Model 1: Fixed

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Data Prep and Cleaning

```r
# Load and clean data
original_tbl <- read.csv("./NBA-BoxScores-2023-2024.csv") |>
  mutate(
    START_POSITION = na_if(START_POSITION, "") |> factor(),
    COMMENT = na_if(COMMENT, "") |> factor(),
    MIN = na_if(MIN, ""),
    MIN = str_replace(MIN, "([0-9]+)\\.[0-9]+:", "\\1:")
  )

# Filter to starters only
starting_dat <- original_tbl |>
  filter(!is.na(START_POSITION))

# Calculate team points per game
team_points <- original_tbl |>
  filter(!is.na(PTS)) |>
  group_by(GAME_ID, TEAM_ID) |>
  summarize(TeamPoints = sum(PTS), .groups = "drop")

# Join with itself to get opponent points
team_vs_opponent <- team_points |>
  inner_join(team_points, by = "GAME_ID", suffix = c("", ".opp")) |>
  filter(TEAM_ID != TEAM_ID.opp) |>
  rename(OPP_TEAM_ID = TEAM_ID.opp, OpponentPoints = TeamPoints.opp)

# Compute average opponent points allowed per team (DRTG)
team_drtg <- team_vs_opponent |>
  group_by(TEAM_ID) |>
  summarize(DRTG_proxy = mean(OpponentPoints), n_games = n(), .groups = "drop")

# Build opponent_map from distinct team-game pairs
game_team_pairs <- original_tbl |>
  select(GAME_ID, TEAM_ID) |>
  distinct()

# Create mapping of TEAM_ID and OPP_TEAM_ID for each game
opponent_map <- game_team_pairs |>
  inner_join(game_team_pairs, by = "GAME_ID") |>
  filter(TEAM_ID.x != TEAM_ID.y) |>
  rename(TEAM_ID = TEAM_ID.x, OPP_TEAM_ID = TEAM_ID.y)
```

```r
# Join with defensive ratings (DRTG)
opponent_map <- opponent_map |>
  left_join(team_drtg |> rename(OPP_TEAM_ID = TEAM_ID, OPP_DRTG = DRTG_proxy), by = "OPP_TEAM_ID")


# Merge opponent info into starting dataset and center DRTG
mean_drtg <- mean(team_drtg$DRTG_proxy)

starting_dat <- starting_dat |>
  left_join(opponent_map, by = c("GAME_ID", "TEAM_ID")) |>
  mutate(centered_OPP_DRTG = OPP_DRTG - mean_drtg)
```

Model 1 implementation

Situation:

$$
\begin{aligned}
y_{ijk} &\sim Binom(n_{ijk}, p_{ik}) \\
n_{ijk} &\sim \text{fixed n, trying mean, median, max} \\
p_{ik} &= p_i \times \exp(\gamma(\text{DRTG}_k - \bar{\text{DRTG}})) \\
p_i &\sim Beta(5, 5)
\end{aligned}
$$

Metropolis Hastings Implementation to Obtain p__i

```r
log_q = function(theta, y, n) {
  if (theta < 0 || theta > 1) return(-Inf)
  sum(dbinom(y, size = n, prob = theta, log = TRUE)) + dbeta(theta, 5, 5, log = TRUE)
}

MH_beta_binom = function(current = 0.5, prop_sd = 0.05, n_vec, y_vec, n_iter = 1000) {
  samps = rep(NA, n_iter)
  for (i in 1:n_iter) {
    proposed = rnorm(1, current, prop_sd)
    logr = log_q(proposed, y = y_vec, n = n_vec) - log_q(current, y = y_vec, n = n_vec)
    if (log(runif(1)) < logr) current = proposed
    samps[i] = current
  }
  return(samps)
}

MH_beta_grid_search = function(current = 0.5, n_vec, y_vec, n_iter = 1000) {
  vals <- seq(0.005, 1, by = 0.005)
  effect_sizes <- data.frame(sd = vals, ess = NA)

  for (i in seq_along(vals)) {
    samps <- MH_beta_binom(current = current, prop_sd = vals[i],
                           n_vec = n_vec, y_vec = y_vec, n_iter = 5000)
    effect_sizes$ess[i] <- effectiveSize(samps)
  }

  best_sd <- effect_sizes$sd[which.max(effect_sizes$ess)]
  final_samps <- MH_beta_binom(current = current, prop_sd = best_sd,
                               n_vec = n_vec, y_vec = y_vec, n_iter = n_iter)

  return(list(
    samples = final_samps,
    best_sd = best_sd,
    ess_table = effect_sizes
  ))
}
```

