# Reinforcement Learning for Improving the Performance of Recovery Algorithm in Noisy Group Testing

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# Tasks Done by Each Team Member

## Seyedeh Nahid Esmati:

- Implementing advantage actor-critic policy gradient with four different notions of states, and four different notions of rewards as stated in the slides.
- For the baseline work on GT and LDPC: Implementing the BP recovery algorithm (without RL) with flooding and maximum-residual node scheduling policies for both GT and LDPC, in order to reproduce the benchmarks.
- For the baseline paper on LDPC: Implementing Q-learning with direct update rule (no function approximation)+clustering+quantization.

## Arghamitra Talkuder:

- Implementing DQN with multiple agents (NNs) to learn multiple action-value functions in parallel (one agent for each BP iteration) for the state and reward definitions stated in the slides.
- Monte-Carlo simulation to reproduce the benchmark results for the BP algorithm with maximal-residual node scheduling policy for GT.

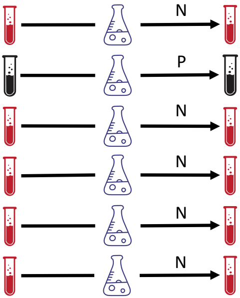
## Ya Mei:

- Implementing DQN with a single agent (NN) to learn a universal action-value function (one agent for all BP iterations) with two different notions of states, and two different notions of rewards as stated in the slides.
- Monte-Carlo simulation to reproduce the benchmark results for the BP algorithm with flooding for GT.

# Group Testing

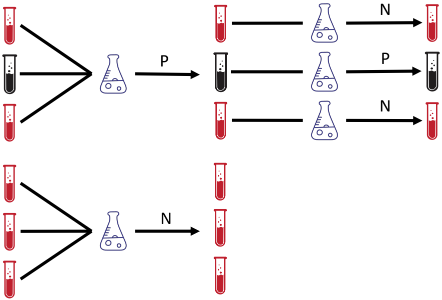
Design testing algorithms by using (binary) tests to identify all defective items among a much larger group of items.

Individual Testing




$\#$ tests (per item) $= 1$

Dorfman's Testing (1943)




$\#$ tests (per item) $= \frac{5}{6}$

How to minimize $\#$ tests for an unknown config. of def. items?

# Applications of Group Testing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Series on Applied Mathematics: Volume 18
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Important Tools for DNA Sequencing
https://doi.org/10.1142/6122 | June 2006
Pages: 248
By (author): Ding-Zhu Du (*University of Texas at Dallas, USA & Xi'an Jiaotong University, China*) and Frank K Hwang (*National Chiao Tung University, Taiwan, ROC*)


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Genetic Mapping and DNA Sequencing pp 133-154 | Cite as
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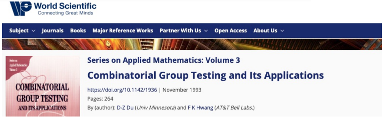
Authors Authors and affiliations
D. J. Balding, W. J. Bruno, D.C Torney, E. Knill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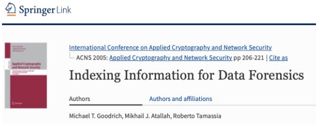
Combinatorial Group Testing and Its Applications
https://doi.org/10.1142/1936 | November 1993
Pages: 264
By (author): D-Z Du (*Univ Minnesota*) and F K Hwang (*AT&T Bell Labs.*)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International Conference on Applied Cryptography and Network Security
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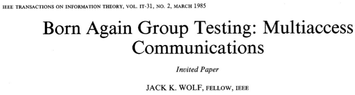
Michael T. Goodrich, Mikhail J. Atallah, Roberto Tamassia


IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. IT-31, NO. 2, MARCH 1985
Born Again Group Testing: Multiaccess Communications
*Invited Paper*
JACK K. WOLF, FELLOW, IEEE


