IIT Bombay - Krittika Summer Projects 3.0

Generating Gravitational Waveforms

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What are Gravitational Waves?

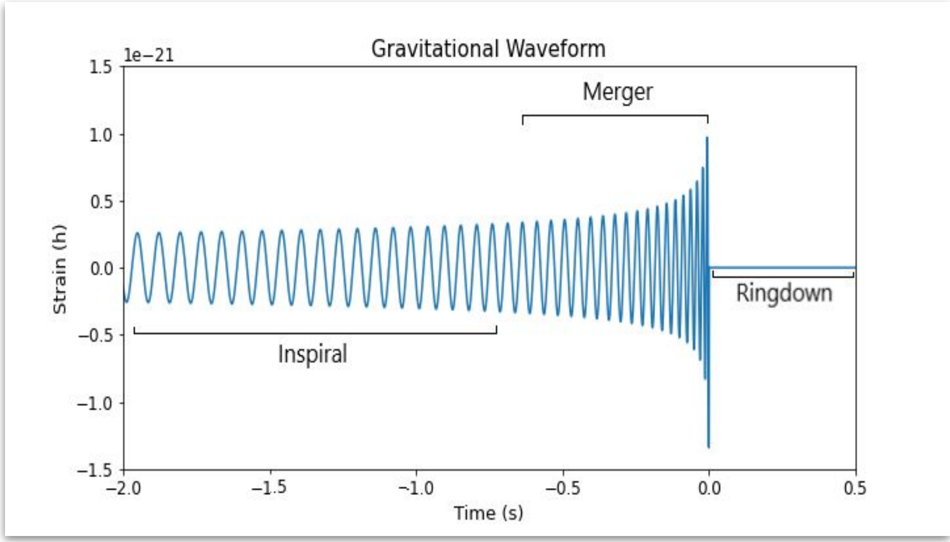
- These are ripples in the fabric of spacetime.
- Produced by violent cataclysmic events in the Universe like supernova, rotating neutron stars, or merging of black holes/ neutron stars.
- First predicted by Albert Einstein in 1916
- First experimental observations of gravitational waves were in 2015 by the Laser Interferometer Gravitational wave Observatory (LIGO).

Compact Binary Coalescence (CBC)

- Primary sources of gravitational waves detected by LIGO.
- It consists of a binary system of compact objects like Black Holes and Neutron Stars.
- These compact objects emit gravitational waves as they orbit around each other.
- They lose energy to these gravitational radiations and inspiral into each other till they merge together.

Gravitational Waveforms

- A gravitational waveform can be split into three phases : Inspiral, Merger and Ringdown.
- The inspiral phase is characterized by a steady frequency and amplitude as the binary components orbit around each other.
- Merger phase is marked by increasing frequency and amplitude as the two components spiral closer into each other and finally merge. This is followed by a Ringdown phase.



Mathematics of Gravitational Waves

- The gravitational wave equation is obtained by solving the Einstein Field Equations of General Relativity.
- Various approximation methods are used to find such solutions. For e.g. Quadrupole Approximation, Post Newtonian Theory, Numerical Relativity, etc.
- In this project we used the Quadrupole Approximation and Post Newtonian Theory.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\bar{h}_{\mu\nu} = 0$$

Quadrupole Formula and Newtonian Approximation

- Quadrupole Formula is used to calculate the gravitational wave strain components using :

$$h_{ij} = \frac{2G}{c^4 d} \cdot \frac{d^2 Q_{ij}}{dt^2}$$

- Q_{ij} is the 2nd rank Quadrupole Moment Tensor.
- The Newtonian Approximation gives approximate terms for frequency and phase evolution of the gravitational waveforms :

$$f^{-8/3} = \frac{(8\pi)^{8/3}}{5} \left(\frac{G\mathcal{M}}{c^3} \right)^{5/3} t$$

Post Newtonian Theory

- Post Newtonian Theory or PN Expansions is used to find approximate solutions for moderately relativistic and weakly self-gravitating sources.
- It involves expanding the Einstein Equations in terms of n orders of the parameter and equating terms of the same order.:

$$\varepsilon \sim \sqrt{\frac{R_s}{d}} \sim \frac{v}{c}$$

- The resulting terms are called n/2 PN corrections or terms.
- Post Newtonian Theory is effective mainly during the inspiral phase of the gravitational waveforms.