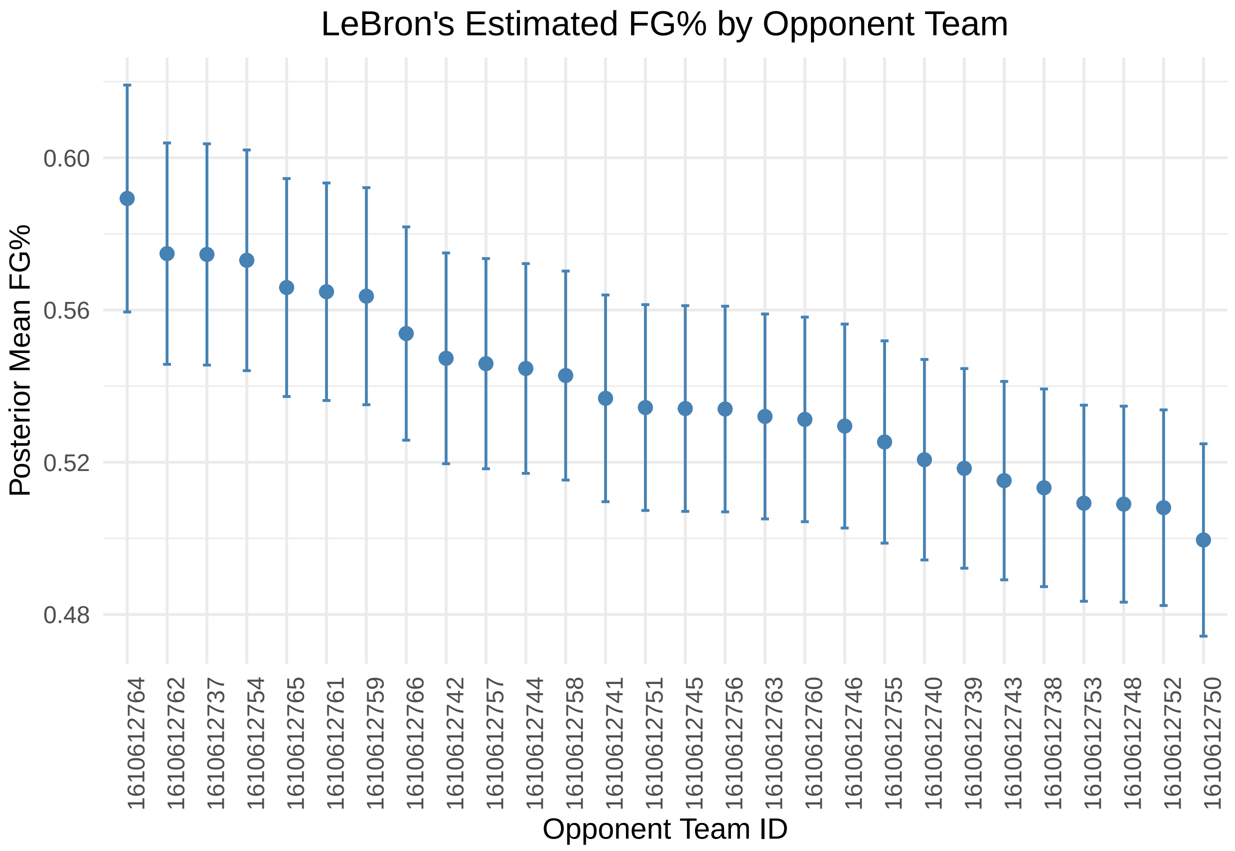
Data Prep for Exploring LeBron James vs GSW (Golden State Warriors)

```
# Filter to LeBron James's games
# LeBron PLAYER_ID is 2544
lebron_dat <- starting_dat |>
  filter(PLAYER_ID == 2544)

# Centered DRTG values for each opponent team LeBron faced
# Only has 28 rows because LeBron only played against 28 out of 30 NBA teams
# LeBron did not play against Milwaukee Bucks (or the Lakers, his own team)
lebron_team_drtg <- lebron_dat |>
  group_by(OPP_TEAM_ID) |>
  summarize(centered_OPP_DRTG = first(centered_OPP_DRTG), .groups = "drop") |>
  arrange(OPP_TEAM_ID)

# Store response and trial vectors
Y <- lebron_dat$FGM
N_vec <- lebron_dat$FGA
n_iter = 100000

mh_result <- MH_beta_grid_search(current = 0.5,
                                 y_vec = Y,
                                 n_vec = N_vec,
                                 n_iter = n_iter)

p_samples <- mh_result$samples
mh_result$best_sd  # tuning parameter chosen
```

```
## [1] 0.03
```

```
drtg_vec <- lebron_team_drtg$centered_OPP_DRTG
gamma_val <- 0.01 # fixed tuning value

# Compute p_ik matrix: rows = posterior draws, cols = opponent teams
p_ik_matrix <- outer(
  p_samples,
  drtg_vec,
  FUN = function(p, drtg) p * exp(gamma_val * drtg)) |> pmin(1)  # clip to max 1

# Posterior mean FG% per opponent
p_ik_mean <- colMeans(p_ik_matrix)
```

```r
# Credible intervals
p_ik_CI <- apply(p_ik_matrix, 2, quantile, probs = c(0.025, 0.975))

# Combine into a table
p_ik_summary <- data.frame(
  OPP_TEAM_ID = lebron_team_drtg$OPP_TEAM_ID,
  p_ik_mean = p_ik_mean,
  p_ik_lower = p_ik_CI[1,],
  p_ik_upper = p_ik_CI[2,]
)

ggplot(p_ik_summary, aes(x = reorder(as.factor(OPP_TEAM_ID), -p_ik_mean), y = p_ik_mean)) +
  geom_point(color = "steelblue", size = 2) +
  geom_errorbar(aes(ymin = p_ik_lower, ymax = p_ik_upper), width = 0.2, color = "steelblue") +
  labs(
    title = "LeBron's Estimated FG% by Opponent Team",
    x = "Opponent Team ID",
    y = "Posterior Mean FG%"
  ) +
  theme_minimal() +
  theme(
    axis.text.x = element_text(angle = 90, hjust = 1),
    plot.title = element_text(hjust = 0.5)
  )
```



