

Model 3 Neg Binom

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Beginning Data preparing

```
## Ben's DRTG code!  
# Calculate total points per team per game  
# here, datatest2 is the entire data frame that is not filtered for starters  
# filtering dataset to remove NAs which arise a player doesnt record any minutes in the game (sitting o  
team_points <- na.omit(original_tbl) %>%  
  group_by(GAME_ID, TEAM_ID) %>%  
  summarize(TeamPoints = sum(PTS), .groups = "drop")  
  
#  
team_points_opponent <- team_points %>%  
  rename(OPP_TEAM_ID = TEAM_ID, OpponentPoints = TeamPoints)  
  
# join and filter  
team_vs_opponent <- team_points %>%  
  inner_join(team_points_opponent, by = "GAME_ID") %>%  
  filter(TEAM_ID != OPP_TEAM_ID)  
  
# calculate average opponent points per team (our DRTG)  
team_drtg <- team_vs_opponent %>%  
  group_by(TEAM_ID) %>%  
  summarize(DRTG_proxy = mean(OpponentPoints), n_games = n(), .groups = "drop")  
  
range(team_drtg$DRTG_proxy)
```

```
## [1] 106.5244 123.0366
```

```
mean(team_drtg$DRTG_proxy)
```

```
## [1] 114.2114
```

DRTG data being joined to the starting data

```
## NOW ADDING DRTG vars to original_tbl  
  
# Each team plays one opponent per game, so we pair them like this:  
game_team_pairs <- original_tbl %>%  
  select(GAME_ID, TEAM_ID) %>%  
  distinct()
```

```

# Join to get each game twice: once for each team, with their opponent
opponent_map <- game_team_pairs %>%
  inner_join(game_team_pairs, by = "GAME_ID") %>%
  filter(TEAM_ID.x != TEAM_ID.y) %>%
  rename(TEAM_ID = TEAM_ID.x, OPP_TEAM_ID = TEAM_ID.y)

# Add opponent DRTG to the mapping
opponent_map <- opponent_map %>%
  left_join(team_drtg %>% rename(OPP_TEAM_ID = TEAM_ID, OPP_DRTG = DRTG_proxy),
    by = "OPP_TEAM_ID")

# Join to add opponent DRTG column to the full data
starting_dat <- starting_dat %>%
  left_join(opponent_map, by = c("GAME_ID", "TEAM_ID"))

mean_drtg <- mean(team_drtg$DRTG_proxy)
starting_dat <- starting_dat %>%
  mutate(centered_OPP_DRTG = OPP_DRTG - mean_drtg)

```

Model 3 implementation

$$\begin{aligned}
 y_{ikj} &\sim \text{Binom}(n_{ikj}, p_{ik}) \\
 p_{ik} &\sim \phi \times \text{Beta}(a, b) \\
 n_{ijk} &\sim \text{NegBinom}(r, \theta) \\
 r &\sim \text{Gamma}(2, 0.1) \\
 \theta &\sim \text{Beta}(2, 2)
 \end{aligned}$$

Next we write out the full conditionals

$$\begin{aligned}
 y_{ikj} | \dots &\sim \text{Bin}(n_{ikj}, p_{ik}) \\
 p_{ik} | \dots &\propto \phi p(y|p_{ik}, n_{ikj}) p(p_{ik}|a, b) \\
 n_{ikj} | \dots &\propto p(y_{ikj}|p_{ik}, n_{ikj}) p(n_{ikj}|r, \theta) \\
 a, b | \dots &\propto p(p_{ik}|a, b) p(a, b) \\
 r, \theta | \dots &\propto p(n_{ikj}|r, \theta) p(r, \theta)
 \end{aligned}$$

So for this, we kinda have a full thingy set up?

```

# Warning: Produces lots of NaNs!
# Data and prior
lebron_dat = starting_dat[starting_dat$PLAYER_ID %in% 2544, ]
y = lebron_dat$FGM
n = lebron_dat$FGM
a = 1
b = 1
r = 20
theta = 0.5
eta = 2
DRTG = lebron_dat$centered_OPP_DRTG
psi=0.2

