Multivariate Analysis for the Behavioral Sciences, Second Edition (Chapman and Hall/CRC, 2019)

Examples of Chapter 14: Multidimensional Scaling and Correspondence Analysis

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10 December 2018

Contents

Examples

Table 14.1: Birth and Death Rates for Seven Countries

Source of data: https://population.un.org/wpp/Download/Standard/Population/

Countries selected manually to be the same ones as in **Chapter 17**.

```
country7 <- read.table("data/country7.txt", sep = '\t', header = TRUE)
country7
```

```
##                BirthR DeathR
## Japan             8.4    9.9
## Italy             8.6   10.2
## Spain             9.4    8.6
## United Kingdom   12.4    9.0
## Finland          10.8    9.5
## Cuba             11.2    7.6
## United States    12.5    8.2
```

Table 14.2

```
options(digits = 2)
dist7 <- dist(country7)
dist7
```

```
##                 Japan          Italy          Spain
## Italy                     0.36
## Spain                     1.64           1.79
## United Kingdom            4.10           3.98           3.03
## Finland                   2.43           2.31           1.66
## Cuba                      3.62           3.68           2.06
## United States             4.44           4.38           3.13
##                 United Kingdom  Finland        Cuba
## Italy
## Spain
## United Kingdom
## Finland                   1.68
## Cuba                      1.84           1.94
## United States             0.81           2.14           1.43
```

Table 14.4

```r
cscal7 <- cmdscale(dist7, k = 2)
cscal7
```

```
##                 [,1]   [,2]
## Japan          -2.26  0.031
## Italy          -2.19  0.385
## Spain          -0.83 -0.782
## United Kingdom  1.78  0.744
## Finland         0.11  0.588
## Cuba            1.21 -1.011
## United States   2.18  0.045
```

Figure 14.1

```
country7 <- cbind(country7, cscal7)
names(country7) <- c("BirthR", "DeathR", "Dim1", "Dim2")
countries <- trimws(rownames(country7))

library(ggplot2)

p1 <- ggplot(country7, aes(x = Dim1, y = Dim2))
p2 <- p1 + geom_point() + geom_text(aes(label = countries),
                                    position = position_nudge(y = +0.20), size=3)
p3 <- p2 + scale_x_continuous(name = "Dimension 1", limits = c(-3, 3))
p4 <- p3 + scale_y_continuous(name = "Dimension 2", limits = c(-1.25, 1.25))
p5 <- p4 + theme_bw()
p6 <- p5 + theme(panel.grid.major = element_blank(), panel.grid.minor = element_blank())
p7 <- p6 + coord_fixed(ratio = 1)
p7
```

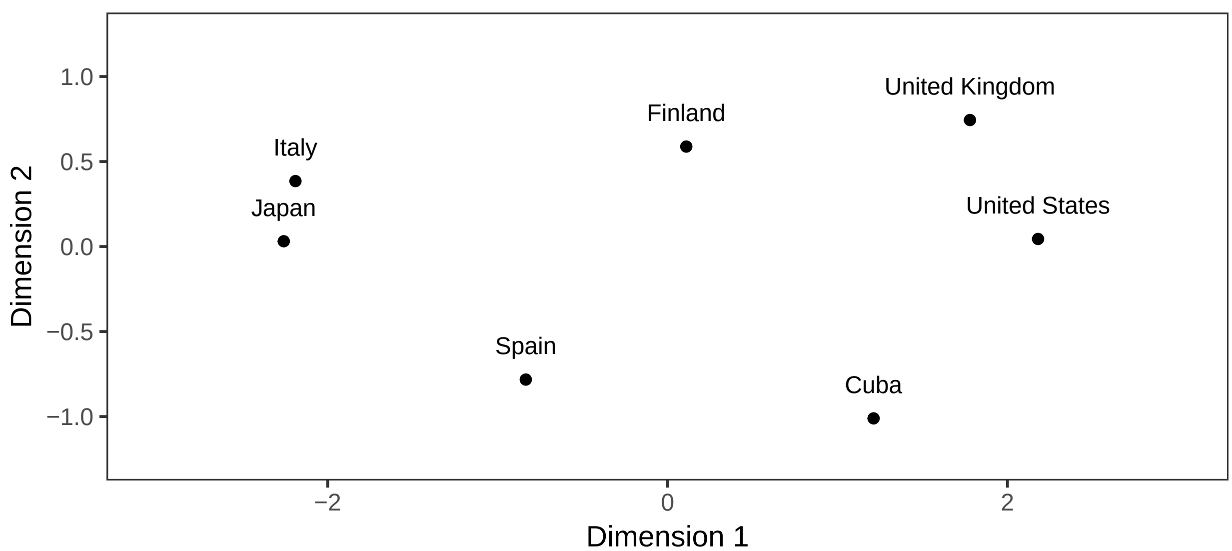


Table 14.3: Dissimilarity Data for All Pairs of 10 Colas for 2 Subjects

These data give an example of (symmetric) dissimilarity (proximity) matrices. There are no analyses involved here, but see some of the further examples (like the classical music composers) for possible ways to visualize and analyze these types of data sets.

Subject 1:

Cola Number

	1	2	3	4	5	6	7	8	9	10
1	0									
2	16	0								
3	81	47	0							
4	56	32	71	0						
5	87	68	44	71	0					
6	60	35	21	98	34	0				
7	84	94	98	57	99	99	0			
8	50	87	79	73	19	92	45	0		
9	99	25	53	98	52	17	99	84	0	
10	16	92	90	83	79	44	24	18	98	0

Subject 2:

Cola Number

	1	2	3	4	5	6	7	8	9	10
1	0									
2	20	0								
3	75	35	0							
4	60	31	80	0						
5	80	70	37	70	0					
6	55	40	20	89	30	0				
7	80	90	90	55	87	88	0			
8	45	80	77	75	25	86	40	0		
9	87	35	50	88	60	10	98	83	0	
10	12	90	96	89	75	40	27	14	90	0