- Public health
- Cybersecurity
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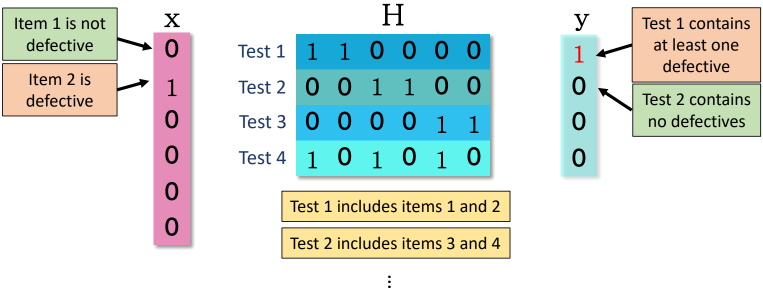
Four charts show how pooling samples from many people can save time or resources.

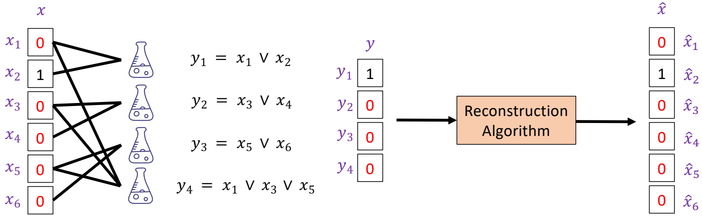
# Problem Setup

- Population of $n$ items (labeled $1, \ldots, n$)
- $k \ll n$ are defective (i.e., probability of defective $= q = \frac{k}{n}$)
- $\mathrm{x} = [x_1, \ldots, x_n]$: binary vector representing status of the items
- $H \in \{0, 1\}^{m \times n}$: binary matrix representing the testing matrix

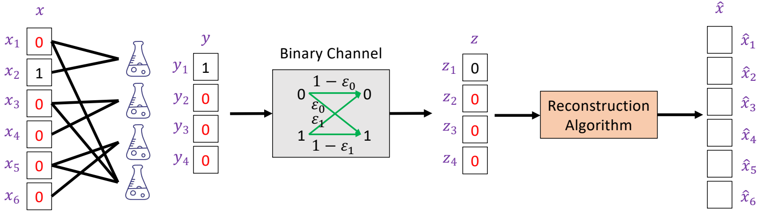


# Noiseless Setting vs. Noisy Setting

## Noiseless:




## Noisy:




**BSC** ($\epsilon_0 = \epsilon_1 = \delta$); Z-channel ($\epsilon_0 = 0$); reverse Z-channel ($\epsilon_1 = 0$).

# Exact Recovery vs. Partial Recovery

**Exact Recovery:** (no false negatives/positives)

- **Requirement:** $\hat{x} = x$;
- **Success probability:** $\Pr(\hat{x} = x)$.

Partial Recovery: ($\leq d_{\max}$ false neg.'s and $\leq d_{\max}$ false pos.'s)

- Requirement: $d(x, \hat{x}) \leq d_{\max}$, where

  $$d(x, \hat{x}) = \max\{|\mathrm{supp}(x) \setminus \mathrm{supp}(\hat{x})|, |\mathrm{supp}(\hat{x}) \setminus \mathrm{supp}(x)|\}$$

  and

  $$\mathrm{supp}(x) = \text{index set of all nonzero components of } x.$$

- Success probability: $\Pr(d(x, \hat{x}) \leq d_{\max})$.

# Recovery Algorithm: Belief Propagation (BP)

Check-to-variable messages: $m^{(l)}_{c\to v}(u_v)$

If $z_c = 0$:

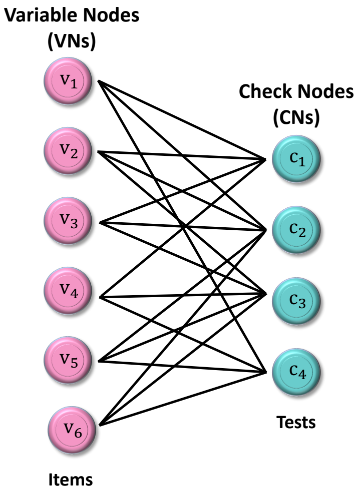
$$\propto \begin{cases} \delta & u_v = 1 \\ \delta + (1-2\delta) \prod_{v' \in \mathcal{N}(c)\setminus\{v\}} m^{(l-1)}_{v'\to c}(0) & u_v = 0 \end{cases}$$

