Pune Vidyarthi Griha's

COLLEGE OF ENGINEERING, NASHIK – 3.

"Number System"

By

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Chapter I – Number System

- **Introduction to digital signal, Advantages of Digital System over analog systems**
  - ✓ Number Systems: Different types of number systems(Binary, Octal, Hexadecimal), conversion of number systems,
  - ✓ Binary arithmetic: Addition, Subtraction, Multiplication, Division.
  - ✓ Subtraction using 1's complement and 2's complement
- **Codes**
  - ✓ Codes -BCD, Gray Code, Excess-3, ASCII code
  - ✓ BCD addition, BCD subtraction using 9's and 10' complement
- **(Numericals based on above topic)**.

Chapter - I
Number System

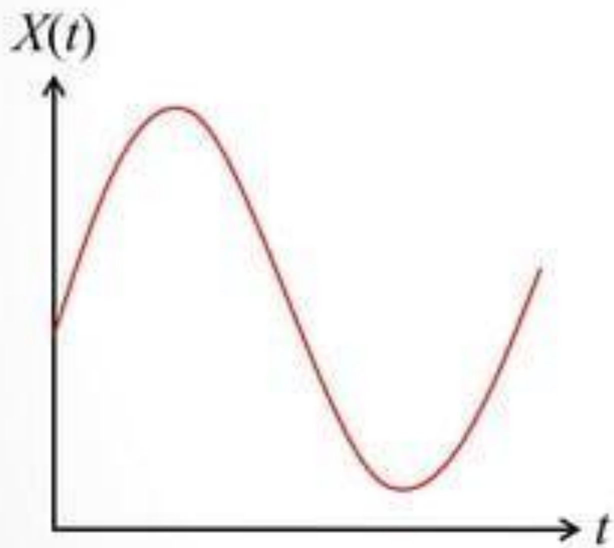
Chapter I – Number System

- **Introduction to digital signal, Advantages of Digital System over analog systems** (8 Marks)
  - ✓ Number Systems: Different types of number systems(Binary, Octal, Hexadecimal), conversion of number systems,
  - ✓ Binary arithmetic: Addition, Subtraction, Multiplication, Division.
  - ✓ Subtraction using 1's complement and 2's complement
- **Codes** (4 Marks)
  - ✓ Codes -BCD, Gray Code, Excess-3, ASCII code
  - ✓ BCD addition, BCD subtraction using 9's and 10' complement
- **(Numericals based on above topic)**.

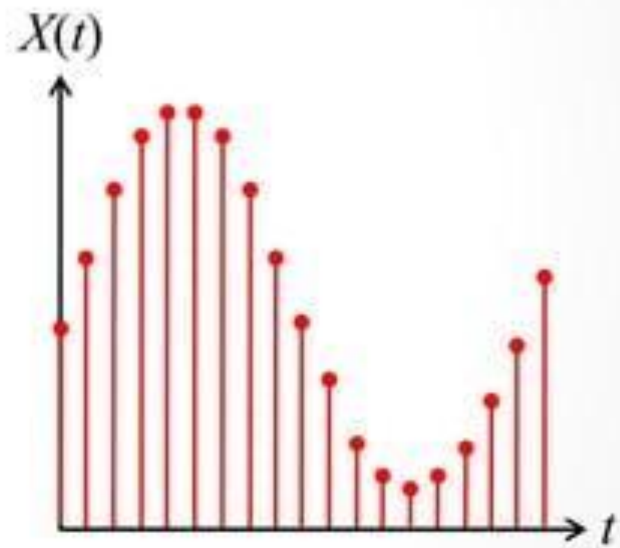
Analog Vs Digital

- Analog system
  - ✓ The physical quantities or signals may vary continuously over a specified range.
- Digital system
  - ✓ The physical quantities or signals can assume only discrete values.
  - ✓ Greater accuracy

Analog Vs Digital



Analog signal



Digital signal

Advantages of Digital System over Analog System

- ✓ Digital Systems are easier to design
- ✓ Information storage is easy
- ✓ Accuracy & Precision are greater
- ✓ Digital systems are more versatile
- ✓ Digital circuits are less affected by noise
- ✓ More digital circuitry can be fabricated on IC chips

- ✓ Reliability is more

Chapter I – Number System

- Introduction to digital signal, Advantages of Digital System over analog systems
- **Number Systems: Different types of number systems(Binary, Octal, Hexadecimal),** conversion of number systems,
  - ✓ Binary arithmetic: Addition, Subtraction, Multiplication, Division.
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- Codes
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  - ✓ BCD addition, BCD subtraction using 9's and 10' complement
- (Numericals based on above topic).

Number System

- ✓ A number system defines a set of values used to represent quantity.

Different Number Systems

- ✓ Decimal Number System

  - Base 10

- ✓ Binary Number System

  - Base 2

- ✓ Octal Number System

  - Base 8

- ✓ Hexadecimal Number System

  - Base 16

Decimal Number System

- ✓ Decimal number system contains ten unique symbols 0,1,2,3,4,5,6,7,8 and 9
- ✓ Since counting in decimal involves ten symbols, we can say that its base or radix is ten.
- ✓ It is a positional weighted system

Decimal Number System

- ✓ In this system, any number (integer, fraction or mixed) of any magnitude can be represented by the use of these ten symbols only
- ✓ Each symbols in the number is called a “Digit”

Decimal Number System

Structure:

		MSD						LSD	
Decimal No.	......	d_3	d_2	d_1	d_0	.	d_{-1}	d_{-2}	....
Positional Weights		10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	

Decimal Point

Decimal Number System

- ✓ MSD: The leftmost digit in any number representation, which has the greatest positional weight out of all the digits present in that number is called the "Most Significant Digit" (MSD)

- ✓ LSD: The rightmost digit in any number representation, which has the least positional weight out of all the digits present in that number is called the "Least Significant Digit" (MSD)

Decimal Number System

- Examples

  1214

  1897

  9875.54

Binary Number System

- ✓ Binary number system is a positional weighted system
- ✓ It contains two unique symbols 0 and 1
- ✓ Since counting in binary involves two symbols, we can say that its base or radix is two.

Binary Number System

- ✓ A binary digit is called a "Bit"
- ✓ A binary number consists of a sequence of bits, each of which is either a 0 or a 1.
- ✓ The binary point separates the integer and fraction parts

Binary Number System

Structure:

		MSB						LSB	
Binary No.	......	b_3	b_2	b_1	b_0	.	b_{-1}	b_{-2}	....
Positional Weights		2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	

Binary Point

Binary Number System

- ✓ MSB: The leftmost bit in a given binary number with the highest positional weight is called the “Most Significant Bit” (MSB)
- ✓ LSB: The rightmost bit in a given binary number with the lowest positional weight is called the “Least Significant Bit” (LSB)

Binary Number System

Decimal No.	Binary No.
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Decimal No.	Binary No.
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Terms related to Binary Numbers

✓ BIT: The binary digits (0 and 1) are called bits.

- Single unit in binary digit is called "Bit"
- Example 1
  0

Terms related to Binary Numbers

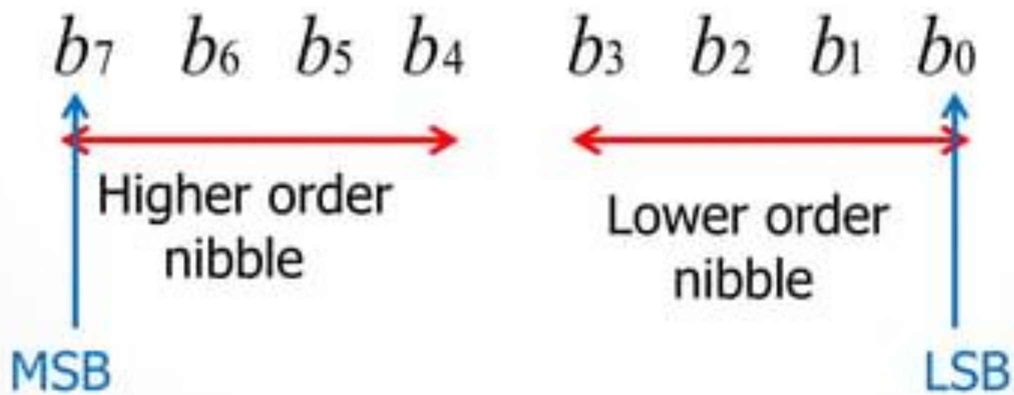
✓ NIBBLE: A nibble is a combination of 4 binary bits.

Examples, 1110
0000
1001
0101

Terms related to Binary Numbers

- ✓ **BYTE:** A byte is a combination of 8 binary bits.

  - ✓ The number of distinct values represented by a byte is 256 ranging from 0000 0000 to 1111 1111.



Terms related to Binary Numbers

- ✓ WORD: A word is a combination of 16 binary bits. Hence it consists of two bytes.

b_{15} b_{14} b_{13} b_{12} b_{11} b_{10} b_9 b_8 b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0

Higher order byte

Lower order byte

MSB

LSB

Terms related to Binary Numbers

✓ DOUBLE WORD: A double word is exactly what its name implies, two words

-It is a combination of 32 binary bits.