Results

Creating the Plots

```
def timeAtFreqN(f, f0, t0):
    return t0 + 5/np.power(8*pi,8/3)*np.power(c**3/G/Mc, 5/3)*(np.power(f0, -8/3) - np.power(f, -8/3))
```

```
def timeAtFreqPN(f, f0, t0):

    f_rel = abs(f)/f0
    return (tau0(f0)*(1-np.power(f_rel, -8/3))
        + tau1(f0)*(1-np.power(f_rel, -2))
        - tau1_5(f0)*(1-np.power(f_rel, -5/3))
        + tau2(f0)*(1-np.power(f_rel, -4/3))
        + t0)
```

1. Choose a starting frequency f_0 and time t_0
2. Obtain frequency and time arrays until f_ISCO is reached.
3. Calculate phase and amplitude of the strain.
4. Vary parameters and compare results

The post-newtonian correction terms have coefficients depending on choice of f_0

What is f_ISCO?

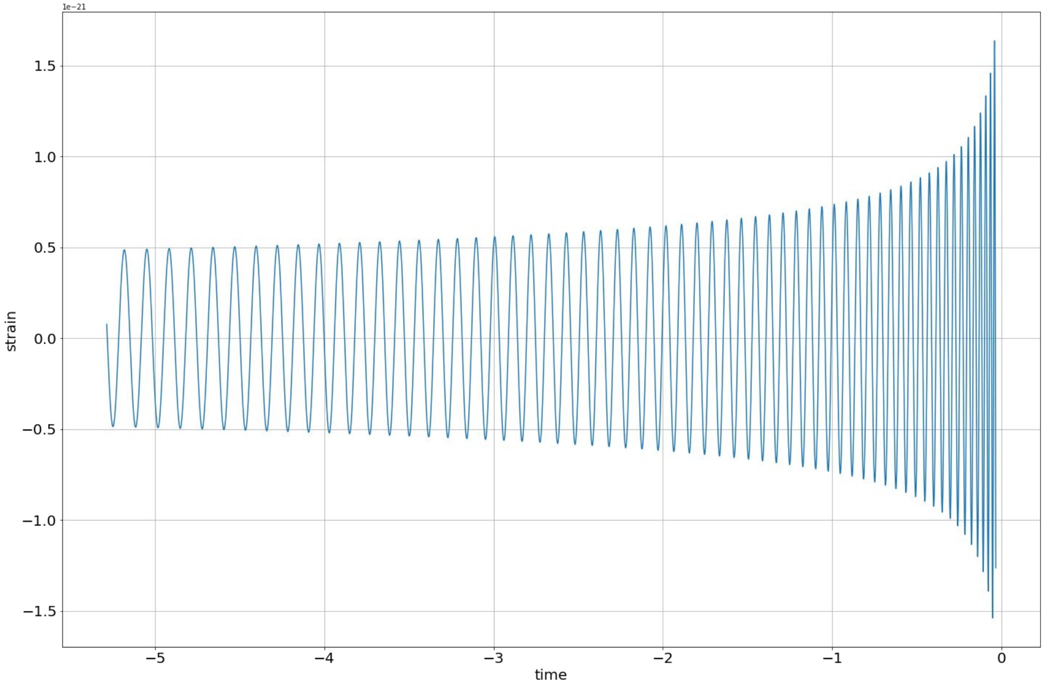
R_ISCO is the Innermost Stable Circular Orbit (for non-spinning black holes).

We estimate the frequency at this radius with Keplerian mechanics and assuming the black holes to be point particles.

Using frequency-time relations, we calculate t_ISCO, and use that as an estimate for the time of coalescence, t_c.