If $z_c = 1$:

$$\propto \begin{cases} 1-\delta & u_v = 1 \\ 1-\delta-(1-2\delta) \prod_{v' \in \mathcal{N}(c)\setminus\{v\}} m^{(l-1)}_{v'\to c}(0) & u_v = 0 \end{cases}$$

Variable-to-check messages: $m^{(l)}_{v\to c}(u_v)$

$$\propto \begin{cases} q \prod_{c' \in \mathcal{N}(v)\setminus\{c\}} m^{(l)}_{c'\to v}(u_v) & u_v = 1 \\ (1-q) \prod_{c' \in \mathcal{N}(v)\setminus\{c\}} m^{(l)}_{c'\to v}(u_v) & u_v = 0 \end{cases}$$



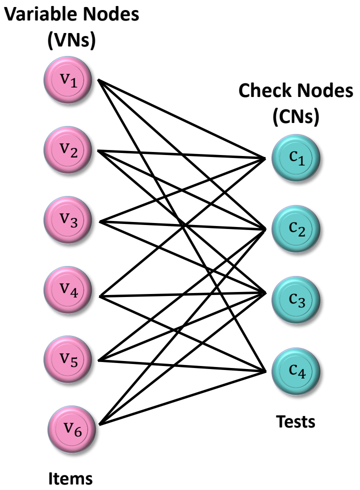
# Recovery Algorithm: Belief Propagation (BP)

Log Likelihood Ratio (LLR): $L_v^{(l)}$

$$= \begin{cases} \ln \dfrac{q}{1-q} & l = 0 \\ \ln \dfrac{q}{1-q} + \sum\limits_{c \in \mathcal{N}(v)} \ln \dfrac{m_{c \to v}^{(l)}(1)}{m_{c \to v}^{(l)}(0)} & l \geq 1 \end{cases}$$

Decoding Rule:

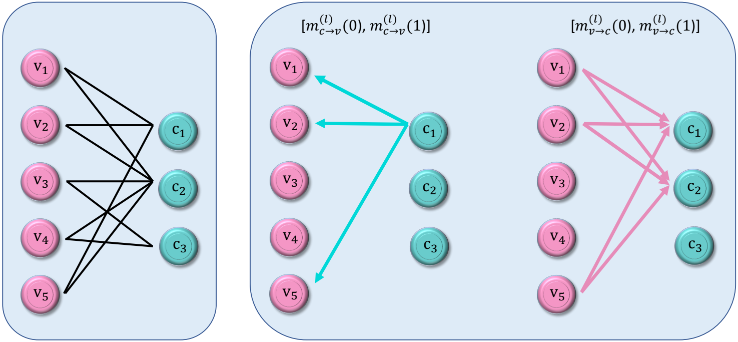
- If $L_v^{(l)} > 0$: $\hat{x}_v^{(l)} = 1$
- If $L_v^{(l)} = 0$: $\hat{x}_v^{(l)} = 1$ or $0$ w.p. $\frac{1}{2}$
- If $L_v^{(l)} < 0$: $\hat{x}_v^{(l)} = 0$



# Flooding vs. Node Scheduling

Flooding: In each iteration, all CNs send messages to their neighboring VNs, and all VNs send messages to their neighboring CNs.

Node Scheduling: In each iteration, a single CN sends messages to its neighboring VNs, and these VNs send messages to their neighboring CNs.




**Goal:** Given testing matrix $H$, # defectives $k$, and crossover prob. $\delta$, find an optimal node scheduling policy that maximizes the success probability.