Octal Number System

- ✓ Octal number system is a positional weighted system
- ✓ It contains eight unique symbols 0,1,2,3,4,5,6 and 7
- ✓ Since counting in octal involves eight symbols, we can say that its base or radix is eight.

Octal Number System

- ✓ The largest value of a digit in the octal system will be 7.
- ✓ That means the octal number higher than 7 will not be 8, instead of that it will be 10.

Octal Number System

Structure:

	......	MSD						LSD	....
Octal No.	......	O_3	O_2	O_1	O_0	$\cdot$	O_{-1}	O_{-2}	....
Positional Weights		8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	

Radix Point

Octal Number System

- ✓ Since its base $8 = 2^3$, every 3 bit group of binary can be represented by an octal digit.
- ✓ An octal number is thus 1/3rd the length of the corresponding binary number

Octal Number System

Decimal No.	Binary No.	Octal No.
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	10
9	1001	11
10	1010	12
11	1011	13
12	1100	14
13	1101	15

Hexadecimal Number System (HEX)

- ✓ Binary numbers are long. These numbers are fine for machines but are too lengthy to be handled by human beings. So there is a need to represent the binary numbers concisely.

- ✓ One number system developed with this objective is the hexadecimal number system (or Hex)

Hexadecimal Number System (HEX)

- ✓ Hex number system is a positional weighted system
- ✓ It contains sixteen unique symbols 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E and F.
- ✓ Since counting in hex involves sixteen symbols, we can say that its base or radix is sixteen.

Hexadecimal Number System (HEX)

Structure:

	MSD						LSD	
Hex No.	……H_3	H_2	H_1	H_0	$\cdot$	H_{-1}	H_{-2}	….
Positional Weights	16^3	16^2	16^1	16^0		16^{-1}	16^{-2}	
					Radix Point			

Hexadecimal Number System (HEX)

- ✓ Since its base $16 = 2^4$, every 4 bit group of binary can be represented by an hex digit.
- ✓ An hex number is thus 1/4th the length of the corresponding binary number
- ✓ The hex system is particularly useful for human communications with computer

Hexadecimal Number System (HEX)

Decimal No.	Binary No.	Hex No.
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7

Decimal No.	Binary No.	Hex No.
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

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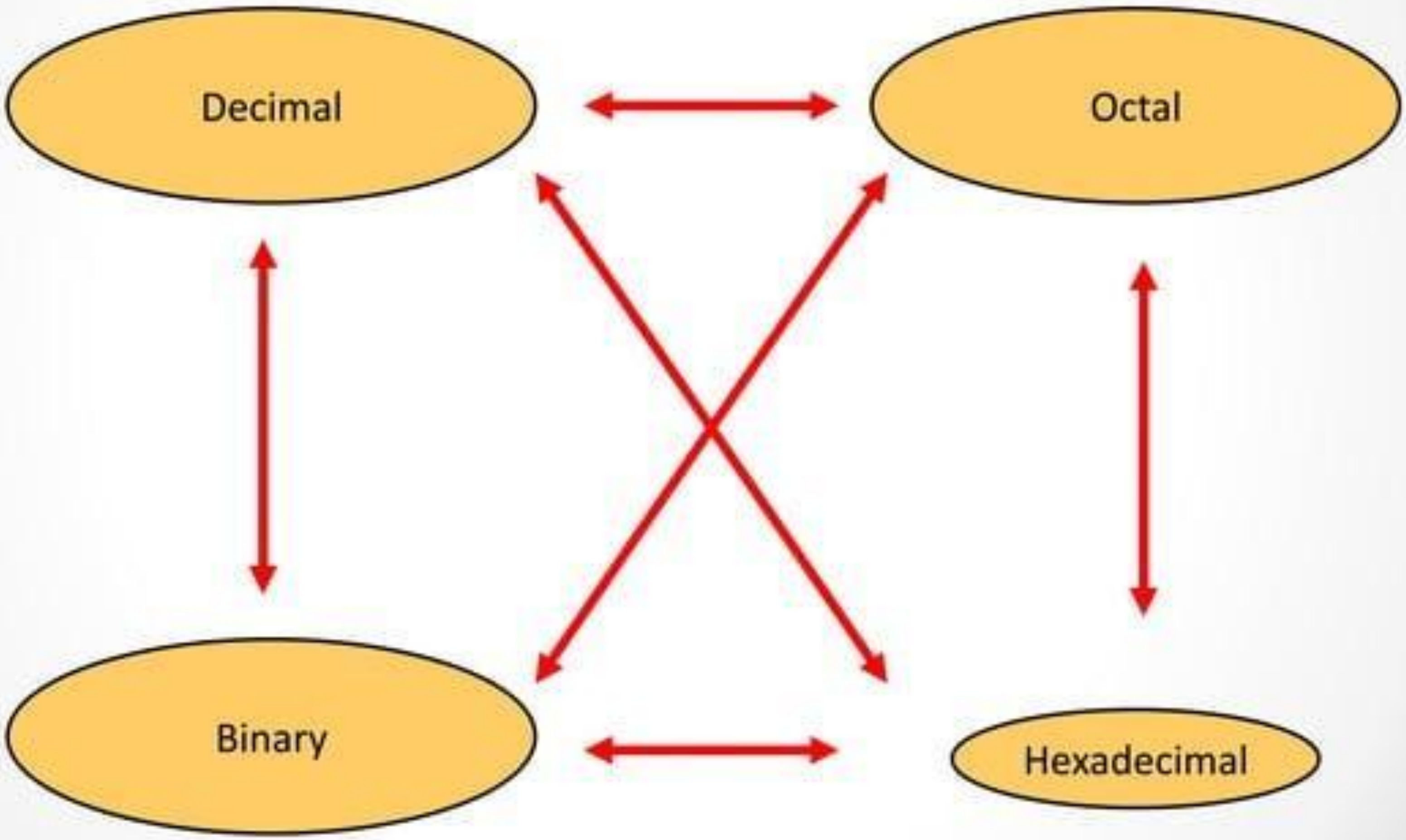
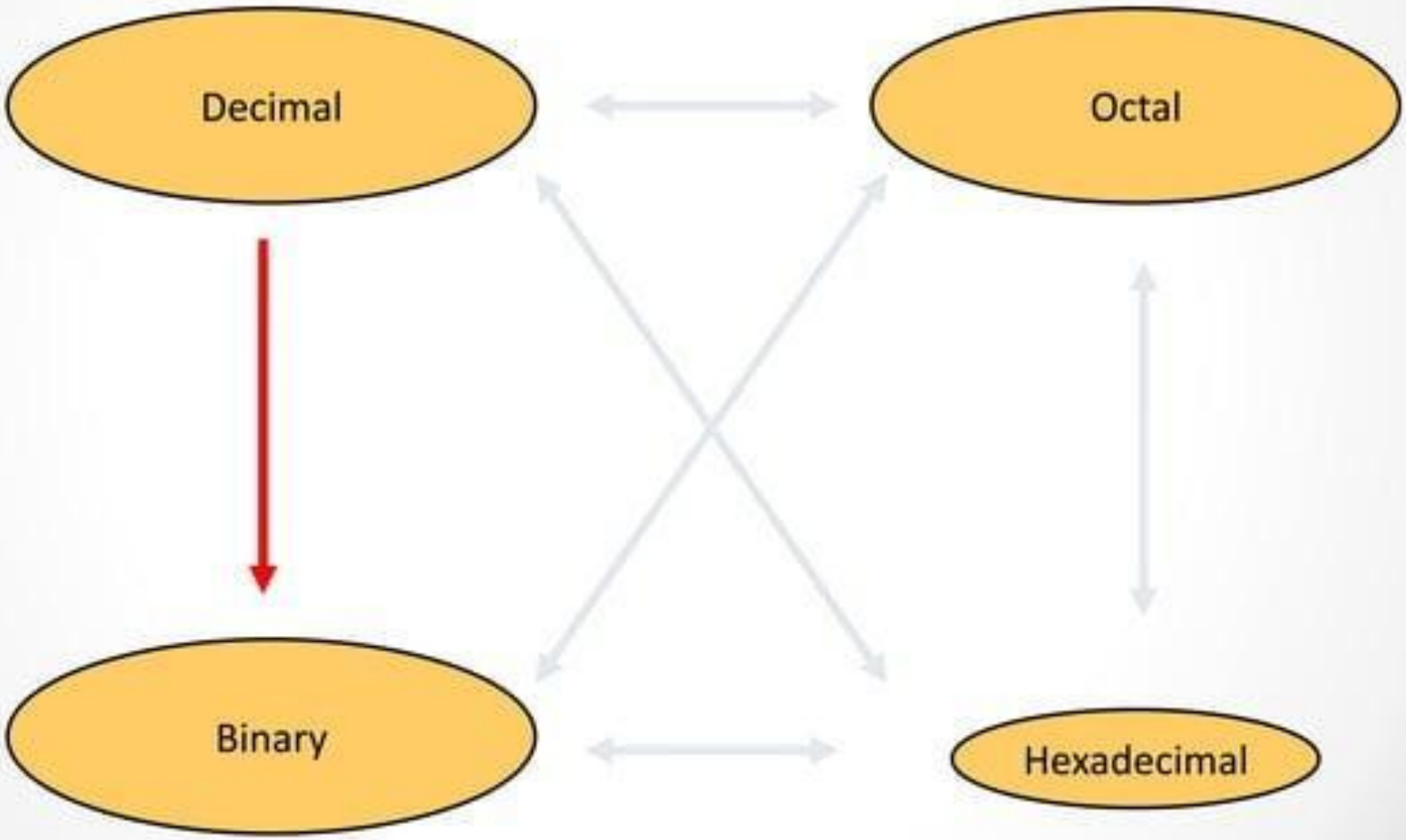
- **(Numericals based on above topic)**.

