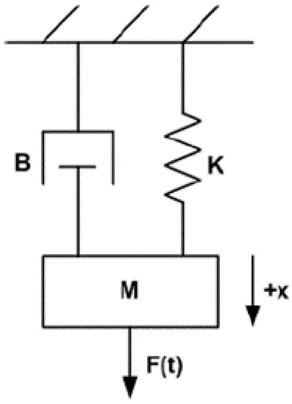
ENG2009 – Modelling of Engineering Systems

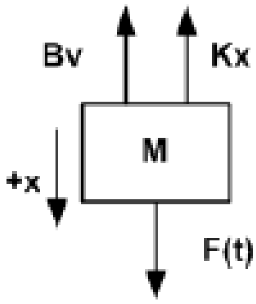
Tutorial 1

Mechanical Systems (Translational)

Example: Write the mathematical equation for the following mechanical system



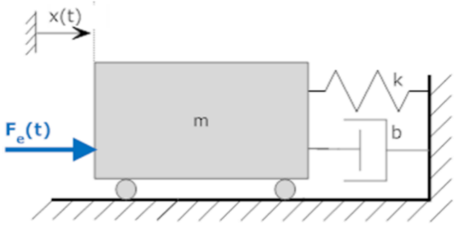
Answer:
Free body diagram:



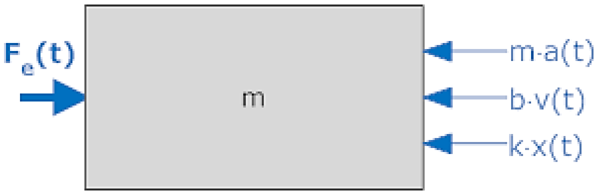
Math model:

$$\sum F_y = Ma$$
$$-Bv - Kx + F = Ma$$
$$a = \frac{1}{M}(-Bv - Kx + F)$$
Let $a = \ddot{x},\ v = \dot{x}$
$$\ddot{x} = \frac{1}{M}(-B\dot{x} - Kx + F)$$

Example: (Mass spring damper from Lecture 2)
Write the mathematical equation for the following mechanical system, where the input is the external force F_e and the outpus is position $x(t)$.



Answer:
Free body diagram:



Math model:

$$\sum_{all} F = 0$$

$$F_e(t) - ma(t) - bv(t) - kx(t) = 0$$

or

$$m\frac{d^2x(t)}{dt^2} + b\frac{dx(t)}{dt} + kx(t) = F_e(t)$$

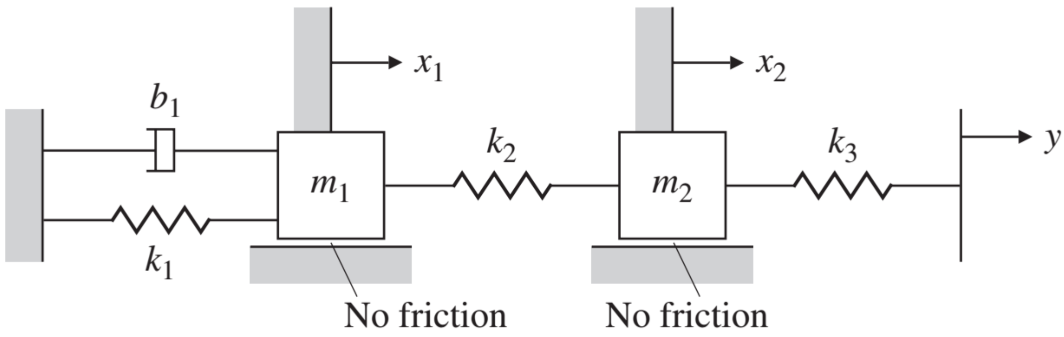
Or dot notation

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F_e(t)$$

Remark:
This equation is in our standard form (input-output notation):
Left hand side: system outputs (the unknown variables)
right hand side: system inputs (the known variables)

Excercise:

Q1) Write the differential equations for the mechanical systems shown below. Assume that there are nonzero initial conditions for both masses and there is no input.



Electrical systems

Notes:

Kirchhoff's voltage law: *the algebraic sum of all voltages taken around any closed path in a circuit is zero*.

$$\sum_j v_j = 0$$

where v_j denotes the voltage across the j_{th} element in the loop

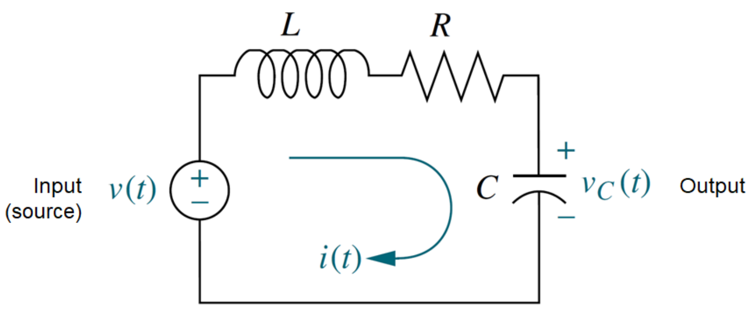
Kirchhoff's current law: *the algebraic sum of the currents at **any junction** is zero*.

$$\sum_j i_j = 0$$

where i_j denotes the current at the j_{th} node.

Example:

Find the differential equation relating the capacitor voltage $v_c(t)$ (output voltage), to the input voltage, $v(t)$



Answer:

- Summing the voltages around the loop, assuming zero initial conditions, yields the integral-differential equation:

$$v_{inductor} + v_{resistor} + v_{capacitor} = v_{in}$$

$$L\frac{di(t)}{dt} + i(t)R + \frac{1}{C}\int_0^t i(\tau)\, d\tau = v(t)$$

- Changing variables from current to charge using

$$i = \frac{dq}{dt}$$

(see Table 1 in lecture notes) yields:

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = v(t)$$

- Changing variables from voltage to charge using

$$q = Cv_c$$

(see Table 1 in lecture notes) yields:

$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = v(t)$$

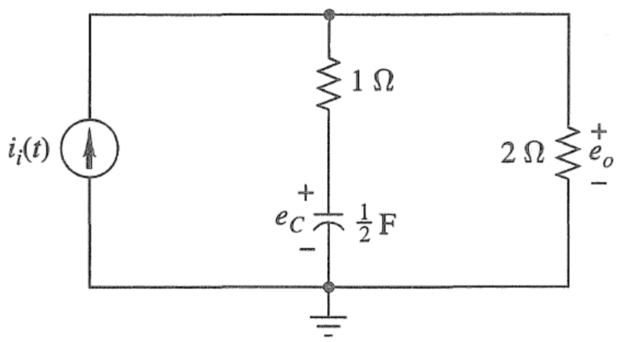
Where $v(t)$ is the input voltage (source)

and $v_c(t)$ is the output voltage

Excercises:

Q1) Find the input-output differential equation relating the output voltage $e_o(t)$, to the input current, $i_i(t)$ for the following circuit

Remark: change of notation as compared to lecture notes. Iinstead of voltage v here voltage is represented by e



Q2) Find the input-output differential equation relating the output voltage $e_o(t)$, to the input voltage, $e_i(t)$ for the following circuit

Remark: change of notation as compared to lecture notes. Iinstead of voltage v here voltage is represented by e