ENG2009

UNIVERSITY OF EXETER

FACULTY OF ENVIRONMENT, SCIENCE AND ECONOMY

ENGINEERING

2023/2024

Modelling of Engineering Systems Coursework

Module Convenor: Dr Halim Alwi

Date Set: 21st September 2023
Date Due: 15th December 2023, 12:00 ~~(noon)~~ (TBC)
Method of submission and file type: ELE2 (zip)

Answer all questions.

The actual number of marks associated with this coursework is 100, which translates to 50% of the overall module assessment.

This coursework tests your understanding of modelling and simulation of engineering systems.

Instructions

- This coursework has two parts: Analytical solutions for ODE - Hand Calculations
  1) A PDF file for Question (1) to (7).
  2) A Python program (py file) to implement simulations for Question (8). Euler & Runge Kutta

- Both files (**PDF and py**) must be zipped and must be submitted as a single **zip**, using **ELE2** (see here and here for detail).
- Submission must be made by 12:00 (noon) on the date indicated.
- Hand written work must be on lined A4 paper, scanned and converted into PDF.
- Your solutions must include neatly drawn labelled diagrams (where applicable), correct use of mathematical notation, sufficient comments and workings for the marker(s) to easily follow your process. Where practicable, you should also show how you have checked your answers.
- Python program must be clearly described with sufficient comments to explain the steps in the program.
- Marks are awarded for clear, legible presentation of work.

Week 2 Tutorial

System description: Mechanical mass-spring-damper system

The coursework deals with modelling and analysis of the mechanical mass-spring-damper system shown in Figure 1.

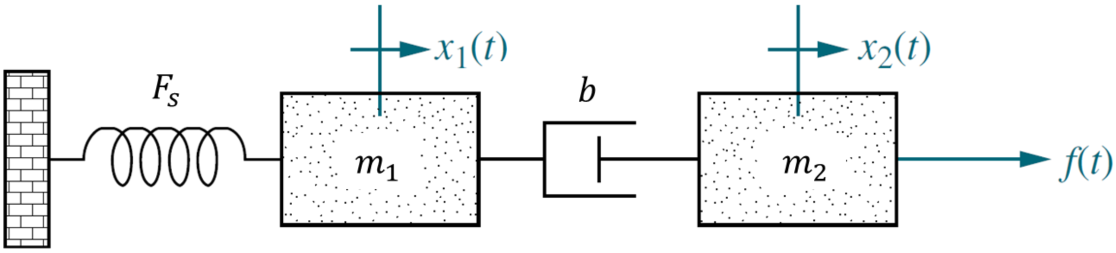

Figure 1: Nonlinear mass-spring-damper system

The variables $x_1(t)$ and $x_2(t)$ represent the positions of mass m_1 and m_2. The force $f(t)$ is the system input, whilst the output is the position $x_2(t)$.

Here the masses are $m_1 = 1\,kg$ and $m_2 = 1\,kg$ and the damper $b = 1\,Ns/m$. The spring is nonlinear, and the force (N), $F_s(t)$, required to stretch the spring is:

$$F_s(t) = 2x_1^2(t) \qquad \textbf{Eq.(1)}$$

Non linear

PART A (Modelling)

Question 1 (2 marks)
Provide a real-life example where a general mass-spring-damper system can be found.

Question 2 (10 marks) Tutorial 2 & 5

x2

Using a free-body-diagram, show that the differential equations representing the system in Figure 1 are given by:

Derivation

Non linear

$$\frac{d^2x_1(t)}{dt^2} + \frac{dx_1(t)}{dt} + 2x_1^2(t) - \frac{dx_2(t)}{dt} = 0 \qquad \textbf{Eq.(2)}$$

$$\frac{d^2x_2(t)}{dt^2} + \frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt} = f(t) \qquad \textbf{Eq.(3)}$$

Question 3 (10 marks)
Linearise the system for the point $x_1(0) = 1, x_2(0) = 0$, $f(0) = 0$, and show that the linearised differential equations are given by:

$$\frac{d^2\delta x_1(t)}{dt^2} + \frac{d\delta x_1(t)}{dt} + 2(1 + 2\delta x_1(t)) - \frac{d\delta x_2(t)}{dt} = 0 \qquad \textbf{Eq.(4)}$$

$$\frac{d^2\delta x_2(t)}{dt^2} + \frac{d\delta x_2(t)}{dt} - \frac{d\delta x_1(t)}{dt} = \delta f(t) \qquad \textbf{Eq.(5)}$$

Note: $\delta x_1(t) = x_1(t) - 1$, where 1 is the linearisation point.