Table 14.5: Road Distances of 15 Selected Places in Finland

```r
fitowns <- c("Helsinki", "Joensuu", "Jyväskylä", "Kilpisjärvi", "Kokkola", "Kotka",
             "Kuopio", "Kuusamo", "Lappeenranta", "Nuorgam", "Oulu", "Rovaniemi",
             "Tampere", "Turku", "Vaasa")
ftwns <- abbreviate(fitowns, 3, strict = TRUE)
```

```
## Warning in abbreviate(fitowns, 3, strict = TRUE): abbreviate used with non-
## ASCII chars
```

```r
fidist <- matrix(
  c(   0, 438, 272,1202, 491, 134, 383, 802, 223,1328, 612, 832, 174, 166, 419,
     438,   0, 245, 979, 429, 343, 136, 459, 236,1045, 393, 550, 394, 549, 494,
     272, 245,   0, 931, 241, 244, 144, 551, 219,1056, 339, 561, 151, 304, 282,
    1202, 979, 931,   0, 789,1177, 878, 620,1144, 649, 592, 428,1079,1226, 910,
     491, 429, 241, 789,   0, 488, 315, 411, 460, 916, 196, 419, 324, 436, 121,
     134, 343, 244,1177, 488,   0, 316, 735, 108,1303, 585, 806, 243, 295, 484,
     383, 136, 144, 878, 315, 316,   0, 417, 264,1006, 286, 509, 293, 448, 379,
     802, 459, 551, 620, 411, 735, 417,   0, 682, 617, 212, 191, 702, 848, 533,
     223, 236, 219,1144, 460, 108, 264, 682,   0,1271, 551, 774, 275, 364, 501,
    1328,1045,1056, 649, 916,1303,1006, 617,1271,   0, 719, 495,1207,1353,1035,
     612, 393, 339, 592, 196, 585, 286, 212, 551, 719,   0, 222, 491, 633, 318,
     832, 550, 561, 428, 419, 806, 509, 191, 774, 495, 222,   0, 712, 856, 541,
     174, 394, 151,1079, 324, 243, 293, 702, 275,1207, 491, 712,   0, 153, 244,
     166, 549, 304,1226, 436, 295, 448, 848, 364,1353, 633, 856, 153,   0, 331,
     419, 494, 282, 910, 121, 484, 379, 533, 501,1035, 318, 541, 244, 331,   0
   ), nrow = 15, ncol = 15, byrow = TRUE,
      dimnames = list(fitowns, ftwns))

fidist
```

```
##               Hls  Jns  Jyv  Klp Kkk  Ktk  Kup Ksm  Lpp  Nrg Oul Rvn  Tmp
## Helsinki        0  438  272 1202 491  134  383 802  223 1328 612 832  174
## Joensuu       438    0  245  979 429  343  136 459  236 1045 393 550  394
## Jyväskylä     272  245    0  931 241  244  144 551  219 1056 339 561  151
## Kilpisjärvi  1202  979  931    0 789 1177  878 620 1144  649 592 428 1079
## Kokkola       491  429  241  789   0  488  315 411  460  916 196 419  324
## Kotka         134  343  244 1177 488    0  316 735  108 1303 585 806  243
## Kuopio        383  136  144  878 315  316    0 417  264 1006 286 509  293
## Kuusamo       802  459  551  620 411  735  417   0  682  617 212 191  702
## Lappeenranta  223  236  219 1144 460  108  264 682    0 1271 551 774  275
## Nuorgam      1328 1045 1056  649 916 1303 1006 617 1271    0 719 495 1207
## Oulu          612  393  339  592 196  585  286 212  551  719   0 222  491
## Rovaniemi     832  550  561  428 419  806  509 191  774  495 222   0  712
## Tampere       174  394  151 1079 324  243  293 702  275 1207 491 712    0
## Turku         166  549  304 1226 436  295  448 848  364 1353 633 856  153
## Vaasa         419  494  282  910 121  484  379 533  501 1035 318 541  244
##               Trk  Vas
## Helsinki      166  419
## Joensuu       549  494
## Jyväskylä     304  282
## Kilpisjärvi  1226  910
## Kokkola       436  121
## Kotka         295  484
```

```
## Kuopio        448  379
## Kuusamo       848  533
## Lappeenranta  364  501
## Nuorgam      1353 1035
## Oulu          633  318
## Rovaniemi     856  541
## Tampere       153  244
## Turku           0  331
## Vaasa         331    0
```

Table 14.6

```r
n <- dim(fidist)[1]

fiscal <- cmdscale(d = fidist, k = n - 1, eig = TRUE, list. = TRUE)
```

```
## Warning in cmdscale(d = fidist, k = n - 1, eig = TRUE, list. = TRUE): only
## 8 of the first 14 eigenvalues are > 0
```

```r
fiscal$eig
```

```
##  [1]  2.4e+06  2.9e+05  2.0e+05  5.7e+04  1.6e+04  1.5e+04  7.8e+03
##  [8]  7.2e+02 -1.1e-10 -9.9e+02 -5.9e+03 -1.3e+04 -2.1e+04 -3.7e+04
## [15] -5.6e+04
```

```r
as.matrix(format(fiscal$eig, scientific = FALSE, justify = "right", nsmall = 0L, digits = 0))
```

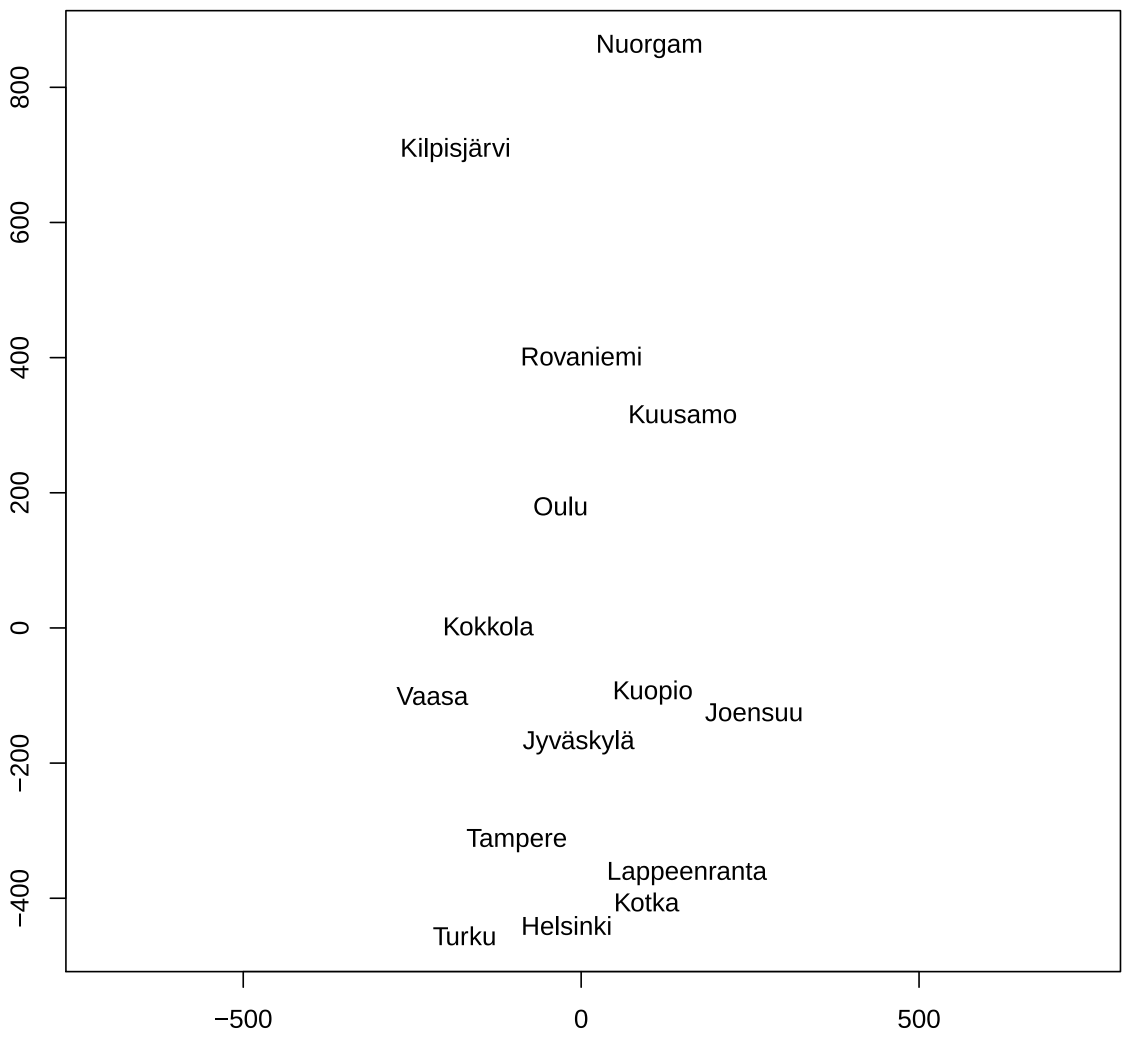
```
##       [,1]
##  [1,] "2395788"
##  [2,] " 285542"
##  [3,] " 198140"
##  [4,] "  56660"
##  [5,] "  16092"
##  [6,] "  15119"
##  [7,] "   7806"
##  [8,] "    724"
##  [9,] "     -0"
## [10,] "   -995"
## [11,] "  -5886"
## [12,] " -13245"
## [13,] " -21336"
## [14,] " -37055"
## [15,] " -56468"
```

```r
fiscal$points[, 1:2]
```

```
##                [,1]    [,2]
## Helsinki     -440.7  -21.46
## Joensuu      -124.5  255.86
## Jyväskylä    -169.6   -3.75
## Kilpisjärvi   707.1 -185.96
## Kokkola         2.6 -137.40
## Kotka        -405.8   96.89
## Kuopio        -95.7  106.04
## Kuusamo       316.6  150.25
## Lappeenranta -362.9  156.52
## Nuorgam       860.8  100.99
## Oulu          180.5  -30.44
## Rovaniemi     402.0    0.45
## Tampere      -314.1  -95.33
## Turku        -455.9 -172.48
## Vaasa        -100.4 -220.18
```

