ENG2009 – Modelling of Engineering Systems

Tutorial 6

Simultaneous Linear Equations 1:

Naïve Gauss elimination:

Example :

Example 1

The upward velocity of a rocket is given at three different times

Table 1 Velocity vs. time data.

Time t (sec)	Velocity v (m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as:

$$v(t) = a_1 t^2 + a_2 t + a_3 , \qquad 5 \le t \le 12.$$

Find the velocity at $t = 6$ seconds .

Answer:

Example 1 Cont.

Rewrite as

$$t^2 a_1 + t a_2 + a_3 = v(t), \quad 5 \le t \le 12$$

Results in a matrix template of the form:

$$\begin{bmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Using data from Table 1, the matrix becomes:

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 64 & 8 & 1 & \vdots & 177.2 \\ 144 & 12 & 1 & \vdots & 279.2 \end{bmatrix}$$

1.Forward Elimination
2.Back Substitution

Number of Steps of Forward Elimination

From

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

- 3 equations, 3 unknown
- Solve unknown: a_1, a_2, a_3
- Number of steps of forward elimination is
  $$(n-1) = (3-1) = 2$$

Forward Elimination: Step 1

Idea: eliminate 64 from Equation 2

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 64 & 8 & 1 & \vdots & 177.2 \\ 144 & 12 & 1 & \vdots & 279.2 \end{bmatrix}$$

Divide Equation 1 by 25 and multiply it by 64, i.e. $\frac{64}{25} = 2.56$

$$[25 \quad 5 \quad 1 \quad \vdots \quad 106.8] \times 2.56 = [64 \quad 12.8 \quad 2.56 \quad \vdots \quad 273.408]$$

Subtract the result from Equation 2

$$\begin{array}{r} [64 \quad 8 \quad 1 \quad \vdots \quad 177.2] \\ -[64 \quad 12.8 \quad 2.56 \quad \vdots \quad 273.408] \\ \hline [0 \quad -4.8 \quad -1.56 \quad \vdots \quad -96.208] \end{array}$$

Substitute new equation for Equation 2

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 0 & -4.8 & -1.56 & \vdots & -96.208 \\ 144 & 12 & 1 & \vdots & 279.2 \end{bmatrix}$$

Forward Elimination: Step 1 (cont.)

Idea: eliminate 144 from Equation 3

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 0 & -4.8 & -1.56 & \vdots & -96.208 \\ 144 & 12 & 1 & \vdots & 279.2 \end{bmatrix}$$

Divide Equation 1 by 25 and multiply it by 144, i.e. $\frac{144}{25} = 5.7$

$$[25 \quad 5 \quad 1 \quad \vdots \quad 106.8] \times 5.76 = [144 \quad 28.8 \quad 5.76 \quad \vdots \quad 615.168]$$

Subtract the result from Equation 3

$$\begin{array}{r} [144 \quad 12 \quad 1 \quad \vdots \quad 279.2] \\ -[144 \quad 28.8 \quad 5.76 \quad \vdots \quad 615.168] \\ \hline [0 \quad -16.8 \quad -4.76 \quad \vdots \quad -335.968] \end{array}$$

Substitute new equation for Equation 3

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 0 & -4.8 & -1.56 & \vdots & -96.208 \\ 0 & -16.8 & -4.76 & \vdots & -335.968 \end{bmatrix}$$

Forward Elimination: Step 2

Idea: eliminate -16.8 from Equation 3

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 0 & -4.8 & -1.56 & \vdots & -96.208 \\ 0 & -16.8 & -4.76 & \vdots & -335.968 \end{bmatrix}$$

Divide Equation 2 by -4.8 and multiply it by -16.8, i.e. $\frac{-16.8}{-4.8} = 3.5$.

$$[0 \quad -4.8 \quad -1.56 \quad \vdots \quad -96.208] \times 3.5 = [0 \quad -16.8 \quad -5.46 \quad \vdots \quad -336.728]$$

Subtract the result from Equation 3

$$\begin{array}{r} [0 \quad -16.8 \quad -4.76 \quad \vdots \quad 335.968] \\ -[0 \quad -16.8 \quad -5.46 \quad \vdots \quad -336.728] \\ \hline [0 \quad 0 \quad 0.7 \quad \vdots \quad 0.76] \end{array}$$

Substitute new equation for Equation 3

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 0 & -4.8 & -1.56 & \vdots & -96.208 \\ 0 & 0 & 0.7 & \vdots & 0.76 \end{bmatrix}$$

Back Substitution

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 0 & -4.8 & -1.56 & \vdots & -96.2 \\ 0 & 0 & 0.7 & \vdots & 0.7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.208 \\ 0.76 \end{bmatrix}$$

Solving for a_3

$0.7a_3 = 0.76$

Therefore

$$a_3 = \frac{0.76}{0.7} = 1.08571$$

Back Substitution (cont.)

