Multivariate Analysis for the Behavioral Sciences, Second Edition (Chapman and Hall/CRC, 2019)

Examples of Chapter 13: Principal Components Analysis

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Examples

Table 13.1: Correlations of Blood Chemistry Variables and Their Standard Deviations

```
blood_chem <- c(1.000, 0.290, 0.202, -0.055, -0.105, -0.252, -0.229, 0.058,
                0.290, 1.000, 0.415, 0.285, -0.376, -0.349, -0.164, -0.129,
                0.202, 0.415, 1.000, 0.419, -0.521, -0.441, -0.145, -0.076,
               -0.055, 0.285, 0.419, 1.000, -0.877, -0.076, 0.023, -0.131,
               -0.105, -0.376, -0.521, -0.877, 1.000, 0.206, 0.034, 0.151,
               -0.252, -0.349, -0.441, -0.076, 0.206, 1.000, 0.192, 0.077,
               -0.229, -0.164, -0.145, 0.023, 0.034, 0.192, 1.000, 0.423,
                0.058, -0.129, -0.076, -0.131, 0.151, 0.077, 0.423, 1.000
blood_corr <- matrix(blood_chem, ncol = 8)</pre>
sds <- c(0.371, 41.253, 1.935, 0.077, 0.071, 4.037, 2.732, 0.297)
names <- c("rBlood", "Plate", "wBlood", "Neut.", "Lymph", "Bilir.", "Sodium", "Potass.")</pre>
dimnames(blood_corr) <- list(names, names)</pre>
blood_corr
       rBlood Plate wBlood Neut. Lymph Bilir. Sodium Potass.
        1.000 0.290 0.202 -0.055 -0.105 -0.252 -0.229
rBlood
         0.290 1.000 0.415 0.285 -0.376 -0.349 -0.164
Plate
                                                          -0.129
       0.202 0.415 1.000 0.419 -0.521 -0.441 -0.145
wBlood
                                                          -0.076
Neut.
       -0.055 0.285 0.419 1.000 -0.877 -0.076 0.023
                                                          -0.131
       -0.105 -0.376 -0.521 -0.877 1.000 0.206 0.034
                                                           0.151
Lymph
Bilir. -0.252 -0.349 -0.441 -0.076 0.206 1.000 0.192
                                                           0.077
Sodium -0.229 -0.164 -0.145 0.023 0.034 0.192 1.000
                                                           0.423
Potass. 0.058 -0.129 -0.076 -0.131 0.151 0.077 0.423
                                                           1.000
sds
```

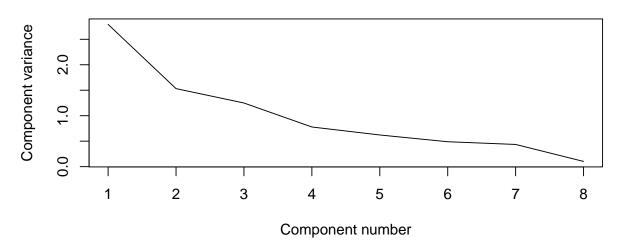
[1] 0.371 41.253 1.935 0.077 0.071 4.037 2.732 0.297

```
# Calculate the covariance matrix:
blood_cov <- diag(sds) %*% blood_corr %*% diag(sds)</pre>
dimnames(blood_cov) <-list(names, names)</pre>
blood_pcacov <- princomp(covmat = blood_cov)</pre>
summary(blood_pcacov, loadings = TRUE)
## Importance of components:
                              Comp.1
                                           Comp.2
                                                      Comp.3
                                                                  Comp.4
## Standard deviation
                          41.2877486 3.880212624 2.64197339 1.624583979
## Proportion of Variance 0.9856182 0.008705172 0.00403574 0.001525986
                           0.9856182 0.994323381 0.99835912 0.999885108
## Cumulative Proportion
                                Comp.5
                                             Comp.6
                                                          Comp.7
## Standard deviation
                          0.353951757 2.561722e-01 8.510631e-02 2.372715e-02
## Proportion of Variance 0.000072436 3.794288e-05 4.187837e-06 3.255049e-07
## Cumulative Proportion 0.999957544 9.999955e-01 9.999997e-01 1.000000e+00
##
## Loadings:
##
           Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8
## rBlood
                                         0.943 0.329
## Plate
           -0.999
## wBlood
                  -0.192
                                -0.981
## Neut.
                                                       0.758 0.650
## Lymph
                                                      -0.649 0.760
                   0.961 0.195 -0.191
## Bilir.
## Sodium
                   0.193 -0.979
## Potass.
                                        0.329 -0.942
```

```
## Proportion of Variance 0.3490343 0.1914520 0.1561605 0.09730097 0.07769584
## Cumulative Proportion 0.3490343 0.5404863 0.6966468 0.79394778 0.87164363
##
                             Comp.6
                                       Comp.7
                                                  Comp.8
## Standard deviation
                         0.69917350 0.66002394 0.31996216
## Proportion of Variance 0.06110545 0.05445395 0.01279697
## Cumulative Proportion 0.93274908 0.98720303 1.00000000
##
## Loadings:
##
          Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8
## rBlood -0.194 0.417 0.400 0.652 0.175 -0.363 0.176 0.102
## Plate -0.400 0.154 0.168
                                     -0.848 0.230 -0.110
## wBlood -0.459
                         0.168 -0.274  0.251  0.403  0.677
## Neut.
          -0.430 -0.472 -0.171 0.169 0.118
                                                   -0.237
## Lymph
           0.494 0.360
                              -0.180 -0.139 0.136 0.157
                                                           0.724
## Bilir. 0.319 -0.320 -0.277 0.633 -0.162 0.384 0.377
## Sodium 0.177 -0.535 0.410 -0.163 -0.299 -0.513 0.367
## Potass. 0.171 -0.245 0.709
                                      0.198 0.469 -0.376
```