# Apply RL to Learn Optimal Node Scheduling Policy

- Scheduling a single check node at each iteration of the BP algo. can be viewed as a Multi-Armed Bandits (MAB) process.
- State:
  (1) The sum of the LLRs of the VNs connected to each CN,
  $$\left[\sum_{v\in\mathcal{N}(1)} L_v^{(l)}, \sum_{v\in\mathcal{N}(2)} L_v^{(l)}, \ldots, \sum_{v\in\mathcal{N}(m)} L_v^{(l)}\right],$$
  where
  $$L_v^{(l)} = \begin{cases} \ln \frac{q}{1-q} & l = 0 \\ \ln \frac{q}{1-q} + \sum_{c\in\mathcal{N}(v)} \ln \frac{m_{c\to v}^{(l)}(1)}{m_{c\to v}^{(l)}(0)} & l \geq 1 \end{cases}.$$
  (2) The LLRs of the VNs connected to each CN,
  $$\left[\{L_v^{(l)}\}_{v\in\mathcal{N}(1)}, \{L_v^{(l)}\}_{v\in\mathcal{N}(2)}, \ldots, \{L_v^{(l)}\}_{v\in\mathcal{N}(m)}\right].$$
  (3) A quantized version of (1) or (2);
  (4) (1), (2), or (3) + the observed noisy test results $\hat{y}_1, \ldots, \hat{y}_m$.

# Apply RL to Learn Optimal Node Scheduling Policy

- Reward:

(1) Max. change in LLR of a variable node incident to the currently-scheduled check node $c^*$,

$$r_{c^*} = \max_{v \in \mathcal{N}(c^*)} |L^{\mathrm{new}}_{c^* \to v} - L^{\mathrm{old}}_{c^* \to v}|,$$

where

$$L^{\mathrm{new}}_{c \to v} = \ln \frac{m^{\mathrm{new}}_{c \to v}(1)}{m^{\mathrm{new}}_{c \to v}(0)}, \quad L^{\mathrm{old}}_{c \to v} = \ln \frac{m^{\mathrm{old}}_{c \to v}(1)}{m^{\mathrm{old}}_{c \to v}(0)}.$$

(2) Ratio of the max. change in LLR of a variable node incident to the currently-scheduled check node $c^*$ to the max. change in LLR of any variable node when scheduling any check node $c$,

$$R = \frac{r_{c^*}}{\max_{c \in [m]} r_c}.$$

(3) Minus the Hamming distance between the original signal $\mathrm{x} = [x_1, x_2, \ldots, x_n]$ and the currently-estimated signal $\hat{\mathrm{x}}^{(l)}$,

$$\hat{\mathrm{x}}^{(l)} = \left[ \mathbb{1}_{(L_1^{(l)} > 0)}, \mathbb{1}_{(L_2^{(l)} > 0)}, \ldots, \mathbb{1}_{(L_n^{(l)} > 0)} \right].$$

(4) A linear/non-linear function of (1), (2), or (3) to further encourage higher rewards and/or discourage lower rewards.

# Case Study

Problem parameters:

- # items $n = 100$
- # tests $m = 20$
- # defective items $k = 2$
- Testing matrix $H$: A randomly generated $20 \times 100$ Bernoulli matrix with probability $1 = \ln(2)/2 \approx 0.35$
- Crossover probability of noisy channel $\epsilon = 0.05$
- # BP iterations: flooding $= 10$, node scheduling $= 100$

Benchmarks:

| | Flooding | Maximum-Residual Node Scheduling |
|---|---|---|
| Success Probability (%) (Unknown $k$) | 32.99 | 51.22 |
| Success Probability (%) (Known $k$) | 50.00 | 61.34 |

# Policy Gradient (Advantage Actor-Critic)

State: Sum of the LLRs of the VNs connected to each CN,

$$\left[\sum_{v \in \mathcal{N}(1)} L_v^{(l)}, \sum_{v \in \mathcal{N}(2)} L_v^{(l)}, \ldots, \sum_{v \in \mathcal{N}(m)} L_v^{(l)}\right]$$

where

$$L_v^{(l)} = \begin{cases} \ln \frac{q}{1-q} & l = 0 \\ \ln \frac{q}{1-q} + \sum_{c \in \mathcal{N}(v)} \ln \frac{m_{c \to v}^{(l)}(1)}{m_{c \to v}^{(l)}(0)} & l \geq 1 \end{cases}.$$

