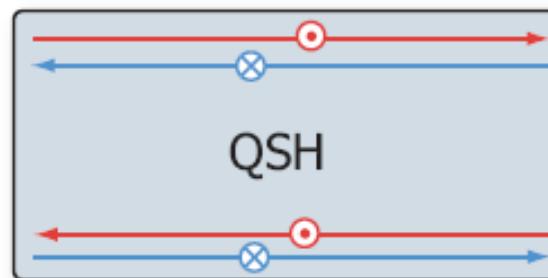
Chiral fermions



Helical SC



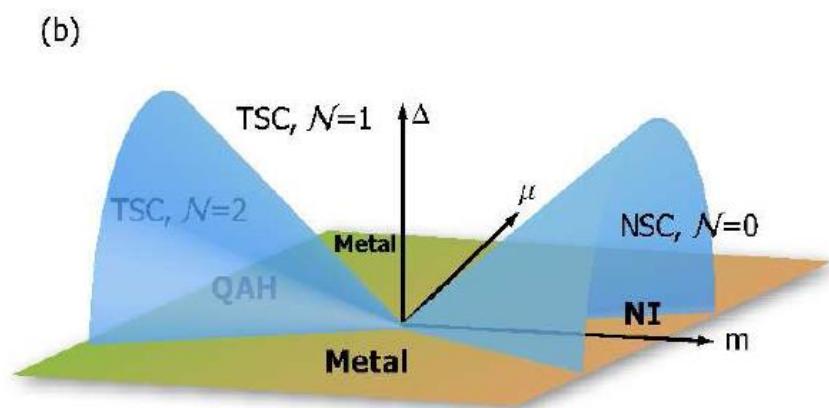
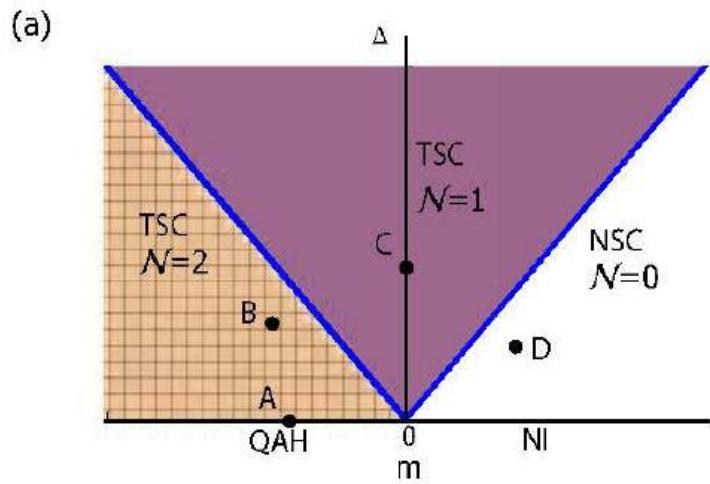
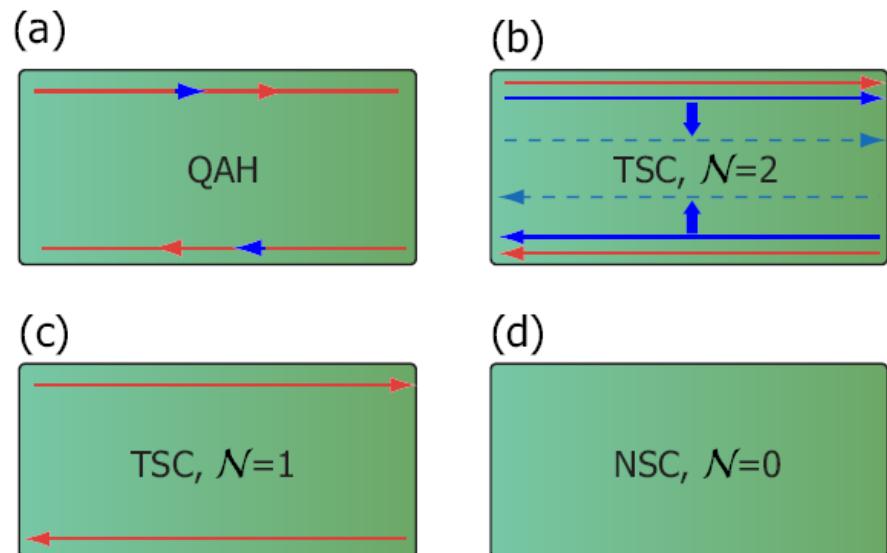
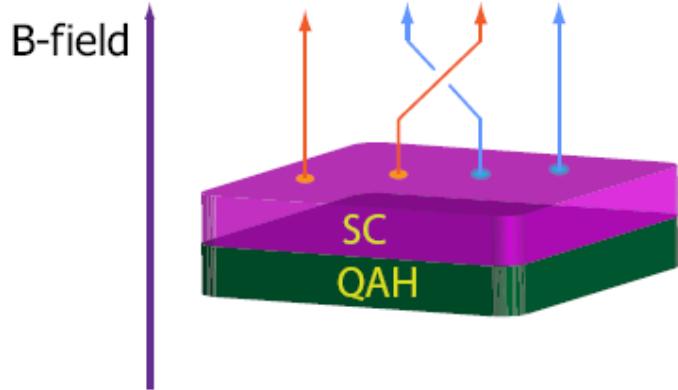
QSH