Figure 13.1

Scree Diagram



Log(Eigenvalue) Diagram

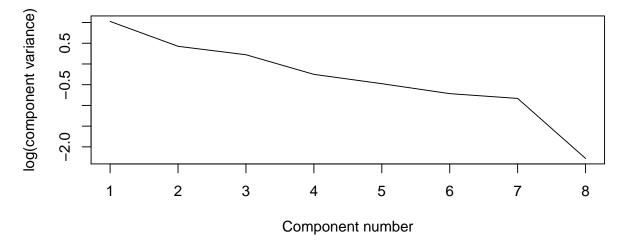


Table 13.4: Head Lengths (in Millimeters) of First and Second Sons of 25 Families

```
head dat <- structure(
  c(191, 195, 181, 183, 176, 208, 189, 197, 188, 192, 179, 183,
    174, 190, 188, 163, 195, 186, 181, 175, 192, 174, 176, 197, 190,
    179, 201, 185, 188, 171, 192, 190, 189, 197, 187, 186, 174,
    185, 195, 187, 161, 183, 173, 182, 165, 185, 178, 176, 200, 187),
  .Dim = c(25L, 2L), .Dimnames = list(NULL, c("HLFS", "HLSS")))
head_dat
##
         HLFS HLSS
##
    [1,]
         191
               179
    [2,]
               201
##
         195
##
   [3,]
         181
               185
   [4,]
##
         183
               188
   [5,]
         176
##
              171
##
    [6,]
         208
               192
##
   [7,]
         189
               190
##
   [8,]
         197
               189
  [9,]
##
         188
               197
## [10,]
         192
               187
## [11,]
         179
               186
## [12,]
         183 174
## [13,]
         174
               185
## [14,]
         190
               195
## [15,]
         188
              187
## [16,]
         163
              161
## [17,]
         195
               183
## [18,]
         186
               173
## [19,]
         181
              182
## [20,]
         175
              165
## [21,]
         192
               185
## [22,]
         174
              178
## [23,]
         176
               176
## [24,]
          197
               200
## [25,] 190 187
# get principal components
head_pc <- princomp(covmat = var(head_dat))</pre>
head_pc
## Call:
## princomp(covmat = var(head_dat))
##
## Standard deviations:
##
      Comp.1
                Comp.2
## 12.952459 5.322951
##
  2 variables and NA observations.
print(summary(head_pc), digits = 3, loadings = TRUE)
## Importance of components:
```

```
##
                               Comp.1
                                        Comp.2
## Standard deviation
                         12.9524588 5.3229513
## Proportion of Variance 0.8555135 0.1444865
## Cumulative Proportion 0.8555135 1.0000000
## Loadings:
        Comp.1 Comp.2
## HLFS 0.693 -0.721
## HLSS 0.721 0.693
# pc scores for first family
head_pc1 <- princomp(head_dat)</pre>
head_pc1$scores[1, ]
##
       Comp.1
                  Comp.2
## 0.1695614 -7.1606738
# check
0.693*(191-185.72)+0.721*(179-183.84)
## [1] 0.1694
-0.721*(191-185.72)+0.693*(179-183.84)
## [1] -7.161
y1 <- 12.9524588 * c(0.692986, 0.720951)
y2 \leftarrow 5.3229513 * c(-0.720951, 0.692986)
X <- cbind(y1, y2)</pre>
X %*% t(X)
##
            [,1]
                      [,2]
## [1,] 95.29336 69.66166
## [2,] 69.66166 100.80662
```