```

```

# Priors for r's gamma
Ga = 0.5
Gb = 0.1

# Priors for theta's beta
Ta = 2
Tb = 2

# Initial values
n = 5
p = 0.1

# Save structures
n_iter = 5000
n_keep = matrix(NA, ncol=length(y), nrow=n_iter)
p_keep = matrix(NA, ncol=length(y), nrow=n_iter)
a_keep = numeric(n_iter)
b_keep = numeric(n_iter)
r_keep = numeric(n_iter)
theta_keep = numeric(n_iter)

# proposal variances
prop_sd_a = 0.4
prop_sd_b = 0.4
prop_sd_eta = 0.4
prop_sd_r = 0.4

#pre-calculations
n = length(y)
sum_y = sum(y)

# full log conditionals
log_fc_a = function(p, a, b) {
  if (a<0) return(-Inf)
  n = length(p)
  (a-1)*sum(log(p))-n*lbeta(a,b)-5/2*(a+b)
}
log_fc_b = function(p, a, b) {
  if (b<0) return(-Inf)
  n = length(p)
  (b-1)*sum(log(1-p))-n*lbeta(a,b)-5/2*(a+b)
}

log_fc_r = function(n, r, theta, Ga, Gb) {
  N = length(theta)
  return(sum(log(gamma(n+r))-log(r)) +
    r^(Ga-1)*N*log(theta)+log(r)-Gb*r)
}

log_fc_eta = function(N, r, theta, prop_sd_eta, eta) {
  sum(dnbinom(N, size=r, prob=theta, log=TRUE)) +