Last slide: $a_3 = 1.08571$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.208 \\ 0.76 \end{bmatrix}$$

Solving for a_2

$-4.8a_2 - 1.56a_3 = -96.208$

Therefore

$$a_2 = \frac{-96.208 + 1.56a_3}{-4.8} = \frac{-96.208 + 1.56 \times 1.08571}{-4.8} = 19.6905$$

Back Substitution (cont.)

Last slide: $a_3 = 1.08571$, $a_2 = 19.6905$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.2 \\ 0.76 \end{bmatrix}$$

Solving for a_1

$$25a_1 + 5a_2 + a_3 = 106.8$$

$$a_1 = \frac{106.8 - 5a_2 - a_3}{25} = \frac{106.8 - 5 \times 19.6905 - 1.08571}{25} = 0.290472$$

Example 1 solution

Original problem formulation:

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

The solution vector is $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0.290472 \\ 19.6905 \\ 1.08571 \end{bmatrix}$

The polynomial that passes through the three data points is then:

$$\begin{aligned} v(t) &= a_1 t^2 + a_2 t + a_3 \\ &= 0.290472t^2 + 19.6905t + 1.08571, \qquad 5 \le t \le 12 \end{aligned}$$

Therefore at 6 sec:

$$\begin{aligned} v(6) &= 0.290472(6)^2 + 19.6905(6) + 1.08571 \\ &= 129.686 \text{ m/s.} \end{aligned}$$

Gaussian elimination: Partial pivoting example

Example:

Recall: Example

The upward velocity of a rocket is given at three different times

Table 1 Velocity vs. time data.

Time t (sec)	Velocity v (m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as:

$$v(t) = a_1 t^2 + a_2 t + a_3 \,, \qquad 5 \le t \le 12.$$

Find the velocity at $t = 6$ seconds,

using Gaussian elimination with partial pivoting.

Answer:

Recall: Example

Rewrite as

$$t^2 a_1 + t a_2 + a_3 = v(t), \quad 5 \le t \le 12$$

Results in a matrix template of the form:

$$\begin{bmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Using data from Table 1, the matrix becomes:

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 64 & 8 & 1 & \vdots & 177.2 \\ 144 & 12 & 1 & \vdots & 279.2 \end{bmatrix}$$

1. Forward Elimination (switch row if required)
2. Back Substitution

Recall: Number of Steps of Forward Elimination

From

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

- 3 equations, 3 unknown
- Solve unknown: a_1, a_2, a_3
- Number of steps of forward elimination is

$$(n-1) = (3-1) = 2$$

Forward Elimination: Step 1

From

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Examine absolute values of first column, first row and below.

$$|25|, |64|, |144|$$

- Largest absolute value is 144 and exists in row 3.
- Switch row 1 and row 3.

Therefore

$$\begin{bmatrix} 25 & 5 & 1 & \vdots & 106.8 \\ 64 & 8 & 1 & \vdots & 177.2 \\ 144 & 12 & 1 & \vdots & 279.2 \end{bmatrix} \Rightarrow \begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 64 & 8 & 1 & \vdots & 177.2 \\ 25 & 5 & 1 & \vdots & 106.8 \end{bmatrix}$$

Forward Elimination: Step 1 (cont.)

Idea: eliminate 64 from Equation 2

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 64 & 8 & 1 & \vdots & 177.2 \\ 25 & 5 & 1 & \vdots & 106.8 \end{bmatrix}$$

Divide Equation 1 by 144 and multiply it by 64, i.e. $\frac{64}{144} = 0.4444$

$$[144 \quad 12 \quad 1 \quad \vdots \quad 279.2] \times 0.4444 = [63.99 \quad 5.333 \quad 0.4444 \quad \vdots \quad 124.1]$$

Subtract the result from Equation 2

$$\begin{array}{r} [64 \quad 8 \quad 1 \quad \vdots \quad 177.2] \\ -[63.99 \quad 5.333 \quad 0.4444 \quad \vdots \quad 124.1] \\ \hline [0 \quad 2.667 \quad 0.5556 \quad \vdots \quad 53.10] \end{array}$$

Substitute new equation for Equation 2

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.667 & 0.5556 & \vdots & 53.10 \\ 25 & 5 & 1 & \vdots & 106.8 \end{bmatrix}$$

Forward Elimination: Step 1 (cont.)