Figure 13.2

```
a1 <- 183.84-0.721*185.72/0.693
b1 <- 0.721/0.693
a2 <- 183.84-(-0.693*185.72/0.721)
b2 <- -0.693/0.721
plot(head_dat, xlab = "First Son's Head Length (mm)", ylab = "Second Son's Head Length")
abline(a1, b1)
abline(a2, b2, lty=2)
```

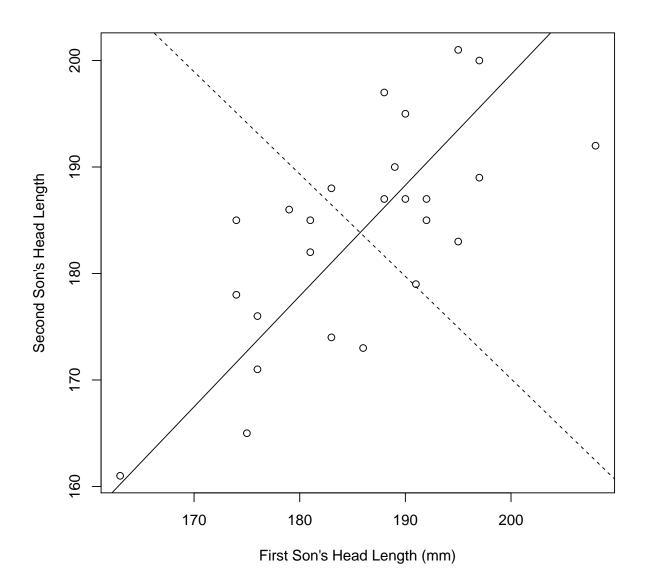


Figure 13.3

```
xlim <- range(head_pc1$scores[, 1])
plot(head_pc1$scores, xlim = xlim, ylim = xlim)</pre>
```

