PROBLEM DESCRIPTION

Consider a group of different items with each item having a different demand or requested quantity. A buyer wants to purchase these items from a set of vendors. Each vendor provides a unit price and the quantity (less than the demand) for some or all the items that the vendor can provide. Note that the price and quantity are quoted separately for each item in the group. The vendors can also provide two types of discounts as follows:

Spend discount: A vendor provides a discount of 'x1' % on the total spend if the total spend allocated to the vendor is greater than or equal to 'S1' and 'x2' % if the total spend allocated to the vendor is greater than or equal to 'S2'. S1 and S2 are less than the total bid price of the vendor. The total bid price for the vendor is calculated by the summation of 'the price multiplied by quantity' over all the items quoted by that vendor. The total allocated spend is given by the summation of 'quantity multiplied by the price' for the items allocated to the vendor.

Shipment discount: A vendor provides a discount of 'y1' % on the total spend if the total allocated quantity to the vendor is greater than or equal to 'Q1' and 'y2' % if the total allocated quantity to the vendor is greater than or equal to 'Q2'. Q1 and Q2 are less than the total quantity (for all items together) quoted by the vendor. The total allocated quantity is given by the summation of quantities of all the items allocated to the vendor.

Note that an item can be allocated to more than one vendor. However, the total allocation should not exceed the demand of that item. Also consider the possibility that some demand of items can remain unfulfilled. Formulate the problem to minimize the cost of allocation and maximize the fulfilment.

SETS AND INDICES

Sets	***Description***	***Indices***
I	*Set of items*	$i \in I$
V	*Set of vendors*	$s \in S$
SD	*Set of spend discount slabs (0,1,2)*	$sd \in SD$
QD	*Set of shipment discount slabs (0,1,2)*	$qd \in QD$

PARAMETERS

Parameter	***Description***	***Unit***
$\boldsymbol{Demand_i}$	*Total demand of item i*	*units*
$\boldsymbol{Quantity_{vi}}$	*Quantity of item i with vendor v*	*units*
$\boldsymbol{Price_{vi}}$	*Price of each item i with vendor v*	*cost/unit*
$\boldsymbol{BidPrice_v}$	*Total Bid Price of vendor v*	*cost*
$\boldsymbol{TotalQ_V}$	*Total quantity of items with vendor v*	*units*
S1	*Minimum spend for spend discount slab 1*	*cost*
S2	*Minimum spend for spend discount slab 2*	*cost*
x1	*Discount if cost falls between S1 and S2 (slab 1)*	-
x2	*Discount if cost falls above S2 (slab 2)*	-
Q1	*Minimum quantity for shipment discount slab 1*	*units*
Q2	*Minimum quantity for shipment discount slab 2*	*units*
y1	*Discount if quantity falls between Q1 and Q2 (slab 1)*	-
y2	*Discount if quantity falls above Q2 (slab 2)*	-
$\boldsymbol{PenaltyCost_i}$	*Penalty cost for unfulfilled demand of item i*	*cost/unit*

DECISION VARIABLES

Decision Variable	***Description***	***Unit***	***Type***	***Bounds***

$\boldsymbol{QFinal_{vi}}$	*Quantity of item i taken from vendor v*	*units*	*Continuous*	*[0,Demand$_i$)*
$\boldsymbol{DUnfulfilled_i}$	*Demand of item i unfulfilled*	*units*	*Continuous*	*[0,Demand$_i$]*
$\boldsymbol{DiscSD_{sd,v}}$	*1 if price for vendor v falls in slab sd of spend discount*	*-*	*Binary*	*{0,1}*
$\boldsymbol{DiscQD_{qd,v}}$	*1 if quantity from vendor v falls in slab qd of shipment discount*	*-*	*Binary*	*{0,1}*
$\boldsymbol{DiscountSD_V}$	*Discount from spend discount criteria*	*-*	*Continuous*	*[0,1]*
$\boldsymbol{DiscountQD_V}$	*Discount from shipment discount criteria*	*-*	*Continuous*	*[0,1]*

OBJECTIVE FUNCTION

$$Total\ cost\ of\ items\ = \sum_{i \in I} (1 - DiscountSD_V) * (1 - DiscountQD_v) * (Price_{vi} * QFinal_{vi})$$

$$Penalty\ of\ unfulfilled\ demand\ = \sum_{i \in I} (PenaltyCost_i * DUnfulfilled_i)$$

Minimise $OBJ\ =\ Total\ cost\ of\ items +\ Penalty\ of\ unfulfilled\ demand$

(weights maybe given as per priority)

CONSTRAINTS

(1) Quantity of each item taken from vendor can't exceed quantity of that item with that vendor

$$Qfinal_{vi} \leq\ Quantity_{vi} \quad \forall\ i \in I, \forall\ V \in V$$

(2) Demand balance for each item – used as a soft constraint – some demand may not be unfulfilled and this unfulfilled demand is taken as a penalty in the objective which will try to maximise the fulfilment required

$$\sum_{v \in V} (Qfinal_{vi}) =\ Demand_i + DUnfulfilled_i \quad \forall\ i \in I$$