Figure 14.2

```
# draft plot:

y <- fiscal$points[, 1] # exchange x and y to show the map
x <- fiscal$points[, 2] # so that Helsinki is in the South

plot(x, y, type = "n", xlab = "", ylab = "", asp = 1, axes = TRUE)
text(x, y, rownames(fiscal$points), cex = 1.0)
```

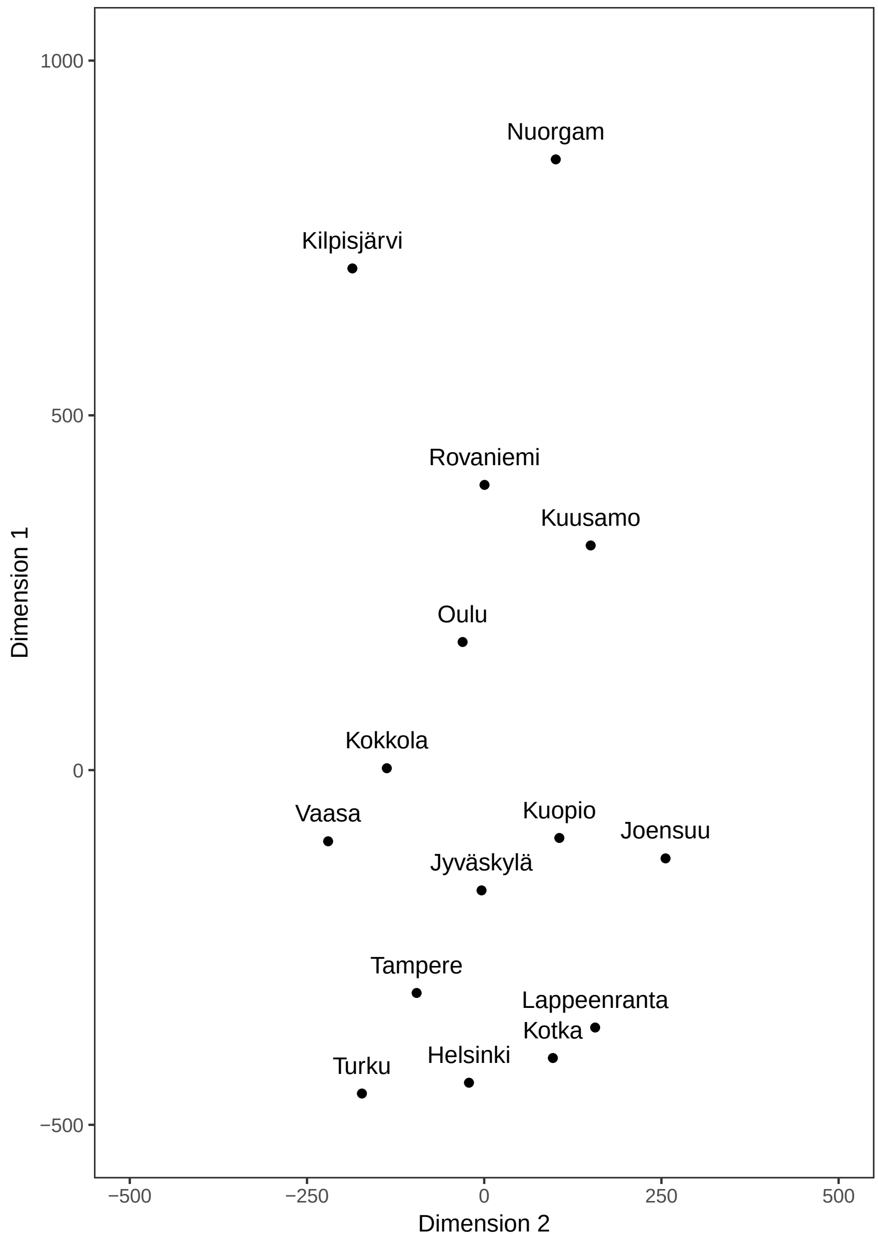



```
#install.packages("vegan") # by Jari Oksanen, Finland
library(vegan)
```

```
## Loading required package: permute
```

```
## Loading required package: lattice
## This is vegan 2.5-2
```

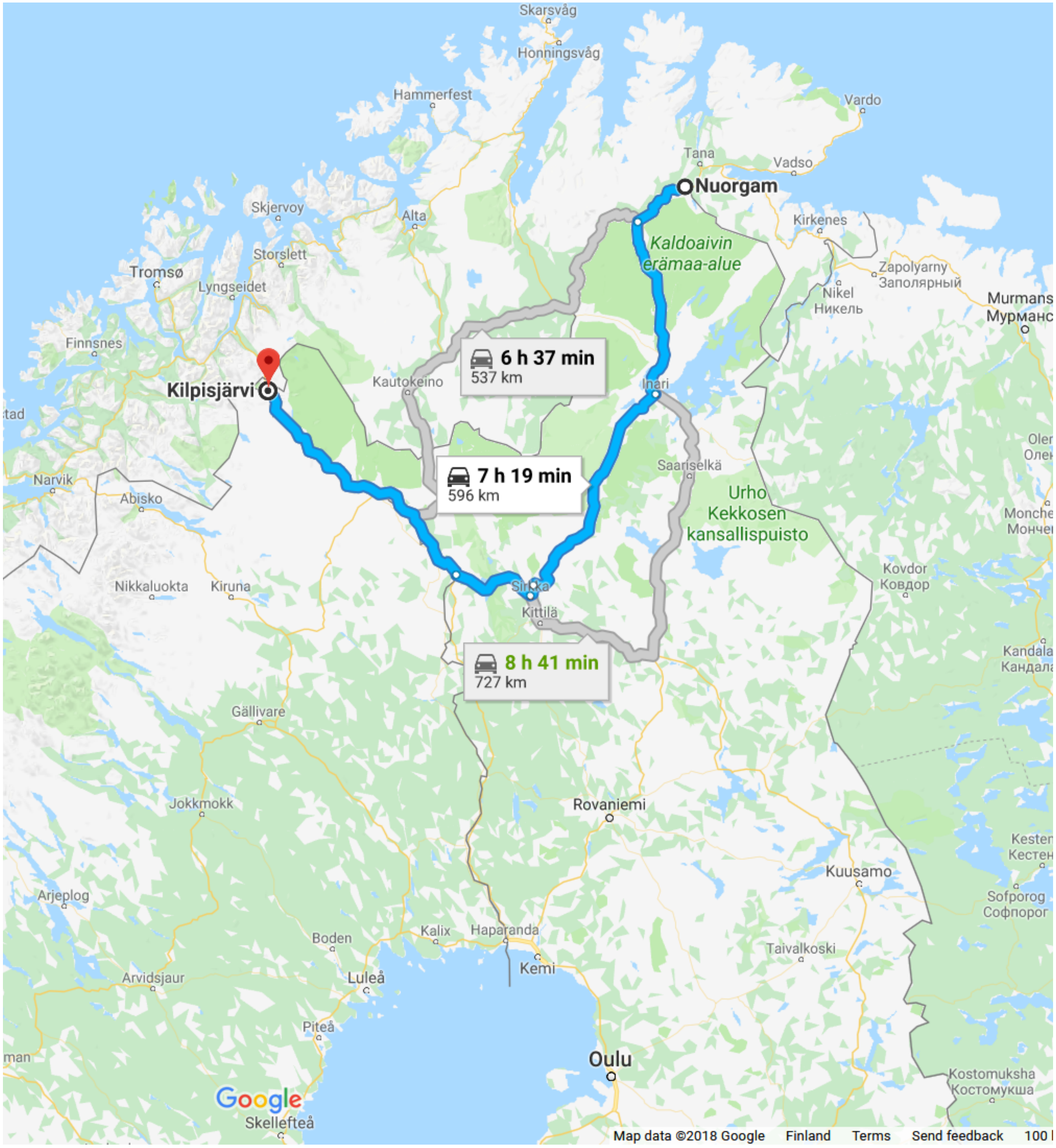
```r
finland <- as.data.frame(scores(fiscal$points[, 1:2])) # adds names Dim1, Dim2

library(ggplot2)

p1 <- ggplot(finland, aes(x = Dim2, y = Dim1)) # OBS! rotation
p2 <- p1 + geom_point() + geom_text(aes(label = fitowns),
                                    position = position_nudge(y = +40), size=4)
p3 <- p2 + scale_x_continuous(name = "Dimension 2", limits = c(-500,  +500))
p4 <- p3 + scale_y_continuous(name = "Dimension 1", limits = c(-500, +1000))
p5 <- p4 + theme_bw()
p6 <- p5 + theme(panel.grid.major = element_blank(), panel.grid.minor = element_blank())
p7 <- p6 + coord_fixed(ratio = 1)
p7
```



