Model 2: Poisson

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Data Prep and Cleaning

```r
# Load and clean data
original_tbl <- read.csv("./NBA-BoxScores-2023-2024.csv") |>
  mutate(
    START_POSITION = na_if(START_POSITION, "") |> factor(),
    COMMENT = na_if(COMMENT, "") |> factor(),
    MIN = na_if(MIN, ""),
    MIN = str_replace(MIN, "([0-9]+)\\.[0-9]+:", "\\1:")
  )

# Filter to starters only
starting_dat <- original_tbl |>
  filter(!is.na(START_POSITION))

# Calculate team points per game
team_points <- original_tbl |>
  filter(!is.na(PTS)) |>
  group_by(GAME_ID, TEAM_ID) |>
  summarize(TeamPoints = sum(PTS), .groups = "drop")

# Join with itself to get opponent points
team_vs_opponent <- team_points |>
  inner_join(team_points, by = "GAME_ID", suffix = c("", ".opp")) |>
  filter(TEAM_ID != TEAM_ID.opp) |>
  rename(OPP_TEAM_ID = TEAM_ID.opp, OpponentPoints = TeamPoints.opp)

# Compute average opponent points allowed per team (DRTG)
team_drtg <- team_vs_opponent |>
  group_by(TEAM_ID) |>
  summarize(DRTG_proxy = mean(OpponentPoints), n_games = n(), .groups = "drop")

# Build opponent_map from distinct team-game pairs
game_team_pairs <- original_tbl |>
  select(GAME_ID, TEAM_ID) |>
  distinct()

# Create mapping of TEAM_ID and OPP_TEAM_ID for each game
opponent_map <- game_team_pairs |>
  inner_join(game_team_pairs, by = "GAME_ID") |>
  filter(TEAM_ID.x != TEAM_ID.y) |>
  rename(TEAM_ID = TEAM_ID.x, OPP_TEAM_ID = TEAM_ID.y)
```

```r
# Join with defensive ratings (DRTG)
opponent_map <- opponent_map |>
  left_join(team_drtg |> rename(OPP_TEAM_ID = TEAM_ID, OPP_DRTG = DRTG_proxy), by = "OPP_TEAM_ID")


# Merge opponent info into starting dataset and center DRTG
mean_drtg <- mean(team_drtg$DRTG_proxy)

starting_dat <- starting_dat |>
  left_join(opponent_map, by = c("GAME_ID", "TEAM_ID")) |>
  mutate(centered_OPP_DRTG = OPP_DRTG - mean_drtg)
```

Model 2 full conditionals

$$
\begin{aligned}
p(p_{ik} \mid \ldots) &\propto p(y_{ijk} \mid p_{ik}, n_i)\, p(p_{ik}) \\
&\propto \left[\prod_j p(y_{ijk} \mid p_{ik}, n_i)\right] \left[\prod_j p(p_{ik})\right] \\
&\propto \prod_j \binom{n_i}{y_{ijk}} p_{ik}^{y_{ijk}} (1-p_{ik})^{n_i - y_{ijk}} \prod_j \frac{\Gamma(5)\Gamma(5)}{\Gamma(10)} p_{ik}^{\alpha-1} (1-p_{ik})^{\beta-1} \\
&\propto \prod_j \left(\binom{n_i}{y_{ijk}}\right) p_{ik}^{\alpha + \sum_j y_{ijk} - 1} (1-p_{ik})^{\beta + \sum_j (n_i - y_{ijk}) - 1} \\
&\propto \text{Beta}\left(\alpha + \sum_j y_{ijk},\ \beta + \sum_j (n_i - y_{ijk})\right) \\
p(n_{ijk} \mid \ldots) &\propto p(y_{ijk} \mid p_{ik}, n_{ijk})\, p(n_{ijk} \mid \lambda_i) \\
&\propto \left[\prod_j p(y_{ijk} \mid p_{ik}, n_{ijk})\right] \left[\prod_j p(n_{ijk} \mid \lambda_i)\right] \\
&\propto \prod_j \binom{n_{ijk}}{y_{ijk}} p_{ik}^{y_{ijk}} (1-p_{ik})^{n_{ijk} - y_{ijk}} \cdot \frac{\lambda_i^{n_{ijk}} e^{-\lambda_i}}{n_{ijk}!} \\
&\propto \prod_j \left(p_{ik}^{y_{ijk}} (1-p_{ik})^{-y_{ijk}} e^{-\lambda_i} \cdot \frac{n_{ijk}!}{y_{ijk}!(n_{ijk} - y_{ijk})!} \cdot \frac{(1-p_{ik})\lambda_i)^{n_{ijk}}}{n_{ijk}!} \right) \\
&\propto \prod_j \frac{((1-p_{ik})\lambda_i)_{ijk}^{n}}{(n_{ijk} - y_{ijk})!} \\
&\propto \frac{((1-p_{ik})\lambda_i)^{\sum_j n_{ijk}}}{(n_{ijk} - y_{ijk})!} \\
p(\lambda_i \mid \ldots) &\propto p(y_{ijk} \mid p_{ik}, n_{ijk})\, p(n_{ijk} \mid \lambda_i)\, p(\lambda_i) \\
&\propto p(n_{ijk} \mid \lambda_i)\, p(\lambda_i) \\
&\propto \prod_j \frac{\lambda_i^{n_{ijk}} e^{-\lambda_i}}{n_{ijk}!} \cdot \lambda_i^{-1/2} \\
&\propto \left(\prod_j \frac{1}{n_{ijk}!}\right) \lambda_i^{\sum_j n_{ijk} - 1/2} e^{-\sum_j \lambda_i} \\
&\propto \lambda_i^{\sum_j n_{ijk} - 1/2} e^{-\sum_j \lambda_i} \\
&\sim \text{Gamma}\left(\text{shape} = \sum_j n_{ijk} - \tfrac{1}{2},\ \text{rate} = J_k\right)
\end{aligned}
$$