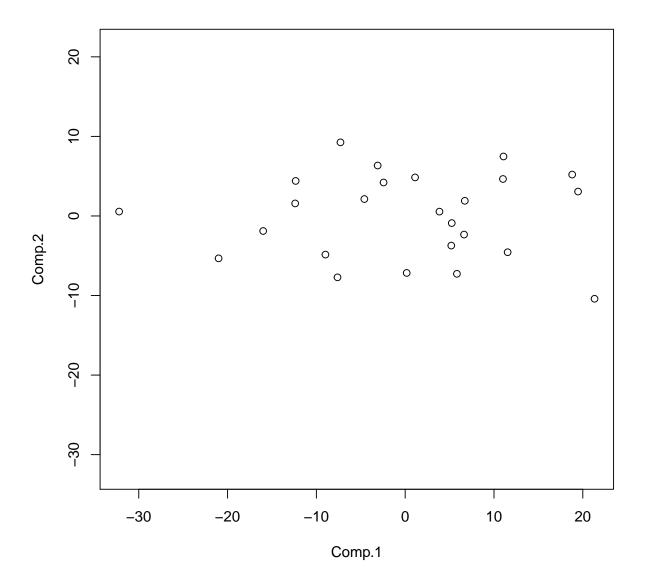


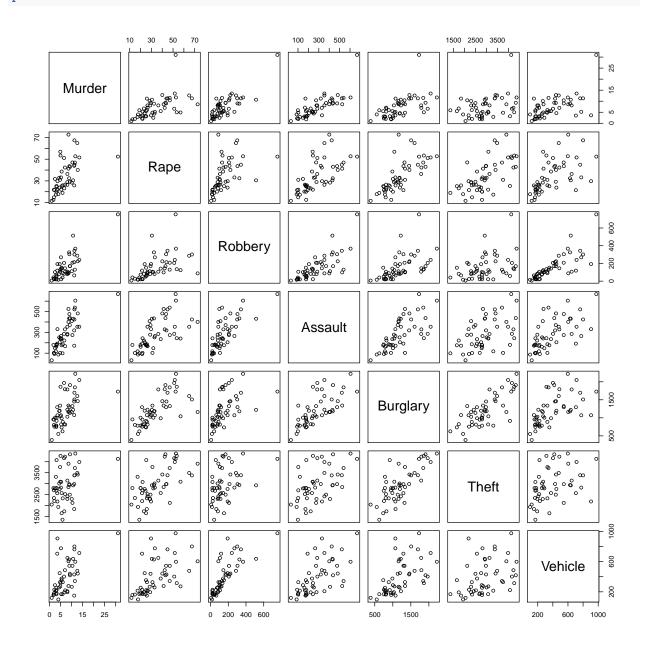
Table 13.5: Crime Rates in the United States

```
crime <- read.table("data/crime.txt", sep = '\t')</pre>
str(crime)
## 'data.frame':
                     51 obs. of 7 variables:
    $ Murder
              : num
                      2 2.2 2 3.6 3.5 4.6 10.7 5.2 5.5 5.5 ...
##
                      14.8 21.5 21.8 29.7 21.4 23.8 30.5 33.2 25.1 38.6 ...
    $ Rape
               : num
##
    $ Robbery : int
                      28 24 22 193 119 192 514 269 152 142 ...
##
    $ Assault : int
                      102 92 103 331 192 205 431 265 176 235 ...
    $ Burglary: int
                      803 755 949 1071 1294 1198 1221 1071 735 988 ...
                      2347 2208 2697 2189 2568 2758 2924 2822 1654 2574 ...
##
    $ Theft
               : int
    $ Vehicle : int
                     164 228 181 906 705 447 637 776 354 376 ...
crime
##
      Murder Rape Robbery Assault Burglary Theft Vehicle
## ME
         2.0 14.8
                         28
                                102
                                          803
                                               2347
                                                         164
## NH
         2.2 21.5
                         24
                                 92
                                          755
                                               2208
                                                         228
## VT
         2.0 21.8
                        22
                                103
                                          949
                                               2697
                                                         181
## MA
         3.6 29.7
                        193
                                331
                                         1071
                                               2189
                                                         906
## RI
         3.5 21.4
                                         1294
                                               2568
                                                         705
                        119
                                192
## CT
         4.6 23.8
                                         1198
                                               2758
                       192
                                205
                                                         447
        10.7 30.5
                                         1221
## NY
                       514
                                431
                                               2924
                                                         637
## NJ
         5.2 33.2
                        269
                                265
                                         1071
                                               2822
                                                         776
## PA
         5.5 25.1
                                          735
                                               1654
                        152
                                176
                                                         354
## OH
         5.5 38.6
                        142
                                235
                                          988
                                               2574
                                                         376
## IN
         6.0 25.9
                                          887
                                               2333
                        90
                                186
                                                         328
## IL
         8.9 32.4
                       325
                                434
                                         1180
                                               2938
                                                         628
## MI
        11.3 67.4
                       301
                                424
                                         1509
                                               3378
                                                         800
## WI
         3.1 20.1
                        73
                                          783
                                               2802
                                                         254
                                162
         2.5 31.8
## MN
                        102
                                148
                                         1004
                                               2785
                                                         288
## IA
         1.8 12.5
                        42
                                179
                                          956
                                               2801
                                                         158
## MO
         9.2 29.2
                        170
                                370
                                         1136
                                               2500
                                                         439
         1.0 11.6
## ND
                         7
                                 32
                                          385
                                               2049
                                                         120
## SD
         4.0 17.7
                         16
                                 87
                                          554
                                               1939
                                                          99
## NE
         3.1 24.6
                        51
                                184
                                          748
                                               2677
                                                         168
## KS
         4.4 32.9
                        80
                                252
                                         1188
                                               3008
                                                         258
## DE
         4.9 56.9
                                         1042
                                               3090
                                                         272
                        124
                                241
  MD
         9.0 43.6
                                         1296
                                               2978
##
                        304
                                476
                                                         545
## DC
        31.0 52.4
                       754
                                668
                                         1728
                                               4131
                                                         975
## VA
         7.1 26.5
                        106
                                167
                                          813
                                               2522
                                                         219
## WV
         5.9 18.9
                                          625
                                               1358
                        41
                                 99
                                                         169
         8.1 26.4
## NC
                        88
                                354
                                         1225
                                               2423
                                                         208
## SC
         8.6 41.3
                        99
                                525
                                         1340
                                               2846
                                                         277
        11.2 43.9
                                         1453
## GA
                       214
                                319
                                               2984
                                                         430
        11.7 52.7
                                         2221
                                               4373
## FL
                       367
                                605
                                                         598
## KY
         6.7 23.1
                        83
                                222
                                          824
                                               1740
                                                         193
## TN
        10.4 47.0
                       208
                                274
                                         1325
                                               2126
                                                         544
## AL
        10.1 28.4
                        112
                                408
                                         1159
                                               2304
                                                         267
## MS
        11.2 25.8
                        65
                                172
                                         1076
                                               1845
                                                         150
## AR
         8.1 28.9
                        80
                                         1030
                                               2305
                                                         195
                                278
## LA
        12.8 40.1
                                         1461
                                               3417
                        224
                                482
                                                         442
         8.1 36.4
## OK
                        107
                                285
                                         1787
                                               3142
                                                         649
```