```

```

    dnorm(eta, mean=0, sd = prop_sd_eta, log=TRUE)
}

for (i in 1:n_iter){
  # generate p based on a and b
  p = pmin(1, exp(psi * DRTG) * rbeta(length(y), a, b))

  #sample a from full conditional
  a_prop = rnorm(1, a, prop_sd_a)
  logr = log_fc_a(p, a_prop, b) - log_fc_a(p, a, b)
  if (is.finite(logr) && log(runif(1)) < logr) {
    a <- a_prop
  }

  # sample b from full conditional
  b_prop = rnorm(1, b, prop_sd_b)
  logr = log_fc_b(p, a, b_prop) - log_fc_b(p, a, b)
  if (is.finite(logr) && log(runif(1)) < logr) {
    b <- b_prop
  }

  # generate n based on r and theta
  n = rbinom(length(y), r, theta)

  # sample theta from full conditional using a weird logit function
  eta_prop = rnorm(1, b, prop_sd_r)
  theta_prop = 1/(1+exp(-eta_prop))
  logr = log_fc_eta(length(y), r, theta_prop, prop_sd_eta, eta_prop) - log_fc_eta(length(y), r, theta,
  if (is.finite(logr) && log(runif(1)) < logr) {
    eta <- eta_prop
    theta <- theta_prop
  }

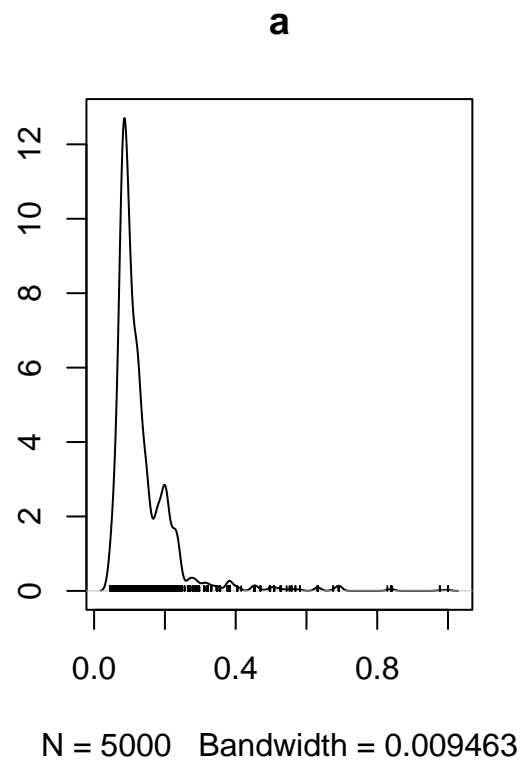
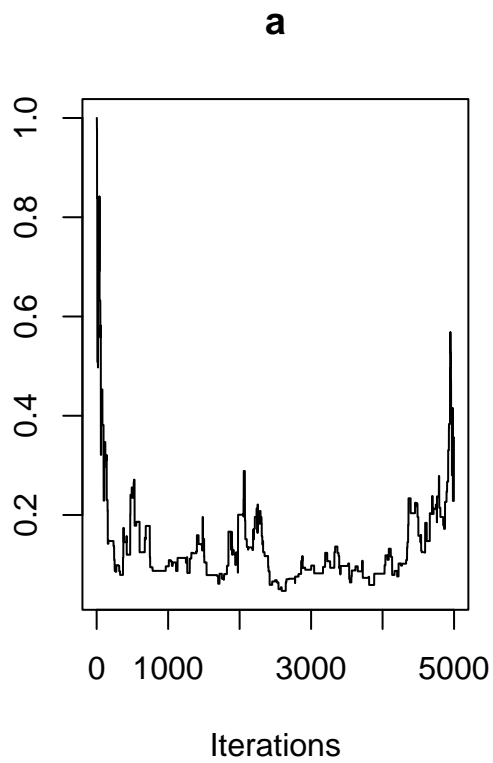
  # sample r from full conditional
  r_prop = rnorm(1, b, prop_sd_r)
  logr = log_fc_r(n, r_prop, theta, Ga, Gb) - log_fc_r(n, r, theta, Ga, Gb)
  if (is.finite(logr) && log(runif(1)) < logr) {
    r <- r_prop
  }

  # Save our samples
  n_keep[i,] = n
  p_keep[i,] = p
  a_keep[i] = a
  b_keep[i] = b
  r_keep[i] = r
  theta_keep[i] = theta
}

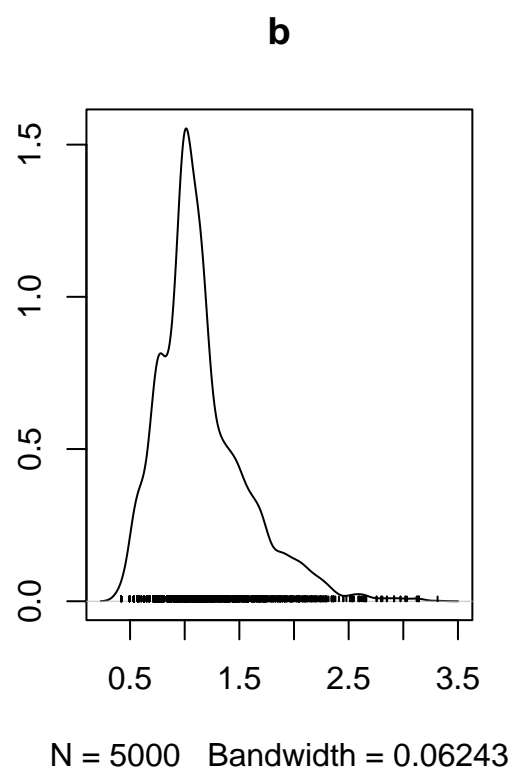
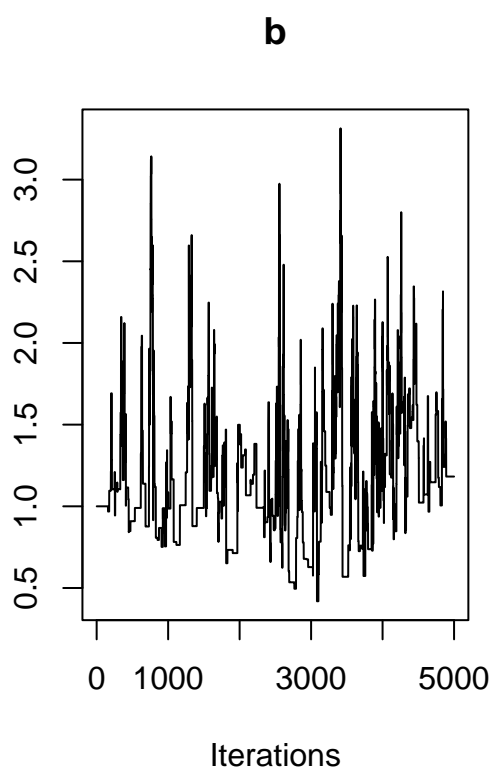
```