Shortened

$$
\begin{aligned}
p(p_{ik} \mid \dots) &\propto p(y_{ijk} \mid p_{ik}, n_i)\, p(p_{ik}) \\
&\propto \text{Beta}\left(\alpha + \sum_j y_{ijk},\ \beta + \sum_j (n_i - y_{ijk})\right) \\
p(n_{ijk} \mid \dots) &\propto p(y_{ijk} \mid p_{ik}, n_{ijk})\, p(n_{ijk} \mid \lambda_i) \\
&\propto \frac{((1-p_{ik})\lambda_i)^{\sum_j n_{ijk}}}{(n_{ijk} - y_{ijk})!} \\
p(\lambda_i \mid \dots) &\propto p(y_{ijk} \mid p_{ik}, n_{ijk})\, p(n_{ijk} \mid \lambda_i)\, p(\lambda_i) \\
&\sim \text{Gamma}\left(\text{shape} = \sum_j n_{ijk} - \tfrac{1}{2},\ \text{rate} = J_k\right)
\end{aligned}
$$

Bayes factor stuff

$$
p(y \mid M_2) = \frac{\Gamma(a_i)\Gamma(b_i)}{\Gamma(a_i+b_i)} \sum_{n_i} \prod_j \binom{n_i}{y_{ijk}} \cdot \left(\frac{1}{n_{ijk}!}\Gamma(n_{ijk}+1/2)\right) \cdot \frac{\Gamma\left(\sum_j y_{ijk} + a_i - 1\right)\Gamma\left(\sum_j (n_i - y_{ijk}) + b_i - 1\right)}{\Gamma\left(\sum_j n_i + a_i + b_i - 2\right)} \hat{\Pi}_{M_2} = P(y|M_2
$$