(3) Bounds on S1 and S2 (should be lesser than the total bid price of each vendor)

$$BidPrice_v = \sum_{i \in I} (Quantity_{vi} * Price_{vi}) \quad \forall\ v \in V$$

$$S1 \leq BidPrice_v \quad \forall\ v \in V$$

$$S2 \leq BidPrice_v \quad \forall\ v \in V$$

(4) Bounds on Q1 and Q2 (should be lesser than the total quantity with each vendor)

$$TotalQ_v = \sum_{i \in I} (Quantity_{vi}) \quad \forall\ v \in V$$

$$Q1 \leq TotalQ_v \quad \forall\ v \in V$$

$$Q2 \leq TotalQ_v \quad \forall\ v \in V$$

(5) Total price of each vendor falls in any one slab of the spend discount slabs (it can fall either in <S1, S1-S2 or >=S2), we make value for that slab equal to 1 and rest to 0

$$\sum_{sd \in SD} (DiscSD_{sd,v}) = 1 \quad \forall v \in V$$

(6) Total quantity from each vendor falls in any one slab of the shipment discount slabs (it can fall either in <Q1, Q1-Q2 or >=Q2), we make value for that slab equal to 1 and rest to 0

$$\sum_{qd \in QD} (DiscQD_{qd,v}) = 1 \quad \forall v \in V$$

(7) Check in which spend discount slabs total price of each vendor lies

7.1 Spend discount slab 0 (<S1) – assures that DiscSD value is 1 if price < S1

$$\sum_{i \in I} (Price_{vi} * QFinal_{vi}) - (S1 - 1) \leq M * (1 - DiscSD_{0,v}) \quad \forall v \in V$$

$$S1 - \sum_{i \in I} (Price_{vi} * QFinal_{vi}) \leq M * DiscSD_{0,v} \quad \forall v \in V$$

7.2 Spend discount slab 1 (between S1, S2) – assures that DiscSD value is 1 if S1<=price< S2

$$\sum_{i \in I} (Price_{vi} * QFinal_{vi}) - (S2 - 1) \leq M * (1 - DiscSD_{1,v}) \quad \forall v \in V$$

$$S2 - \sum_{i \in I} (Price_{vi} * QFinal_{vi}) \leq M * DiscSD_{1,v} \quad \forall v \in V$$

$$\sum_{i \in I} (Price_{vi} * QFinal_{vi}) - (S1 - 1) \leq M * DiscSD_{1,v} \quad \forall v \in V$$

$$S1 - \sum_{i \in I} (Price_{vi} * QFinal_{vi}) \leq M * (1 - DiscSD_{1,v}) \quad \forall v \in V$$

7.3 Spend discount slab 2 (>=S2) – assures that DiscSD value is 1 if price >= S2

$$\sum_{i \in I} (Price_{vi} * QFinal_{vi}) - (S2 - 1) \leq M * DiscSD_{2,v} \quad \forall v \in V$$

$$S2 - \sum_{i \in I} (Price_{vi} * QFinal_{vi}) \leq M * (1 - DiscSD_{2,v}) \quad \forall v \in V$$

(8) Check in which shipment discount slabs total quantity from each vendor lies

8.1 Shipment discount slab 0 (<Q1) – assures that DiscQD value is 1 if quantity< Q1

$$\sum_{i \in I} (QFinal_{vi}) - (Q1 - 1) \leq M * (1 - DiscQD_{0,v}) \quad \forall v \in V$$

$$Q1 - \sum_{i \in I} (QFinal_{vi}) \leq M * DiscQD_{0,v} \quad \forall v \in V$$

8.2 Shipment discount slab 1 (between Q1, Q2) – assures that DiscQD value is 1 if Q1<=quantity< Q2

$$\sum_{i \in I} (QFinal_{vi}) - (Q2 - 1) \leq M * (1 - DiscQD_{1,v}) \quad \forall v \in V$$

$$Q2 - \sum_{i \in I} (QFinal_{vi}) \leq M * DiscQD_{1,v} \quad \forall\, v \in V$$

$$\sum_{i \in I} (QFinal_{vi}) - (Q1 - 1) \leq M * DiscQD_{1,v} \quad \forall\, v \in V$$

$$Q1 - \sum_{i \in I} (QFinal_{vi}) \leq M * (1 - DiscQD_{1,v}) \quad \forall\, v \in V$$

8.3 Shipment discount slab 2 (>=Q2) – assures that DiscQD value is 1 if quantity >= Q2

$$\sum_{i \in I} (QFinal_{vi}) - (Q2 - 1) \leq M * DiscQD_{2,v} \quad \forall\, v \in V$$

$$Q2 - \sum_{i \in I} (QFinal_{vi}) \leq M * (1 - DiscQD_{2,v}) \quad \forall\, v \in V$$

(9) Discount from spend discount criteria

$$DiscountSD_v = DiscSD_{0,v} * 0 + DiscSD_{1,v} * x1 + DiscSD_{2,v} * x2 \quad \forall\, v \in V$$

(10) Discount from shipment discount criteria

$$DiscountQD_v = DiscQD_{0,v} * 0 + DiscQD_{1,v} * y1 + DiscQD_{2,v} * y2 \quad \forall\, v \in V$$