Now we will look at LeBron against Stephen Curry predictions (against Golden State Warriors, TEAM_ID = 1610612744)

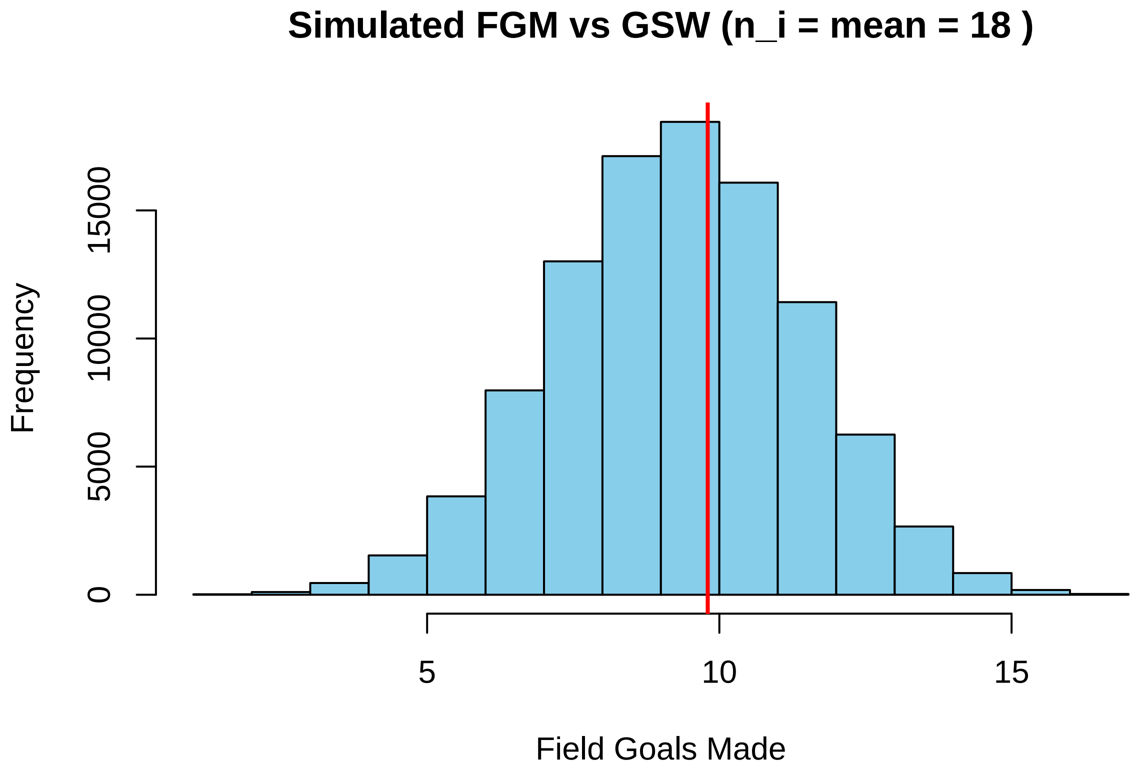
```
# Get centered DRTG for GSW (Golden State Warriors)
drtg_gsw <- lebron_team_drtg$centered_OPP_DRTG[lebron_team_drtg$OPP_TEAM_ID == 1610612744]
gamma_val <- 0.01  # fixed tuning value
p_ik_gsw <- p_samples * exp(gamma_val * drtg_gsw)
p_ik_gsw <- pmin(p_ik_gsw, 1)  # ensure FG% stays within [0,1]

mean(p_ik_gsw)
```

```
## [1] 0.5446406
```