1000
500
0
−500
−500
−250
0
250
500
Dimension 1
Dimension 2
Nuorgam
Kilpisjärvi
Rovaniemi
Kuusamo
Oulu
Kokkola
Vaasa
Kuopio
Joensuu
Jyväskylä
Tampere
Lappeenranta
Kotka
Turku
Helsinki

Figure 14.3

For an interactive map, see https://goo.gl/maps/5Yp6nG31HeQ2



For an interactive map, see https://goo.gl/maps/5Yp6nG31HeQ2

Table 14.7: Proximity Matrix of Ten Remarkable Classical Music Composers Selected and Compared by Olli Mustonen

```
composers <- c("Bach", "Haydn", "Mozart", "Beethoven", "Schubert", "Brahms",
               "Sibelius", "Debussy", "Bartok", "Sostakovits")
OMD <- matrix(
c( 0,  50,  30,  20,  40,  40,  40,  50,  30,  30,
  50,   0,  10,  15,  30,  70,  90,  50,  80,  40,
  30,  10,   0,  20,  25,  40,  70,  50,  80,  50,
  20,  15,  20,   0,  10,  20,  25,  80,  60,  40,
  40,  30,  25,  10,   0,  15,  60,  50,  70,  60,
  40,  70,  40,  20,  15,   0,  20,  70,  70,  70,
  40,  90,  70,  25,  60,  20,   0,  35,  35,  20,
  50,  50,  50,  80,  50,  70,  35,   0,  15,  40,
  30,  80,  80,  60,  70,  70,  35,  15,   0,  20,
  30,  40,  50,  40,  60,  70,  20,  40,  20,   0
 ), nrow = 10, ncol = 10, byrow = TRUE, dimnames = list(composers, composers))

n <- dim(OMD)[1]
OMDS <- cmdscale(d = OMD, k = n-1, eig = TRUE, list. = TRUE)
```

```
## Warning in cmdscale(d = OMD, k = n - 1, eig = TRUE, list. = TRUE): only 5
## of the first 9 eigenvalues are > 0
```

```
as.matrix(format(OMDS$eig, scientific = FALSE, justify = "right", nsmall = 0L, digits = 0))
```

```
##       [,1]
##  [1,] " 7459"
##  [2,] " 4830"
##  [3,] " 2288"
##  [4,] "  752"
##  [5,] "  514"
##  [6,] "    0"
##  [7,] " -661"
##  [8,] " -906"
##  [9,] " -937"
## [10,] "-2912"
```

```
pk1 <- cumsum(abs(OMDS$eig))/sum(abs(OMDS$eig))
pk2 <- cumsum(OMDS$eig^2)/sum(OMDS$eig^2)
pk1
```

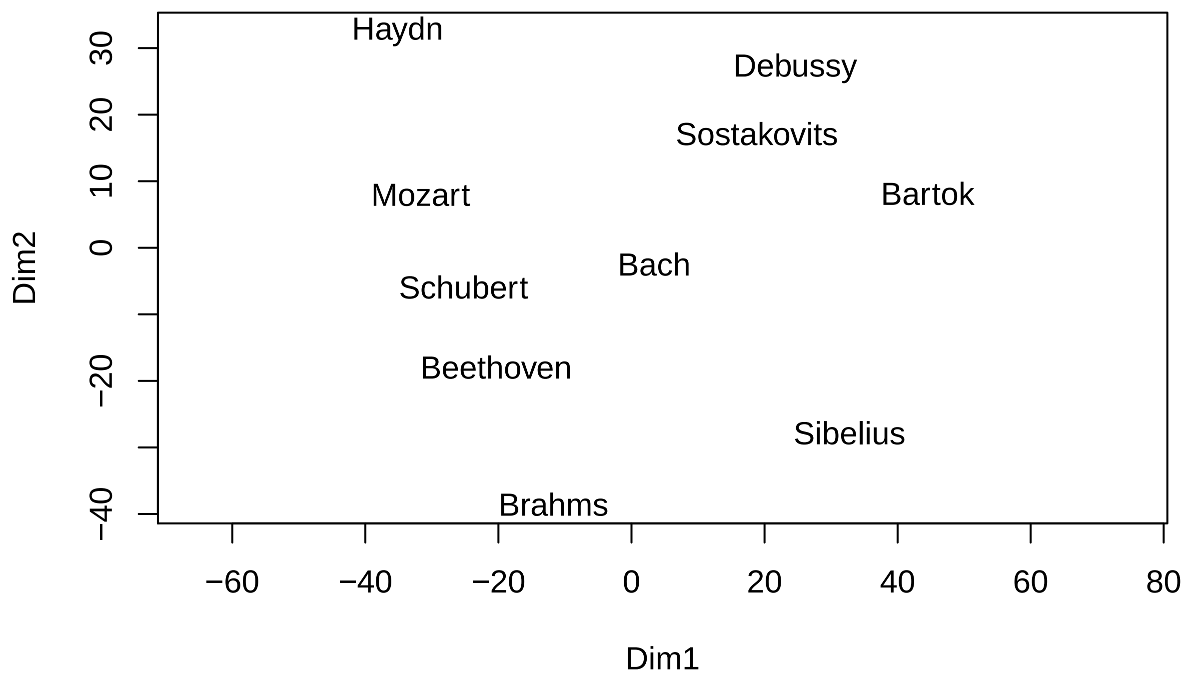
```
##  [1] 0.35 0.58 0.69 0.72 0.75 0.75 0.78 0.82 0.86 1.00
```

```
pk2
```

```
##  [1] 0.58 0.83 0.88 0.89 0.89 0.89 0.89 0.90 0.91 1.00
```