Conversion Among Bases

Possibilities



Conversion from Decimal Number to Binary Number



Conversion of Decimal number into Binary number (Integer Number)

Procedure:

1. Divide the decimal no by the base 2, noting the remainder.
2. Continue to divide the quotient by 2 until there is nothing left, keeping the track of the remainders from each step.
3. List the remainder values in reverse order to find the number's binary equivalent

Example: Convert 105 decimal number in to it's equivalent binary number.

Example: Convert 105 decimal number in to it's equivalent binary number.

$$\begin{array}{r|l} 2 & 105 \\ \hline & \end{array}$$

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105
2	52

1

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105	
2	52	1
2	26	0

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105	
2	52	1
2	26	0
2	13	0

Example: Convert 105 decimal number in to it’s equivalent binary number.

2	105	
2	52	1
2	26	0
2	13	0
2	6	1

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1
	0	1

Example: Convert 105 decimal number in to it's equivalent binary number.

2	105		
2	52	1	LSB
2	26	0	
2	13	0	
2	6	1	
2	3	0	
2	1	1	
	0	1	MSB

$$(105)_{10} = (1101001)_2$$

Conversion of Decimal number into Binary number (Fractional Number)

Procedure:

1. Multiply the given fractional number by base 2.
2. Record the carry generated in this multiplication as MSB.
3. Multiply only the fractional number of the product in step 2 by 2 and record the carry as the next bit to MSB.
4. Repeat the steps 2 and 3 up to 5 bits. The last carry will represent the LSB of equivalent binary number

Example: Convert 0.42 decimal number in to it's equivalent binary number.

Example: Convert 0.42 decimal number in to it's equivalent binary number.

0.42 X 2 = 0.84 0

Example: Convert 0.42 decimal number in to it's equivalent binary number.

0.42 X 2	=	0.84		0
0.84 X 2	=	1.68		1

Example: Convert 0.42 decimal number in to it's equivalent binary number.

0.42 X 2 =	0.84	0
0.84 X 2 =	1.68	1
0.68 X 2 =	1.36	1

Example: Convert 0.42 decimal number in to it's equivalent binary number.

0.42 X 2	=	0.84		0
0.84 X 2	=	1.68		1
0.68 X 2	=	1.36		1
0.36 X 2	=	0.72		0

Example: Convert 0.42 decimal number in to it's equivalent binary number.

0.42 X 2	=	0.84		0
0.84 X 2	=	1.68		1
0.68 X 2	=	1.36		1
0.36 X 2	=	0.72		0
0.72 X 2	=	1.44		1

Example: Convert 0.42 decimal number in to it's equivalent binary number.

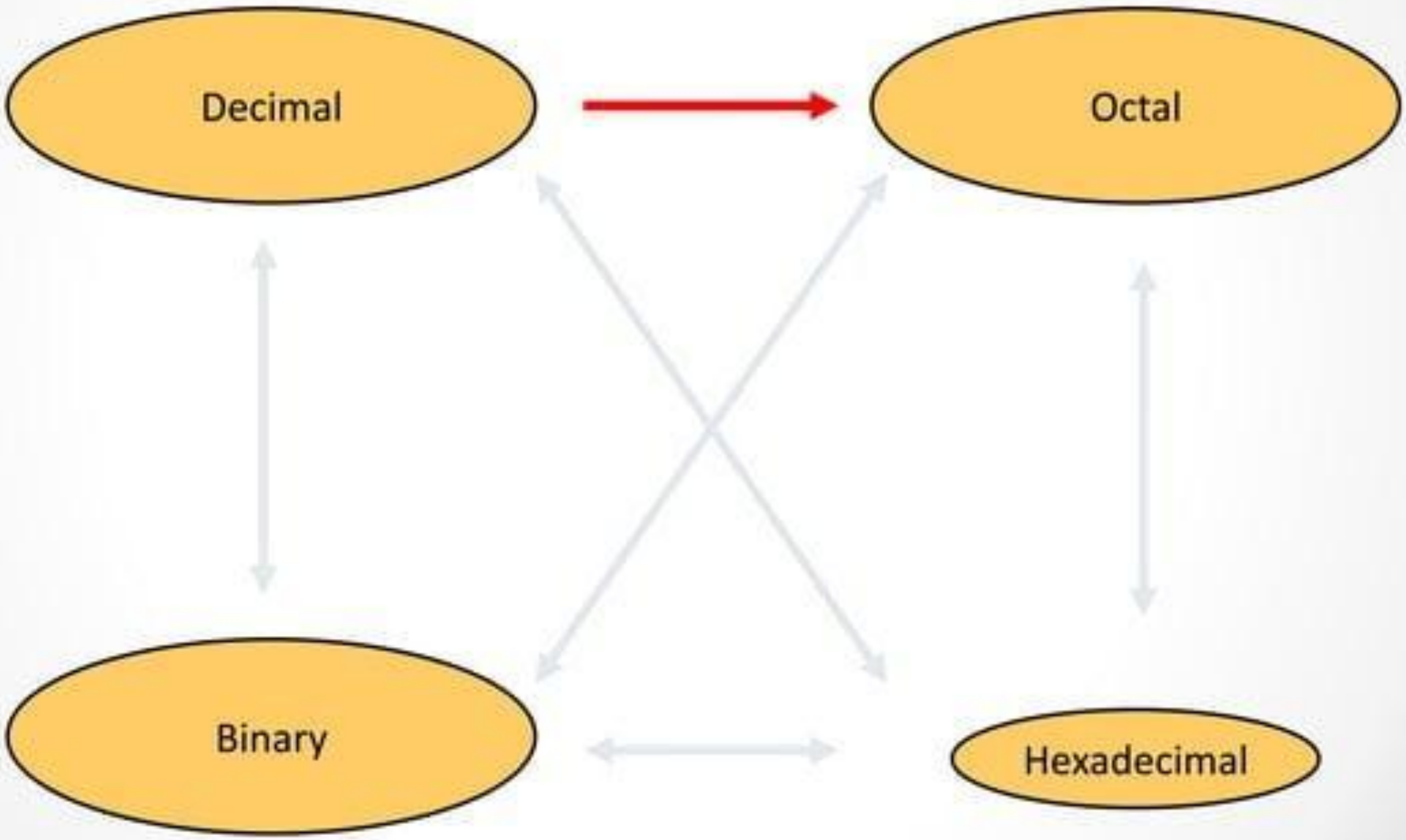
0.42 X 2 = 0.84	0	MSB
0.84 X 2 = 1.68	1	
0.68 X 2 = 1.36	1	
0.36 X 2 = 0.72	0	
0.72 X 2 = 1.44	1	LSB

$$(0.42)_{10} = (0.01101)_2$$

Exercise

- Convert following Decimal Numbers in to its equivalent Binary Number:
    1. $(1248.56)_{10} = (?)_2$
    2. $(8957.75)_{10} = (?)_2$
    3. $(420.6)_{10} = (?)_2$
    4. $(8476.47)_{10} = (?)_2$

Conversion from Decimal Number to Octal Number



Conversion of Decimal Number into Octal Number (Integer Number)

Procedure:

1. Divide the decimal no by the base 8, noting the remainder.
2. Continue to divide the quotient by 8 until there is nothing left, keeping the track of the remainders from each step.
3. List the remainder values in reverse order to find the number’s octal equivalent

Example: Convert 204 decimal number in to it's equivalent octal number.

Example: Convert 204 decimal number in to it’s equivalent octal number.

8	204

Example: Convert 204 decimal number in to it's equivalent octal number.

8	204
8	25

4

```
      2 5
   ┌──────
8  │  204
   - 16
   ────
      44
   -  40
   ─────
       4
```

Example: Convert 204 decimal number in to it's equivalent octal number.

8	204	
8	25	4
8	3	1

$$
\begin{array}{r|l}
 & \;\;3 \\
\hline
8 & \;25 \\
 & -24 \\
\cline{2-2}
 & \;\;\;1
\end{array}
$$

Example: Convert 204 decimal number in to it's equivalent octal number.

8	204	
8	25	4
8	3	1
	0	3

Example: Convert 204 decimal number in to it's equivalent octal number.