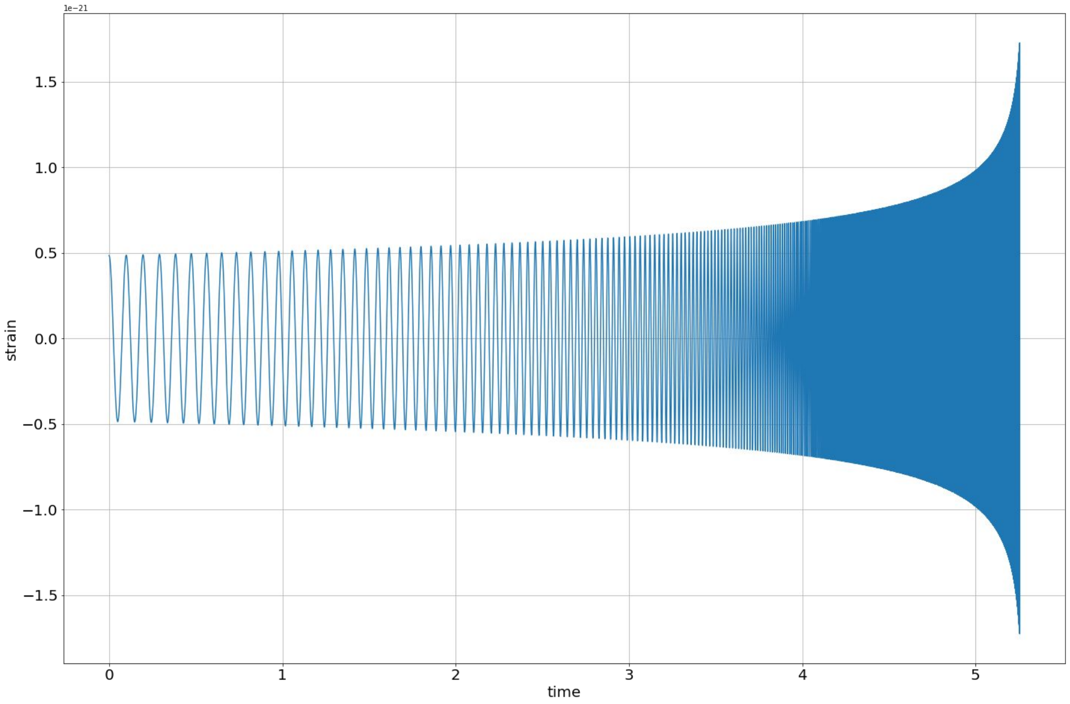
Hence we solve the inspiral and merger-phase expressions only until f_ISCO is reached, as ringdown is expected to start right after.