##	TX	13.5 51.6	240	354	2049	3987	714
##	MT	2.9 17.3	20	118	783	3314	215
##	ID	3.2 20.0	21	178	1003	2800	181
##	WY	5.3 21.9	22	243	817	3078	169
##	CO	7.0 42.3	145	329	1792	4231	486
##	NM	11.5 46.9	130	538	1845	3712	343
##	ΑZ	9.3 43.0	169	437	1908	4337	419
##	UT	3.2 25.3	59	180	915	4074	223
##	NV	12.6 64.9	287	354	1604	3489	478
##	WA	5.0 53.4	135	244	1861	4267	315
##	OR	6.6 51.1	206	286	1967	4163	402
##	CA	11.3 44.9	343	521	1696	3384	762
##	AK	8.6 72.7	88	401	1162	3910	604
##	ΗI	4.8 31.0	106	103	1339	3759	328

Figure 13.4

pairs(crime)



```
crime_pc <- princomp(crime, cor = TRUE)</pre>
summary(crime_pc, loadings = TRUE)
## Importance of components:
##
                             Comp.1
                                      Comp.2
                                                 Comp.3
                                                             Comp.4
                          2.1665871 0.9927836 0.68137811 0.58666059
## Standard deviation
## Proportion of Variance 0.6705856 0.1408028 0.06632516 0.04916724
## Cumulative Proportion 0.6705856 0.8113884 0.87771356 0.92688079
##
                              Comp.5
                                        Comp.6
                                                   Comp.7
## Standard deviation
                          0.48885815 0.42269922 0.30688358
## Proportion of Variance 0.03414033 0.02552495 0.01345393
## Cumulative Proportion 0.96102112 0.98654607 1.00000000
##
## Loadings:
##
           Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
           -0.381 -0.350 -0.538
                                       -0.274 0.370 0.480
## Murder
## Rape
          -0.377 0.279
                                -0.830 -0.250
                                                     -0.151
## Robbery -0.391 -0.420 0.131 0.275 -0.387
                                                      -0.651
## Assault -0.410 -0.124 -0.335
                                        0.564 - 0.620
## Burglary -0.394 0.367
                                 0.162 0.466 0.622 -0.283
                                 0.449 -0.388 -0.282 0.256
## Theft -0.321 0.628
## Vehicle -0.366 -0.282 0.758
                                       0.163
                                                      0.422
```

Figure 13.5

```
plot(1:7, crime_pc$sdev^2, type = "l", xlab = "Component Number", ylab = "Variance")
```