Some diagnostics to consider

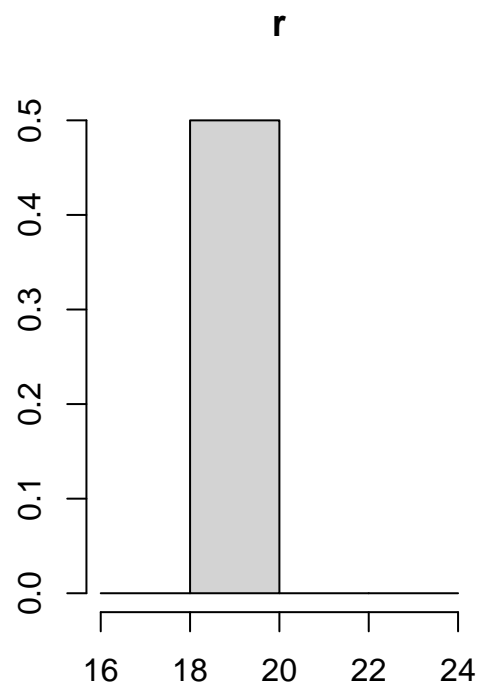
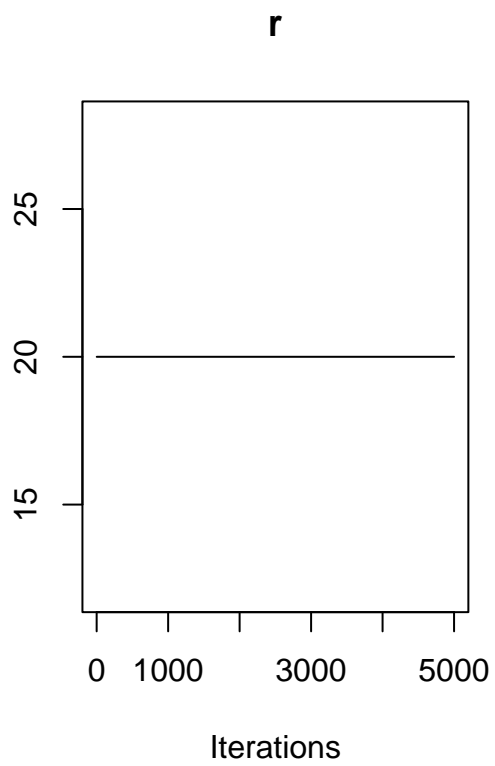
```
par(mfrow=c(4,2))
plot(as.mcmc(a_keep), main="a")
```



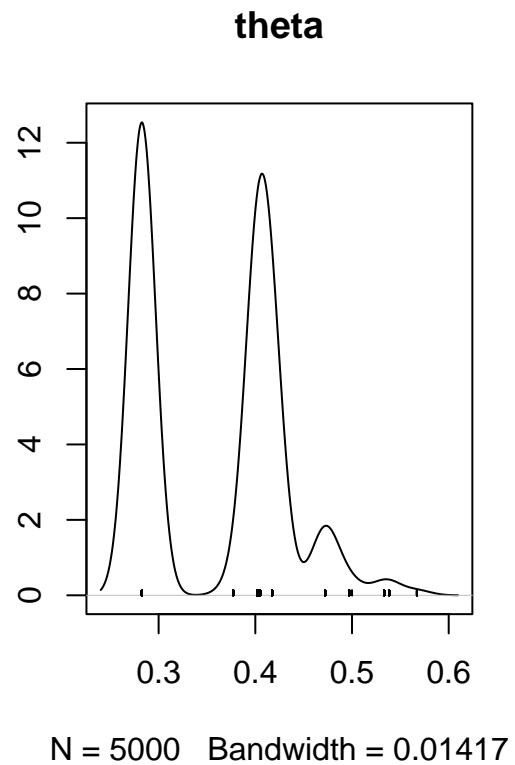
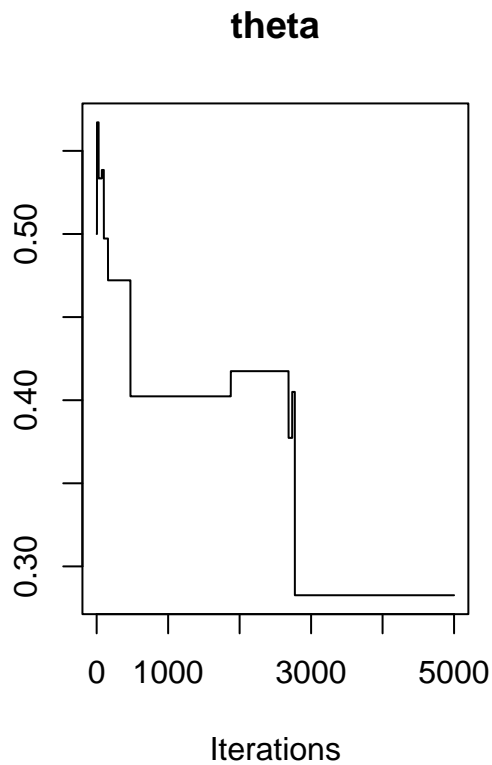
```
plot(as.mcmc(b_keep), main="b")
```



```
plot(as.mcmc(r_keep), main="r")
```