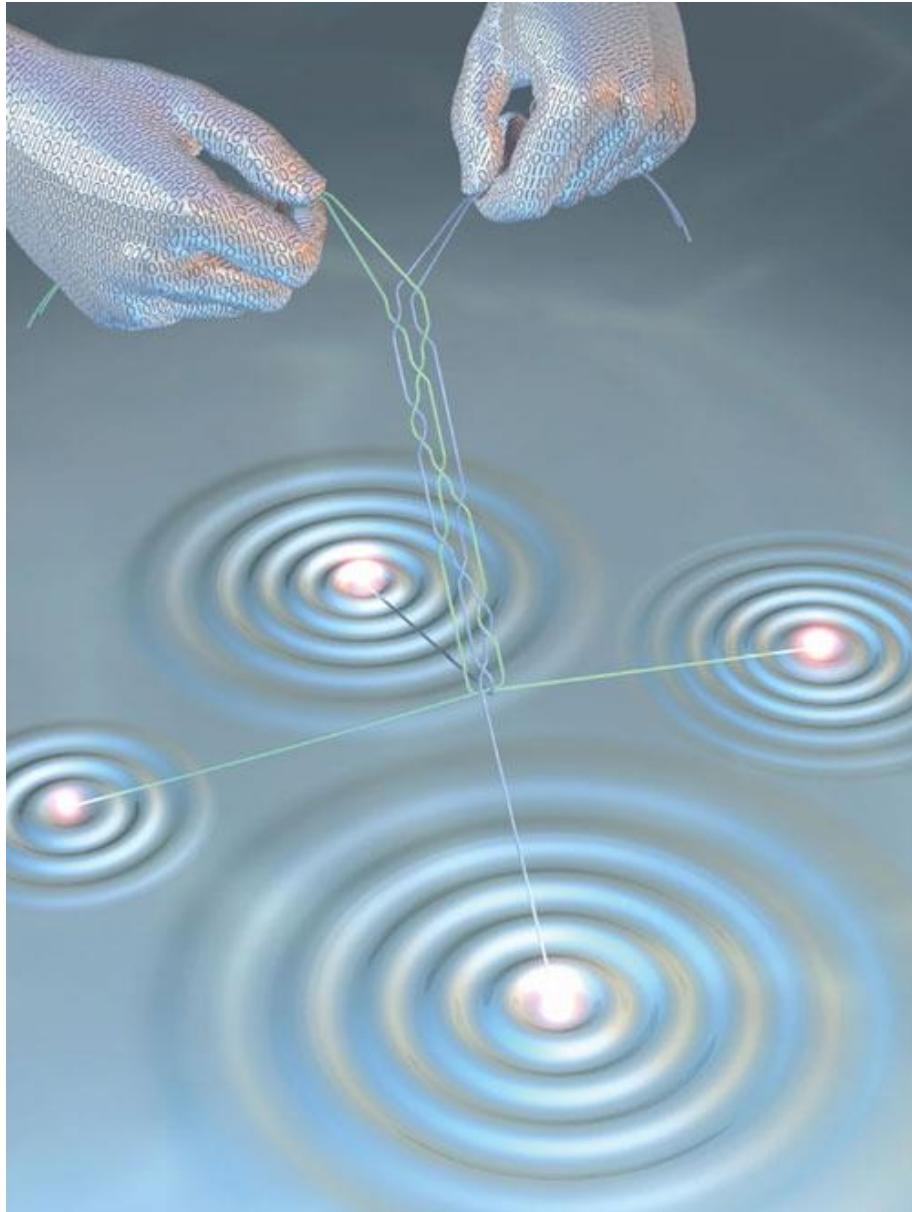
massless Majorana fermions

massless Dirac fermions

# Chiral superconductor obtained from QAH+SC (Qi, Hughes and SCZ, 2010)



# Topological quantum computers



$$15 = 3 \times 5$$

15475027472047264057  
29303484737456393847  
23937392273646483629  
37439327293272927236  
32927292820282739374  
= ?

# Topological insulators



**New material for the future**

**Novel applications in electronics and energy**

**Using Moore's law to save Moore's law!**