Newtonian Approximation



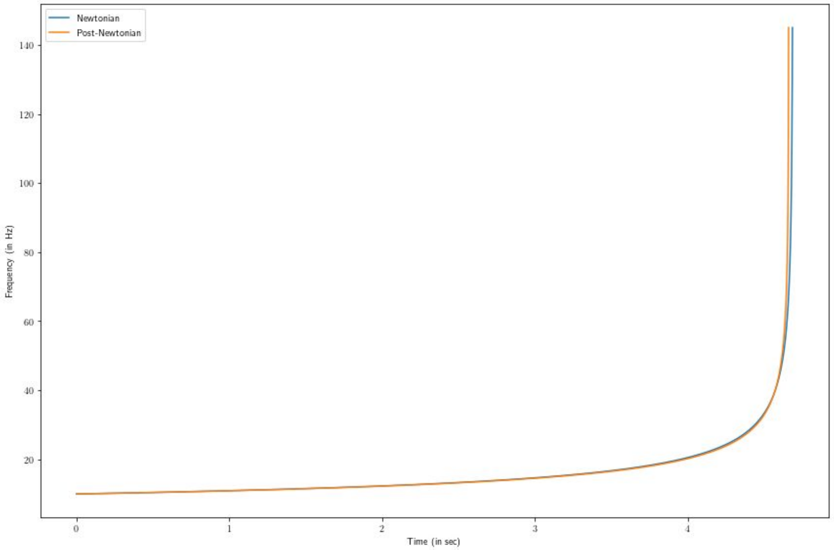
Strain vs Time

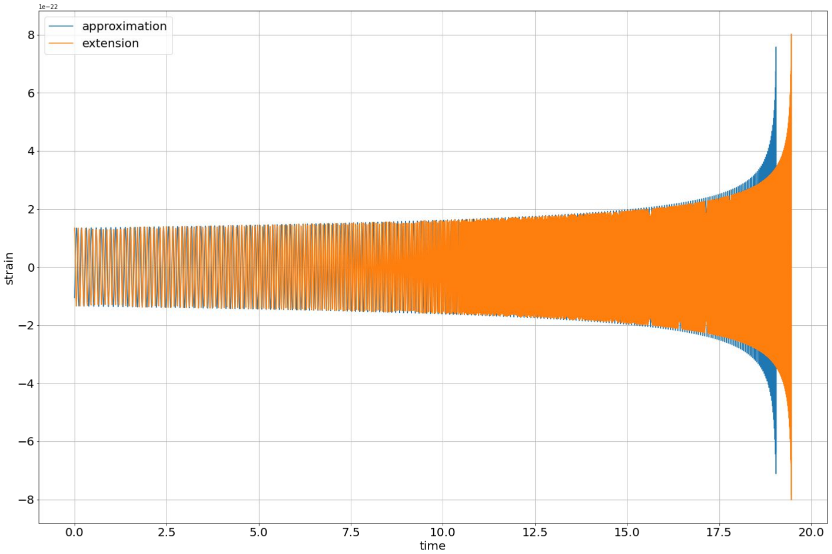
Post-Newtonian Approximation



Strain vs Time

Comparison



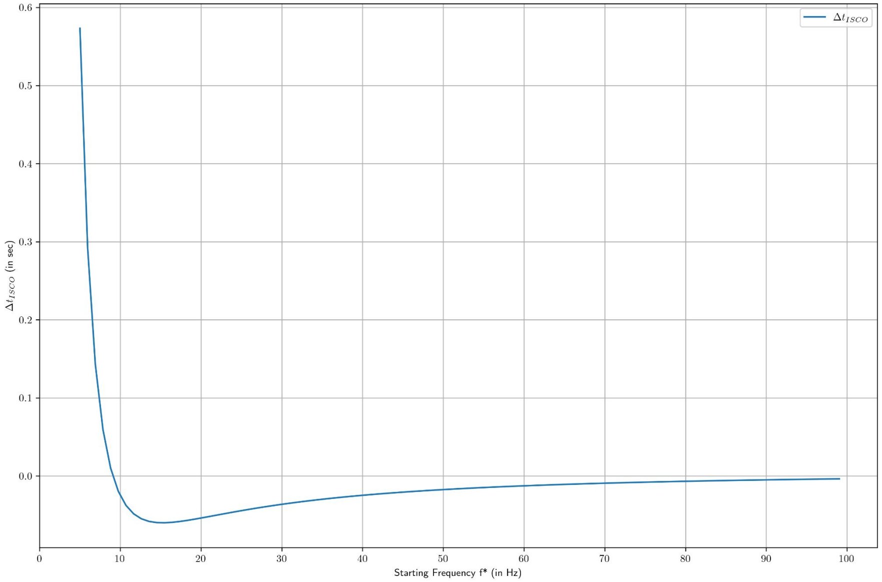


Frequency vs Time

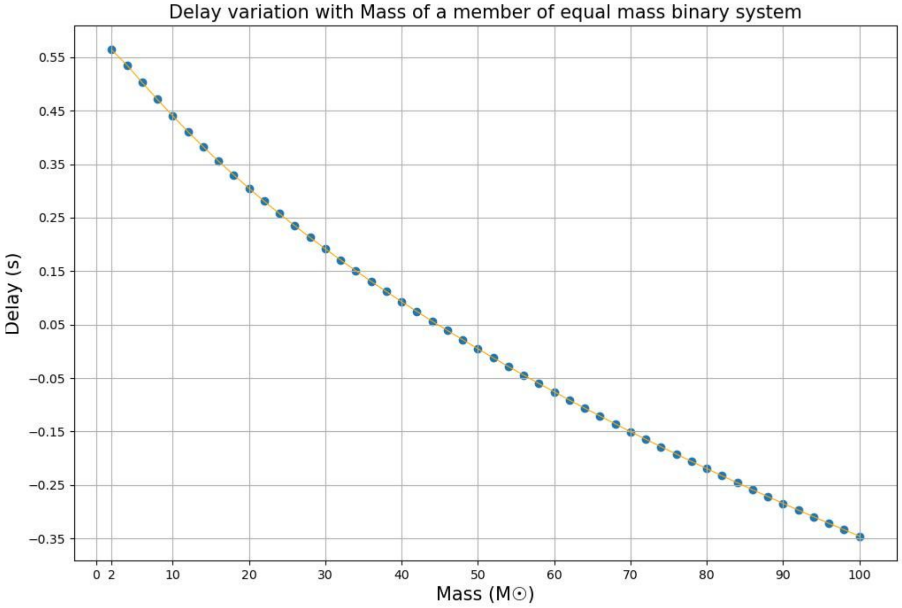
Strain vs Time (both 15 M_0)

Note the time difference between both approximations reaching f_ISCO!

What's this Time Difference?



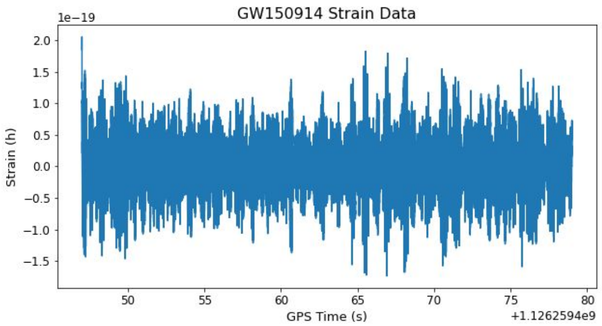
Variation with starting frequency f_0



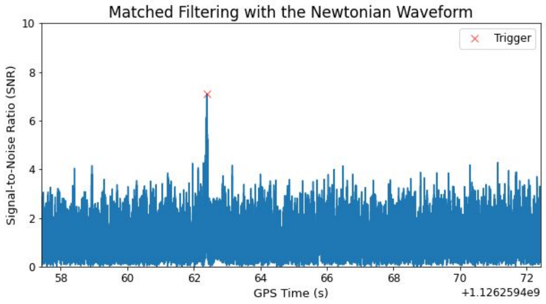
Variation with mass

Matched Filtering

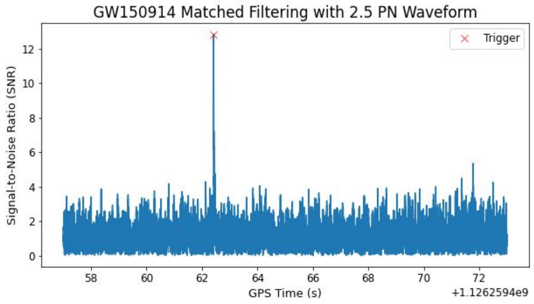
- Strain data recorded by LIGO detectors contain overwhelming amount of noise.
- Our gravitational wave signal is buried in this noise.
- In Matched Filtering, we cross-correlated the LIGO strain data with generated waveform at different time steps to obtain a Signal to Noise Ratio (SNR) time series.
- A spike in this SNR time series can indicate the presence of a Gravitational wave signal.



LIGO Strain Data



Matched Filtering with Newtonian Waveform



Matched Filtering with PN Waveform