8	204		
8	25	4	LSB
8	3	1	↑
	0	3	MSB

$$(204)_{10} = (314)_8$$

Conversion of Decimal Number into Octal Number (Fractional Number)

Procedure:

1. Multiply the given fractional number by base 8.
2. Record the carry generated in this multiplication as MSB.
3. Multiply only the fractional number of the product in step 2 by 8 and record the carry as the next bit to MSB.
4. Repeat the steps 2 and 3 up to 5 bits. The last carry will represent the LSB of equivalent octal number

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

$0.6234 \times 8 = 4.9872 \quad 4$

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

0.6234 X 8	=	4.9872		4
0.9872 X 8	=	7.8976		7

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

0.6234 X 8 = 4.9872 4

0.9872 X 8 = 7.8976 7

0.8976 X 8 = 7.1808 7

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

0.6234 X 8	=	4.9872		4
0.9872 X 8	=	7.8976		7
0.8976 X 8	=	7.1808		7
0.1808 X 8	=	1.4464		1

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

0.6234 X 8	=	4.9872		4
0.9872 X 8	=	7.8976		7
0.8976 X 8	=	7.1808		7
0.1808 X 8	=	1.4464		1
0.4464 X 8	=	3.5712		3

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

0.6234 X 8	=	4.9872	4	MSB
0.9872 X 8	=	7.8976	7	
0.8976 X 8	=	7.1808	7	
0.1808 X 8	=	1.4464	1	
0.4464 X 8	=	3.5712	3	
				LSB

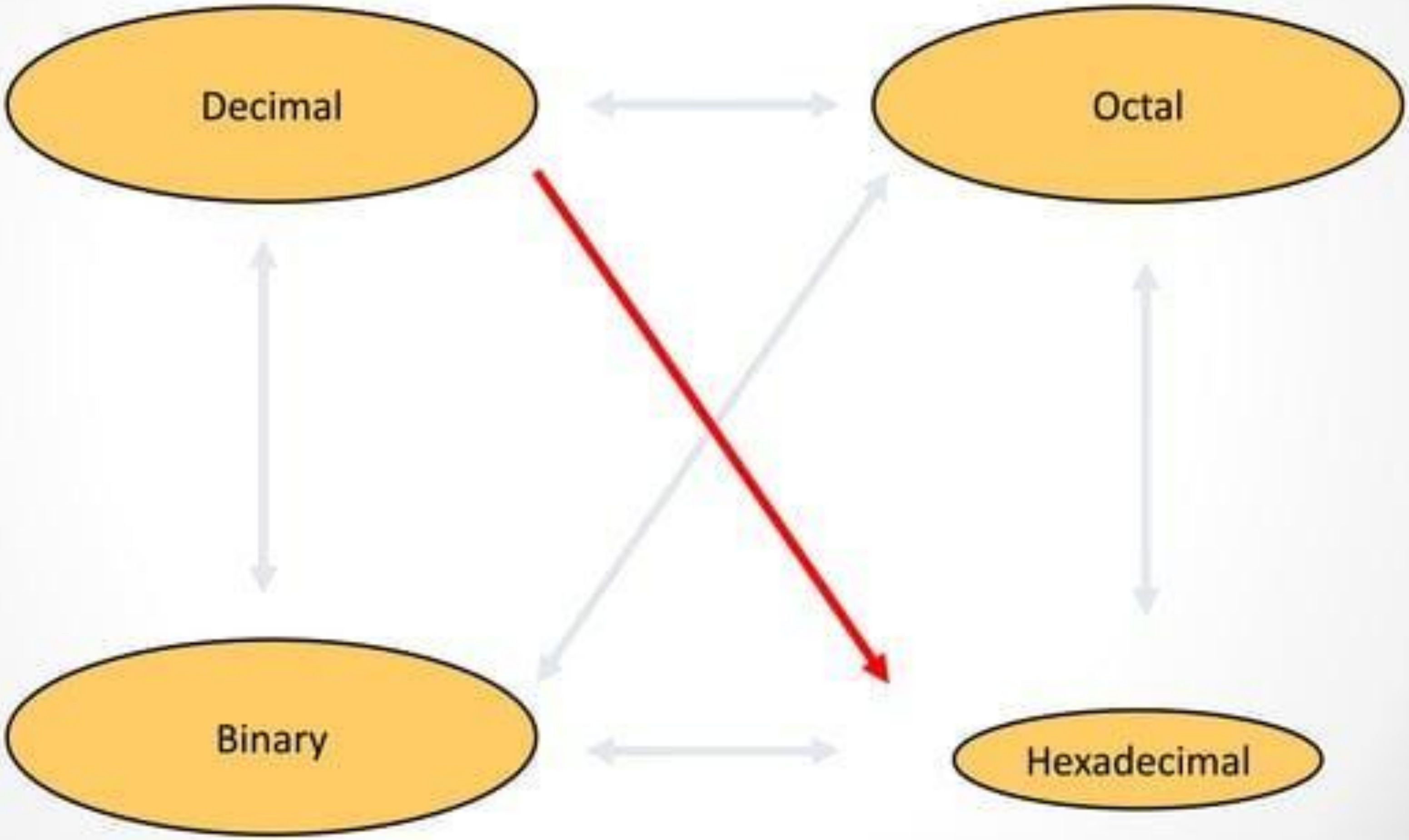
$$(0.6234)_{10} = (0.47713)_8$$

Exercise

- Convert following Decimal Numbers in to its equivalent Octal Number:

    1. $(1248.56)_{10} = (?)_8$
    2. $(8957.75)_{10} = (?)_8$
    3. $(420.6)_{10} = (?)_8$
    4. $(8476.47)_{10} = (?)_8$

Conversion from Decimal Number to Hex Number



Conversion of Decimal Number into Hexadecimal Number (Integer Number)

Procedure:

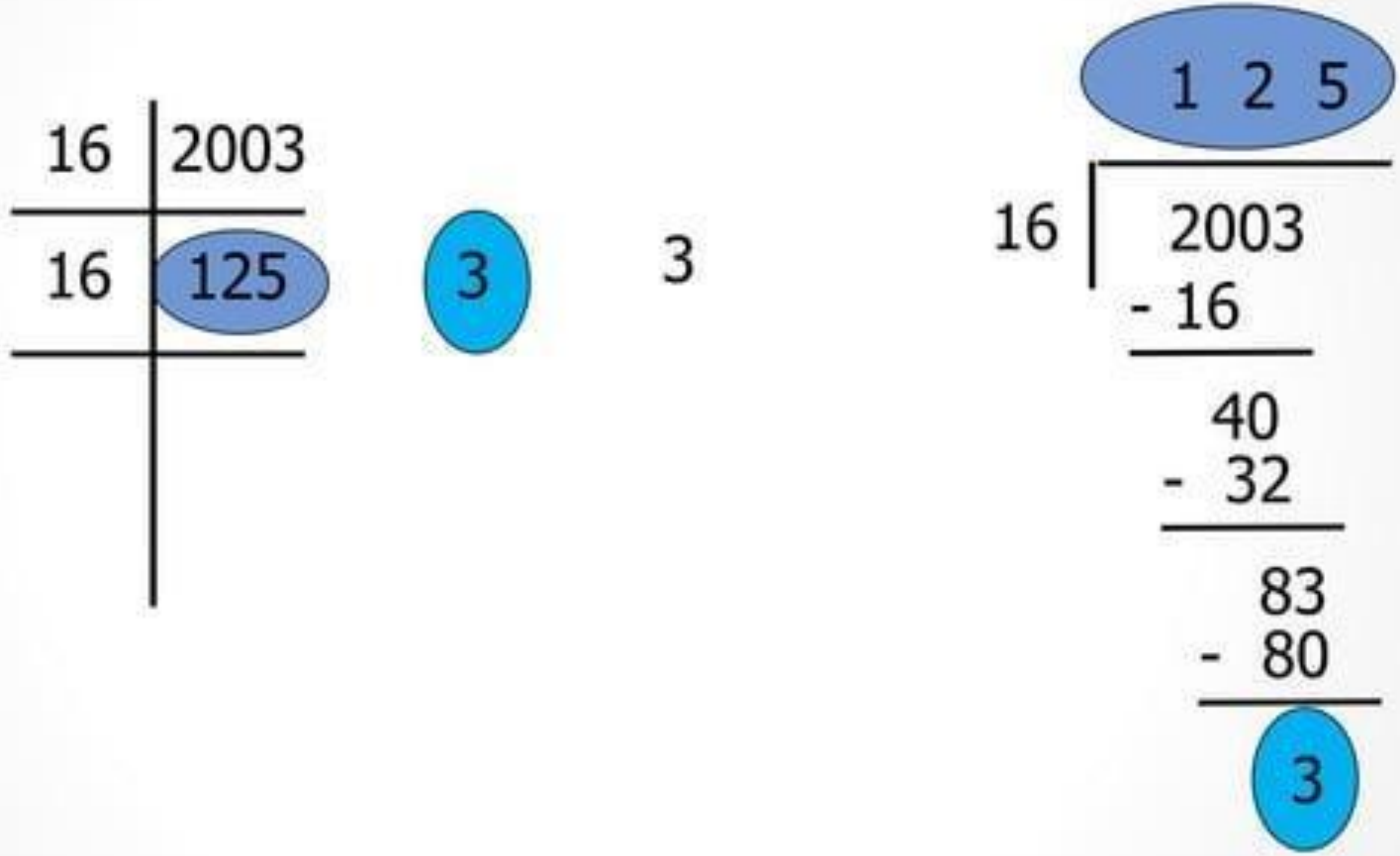
1. Divide the decimal no by the base 16, noting the remainder.
2. Continue to divide the quotient by 16 until there is nothing left, keeping the track of the remainders from each step.
3. List the remainder values in reverse order to find the number's hex equivalent

Example: Convert 2003 decimal number in to it’s equivalent Hex number.