Figure 14.4

```
OwMDS <- wcmdscale(d = OMD, k = n-1, eig = TRUE)
plot(OwMDS, cex = 1.0)
```




```
Owcscal <- as.data.frame(scores(OwMDS$points[, 1:2]))

library(ggplot2)

p1 <- ggplot(Owcscal, aes(x = Dim1, y = Dim2))
p2 <- p1 + geom_point() + geom_text(aes(label = composers),
                                    position = position_nudge(y = +4), size=4)
p3 <- p2 + scale_x_continuous(name = "Dimension 1", limits = c(-40, +50))
p4 <- p3 + scale_y_continuous(name = "Dimension 2", limits = c(-50, +40))
p5 <- p4 + theme_bw()
p6 <- p5 + theme(panel.grid.major = element_blank(), panel.grid.minor = element_blank())
p7 <- p6 + coord_fixed(ratio = 1)
p7
```

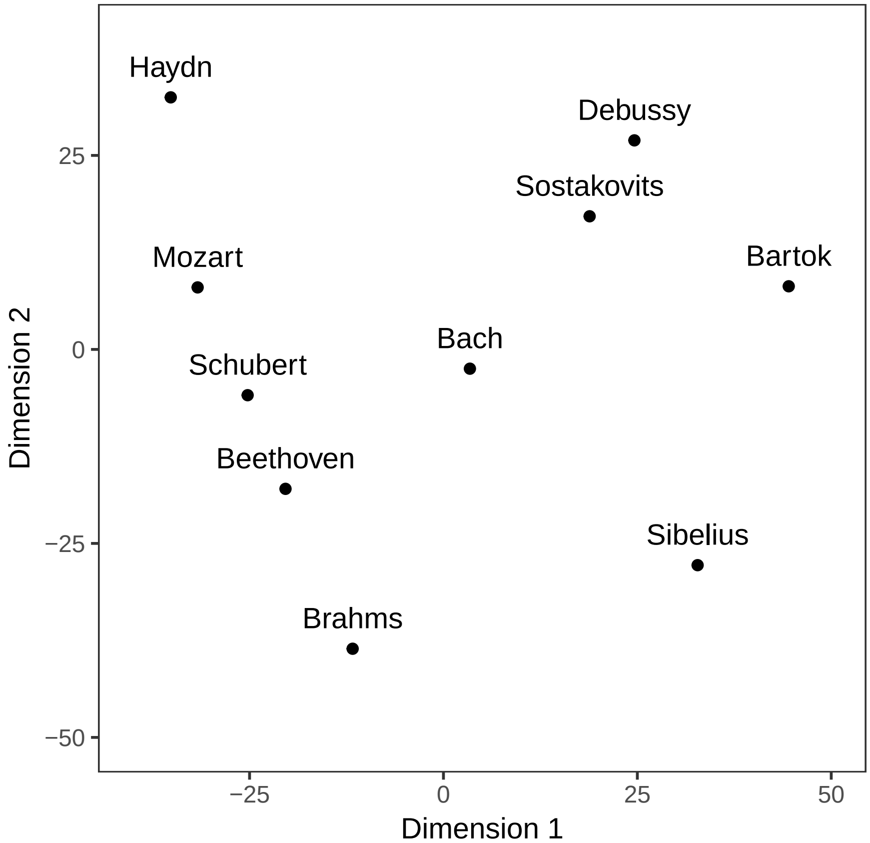
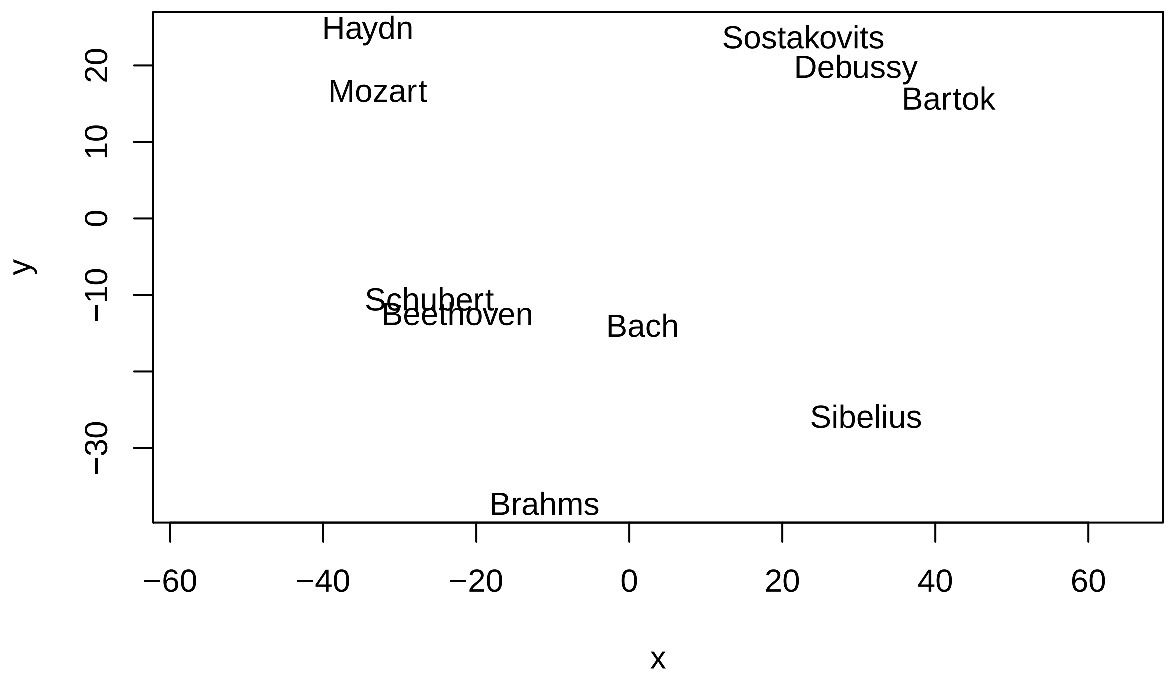

Haydn
Debussy
Sostakovits
Bartok
Mozart
Bach
Schubert
Beethoven
Sibelius
Brahms
25
0
−25
−50
−25
0
25
50
Dimension 1
Dimension 2

Figure 14.5

```
library(vegan)
OmonoMDS <- monoMDS(dist = OMD, y = cmdscale(OMD, 3), k = 3,
                    model = "global", scaling = FALSE, pc = FALSE)

x <- OmonoMDS$points[, 1]
y <- OmonoMDS$points[, 2]
plot(x, y, type = "n", asp = 1, axes = TRUE)
text(x, y, composers, cex = 1.0)
```




```
Omono <- as.data.frame(scores(OmonoMDS$points[, 1:3])) # adds names MDS1,2,3

p1 <- ggplot(Omono, aes(x = MDS1, y = MDS2))
p2 <- p1 + geom_point() + geom_text(aes(label = composers),
                                    position = position_nudge(y = +4), size=4)
p3 <- p2 + scale_x_continuous(name = "Dimension 1", limits = c(-40, +50))
p4 <- p3 + scale_y_continuous(name = "Dimension 2", limits = c(-50, +40))
p5 <- p4 + theme_bw()
p6 <- p5 + theme(panel.grid.major = element_blank(), panel.grid.minor = element_blank())
p7 <- p6 + coord_fixed(ratio = 1)
p7
```