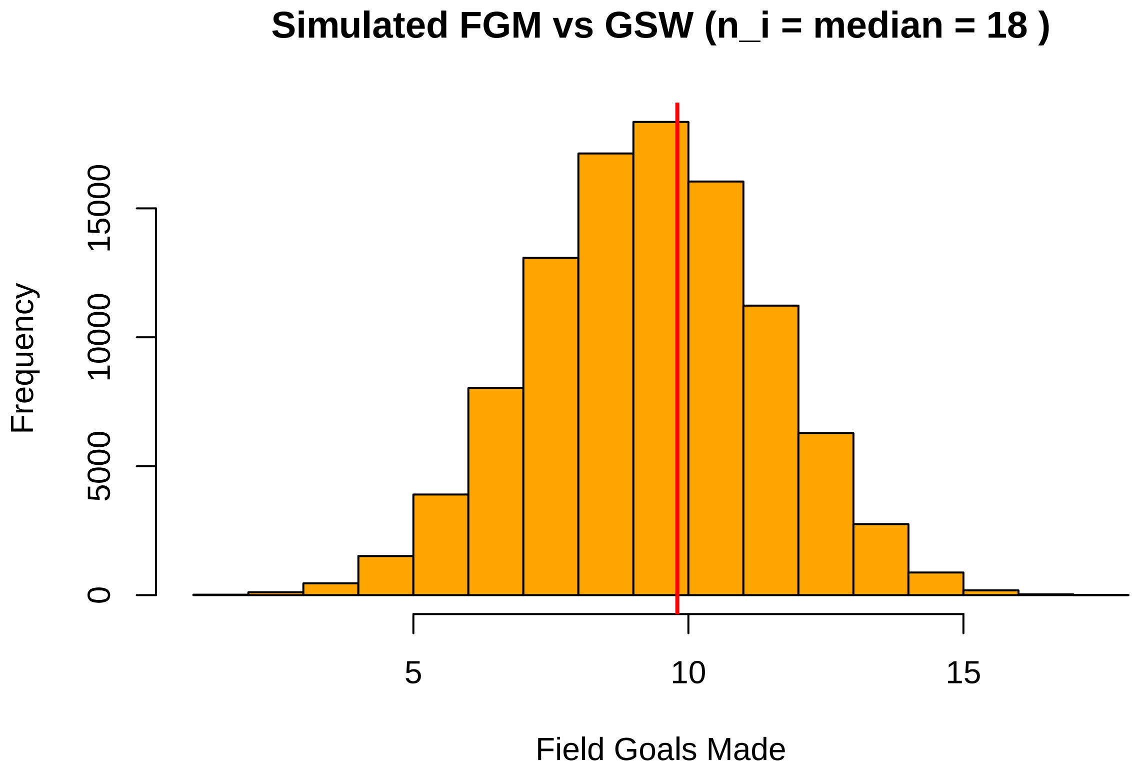
```
quantile(p_ik_gsw, c(0.025, 0.975))
```

```
##      2.5%     97.5%
## 0.5170887 0.5721842
```