Example: Convert 2003 decimal number in to it's equivalent Hex number.

16	2003

Example: Convert 2003 decimal number in to it's equivalent Hex number.



Example: Convert 2003 decimal number in to it's equivalent Hex number.

16	2003		
16	125	3	3
16	7	13	D

7

16 | 125

- 112

13

Example: Convert 2003 decimal number in to it’s equivalent Hex number.

16	2003		
16	125	3	3
16	7	13	D
	0	7	7

Example: Convert 2003 decimal number in to it's equivalent Hex number.

16	2003			
16	125	3	3	LSB
16	7	13	D	↑
	0	7	7	MSB

$$(2003)_{10} = (7D3)_{16}$$

Conversion of Decimal Number into Hexadecimal Number (Fractional Number)

Procedure:

1. Multiply the given fractional number by base 16.
2. Record the carry generated in this multiplication as MSB.
3. Multiply only the fractional number of the product in step 2 by 16 and record the carry as the next bit to MSB.
4. Repeat the steps 2 and 3 up to 5 bits. The last carry will represent the LSB of equivalent hex number

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

0.122 X 16 = 1.952   1   1

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

0.122 X 16	=	1.952		1	1
0.952 X 16	=	15.232		15	F

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

0.122 X 16	=	1.952		1	1
0.952 X 16	=	15.232		15	F
0.232 X 16	=	3.712		3	3

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

0.122 X 16	= 1.952	1	1
0.952 X 16	= 15.232	15	F
0.232 X 16	= 3.712	3	3
0.712 X 16	= 11.392	11	B

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

0.122 X 16	= 1.952	1	1
0.952 X 16	= 15.232	15	F
0.232 X 16	= 3.712	3	3
0.712 X 16	= 11.392	11	B
0.392 X 16	= 6.272	6	6

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

0.122 X 16 =	1.952	1	1	MSB
0.952 X 16 =	15.232	15	F	
0.232 X 16 =	3.712	3	3	
0.712 X 16 =	11.392	11	B	
0.392 X 16 =	6.272	6	6	
				LSB

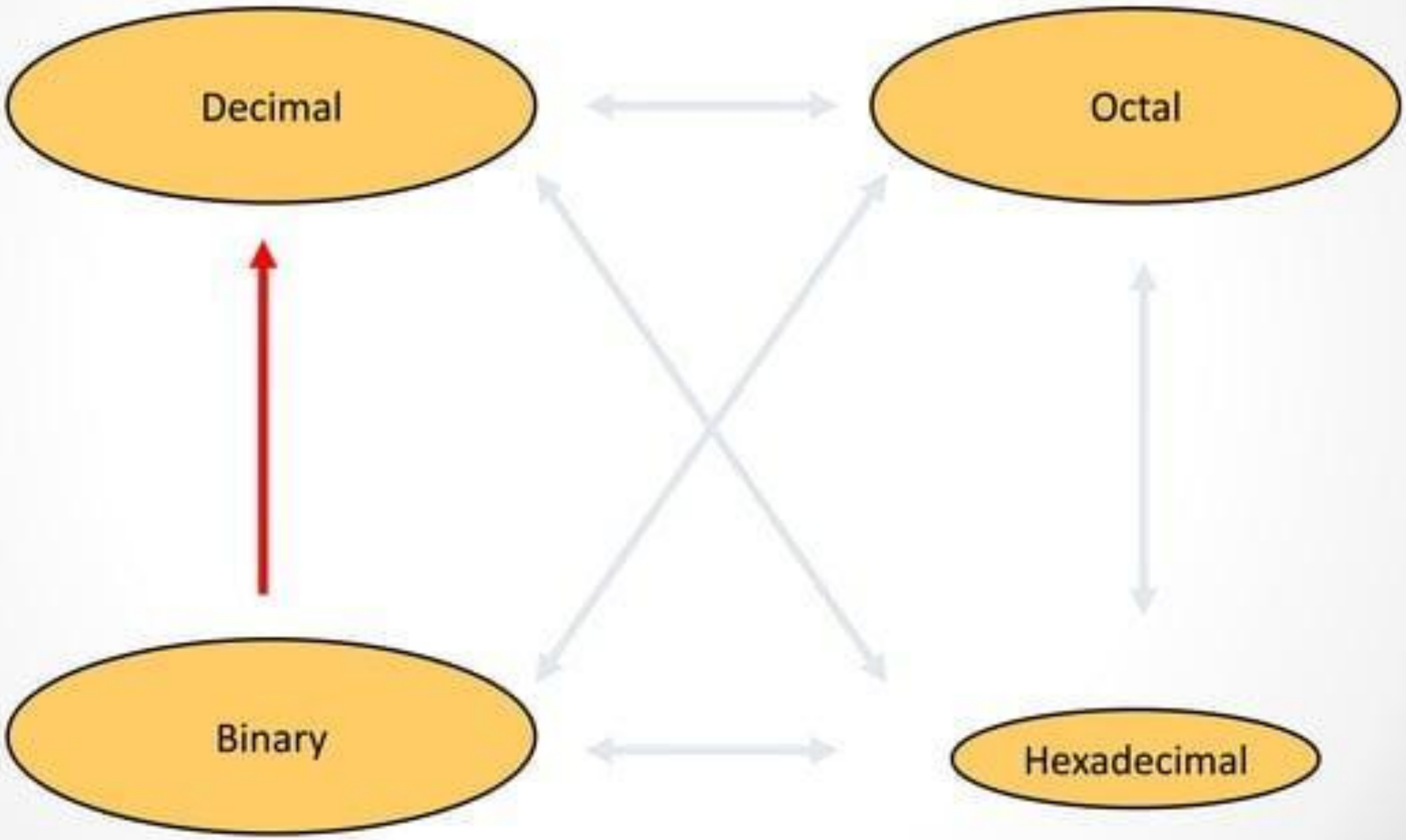
$$(0.122)_{10} = (0.1F3B6)_{16}$$

Exercise

- Convert following Decimal Numbers in to its equivalent Hex Number:

    1. $(1248.56)_{10} = (?)_{16}$
    2. $(8957.75)_{10} = (?)_{16}$
    3. $(420.6)_{10} = (?)_{16}$
    4. $(8476.47)_{10} = (?)_{16}$

Conversion from Binary Number to Decimal Number



Conversion of Binary Number into Decimal Number

Procedure:

1. Write down the binary number.
2. Write down the weights for different positions.
3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.
4. Add all the product numbers to get the decimal equivalent

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Binary No.	1	0	1	1	·	0	1

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Binary No.	1	0	1	1	.	0	1
Positional Weights	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Binary No.	1	0	1	1	.	0	1
Positional Weights	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$=(1\times 2^3)+(0\times 2^2)+(1\times 2^1)+(1\times 2^0).(0\times 2^{-1})+(1\times 2^{-2})$$