Newton Raphson?

PART B (Analytical and numerical methods)

Remark: The remaining questions will be based on the linearised system given in Equation Eq.(6) and Equation Eq.(7).

For the remaining questions, replace the linear variable $\delta x_1(t)$ with $x_1(t)$, $\delta x_2(t)$ in Eq.(4) and Eq.(5) with $x_2(t)$ and $\delta f(t)$ with $f(t)$ (i.e. ignoring the linearisation point). In other words, the remaining questions are based on the result of the linearisation process, which is given by:

$$\frac{d^2x_1(t)}{dt^2} + \frac{dx_1(t)}{dt} + 2(1 + 2x_1(t)) - \frac{dx_2(t)}{dt} = 0 \qquad \textbf{Eq.(6)}$$

$$\frac{d^2x_2(t)}{dt^2} + \frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt} = f(t) \qquad \textbf{Eq.(7)}$$

Question 4 (6 marks)
Apply Laplace transform on the linearised system in equations Eq.(6)-Eq.(7). Assume zero initial conditions, i.e. $x_1(0) = 0, \dot{x}_1(0) = 0, x_2(0) = 0, \dot{x}_2(0) = 0$.

Input-Output Ratio

Question 5 (7 marks)
Using the results from Question (4),

a) Obtain the transfer function for the linearised system (i.e. equations Eq.(6) and Eq.(7)). Note that the input of the system is $f(t)$ and the output is $x_2(t)$.
(5 marks)

b) What is the order of the system? Justify your answer.
(2 marks)

Question 6 (12 marks)
Using the transfer function from Question (5)

a) Obtain and clearly list <u>all the poles and zeros</u> of the linearised system. (**Remark**: there should be two zeros and four poles. Two of the poles are at $s = -0.3522 \pm 1.7214i$ and one at $s = 0$. You need to find the other one real pole. You can use calculator to find these values.)
(**Remark 2:** the location of the remaining pole is between -2 and 0).
(4 marks)

b) Determine if the system is stable. Justify your answer.
(3 marks)

c) Draw the "s-plane".
(5 marks)

Question 7 (3 marks)
Use Final Value Theorem to predict the value of $x_2(t)$ (if available) when the input is a unit impulse. Justify your answer.

Week 3/4

Use files from 1st Yr

Coding available in python files ELE

Question 8 (50 marks)
Using Equation (6) and Equation (7), create a simulation of the system using both:
a) Euler method, and
b) Heun (Runge-Kutta second order) method

Can be editted from Euler

Assume that the input force $f(t)$ is a unit impulse (duration of 0.1 sec from $t = 0$).

For each method (**25 marks** $\times$ **2** $=$ **50 marks**):

i. Write <u>your own Python code</u> that creates the simulation of the system *(10 marks)*

ii. Select an appropriate step size $h = \Delta t$. Explain your reasoning. (**Remark**: As a guidance, this value must be chosen (iteratively?) such that the steady state error is less than 1%). Use profile tab – Spyder *(7 marks)*

iii. Select an appropriate end time for the simulation. Explain your reasoning. *(2 marks)*

iv. Calculate the error (E_t) and percentage error ($|\epsilon_t|\%$) and discuss the results. **Remark**: Use the final value from Question (7) as the exact numerical solution *(3 marks)*

v. Plot the output responses (i.e. plot the output position $x_2(t)$ against time in seconds) and discuss the results in terms of general system behaviour (e.g. underdamped or overdamped etc). *(3 marks)*

Important remark for Question 8:
- Write <u>your own Python code</u> to implement both numerical methods to simulate the system.
- Python program must be clearly described with sufficient comments to explain the steps in the program, especially addressing items (i) to (v) from Question (8).
- Use a single Python file to implement both methods.
- You can use an existing/example program to check your solution.
- However, do not submit the existing/example program as your submission.
- To avoid compatibility issues when marking the Python code, please use Spyder 4.2.5 (Python 3.8). (You can download Spyder through Anaconda navigator – see the webpage: https://www.anaconda.com/products/individual).

Exam: Inperson
closed Book -> Laplace Transform Table
Approved Calculators.

END OF QUESTION PAPER