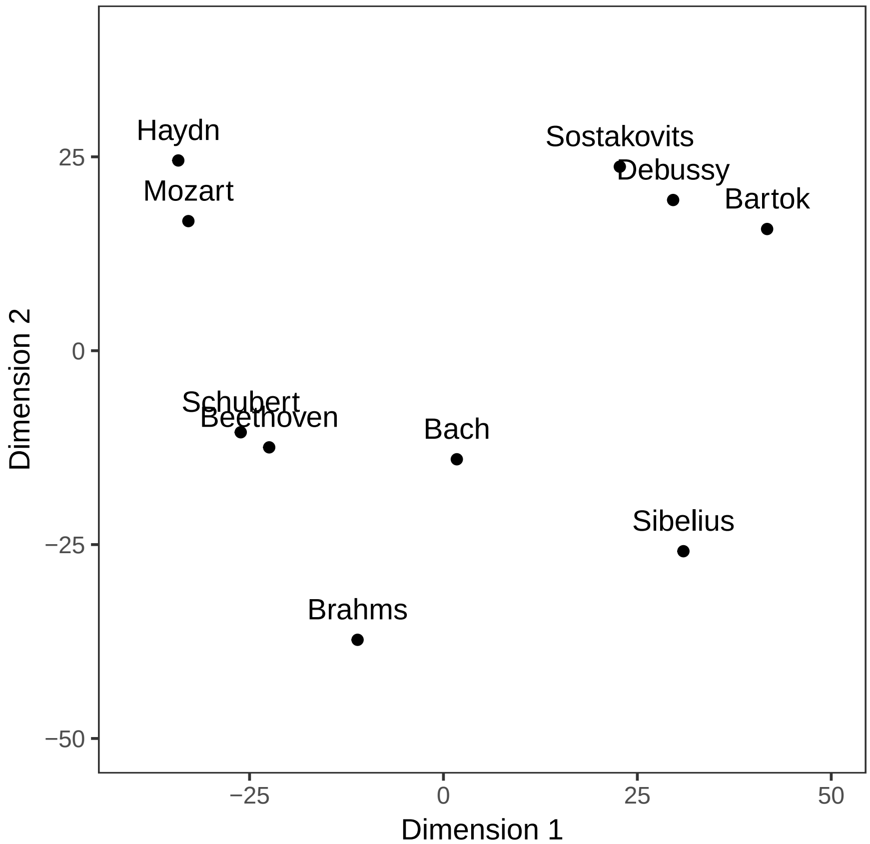

Haydn
Mozart
Sostakovits
Debussy
Bartok
Schubert
Beethoven
Bach
Sibelius
Brahms
25
0
−25
−50
−25
0
25
50
Dimension 1
Dimension 2

Figure 14.6

```
p1 <- ggplot(Omono, aes(x = MDS1, y = MDS3))
p2 <- p1 + geom_point() + geom_text(aes(label = composers),
                              position = position_nudge(y = +4.1), size=4)
p3 <- p2 + scale_x_continuous(name = "Dimension 1", limits = c(-40, +50))
p4 <- p3 + scale_y_continuous(name = "Dimension 3", limits = c(-50, +40))
p5 <- p4 + theme_bw()
p6 <- p5 + theme(panel.grid.major = element_blank(), panel.grid.minor = element_blank())
p7 <- p6 + coord_fixed(ratio = 1)
p7
```

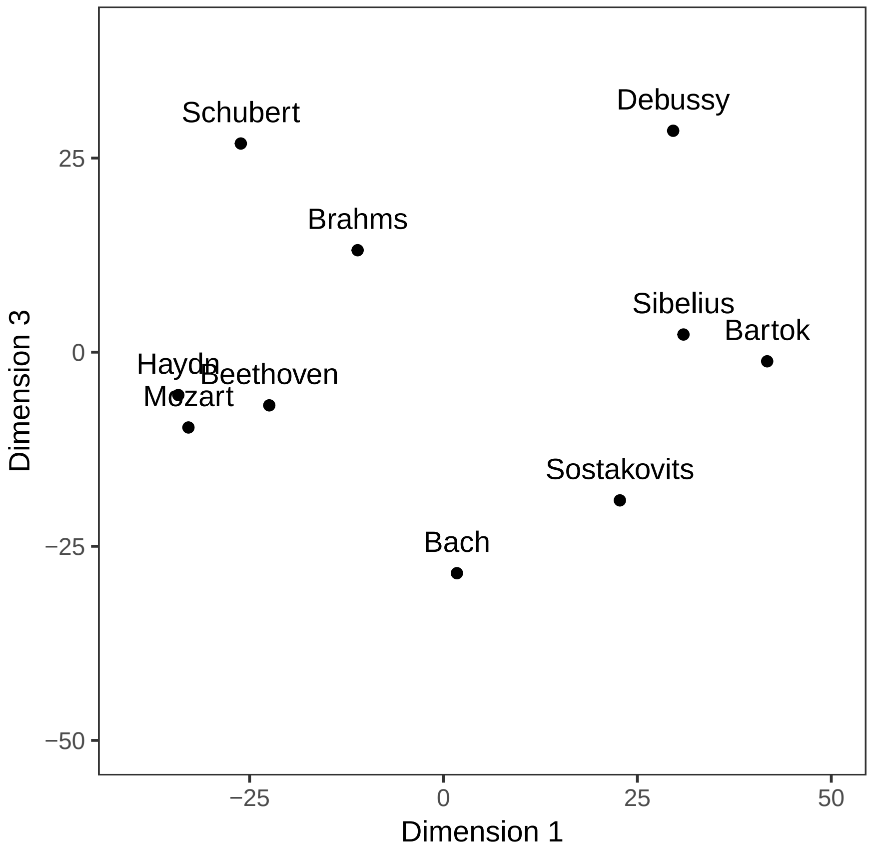
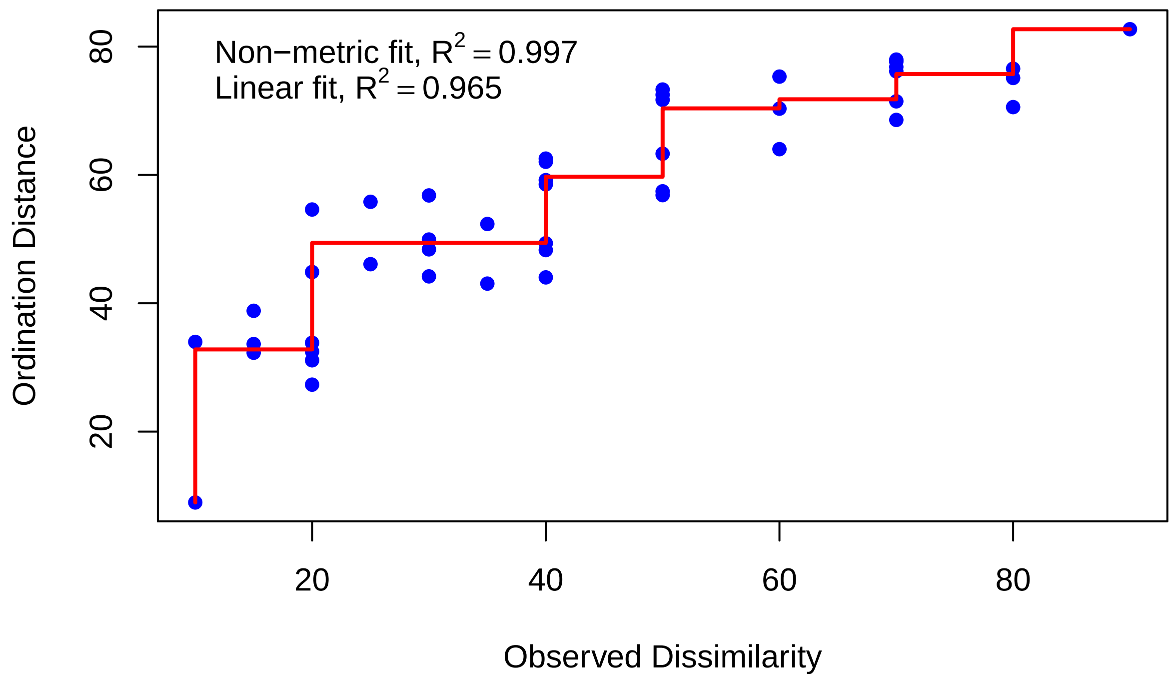