Reward: A non-linear function of the ratio $R$,

$$10 \tanh(R - 0.5)$$

where

$$R = \frac{\max_{v \in \mathcal{N}(c^*)} \left| L_{c^* \to v}^{\mathrm{new}} - L_{c^* \to v}^{\mathrm{old}} \right|}{\max_{c \in [m]} \max_{v \in \mathcal{N}(c)} \left| L_{c \to v}^{\mathrm{new}} - L_{c \to v}^{\mathrm{old}} \right|},$$

$$L_{c \to v}^{\mathrm{new}} = \ln \frac{m_{c \to v}^{\mathrm{new}}(1)}{m_{c \to v}^{\mathrm{new}}(0)}, \quad L_{c \to v}^{\mathrm{old}} = \ln \frac{m_{c \to v}^{\mathrm{old}}(1)}{m_{c \to v}^{\mathrm{old}}(0)}.$$

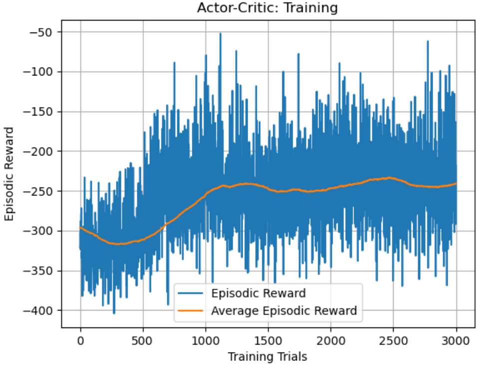
# Results: Policy Gradient (Advantage Actor-Critic)

Learning parameters:

- Learning rate $\alpha = 0.001$
- Discount factor $\gamma = 0.99$

Actor and critic NNs:

- # hidden layers: 2
- # nodes per layer: 60




| | Flooding | Max-Res NS | **RL-based Node Scheduling** | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 500 eps | 1000 eps | 1500 eps | 2000 eps | 2500 eps | 3000 eps |
| Success Probability (%) (Unknown $k$) | 32.99 | 51.22 | **35.4** | **45.5** | **47.1** | **48.3** | **48.9** | **49.2** |
| Success Probability (%) (Known $k$) | 50.00 | 61.34 | **47.2** | **56.4** | **57.6** | **58.1** | **58.5** | **59.0** |

# DQN with Multiple NNs

## (One NN for Each BP Iterations)

**Following the sequential steps as code: Declaring agent**.

Goal: To train $l_{max}$ NNs, each as a nonlinear approx. of the action-value function $Q^{(l)}(\text{state}, \text{action})$ for the BP iteration $1 \leq l \leq l_{\max}$. Individual agent for each iteration

For state (1) and reward (1):

- # hidden layers: 2
- # nodes per layer: $\{32, 64\}$
- Batch size: 64
- Replay buffer size: $1e5$
- Learning rate $\alpha$: $1e-4$
- Discount factor $\gamma$: 0.99
- Decay rate ($\epsilon$-greedy): 0.999
- Iteration per episode: 101

# DQN with Multiple NNs
## (One NN for Each BP Iterations)

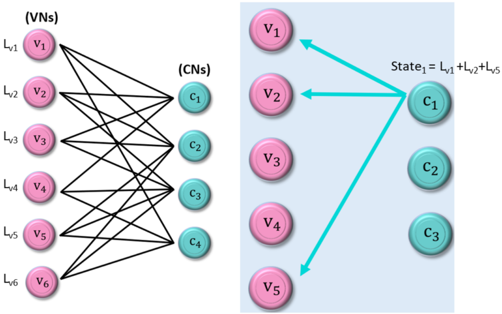
**First observation**

State: The sum of the LLRs of the VNs connected to each CN,

$$\left[\sum_{v\in\mathcal{N}(1)} L_v^{(l)}, \sum_{v\in\mathcal{N}(2)} L_v^{(l)}, \ldots, \sum_{v\in\mathcal{N}(m)} L_v^{(l)}\right],$$