Idea: eliminate 25 from Equation 3

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.667 & 0.5556 & \vdots & 53.10 \\ 25 & 5 & 1 & \vdots & 106.8 \end{bmatrix}$$

Divide Equation 1 by 144 and multiply it by 25, i.e. $\frac{25}{144} = 0.1736$

$$[144 \quad 12 \quad 1 \quad \vdots \quad 279.2] \times 0.1736 = [25.00 \quad 2.083 \quad 0.1736 \quad \vdots \quad 48.47]$$

Subtract the result from Equation 3

$$\begin{array}{r} [25 \quad 5 \quad 1 \quad \vdots \quad 106.8] \\ -[25 \quad 2.083 \quad 0.1736 \quad \vdots \quad 48.47] \\ \hline [0 \quad 2.917 \quad 0.8264 \quad \vdots \quad 58.33] \end{array}$$

Substitute new equation for Equation 3

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.667 & 0.5556 & \vdots & 53.10 \\ 0 & 2.917 & 0.8264 & \vdots & 58.33 \end{bmatrix}$$

Forward Elimination: Step 2

From last step

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.667 & 0.5556 & \vdots & 53.10 \\ 0 & 2.917 & 0.8264 & \vdots & 58.33 \end{bmatrix}$$

Examine absolute values of second column, second row and below:

$$|2.667|, |2.917|$$

- Largest absolute value is 2.917 and exists in row 3.
- Switch row 2 and row 3.

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.667 & 0.5556 & \vdots & 53.10 \\ 0 & 2.917 & 0.8264 & \vdots & 58.33 \end{bmatrix} \Rightarrow \begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.917 & 0.8264 & \vdots & 58.33 \\ 0 & 2.667 & 0.5556 & \vdots & 53.10 \end{bmatrix}$$

Forward Elimination: Step 2 (cont.)

Idea: eliminate 2.667 from Equation 3

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.917 & 0.8264 & \vdots & 58.33 \\ 0 & 2.667 & 0.5556 & \vdots & 53.10 \end{bmatrix}$$

Divide Equation 2 by 2.917 and multiply it by 2.667, i.e. $\frac{2.667}{2.917} = 0.9143$.

$$[0 \quad 2.917 \quad 0.8264 \quad \vdots \quad 58.33] \times 0.9143 = [0 \quad 2.667 \quad 0.7556 \quad \vdots \quad 53.33]$$

Subtract the result from Equation 3

$$\begin{array}{r} [0 \quad 2.667 \quad 0.5556 \quad \vdots \quad 53.10] \\ -[0 \quad 2.667 \quad 0.7556 \quad \vdots \quad 53.33] \\ \hline [0 \quad 0 \quad -0.2 \quad \vdots \quad -0.23] \end{array}$$

Substitute new equation for Equation 3

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.917 & 0.8264 & \vdots & 58.33 \\ 0 & 0 & -0.2 & \vdots & -0.23 \end{bmatrix}$$

Back Substitution

$$\begin{bmatrix} 144 & 12 & 1 & \vdots & 279.2 \\ 0 & 2.917 & 0.8264 & \vdots & 58.33 \\ 0 & 0 & -0.2 & \vdots & -0.23 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.917 & 0.8264 \\ 0 & 0 & -0.2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 279.2 \\ 58.33 \\ -0.23 \end{bmatrix}$$

Solving for a_3

$$-0.2a_3 = -0.23$$

Therefore

$$a_3 = \frac{-0.23}{-0.2} = 1.15$$

Back Substitution (cont.)

Last slide: $a_3 = 1.15$

$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.917 & 0.8264 \\ 0 & 0 & -0.2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 279.2 \\ 58.33 \\ -0.23 \end{bmatrix}$$

Solving for a_2

$$2.917a_2 + 0.8264a_3 = 58.33$$
$$a_2 = \frac{58.33 - 0.8264a_3}{2.917} = \frac{58.33 - 0.8264 \times 1.15}{2.917} = 19.67$$

Back Substitution (cont.)

Last slide: $a_3 = 1.15$, $a_2 = 19.67$

$$\begin{bmatrix} 144 & 12 & 1 \\ 0 & 2.917 & 0.8264 \\ 0 & 0 & -0.2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 279.2 \\ 58.33 \\ -0.23 \end{bmatrix}$$

Solving for a_1

$$144a_1 + 12a_2 + a_3 = 279.2$$
$$a_1 = \frac{279.2 - 12a_2 - a_3}{144} = \frac{279.2 - 12 \times 19.67 - 1.15}{144} = 0.2917$$

Gaussian Elim. with Partial Pivoting Solution

Original problem

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Solution (similar to Naïve Gaussian elimination)

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0.2917 \\ 19.67 \\ 1.15 \end{bmatrix}$$

The polynomial that passes through the three data points is then:

$$v(t) = a_1t^2 + a_2t + a_3$$
$$= 0.2917t^2 + 19.67t + 1.15, \qquad 5 \le t \le 12$$