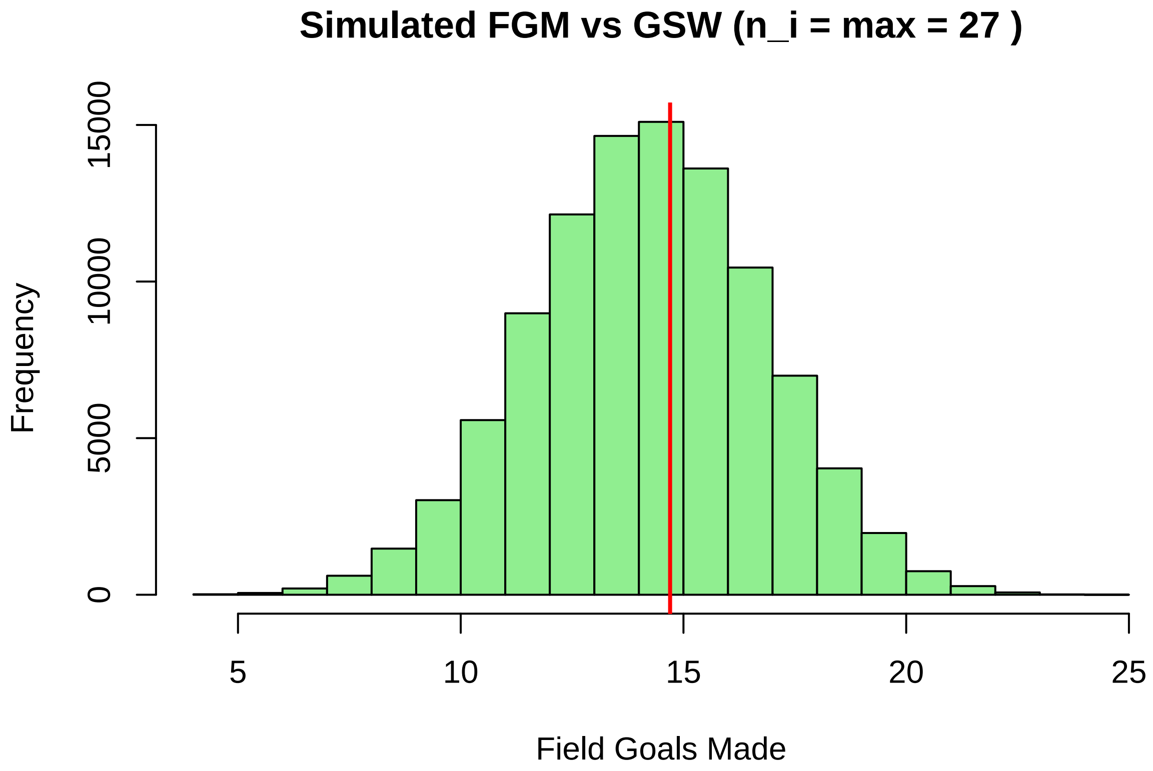
```
# FGAs for LeBron
n_mean <- round(mean(lebron_dat$FGA, na.rm = TRUE))
n_median <- round(median(lebron_dat$FGA, na.rm = TRUE))
n_max <- max(lebron_dat$FGA, na.rm = TRUE)

# Simulations using different n_i values
set.seed(5440)
fgm_sim_mean <- rbinom(length(p_ik_gsw), size = n_mean, prob = p_ik_gsw)
fgm_sim_median <- rbinom(length(p_ik_gsw), size = n_median, prob = p_ik_gsw)
fgm_sim_max <- rbinom(length(p_ik_gsw), size = n_max, prob = p_ik_gsw)

# Plot 1: Mean FGA
hist(fgm_sim_mean,
     main = paste("Simulated FGM vs GSW (n_i = mean =", n_mean, ")"),
     xlab = "Field Goals Made",
     col = "skyblue", breaks = 15)
abline(v = mean(fgm_sim_mean), col = "red", lwd = 2)
```



```r
# Plot 2: Median FGA
hist(fgm_sim_median,
     main = paste("Simulated FGM vs GSW (n_i = median =", n_median, ")"),
     xlab = "Field Goals Made",
     col = "orange", breaks = 15)
abline(v = mean(fgm_sim_median), col = "red", lwd = 2)
```