where

$$L_v^{(l)} = \begin{cases} \ln \frac{q}{1-q} & l = 0 \\ \ln \frac{q}{1-q} + \sum_{c\in\mathcal{N}(v)} \ln \frac{m_{c\to v}^{(l)}(1)}{m_{c\to v}^{(l)}(0)} & l \geq 1 \end{cases}.$$

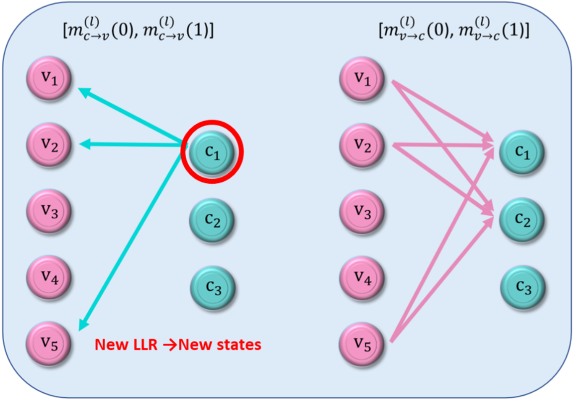


# DQN with Multiple NNs
## (One NN for Each BP Iterations)

**First iteration: Choose action**

$\epsilon$-greedy: For first iteration selecting a random node. Action space is same as state space

**First iteration: New observation based on chosen action**



# DQN with Multiple NNs
## (One NN for Each BP Iterations)

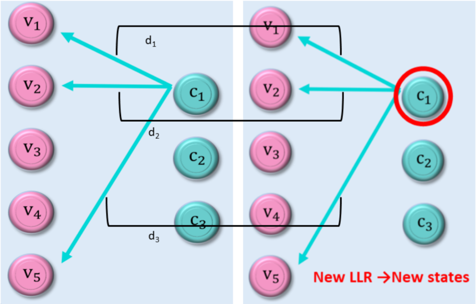
**First iteration: Reward**

Reward: The max. change in LLR of a variable node incident to the currently-scheduled check node $c^*$,

$$\max_{v \in \mathcal{N}(c^*)} \left| L_{c^* \to v}^{\mathrm{new}} - L_{c^* \to v}^{\mathrm{old}} \right|,$$

where

$$L_{c \to v}^{\mathrm{new}} = \ln \frac{m_{c \to v}^{\mathrm{new}}(1)}{m_{c \to v}^{\mathrm{new}}(0)}, \quad L_{c \to v}^{\mathrm{old}} = \ln \frac{m_{c \to v}^{\mathrm{old}}(1)}{m_{c \to v}^{\mathrm{old}}(0)}.$$



# DQN with Multiple NNs
## (One NN for Each BP Iterations)

**First iteration: After reward**

- Store in replay buffer
- Learn: loss, backpropagation, step

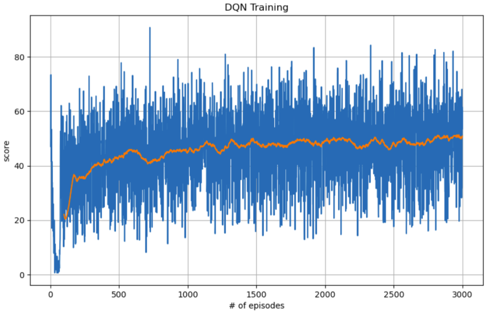



Table: Success probability for different episodes

| Success Probability | 500 | 1500 | 2500 | 3000 |
|---|---|---|---|---|
| % for unknown k | 39.2 | 41.3 | 46.9 | 48.7 |
| % for known k | 50.2 | 53.5 | 58.3 | 60.3 |

- We see a gradual increase in success probability as the number of episodes increases.
- Reward remains almost same, irrespective of episodes (around 55); and a gradual learning can be noticed from episode 0-500.