Model 2 implementation

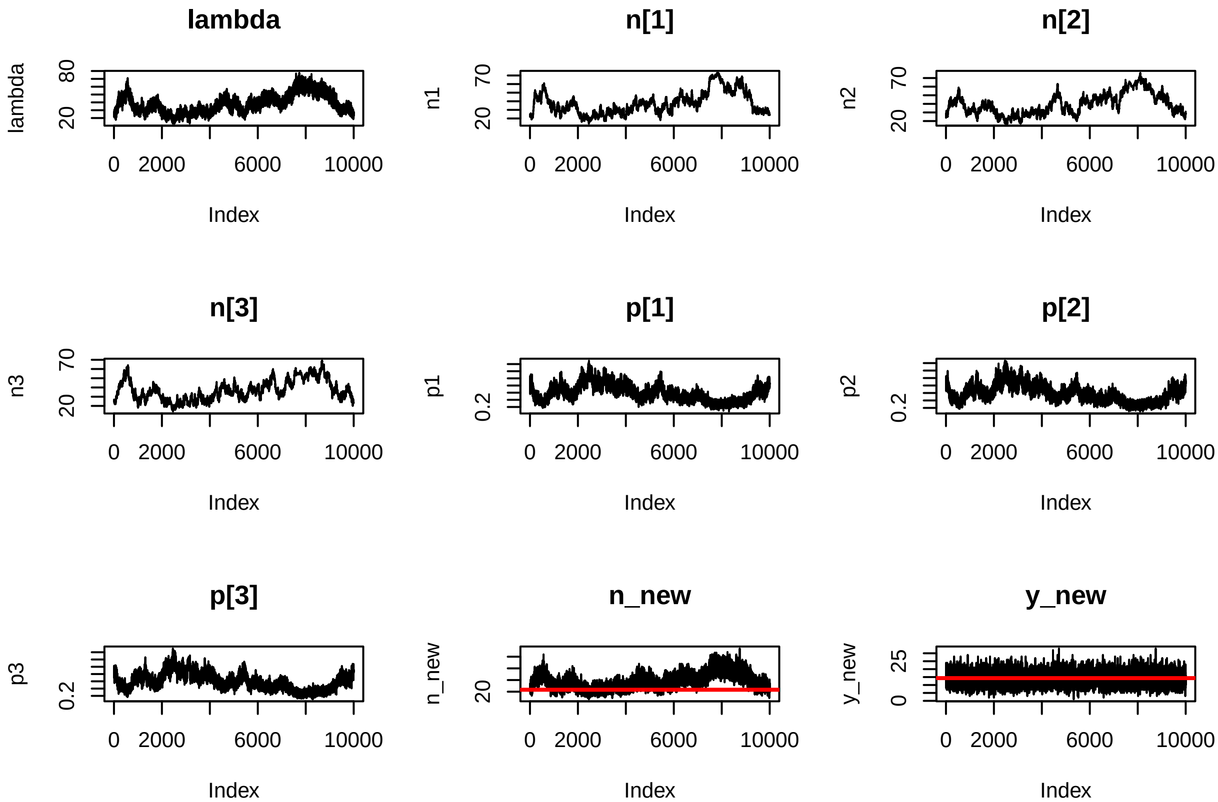
```
### MODEL 2 ###
log_n_con = function(p, lambda, n, y) {
  if (all(is.nan(log( ((1-p)*lambda)^sum(n) ) - sum(log((factorial(n-y)))) ))){
    return(rep(-Inf,length(y)))
  }else{
    log( ((1-p)*lambda)^sum(n) ) - sum(log((factorial(n-y))))
  }
}
mcmc_model_2 = function(data, player_id, opp_team_id, n_iter=5000,
                        init_lambda = c(), init_n = c(), gamma=0.01) {
  # Gather true data
  player_dat = data[data$PLAYER_ID == player_id, ]
  player_dat = player_dat[player_dat$OPP_TEAM_ID == opp_team_id, ]
  y = player_dat$FGM
  true_n = player_dat$FGA
  def_factor<- exp(gamma*(data$centered_OPP_DRTG[1]))
  if(length(init_lambda) ==0) {
    init_lambda = mean(player_dat$FGA)
  }
  if(length(init_n) ==0) {
    init_n = mean(player_dat$FGA)
  }


  big_N<- length(y)
  lambda<- init_lambda
  n<- rep(init_n,big_N)

  # setting up lists/matrices for returning
  p_matrix<- matrix(NA, nrow=n_iter, ncol=big_N)
  lambda_list<- rep(lambda, n_iter)
  n_matrix<- matrix(NA, nrow=n_iter, ncol=big_N)
```

```r
  y_new_list <- rep(NA, n_iter)
  n_new_list <- rep(NA, n_iter)

  for (i in 1:n_iter) {
    # sample p
    p_unscaled<- rbeta(big_N, 5 + sum(y), 5 + sum(n-y))
    p<- p_unscaled*def_factor

    # sample lambda
    lambda<- rgamma(1,shape=sum(n)-1/2,rate=big_N)

    # sample n
    n_prop<- rnorm(big_N, n, 1) # the third 1 is a tuning parameter
    logr<- log_n_con(p, lambda, n_prop, y)-log_n_con(p, lambda, n, y)
    for (j in 1:length(logr)) {
      if (is.finite(logr[j]) && log(runif(1))<logr[j]) {
        n[j]<- n_prop[j]
      }
    }

    # generate new values
    n_new = rpois(1, lambda)
    y_new <- rbinom(1, size = n_new, prob = mean(p))

    # save values
    p_matrix[i,] = p
    n_matrix[i,] = n
    lambda_list[i] = lambda
    n_new_list[i] = n_new
    y_new_list[i] =  y_new
  }
  return(data.frame(iteration=1:n_iter,
                    parameter=rep(c(paste("n[",1:big_N,"]", sep=""), "lambda", "n_new", "y_new", paste(
                    value=c(as.numeric(n_matrix),lambda_list,n_new_list,y_new_list, as.numeric(p_matrix)
}
# running mcmc
n_iter<- 10000
MCMC_model_2 = mcmc_model_2(data = starting_dat,
                            player_id = 2544, #LeBron
                            opp_team_id = 1610612744, #GSW
                            n_iter=10000,
                            gamma=0.01) #Previously found to be good value
```