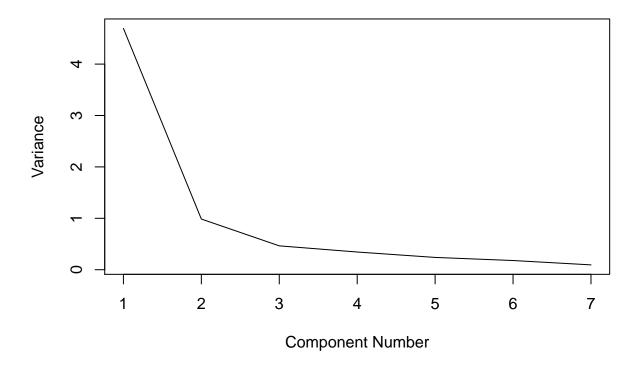
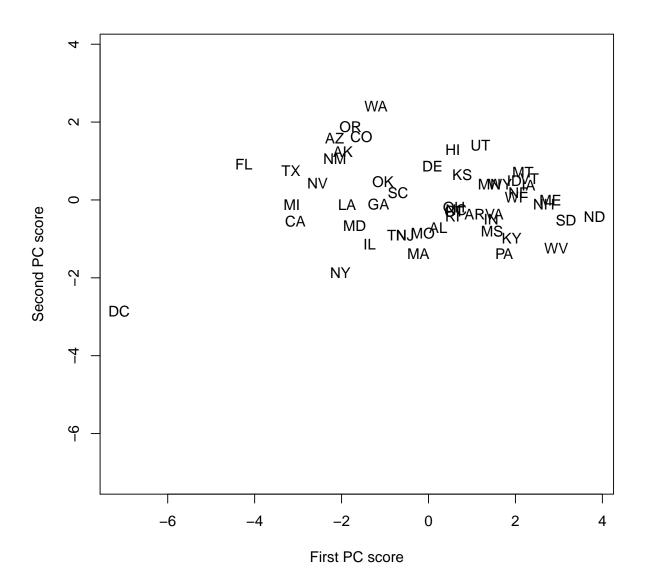


Figure 13.6



```
options(digits=3)
# use all pcas to get original correlation matrix
lambda <- crime_pc$sdev^2</pre>
Astar <- crime_pc$loadings[1:7, 1:7]
R <- Astar %*% diag(lambda) %*% t(Astar)</pre>
##
           Murder Rape Robbery Assault Burglary Theft Vehicle
                          0.804 0.781
                                           0.581 0.361
## Murder
            1.000 0.578
                                                         0.573
            0.578 1.000
                          0.530 0.659
                                           0.721 0.635
## Rape
                                                         0.569
## Robbery 0.804 0.530
                          1.000 0.740
                                           0.551 0.400
                                                         0.786
## Assault 0.781 0.659
                          0.740 1.000
                                           0.710 0.512
                                                         0.638
## Burglary 0.581 0.721
                                           1.000 0.764
                          0.551
                                  0.710
                                                         0.579
## Theft
            0.361 0.635
                          0.400
                                0.512
                                           0.764 1.000
                                                         0.386
## Vehicle 0.573 0.569
                          0.786 0.638
                                           0.579 0.386
                                                         1.000
# predicted correlation matrix based on first two components
lambda2 <- lambda[1:2]</pre>
Astar2 <- Astar[,1:2]</pre>
R2 <- Astar2 %*% diag(lambda2) %*% t(Astar2)
R2
##
           Murder Rape Robbery Assault Burglary Theft Vehicle
## Murder
           0.801 0.578
                          0.843
                                  0.775
                                           0.578 0.357
                                                         0.752
## Rape
            0.578 0.745
                          0.576 0.691
                                           0.799 0.741
                                                         0.571
## Robbery 0.843 0.576
                          0.889 0.802
                                           0.571 0.328
                                                         0.788
## Assault 0.775 0.691
                          0.802 0.803
                                           0.713 0.540
                                                         0.739
## Burglary 0.578 0.799
                                           0.862 0.821
                          0.571
                                  0.713
                                                         0.576
## Theft
            0.357 0.741
                          0.328 0.540
                                           0.821 0.872
                                                         0.377
## Vehicle
           0.752 0.571
                          0.788 0.739
                                          0.576 0.377
                                                         0.708
```