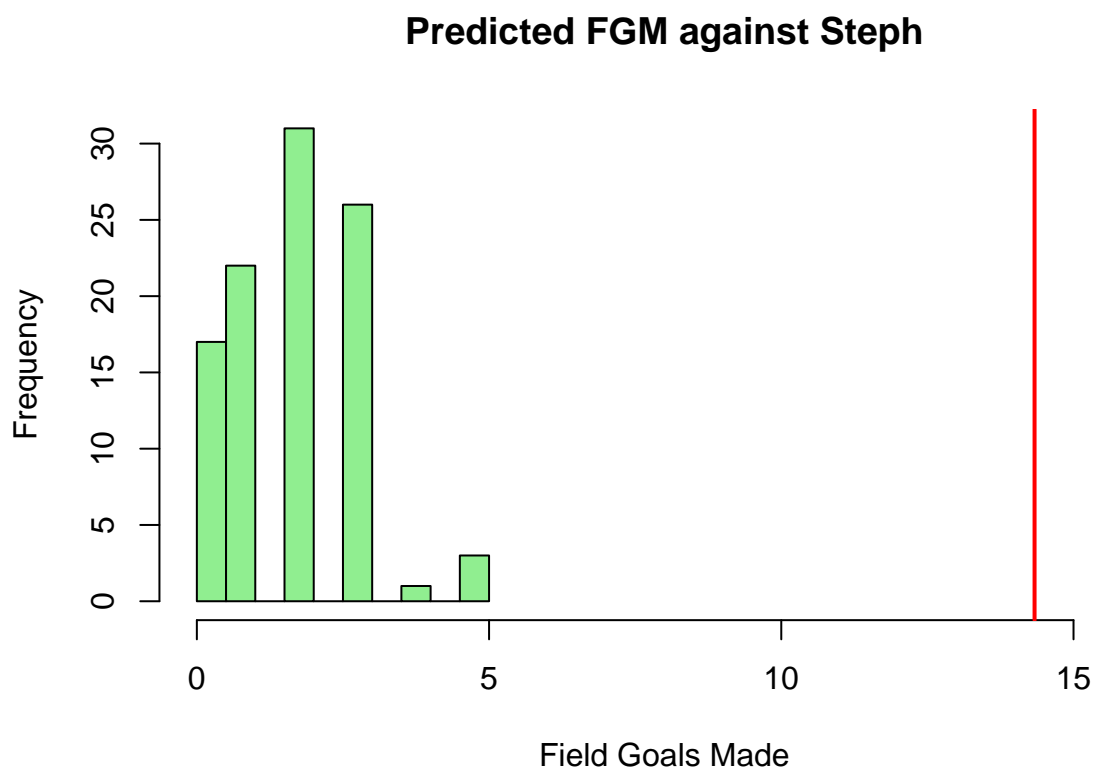


```
plot(as.mcmc(theta_keep), main="theta")
```