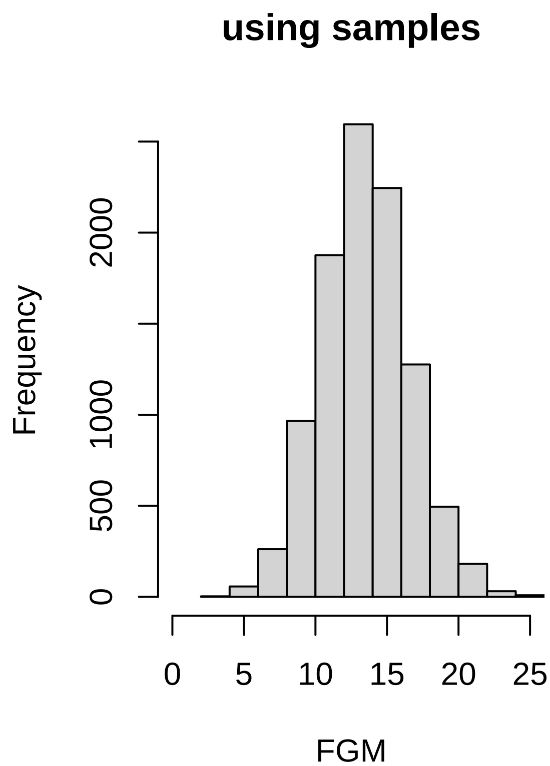
Model Diagnostics

```r
player_dat = starting_dat[starting_dat$PLAYER_ID == 2544, ]
player_dat = player_dat[player_dat$OPP_TEAM_ID == 1610612744, ]

# setup / data
lebron_GSW_n<- player_dat$FGA
lebron_mean_n<- mean(lebron_GSW_n)
```