Therefore at 6 sec:

$$v(6) = 0.2917(6)^2 + 19.67(6) + 1.15$$
$= 129.6712$ m/s (similar to Naïve Gaussian elimination)

LU decomposition

Example:

Using LU Decomposition to solve SLEs

Example:
Solve the following set of linear equations using LU Decomposition

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$
$$[A] \quad [X] = [C]$$

Using the procedure for finding the $[L]$ and $[U]$ matrices

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Answer:

Example

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$
$$[A] \quad [X] = [C]$$

$$[A] = [L][U] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Set $[L][Z] = [C]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Solve for $[Z]$

$$z_1 = 106.8$$
$$2.56z_1 + z_2 = 177.2$$
$$5.76z_1 + 3.5z_2 + z_3 = 279.2$$

Example

Solve for $[Z]$

$$z_1 = 106.8$$
$$2.56z_1 + z_2 = 177.2$$
$$5.76z_1 + 3.5z_2 + z_3 = 279.2$$

Complete the forward substitution to solve for $[Z]$

$$\begin{aligned} z_1 &= 106.8 \\ z_2 &= 177.2 - 2.56z_1 \\ &= 177.2 - 2.56(106.8) \\ &= -96.208 \\ z_3 &= 279.2 - 5.76z_1 - 3.5z_2 \\ &= 279.2 - 5.76(106.8) - 3.5(-96.208) \\ &= 0.76 \end{aligned}$$

Therefore

$$[Z] = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.208 \\ 0.76 \end{bmatrix}$$

Example

Set $[U][X] = [Z]$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.208 \\ 0.76 \end{bmatrix}$$

Solve for $[X]$

The 3 equations become

$$25x_1 + 5x_2 + x_3 = 106.8$$
$$-4.8x_2 - 1.56x_3 = -96.208$$
$$0.7x_3 = 0.76$$

Example

From the 3rd equation

$$0.7x_3 = 0.76$$
$$x_3 = \frac{0.76}{0.7}$$
$$x_3 = 1.0857$$

Substituting in x_3 and using the second equation

$$-4.8x_2 - 1.56x_3 = -96.208$$

$$\begin{aligned} x_2 &= \frac{-96.21 + 1.56x_3}{-4.8} \\ &= \frac{-96.21 + 1.56(1.0857)}{-4.8} \\ &= 19.691 \end{aligned}$$

Example

Substituting in x_3 and x_2 using the first equation

$$25x_1 + 5x_2 + x_3 = 106.8$$

$$\begin{aligned} x_1 &= \frac{106.8 - 5x_2 - x_3}{25} \\ &= \frac{106.8 - 5(19.691) - 1.0857}{25} \\ &= 0.29048 \end{aligned}$$

Hence the Solution Vector is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.29048 \\ 19.691 \\ 1.0857 \end{bmatrix}$$

Exercises

Q 1) (Kaw & Kalu)

A trunnion of diameter $12.363"$ has to be cooled from a room temperature of $80°F$ before it is shrink fitted into a steel hub (Figure 1).

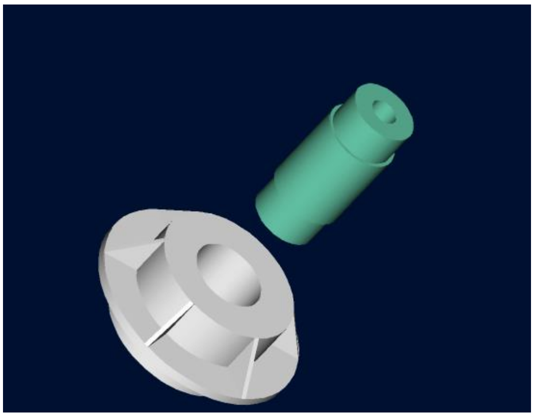

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction ΔD of the trunnion in a dry-ice/alcohol mixture (boiling temperature is $-108°F$) is given by

$$\Delta D = 12.363 \int_{80}^{-108} \alpha(T)dT$$

The equation for the thermal expansion coefficient, $\alpha = a_1 + a_2T + a_3T^2$, is obtained using regression analysis where the constants of the model are found by solving the following simultaneous linear equations.

$$\begin{bmatrix} 24 & -2860 & 7.26\times10^5 \\ -2860 & 7.26\times10^5 & -1.86472\times10^8 \\ 7.26\times10^5 & -1.86472\times10^8 & 5.24357\times10^{10} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1.057\times10^{-4} \\ -1.04162\times10^{-2} \\ 2.56799 \end{bmatrix}$$

Find the values of a_1, a_2, and a_3 using

a) naïve Gaussian elimination,
b) partial-pivoting Gaussian elimination,
c) LU decomposition