# DQN with a Single NN
## (One NN for All BP Iterations)

**State:**

(1) The LLRs of the VNs connected to each CN,

$$\left[\{L_v^{(l)}\}_{v\in\mathcal{N}(1)}, \{L_v^{(l)}\}_{v\in\mathcal{N}(2)}, \ldots, \{L_v^{(l)}\}_{v\in\mathcal{N}(m)}\right].$$

(2) The sum of the LLRs of the VNs connected to each CN,

$$\left[\sum_{v\in\mathcal{N}(1)} L_v^{(l)}, \sum_{v\in\mathcal{N}(2)} L_v^{(l)}, \ldots, \sum_{v\in\mathcal{N}(m)} L_v^{(l)}\right].$$

**Reward:**

(1) Minus the Hamming distance between the original signal $\mathrm{x} = [x_1, x_2, \ldots, x_n]$ and the currently-estimated signal $\hat{\mathrm{x}}^{(l)}$,

$$\hat{\mathrm{x}}^{(l)} = \left[\mathbb{1}_{(L_1^{(l)}>0)}, \mathbb{1}_{(L_2^{(l)}>0)}, \ldots, \mathbb{1}_{(L_n^{(l)}>0)}\right].$$

(2) The max. change in LLR of a variable node incident to the currently-scheduled check node $c^*$,

$$\max_{v\in\mathcal{N}(c^*)} |L_{c^*\to v}^{\mathrm{new}} - L_{c^*\to v}^{\mathrm{old}}|.$$

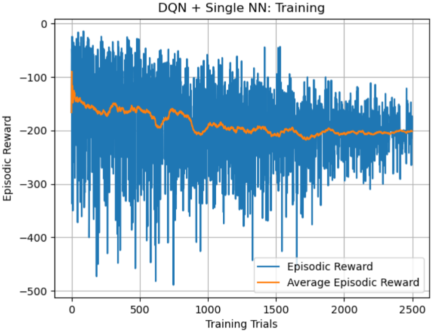
# DQN with a Single NN

## (One NN for All BP Iterations)

Goal: To train a neural network (NN) as a nonlinear approx. of the action-value function $Q(\text{state}, \text{action})$.

For state (1) and reward (1):

- $\#$ hidden layers: $\{1, 2, 3, 4, 5\}$
- $\#$ nodes per layer (3 layers): $\{1000, 500, 100\}$
- Batch size (exp. replay): $\{4, 16, 32\}$
- Update freq. (target net): $\{4, 8, 16\}$
- Learning rate $\alpha$: $\{0.01, 0.005, 0.001, 0.0001\}$
- Discount factor $\gamma$: $\{0.9, 0.95, 0.99\}$
- Decay rate ($\epsilon$-greedy): $\{0.99, 0.999, 0.9999, 0.99999\}$




Training: Not successful!

(Success probability $= 0\%$)

# Summary

**Goal:** To find a node scheduling (NS) policy that maximizes the success probability of the BP recovery algo. for noisy group testing.

Our RL-based NS policies (actor-critic and DQN + multiple NNs):

- outperform the flooding approach in terms of success probability.
- perform closely to the maximum-residual NS policy in terms of success probability, and are less computationally complex.

# Future Work

- Applying variance reducing methods e.g. Averaged-DQN, which is an extension to the DQN based on averaging previously learned Q-values estimates.
- Improving PG by applying a more directed exploration strategy that promotes exploration of under-appreciated rewards.
- Applying other policy gradient algorithms e.g., Natural Policy Gradient (NPG), Proximal Policy Optimization (PPO).
- Implementing DQN + multiple NNs with target network to improve the performance.
- Different variations of the problem:
  - BSC with different cross-over probabilities
  - Z-channel and reverse Z-channel
  - Partial recovery guarantee