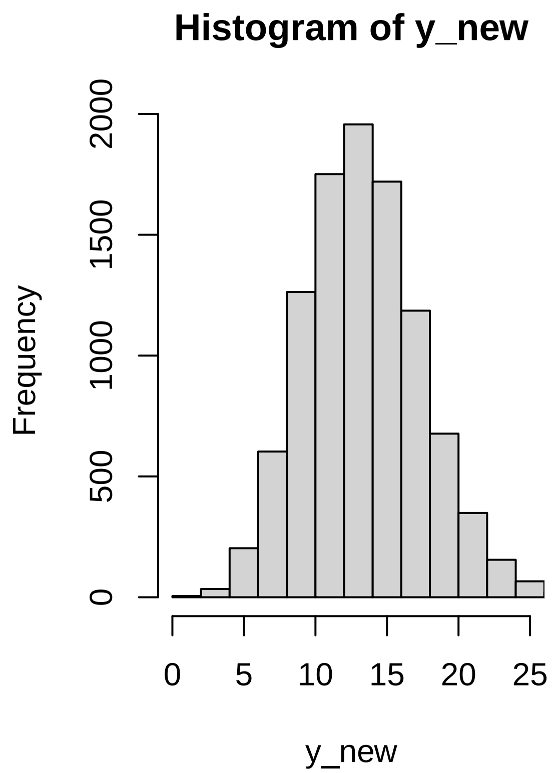
```r
lebron_median_n<- median(lebron_GSW_n)
lebron_max_n<- max(lebron_GSW_n)
lebron_GSW_p<- player_dat$FG_PCT
lebron_GSW_y<- player_dat$FGM



### Traceplots
lambda<- MCMC_model_2$value[which(MCMC_model_2$parameter=="lambda")]
n1<- round(MCMC_model_2$value[which(MCMC_model_2$parameter=="n[1]")])
n2<- round(MCMC_model_2$value[which(MCMC_model_2$parameter=="n[2]")])
n3<- round(MCMC_model_2$value[which(MCMC_model_2$parameter=="n[3]")])
p1<- MCMC_model_2$value[which(MCMC_model_2$parameter=="p[1]")]
p2<- MCMC_model_2$value[which(MCMC_model_2$parameter=="p[2]")]
p3<- MCMC_model_2$value[which(MCMC_model_2$parameter=="p[3]")]

par(mfrow=c(3,3))
plot(lambda,type="l",main="lambda")
plot(n1,type="l",main="n[1]")
plot(n2,type="l",main="n[2]")
plot(n3,type="l",main="n[3]")
plot(p1,type="l",main="p[1]")
plot(p2,type="l",main="p[2]")
plot(p3,type="l",main="p[3]")

####

n_new<- MCMC_model_2$value[which(MCMC_model_2$parameter=="n_new")]
y_new<- MCMC_model_2$value[which(MCMC_model_2$parameter=="y_new")]
plot(n_new,type="l",main="n_new")
abline(h=mean(lebron_GSW_n), col='red', lwd=2)
plot(y_new,type="l",main="y_new")
abline(h=mean(lebron_GSW_y), col='red', lwd=2)
```

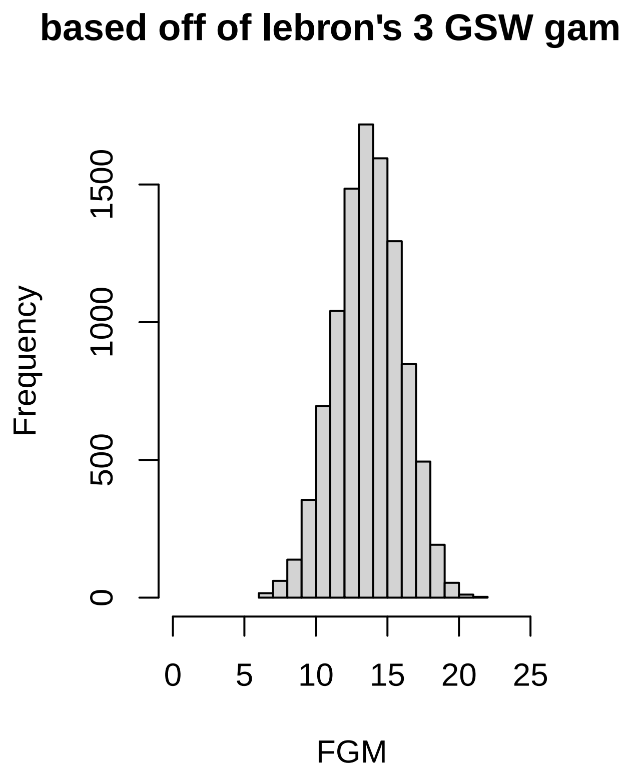



```
### I don't think this is actually the Predictive Posterior
posterior_mean_ps<- apply(data.frame(p1,p2,p3),1,mean)
posterior_mean_ns<- round(apply(data.frame(n1,n2,n3),1,mean))
# making the y's using our posterior sample
y_model_2<- rbinom(n_iter,posterior_mean_ns,posterior_mean_ps)
par(mfrow=c(1,2))
#hist of distribution based off our samples
hist(y_model_2,xlim=c(0,25),main="using samples", xlab="FGM")
#compare to a hist of lebron's games against GSW using observed values
par(mfrow=c(1,2))
```




```
hist(y_new,xlim=c(0,25))
hist(rbinom(10000,round(mean(lebron_GSW_n)),mean(lebron_GSW_p)),xlim=c(0,25), main="based off of lebron
```


Histogram of y_new
Frequency
0
500
1000
1500
2000
0
5
10
15
20
25
y_new


based off of lebron's 3 GSW gam
Frequency
0
500
1000
1500
0
5
10
15
20
25
FGM