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Binary No.	1	0	1	1	·	0	1
	↑	↑	↑	↑		↑	↑
Positional Weights	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$=(1\times 2^3)+(0\times 2^2)+(1\times 2^1)+(1\times 2^0).(0\times 2^{-1})+(1\times 2^{-2})$$

$$= 8 + 0 + 2 + 1 . 0 + 0.25$$

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Binary No.	1	0	1	1	.	0	1
Positional Weights	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$=(1\times 2^3)+(0\times 2^2)+(1\times 2^1)+(1\times 2^0).(0\times 2^{-1})+(1\times 2^{-2})$$

$$= \ 8 \ + \ 0 \ + \ 2 \ + \ 1 \ . \ 0 \ + \ 0.25$$

$$= 11.25$$

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Binary No.	1	0	1	1	.	0	1
Positional Weights	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$=(1\times2^3)+(0\times2^2)+(1\times2^1)+(1\times2^0).(0\times2^{-1})+(1\times2^{-2})$$

$$= 8 + 0 + 2 + 1 . 0 + 0.25$$

$$= 11.25$$

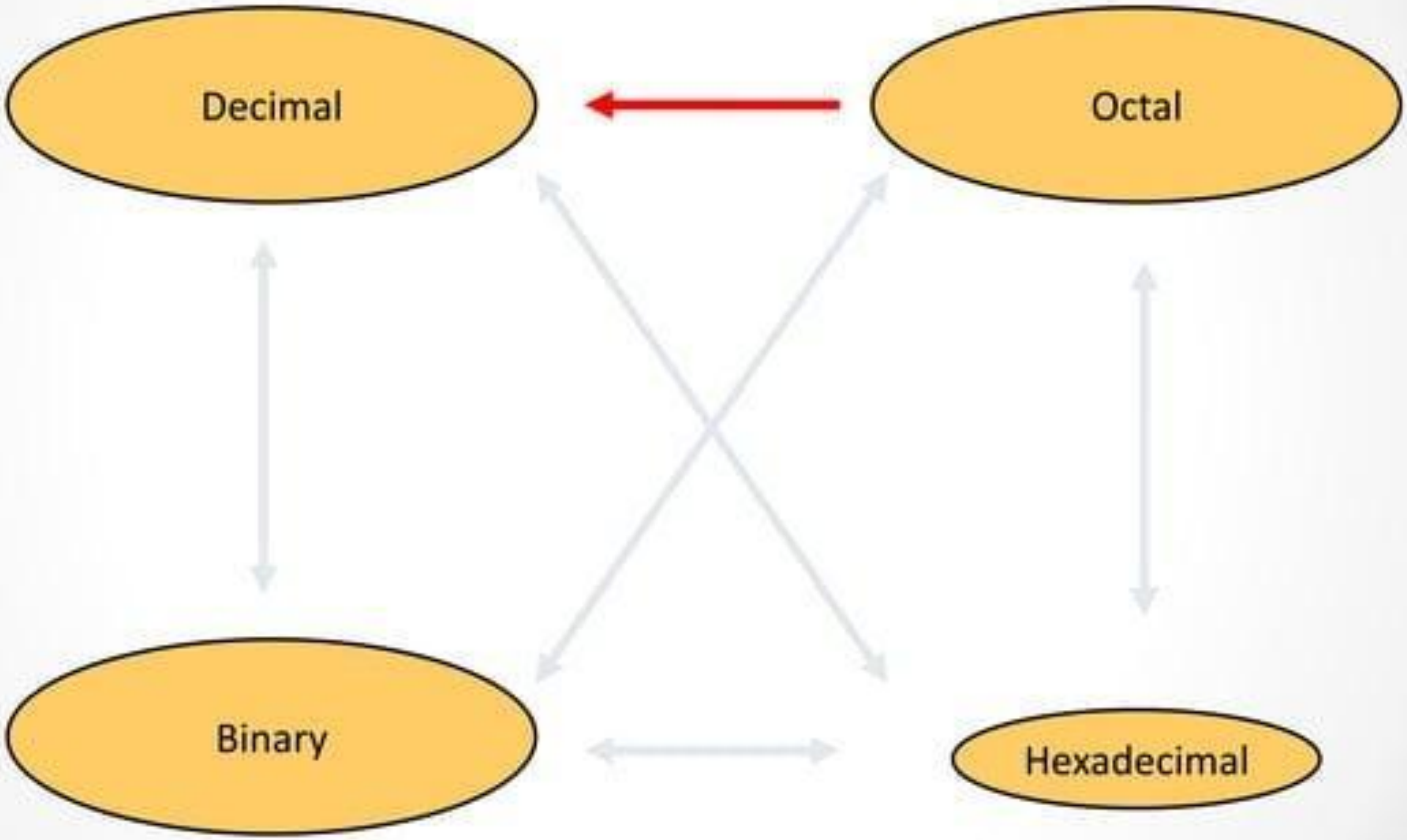
$$(1011.01)_2 = (11.25)_{10}$$

Exercise

- Convert following Binary Numbers in to its equivalent Decimal Number:

1. $(1101110.011)_2 = (\ ?\ )_{10}$
2. $(1101.11)_2 = (\ ?\ )_{10}$
3. $(10001.01)_2 = (\ ?\ )_{10}$

Conversion from Octal Number to Decimal Number



Conversion of Octal Number into Decimal Number

Procedure:

1. Write down the octal number.
2. Write down the weights for different positions.
3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.
4. Add all the product numbers to get the decimal equivalent

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Octal No.	3	6	5	·	2	4

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Octal No.	3	6	5	.	2	4
	↑	↑	↑		↑	↑
Positional Weights	8^2	8^1	8^0		8^{-1}	8^{-2}

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Octal No.	3	6	5	·	2	4
	↑	↑	↑		↑	↑
Positional Weights	8^2	8^1	8^0		8^{-1}	8^{-2}

$$=(3\times8^2)+(6\times8^1)+(5\times8^0).(2\times8^{-1})+(4\times8^{-2})$$

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Octal No.	3	6	5	·	2	4
	↑	↑	↑		↑	↑
Positional Weights	8^2	8^1	8^0		8^{-1}	8^{-2}

$$=(3\times 8^2)+(6\times 8^1)+(5\times 8^0).(2\times 8^{-1})+(4\times 8^{-2})$$

$$= 192 \quad + \quad 48 \quad + \quad 5 \quad . \quad 0.25 \quad +0.0625$$

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Octal No.	3	6	5	.	2	4
	↑	↑	↑		↑	↑
Positional Weights	8^2	8^1	8^0		8^{-1}	8^{-2}

$$=(3\times8^2)+(6\times8^1)+(5\times8^0).(2\times8^{-1})+(4\times8^{-2})$$

$$= 192 + 48 + 5 \;.\; 0.25 + 0.0625$$

$$= 245.3125$$

Example: Convert 365.24 octal number in to it's equivalent decimal number.

Octal No.	3	6	5	·	2	4
	↑	↑	↑		↑	↑
Positional Weights	8^2	8^1	8^0		8^{-1}	8^{-2}

$$=(3\times 8^2)+(6\times 8^1)+(5\times 8^0).(2\times 8^{-1})+(4\times 8^{-2})$$

$$= 192 \quad + \quad 48 \quad + \quad 5 \quad . \quad 0.25 \quad +0.0625$$

$$= 245.3125$$

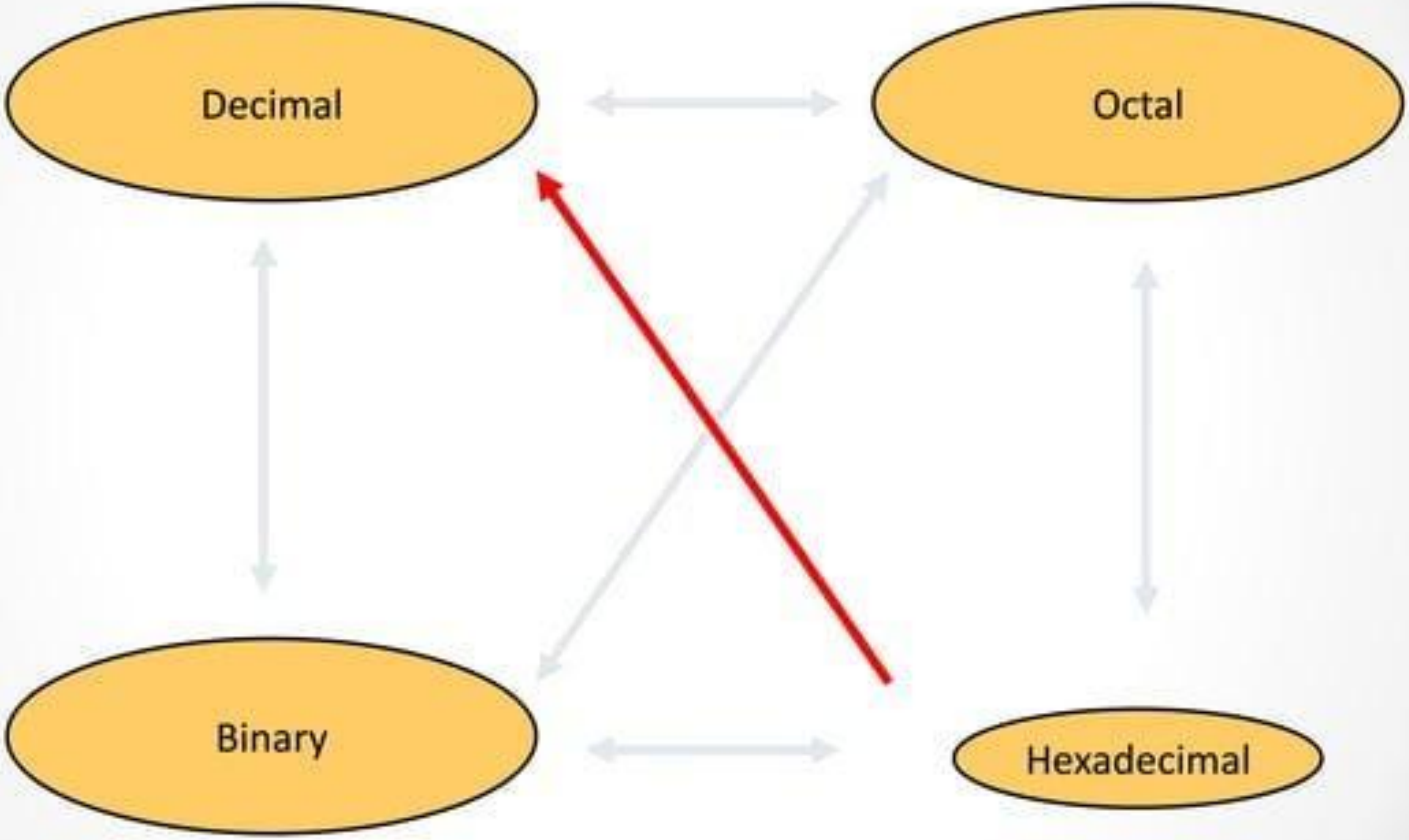
$$\boxed{(365.24)_8 = (245.3125)_{10}}$$