Figure 14.7

```r
# instead of Shepard function, use stressplot from vegan package:
strpl <- stressplot(OmonoMDS, pch = 16)
```




```r
plot(strpl, pch = 16, xlab = "Dissimilarity", ylab = "Distance")
lines(sort(strpl$x), sort(strpl$yf), type = "S")
```

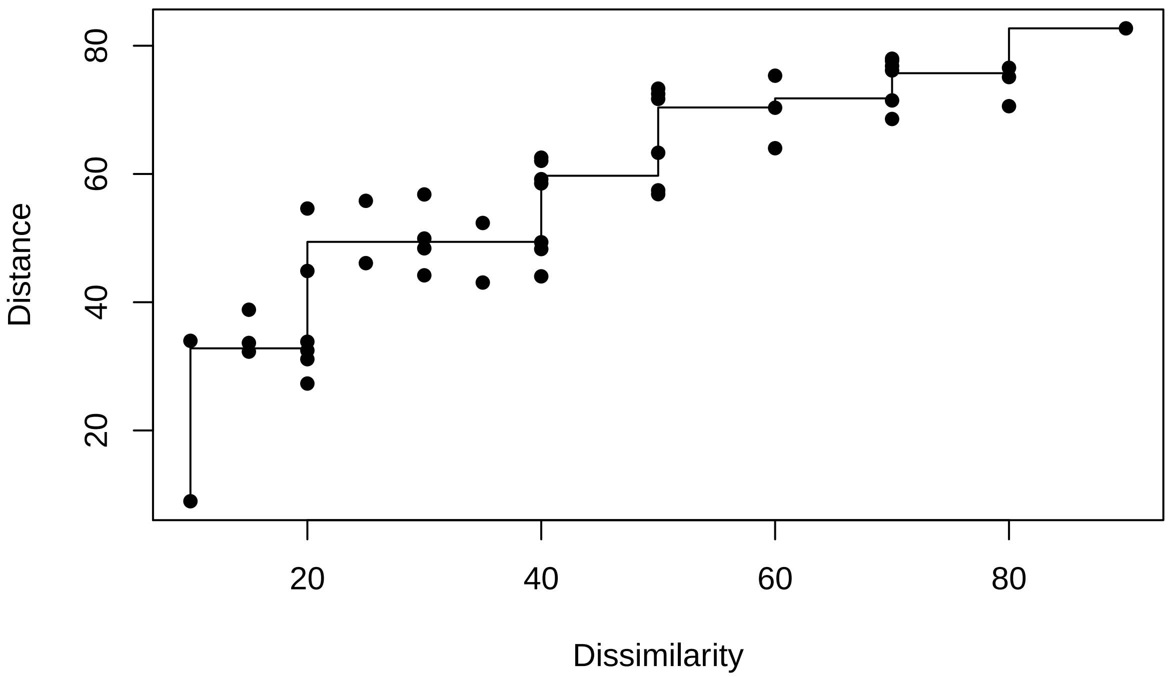

80
60
40
20
Distance
20
40
60
80
Dissimilarity

Table 14.8: The Influence of Age of Relationship Status with Boyfriends

```
Girls <- as.table(rbind(
          c(21, 21, 14, 13, 8),
          c(8, 9, 6, 8, 2),
          c(2, 3, 4, 10, 10)))

dimnames(Girls) <- list(relship = c("No boyfriend",
                                    "Boyfriend: No sex",
                                    "Boyfriend: Sex"),
                       agegroup = c("< 16", "16-17", "17-18", "18-19", "19-20"))
Girls
```

```
##                      agegroup
## relship              < 16 16-17 17-18 18-19 19-20
##   No boyfriend        21    21    14    13     8
##   Boyfriend: No sex    8     9     6     8     2
##   Boyfriend: Sex       2     3     4    10    10
```

```
# observed and expected frequencies, chi-square test, percentages etc.

Xsq <- chisq.test(Girls)  # Prints test summary
```

```
## Warning in chisq.test(Girls): Chi-squared approximation may be incorrect
```

```
Xsq$observed   # observed counts (same as Girls)
```

```
##                      agegroup
## relship              < 16 16-17 17-18 18-19 19-20
##   No boyfriend        21    21    14    13     8
##   Boyfriend: No sex    8     9     6     8     2
##   Boyfriend: Sex       2     3     4    10    10
```

```
Xsq$expected   # expected counts under the null
```

```
##                      agegroup
## relship              < 16 16-17 17-18 18-19 19-20
##   No boyfriend      17.2  18.3  13.3  17.2  11.1
##   Boyfriend: No sex  7.4   7.8   5.7   7.4   4.7
##   Boyfriend: Sex     6.5   6.9   5.0   6.5   4.2
```

```
Xsq
```

```
##
##  Pearson's Chi-squared test
##
## data:  Girls
## X-squared = 20, df = 8, p-value = 0.008
```

```
100*prop.table(Girls, margin = 1)
```

```
##                      agegroup
## relship              < 16 16-17 17-18 18-19 19-20
##   No boyfriend      27.3  27.3  18.2  16.9  10.4
##   Boyfriend: No sex 24.2  27.3  18.2  24.2   6.1
##   Boyfriend: Sex     6.9  10.3  13.8  34.5  34.5
```

```r
100*prop.table(Girls, margin = 2)
```

```
##                     agegroup
## relship               < 16 16-17 17-18 18-19 19-20
##   No boyfriend        67.7  63.6  58.3  41.9  40.0
##   Boyfriend: No sex   25.8  27.3  25.0  25.8  10.0
##   Boyfriend: Sex       6.5   9.1  16.7  32.3  50.0
```

```r
100*prop.table(Girls)
```

```
##                     agegroup
## relship               < 16 16-17 17-18 18-19 19-20
##   No boyfriend        15.1  15.1  10.1   9.4   5.8
##   Boyfriend: No sex    5.8   6.5   4.3   5.8   1.4
##   Boyfriend: Sex       1.4   2.2   2.9   7.2   7.2
```

```r
addmargins(100*prop.table(Girls, margin = 1), margin = c(1,2), FUN = list(Sum = mean, Sum = sum) )
```

```
## Margins computed over dimensions
## in the following order:
## 1: relship
## 2: agegroup
```

```
##                     agegroup
## relship                < 16 16-17 17-18 18-19 19-20   Sum
##   No boyfriend         27.3  27.3  18.2  16.9  10.4 100.0
##   Boyfriend: No sex    24.2  27.3  18.2  24.2   6.1 100.0
##   Boyfriend: Sex        6.9  10.3  13.8  34.5  34.5 100.0
##   Sum                  19.5  21.6  16.7  25.2  17.0 100.0
```