Table 13.8: Correlation Matrix for Drug Usage Data

```
drugs_corr <- structure(c(</pre>
        0.447, 0.442, 0.435, 0.114, 0.203, 0.091, 0.082, 0.513, 0.304, 0.245, 0.101, 0.245,
               0.619, 0.604, 0.068, 0.146, 0.103, 0.063, 0.445, 0.318, 0.203, 0.088, 0.199,
                      0.583, 0.053, 0.139, 0.110, 0.066, 0.365, 0.240, 0.183, 0.074, 0.184,
 0.442, 0.619, 1,
                             0.115, 0.258, 0.122, 0.097, 0.482, 0.368, 0.255, 0.139, 0.293,
 0.435, 0.605, 0.583, 1,
0.114, 0.068, 0.053, 0.115, 1,
                                    0.349, 0.209, 0.321, 0.186, 0.303, 0.272, 0.279, 0.278,
0.203, 0.146, 0.139, 0.258, 0.349, 1,
                                           0.221, 0.355, 0.315, 0.377, 0.323, 0.367, 0.545,
 0.091, 0.103, 0.110, 0.122, 0.209, 0.221, 1,
                                                  0.201, 0.150, 0.163, 0.310, 0.232, 0.232,
                                                          0.154, 0.219, 0.288, 0.320, 0.314,
 0.082, 0.063, 0.066, 0.097, 0.321, 0.355, 0.201, 1,
0.513, 0.445, 0.365, 0.482, 0.186, 0.315, 0.150, 0.154, 1,
                                                                 0.534, 0.301, 0.204, 0.394,
 0.304, 0.318, 0.240, 0.368, 0.303, 0.377, 0.163, 0.219, 0.534, 1,
                                                                        0.302, 0.368, 0.467,
0.245, 0.203, 0.183, 0.255, 0.272, 0.323, 0.310, 0.288, 0.301, 0.302, 1,
                                                                               0.304, 0.392,
0.101, 0.088, 0.074, 0.139, 0.279, 0.367, 0.232, 0.320, 0.204, 0.368, 0.304, 1,
0.245, 0.199, 0.184, 0.293, 0.278, 0.545, 0.232, 0.314, 0.394, 0.467, 0.392, 0.511, 1
), .Dim = c(13L, 13L))
drugs <- c("Cigarettes", "Beer", "Wine", "Liquor", "Cocaine", "Tranquilizers", "Drugstore",</pre>
           "Heroin", "Marijuana", "Hashish", "Inhalants", "Hallucinogenics", "Amphetamine")
dimnames(drugs_corr) <- list(drugs, c(1:13))</pre>
drugs_corr
##
                                   3
                                                            7
                                                                        9
                                                5
                                                      6
                   1.000 0.447 0.442 0.435 0.114 0.203 0.091 0.082 0.513
## Cigarettes
## Beer
                   0.447 1.000 0.619 0.605 0.068 0.146 0.103 0.063 0.445
## Wine
                   0.442 0.619 1.000 0.583 0.053 0.139 0.110 0.066 0.365
                   0.435 0.604 0.583 1.000 0.115 0.258 0.122 0.097 0.482
## Liquor
                   0.114 0.068 0.053 0.115 1.000 0.349 0.209 0.321 0.186
## Cocaine
## Tranquilizers
                   0.203 0.146 0.139 0.258 0.349 1.000 0.221 0.355 0.315
## Drugstore
                   0.091 0.103 0.110 0.122 0.209 0.221 1.000 0.201 0.150
## Heroin
                   0.082 0.063 0.066 0.097 0.321 0.355 0.201 1.000 0.154
                   0.513 0.445 0.365 0.482 0.186 0.315 0.150 0.154 1.000
## Marijuana
## Hashish
                   0.304 0.318 0.240 0.368 0.303 0.377 0.163 0.219 0.534
## Inhalants
                   0.245 0.203 0.183 0.255 0.272 0.323 0.310 0.288 0.301
## Hallucinogenics 0.101 0.088 0.074 0.139 0.279 0.367 0.232 0.320 0.204
                   0.245 0.199 0.184 0.293 0.278 0.545 0.232 0.314 0.394
## Amphetamine
##
                      10
                            11
                                  12
                                         13
## Cigarettes
                   0.304 0.245 0.101 0.245
## Beer
                   0.318 0.203 0.088 0.199
## Wine
                   0.240 0.183 0.074 0.184
## Liquor
                   0.368 0.255 0.139 0.293
                   0.303 0.272 0.279 0.278