Exercise

- Convert following Octal Numbers in to its equivalent Decimal Number:

  1. $(3006.05)_8 = (?)_{10}$
  2. $(273.56)_8 = (?)_{10}$
  3. $(6534.04)_8 = (?)_{10}$

Conversion from Hex Number to Decimal Number



Conversion of Hexadecimal Number into Decimal Number

Procedure:

1. Write down the hex number.
2. Write down the weights for different positions.
3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.
4. Add all the product numbers to get the decimal equivalent

Example: Convert 5826 hex number in to it's equivalent decimal number.

Example: Convert 5826 hex number in to it's equivalent decimal number.

Hex No.	5	8	2	6

Example: Convert 5826 hex number in to it's equivalent decimal number.

Hex No.	5	8	2	6
	↑	↑	↑	↑
Positional Weights	16^3	16^2	16^1	16^0

Example: Convert 5826 hex number in to it's equivalent decimal number.

Hex No.	5	8	2	6
	↑	↑	↑	↑
Positional Weights	16^3	16^2	16^1	16^0

$$= (5 \times 16^3) + (8 \times 16^2) + (2 \times 16^1) + (6 \times 16^0)$$

Example: Convert 5826 hex number in to it's equivalent decimal number.

Hex No.	5	8	2	6
	↑	↑	↑	↑
Positional Weights	16^3	16^2	16^1	16^0

$$=(5\times16^3)+(8\times16^2)+(2\times16^1)+(6\times16^0)$$

$$=20480 \;+\; 2048 \;+\; 32 \;+\; 6$$

Example: Convert 5826 hex number in to it's equivalent decimal number.

Hex No.	5	8	2	6
	↑	↑	↑	↑
Positional Weights	16^3	16^2	16^1	16^0

$$=(5\times16^3)+(8\times16^2)+(2\times16^1)+(6\times16^0)$$

=20480 + 2048 + 32 + 6

= 22566

Example: Convert 5826 hex number in to it's equivalent decimal number.

Hex No.	5	8	2	6
Positional Weights	16^3	16^2	16^1	16^0

$$=(5\times16^3)+(8\times16^2)+(2\times16^1)+(6\times16^0)$$

=20480 + 2048 + 32 + 6

= 22566

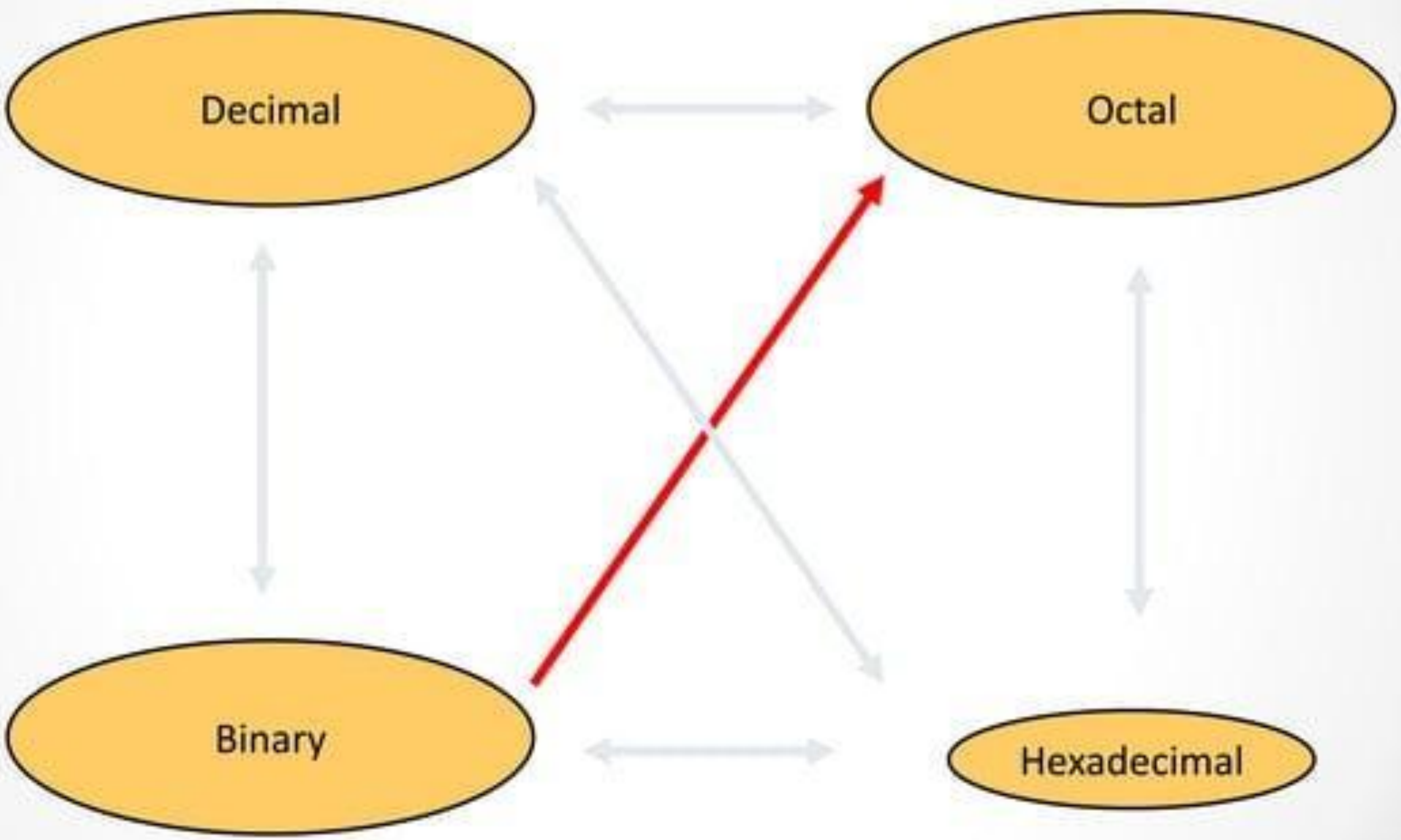
$$(5826)_{16} = (22566)_{10}$$

Exercise

- Convert following Hexadecimal Numbers in to its equivalent Decimal Number:

    1. $(4056)_{16} = (\ ?\ )_{10}$
    2. $(6B7)_{16} = (\ ?\ )_{10}$
    3. $(8E47.AB)_{16} = (\ ?\ )_{10}$

Conversion from Binary Number to Octal Number



Conversion of Binary Number into Octal Number

Procedure:

1. Group the binary bits into groups of 3 starting from LSB.
2. Convert each group into its equivalent decimal. As the number of bits in each group is restricted to 3, the decimal number will be same as octal number

Example: Convert 11010010 binary number in to it's equivalent octal number.

Example: Convert 11010010 binary number in to it’s equivalent octal number.

0 1 1 0 1 0 0 1 0

Example: Convert 11010010 binary number in to it's equivalent octal number.

LSB

0 1 1 0 1 0 0 1 0

Example: Convert 11010010 binary number in to it's equivalent octal number.

0 1 1 0 1 0 0 1 0

3 2 2

Example: Convert 11010010 binary number in to it's equivalent octal number.