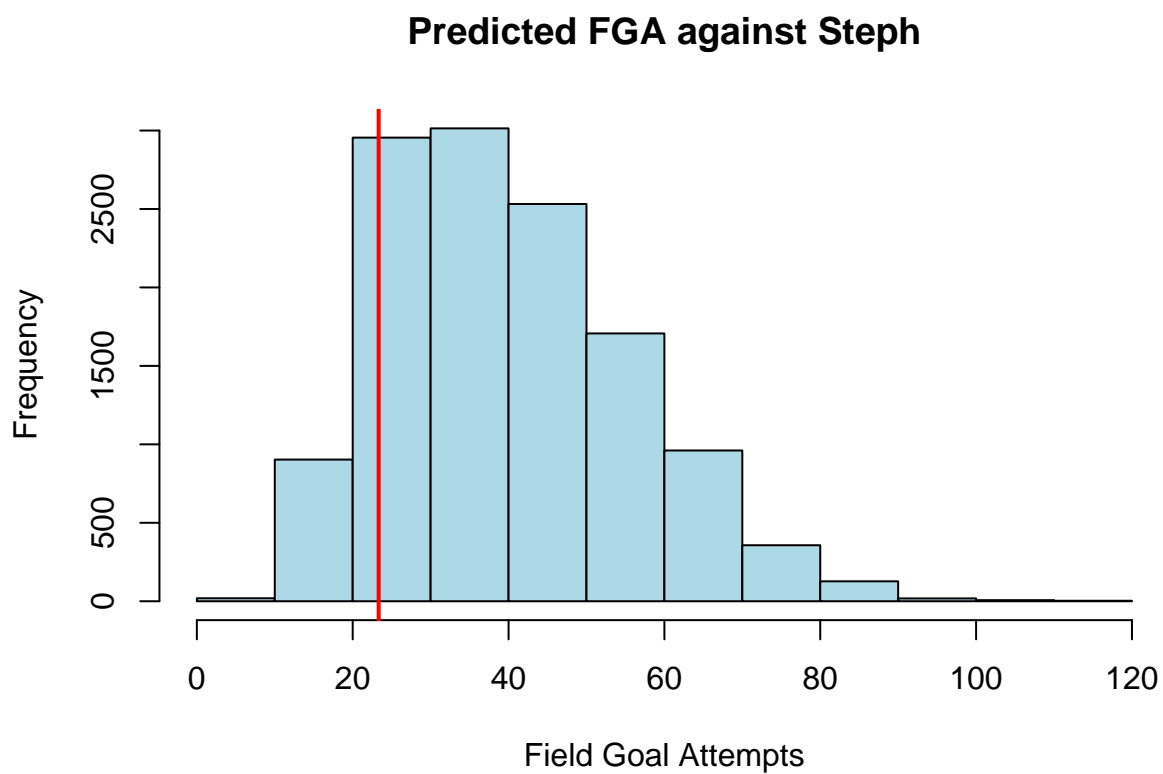


Find the columns pertaining to the steph v lebron games

```
cols_of_interest = which(lebron_dat$GAME_ID %in% lebron_vs_steph_games, TRUE)
n_pred = round(mean(n_keep[n_iter, cols_of_interest]))
p_pred = mean(p_keep[n_iter, cols_of_interest]) # p's are pre scaled
samps = rbinom(100, n_pred, p_pred)
hist(samps,
     main = "Predicted FGM against Steph",
     xlab = "Field Goals Made",
     col = "lightgreen", breaks = 15,
     xlim = c(0, 16))
abline(v = mean(lebron_dat$FGM[lebron_dat$GAME_ID %in% lebron_vs_steph_games]), col = "red", lwd = 2)
```

```
hist(n_keep[800:n_iter, cols_of_interest],  
     main = "Predicted FGA against Steph",  
     xlab = "Field Goal Attempts",  
     col = "lightblue", breaks = 15)  
abline(v = mean(lebron_dat$FGA[lebron_dat$GAME_ID %in% lebron_vs_steph_games]), col = "red", lwd = 2)
```



```
hist(p_keep[800:n_iter, cols_of_interest],  
     main = "Predicted FG_Pct against Steph",  
     xlab = "Field Goal Percent",  
     col = "orange", breaks = 15)  
abline(v = mean(lebron_dat$FG_PCT[lebron_dat$GAME_ID %in% lebron_vs_steph_games]), col = "red", lwd = 2)
```

Predicted FG_Pct against Steph