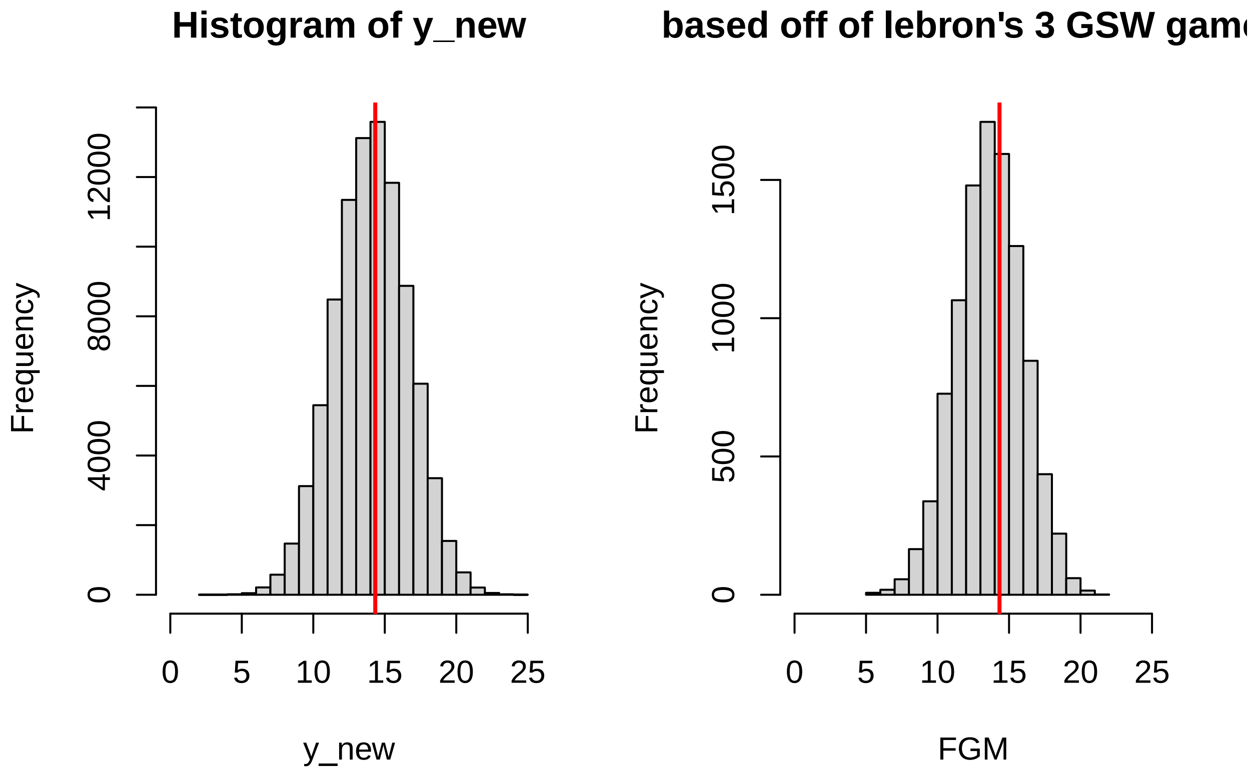


```r
# Plot 3: Max FGA
hist(fgm_sim_max,
     main = paste("Simulated FGM vs GSW (n_i = max =", n_max, ")"),
     xlab = "Field Goals Made",
     col = "lightgreen", breaks = 15)
abline(v = mean(fgm_sim_max), col = "red", lwd = 2)
```



Predictive posterior

```
player_dat = lebron_dat[lebron_dat$OPP_TEAM_ID == 1610612744,]
# setup / data
lebron_GSW_n<- player_dat$FGA
lebron_mean_n<- mean(lebron_GSW_n)
lebron_median_n<- median(lebron_GSW_n)
lebron_max_n<- max(lebron_GSW_n)
lebron_GSW_p<- player_dat$FG_PCT
lebron_GSW_y<- player_dat$FGM

# And go!
burnin = 10000
y_new = rbinom(n_iter-burnin, n_max, p_samples[burnin:n_iter])

par(mfrow=c(1,2))
hist(y_new,xlim=c(0,25))
abline(v=mean(lebron_GSW_y), col='red', lwd=2, xlab="FGM")
hist(rbinom(10000,round(mean(lebron_GSW_n)),mean(lebron_GSW_p)),xlim=c(0,25), main="based off of lebron
abline(v=mean(lebron_GSW_y), col='red', lwd=2, xlab="FGM")
```



```
model_1_sample = rbinom(n_iter-burnin, n_max, p_samples[burnin:n_iter])
```