Figure 14.8

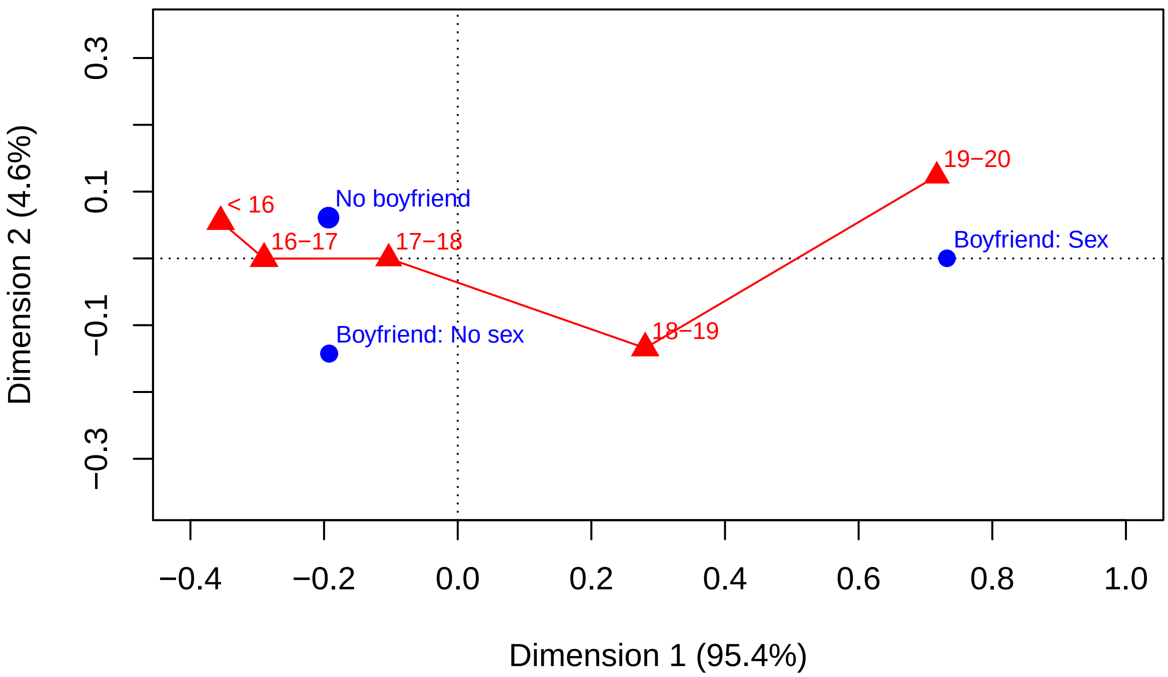
```r
#install.packages("ca")
library(ca)
caGirls <- ca(Girls)
caGirls
```

```
##
##  Principal inertias (eigenvalues):
##            1        2
## Value      0.141348 0.006884
## Percentage 95.36%   4.64%
##
##
##  Rows:
##          No boyfriend Boyfriend: No sex Boyfriend: Sex
## Mass            0.554             0.237          0.209
## ChiDist         0.203             0.239          0.732
## Inertia         0.023             0.014          0.112
## Dim. 1         -0.514            -0.512          1.948
## Dim. 2          0.735            -1.718          0.002
##
##
##  Columns:
##            < 16   16-17   17-18  18-19 19-20
## Mass      0.223  0.2374  0.1727  0.223 0.144
## ChiDist   0.359  0.2897  0.1033  0.311 0.727
## Inertia   0.029  0.0199  0.0018  0.022 0.076
## Dim. 1   -0.944 -0.7706 -0.2747  0.746 1.907
## Dim. 2    0.663 -0.0036 -0.0013 -1.618 1.487
```

```r
summary(caGirls)
```

```
##
## Principal inertias (eigenvalues):
##
##  dim    value      %   cum%   scree plot
##  1      0.141348  95.4  95.4  ************************
##  2      0.006884   4.6 100.0  *
##         -------- -----
##  Total: 0.148232 100.0
##
##
## Rows:
##     name   mass  qlt  inr    k=1  cor ctr    k=2 cor ctr
## 1 | Nbyf |  554 1000  154 | -193  909 146 |   61  91 300 |
## 2 | ByfN |  237 1000   92 | -192  646  62 | -143 354 700 |
## 3 | ByfS |  209 1000  755 |  732 1000 791 |    0   0   0 |
##
## Columns:
##     name   mass  qlt  inr    k=1  cor ctr    k=2 cor ctr
## 1 |   16 |  223 1000  194 | -355  977 199 |   55  23  98 |
## 2 | 1617 |  237 1000  134 | -290 1000 141 |    0   0   0 |
## 3 | 1718 |  173 1000   12 | -103 1000  13 |    0   0   0 |
## 4 | 1819 |  223 1000  146 |  281  814 124 | -134 186 584 |
## 5 | 1920 |  144 1000  514 |  717  971 523 |  123  29 318 |
```

```r
plot(caGirls, lines=c(FALSE, TRUE), mass=c(TRUE,TRUE), xlim = c(-0.4,1))
```



Dimension 2 (4.6%)
Dimension 1 (95.4%)
0.3
0.1
−0.1
−0.3
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
< 16
16−17
17−18
18−19
19−20
No boyfriend
Boyfriend: No sex
Boyfriend: Sex

Table 14.9: Types of Work Activities and Main Advantages of Job from 6933 Survey Respondents

Source of data: Lebart, L., Morineau, A. and Warwick, K. (1984). *Multivariate Descriptive Statistical Analysis: Correspondence Analysis and Related Techniques for Large Matrices.* John Wiley & Sons, Chichester, U.K.

```
jobs <- read.csv(file = "data/jobs.csv", row.names = 1)
str(jobs)
```

```
## 'data.frame':    26 obs. of  23 variables:
##  $ WORKTYPE: Factor w/ 26 levels "Administrative services   ",..: 8 7 6 21 3 26 2 24 16 12 ...
##  $ VARIETY : int  4 1 1 5 2 2 2 3 3 0 ...
##  $ FREEDOM : int  189 13 9 5 7 5 3 18 7 18 ...
##  $ HUMAN   : int  0 3 1 2 1 0 1 0 3 1 ...
##  $ SCHEDULE: int  3 10 0 9 4 4 8 6 6 12 ...
##  $ SALARIES: int  2 17 4 18 15 1 16 16 6 31 ...
##  $ SECURITY: int  2 12 13 5 5 0 17 5 0 7 ...
##  $ FAMILY  : int  9 4 0 3 2 3 1 4 0 0 ...
##  $ INTEREST: int  3 1 2 2 1 0 8 4 2 8 ...
##  $ NEARHOME: int  12 8 2 6 6 2 7 13 6 19 ...
##  $ ATMOSPHE: int  2 3 0 5 1 1 2 4 3 11 ...
##  $ SOC_ADV : int  1 5 2 5 2 1 4 2 3 3 ...
##  $ OWN_BOSS: int  4 1 1 0 2 1 3 3 0 2 ...
##  $ LIKE_IT : int  11 9 4 2 3 1 6 6 2 10 ...
##  $ OTHER   : int  15 5 3 3 0 0 1 2 1 4 ...
##  $ NONE    : int  12 11 6 22 5 3 24 26 8 26 ...
##  $ OUTDOORS: int  8 0 1 0 0 0 0 0 0 0 ...
##  $ NO_ANS  : int  1 0 0 0 1 2 1 2 0 6 ...
##  $ NO_DIPL : int  93 32 9 29 13 10 25 28 11 51 ...
##  $ HIGH_SCH: int  18 15 8 4 13 5 4 4 11 26 ...
##  $ FIRM10  : int  187 12 6 4 3 6 7 24 6 32 ...
##  $ FIRM500 : int  2 40 17 30 14 11 19 38 16 40 ...
##  $ FIRM1000: int  0 11 6 27 25 0 61 6 9 30 ...
```

```
row.names(jobs) <- jobs$WORKTYPE
jobs <- jobs[, 2:23]
# for simplicity, withOUT supplementary variables:
jobs2 <- jobs[, 1:17]
head(jobs2[, c(1:3, 16:17)])
```

```
##                              VARIETY FREEDOM HUMAN OUTDOORS NO_ANS
## Farming-fishing                    4     189     0        8      1
## Farm-food industry                 1      13     3        0      0
## Energy-mines                       1       9     1        1      0
## Steel                              5       5     2        0      0
## Chemical-glass-oil                 2       7     1        0      1
## Wood-paper                         2       5     0        0      2
```

```
chisq.test(jobs2)
```

```
## Warning in chisq.test(jobs2): Chi-squared approximation may be incorrect

## 
##  Pearson's Chi-squared test
## 
```

```
## data:  jobs2
## X-squared = 2000, df = 400, p-value <2e-16
```

Figure 14.9

```r
lebart <- ca(jobs2)
plot(lebart, xlim = c(-1.1, 1.1), ylim = c(-1.1, 0.5), mass=c(TRUE,TRUE))
```