## Cocaine
## Tranquilizers
                   0.377 0.323 0.367 0.545
## Drugstore
                   0.163 0.310 0.232 0.232
                   0.219 0.288 0.320 0.314
## Heroin
## Marijuana
                   0.534 0.301 0.204 0.394
                   1.000 0.302 0.368 0.467
## Hashish
## Inhalants
                   0.302 1.000 0.304 0.392
## Hallucinogenics 0.368 0.304 1.000 0.511
## Amphetamine
                   0.467 0.392 0.511 1.000
```

```
drugs_pc <- princomp(covmat = drugs_corr)</pre>
print(drugs_pc)
## Call:
## princomp(covmat = drugs_corr)
##
## Standard deviations:
  Comp.1 Comp.2 Comp.3 Comp.4
                                 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
##
    2.093
           1.430
                   0.977
                          0.902
                                  0.875
                                         0.831
                                                 0.793
                                                        0.787
                                                                0.757
## Comp.10 Comp.11 Comp.12 Comp.13
           0.626
                          0.597
##
    0.634
                   0.613
##
## 13 variables and NA observations.
summary(drugs_pc, loadings=TRUE)
## Importance of components:
##
                        Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7
                         2.093 1.430 0.9773 0.9022 0.8745 0.8314 0.7933
## Standard deviation
## Proportion of Variance 0.337 0.157 0.0735 0.0626 0.0588 0.0532 0.0484
## Cumulative Proportion
                        Comp.8 Comp.9 Comp.10 Comp.11 Comp.12 Comp.13
## Standard deviation
                        0.7872 0.7572 0.6340 0.6261 0.6127 0.5975
## Proportion of Variance 0.0477 0.0441 0.0309 0.0302 0.0289 0.0275
## Cumulative Proportion 0.8385 0.8826 0.9135 0.9437
                                                    0.9725 1.0000
##
## Loadings:
##
     Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9 Comp.10
    -0.280 -0.283
                               -0.300 -0.387 -0.124 0.137 0.655 -0.139
## 1
## 2 -0.287 -0.394 0.120
                                0.187 0.161 0.114
## 3 -0.267 -0.393 0.207 0.139 0.309 0.141
                                                          0.107 - 0.421
## 4 -0.318 -0.322
                                0.181 0.142
                                                   -0.164 -0.214 0.563
## 5
    -0.208 0.290
                         0.582 -0.432  0.416  0.185 -0.244  0.204
## 6 -0.293 0.262 -0.165
                                0.122
                                            -0.629 -0.399
                                                                -0.124
## 7 -0.176 0.190 0.723 -0.372 -0.178 0.277 -0.309 0.253
## 8 -0.201 0.317 0.153 0.534 0.327 -0.359
                                                   0.525 -0.169
## 9 -0.340 -0.160 -0.228 -0.112 -0.365 -0.129
                                                   0.285 -0.149
## 10 -0.329
                  -0.352 -0.125 -0.256  0.243  0.167  0.274 -0.400 -0.496
## 11 -0.274 0.163 0.330 -0.159 -0.152 -0.531 0.466 -0.417 -0.228
## 13 -0.328  0.235 -0.235 -0.267  0.203
                                            -0.132 -0.177
##
     Comp.11 Comp.12 Comp.13
## 1
      0.136
             0.169
                     0.263
## 2
             -0.695
                     0.410
## 3 -0.210
             0.188 - 0.564
## 4
     0.181
             0.519
                    0.210
## 5
    -0.154
## 6
      0.421 -0.170 -0.138
## 7
## 8
## 9 -0.154 -0.285
                   -0.502
## 10 0.187
             0.240
                     0.152
```

11

12 0.308 -0.159 ## 13 -0.733 0.269

Figure 13.7

```
plot(1:13, drugs_pc$sdev^2, type = "1", xlab = "Component Number", ylab = "Variance")
```