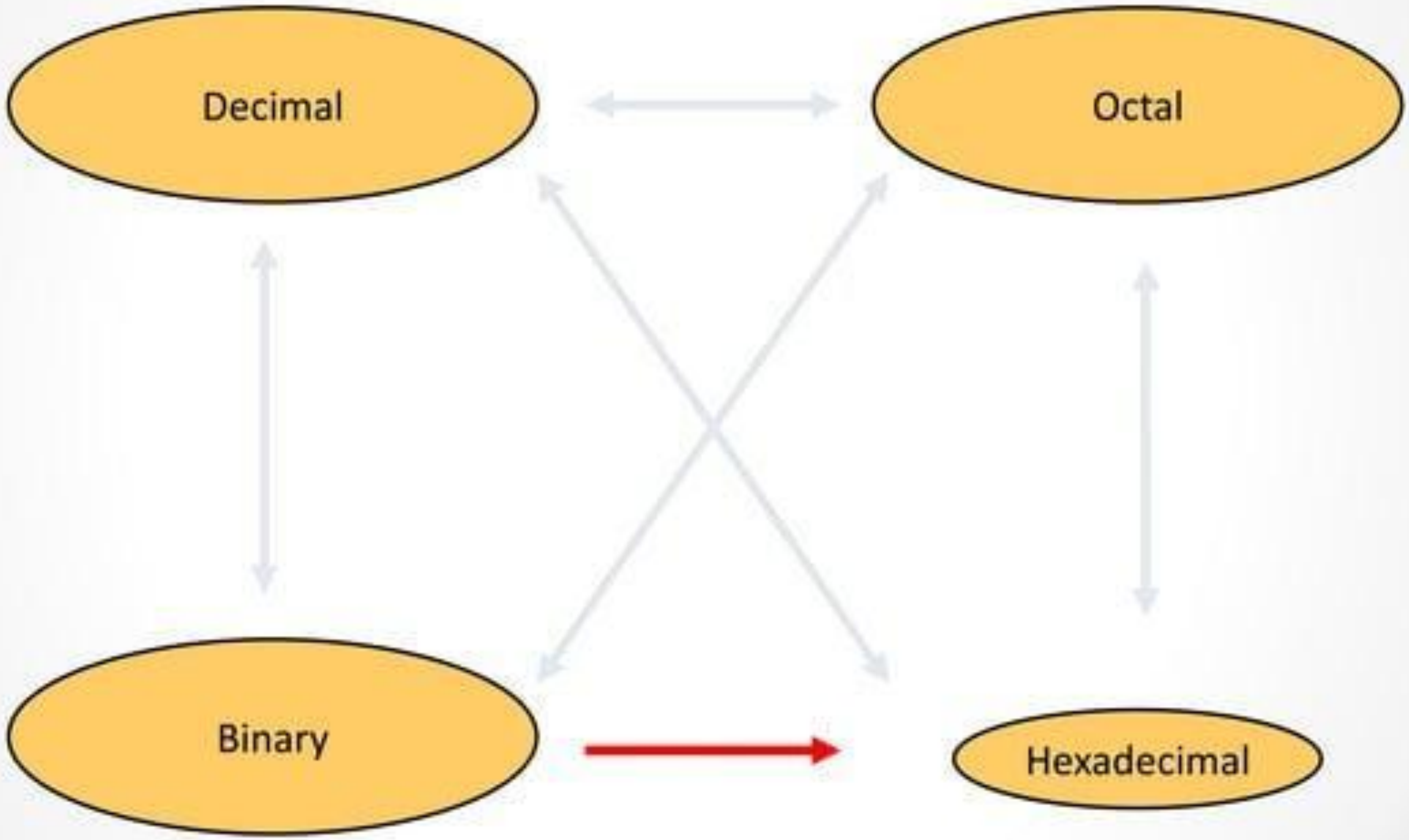
0 1 1	0 1 0	0 1 0
3	2	2

$$(11010010)_2 = (322)_8$$

Exercise

- Convert following Binary Numbers in to its equivalent Octal Number:
    1. $(1101110.011)_2 = (\ ?\ )_8$
    2. $(1101.11)_2 = (\ ?\ )_8$
    3. $(10001.01)_2 = (\ ?\ )_8$

Conversion from Binary Number to Hexadecimal Number



Conversion of Binary Number to Hexadecimal Number

Procedure:

1. Group the binary bits into groups of 4 starting from LSB.
2. Convert each group into its equivalent decimal. As the number of bits in each group is restricted to 4, the decimal number will be same as hex number

Example: Convert 11010010 binary number in to it's equivalent hex number.

Example: Convert 11010010 binary number in to it's equivalent hex number.

LSB

1 1 0 1 0 0 1 0

Example: Convert 11010010 binary number in to it's equivalent hex number.

LSB

1 1 0 1 0 0 1 0

Example: Convert 11010010 binary number in to it's equivalent hex number.

1 1 0 1 0 0 1 0

D 2

Example: Convert 11010010 binary number in to it’s equivalent hex number.

1 1 0 1 0 0 1 0

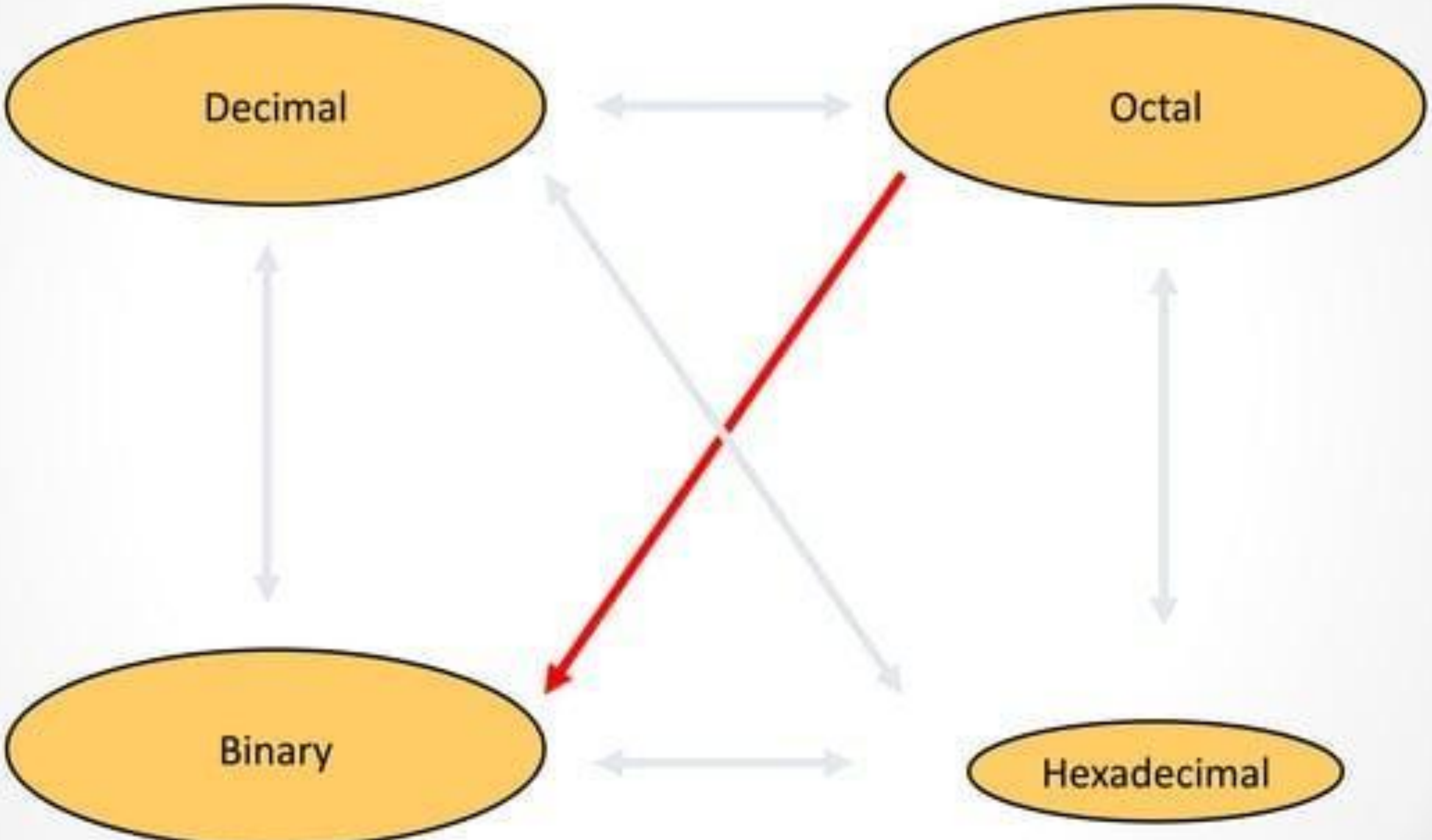
D 2

$$(11010010)_2 = (D2)_{16}$$

Exercise

- Convert following Binary Numbers in to its equivalent Hexadecimal Number:
    1. $(1101110.011)_2 = (\ ?\ )_{16}$
    2. $(1101.11)_2 = (\ ?\ )_{16}$
    3. $(10001.01)_2 = (\ ?\ )_{16}$

Conversion from Octal Number to Binary Number



Conversion of Octal Number into Binary Number

- ✓ To get the binary equivalent of the given octal number we have to convert each octal digit into its equivalent 3 bit binary number

Example: Convert 364 octal number in to it's equivalent binary number.

Example: Convert 364 octal number in to it's equivalent binary number.

3 6 4

Example: Convert 364 octal number in to it's equivalent binary number.

$$\begin{array}{ccc} 3 & 6 & 4 \\ \downarrow & \downarrow & \downarrow \\ 011 & 110 & 100 \end{array}$$

Example: Convert 364 octal number in to it's equivalent binary number.

3	6	4
↓	↓	↓
011	110	100

$$(364)_8 = (011110100)_2$$

Example: Convert 364 octal number in to it's equivalent binary number.

3	6	4
↓	↓	↓
011	110	100

$$(364)_8 = (011110100)_2$$

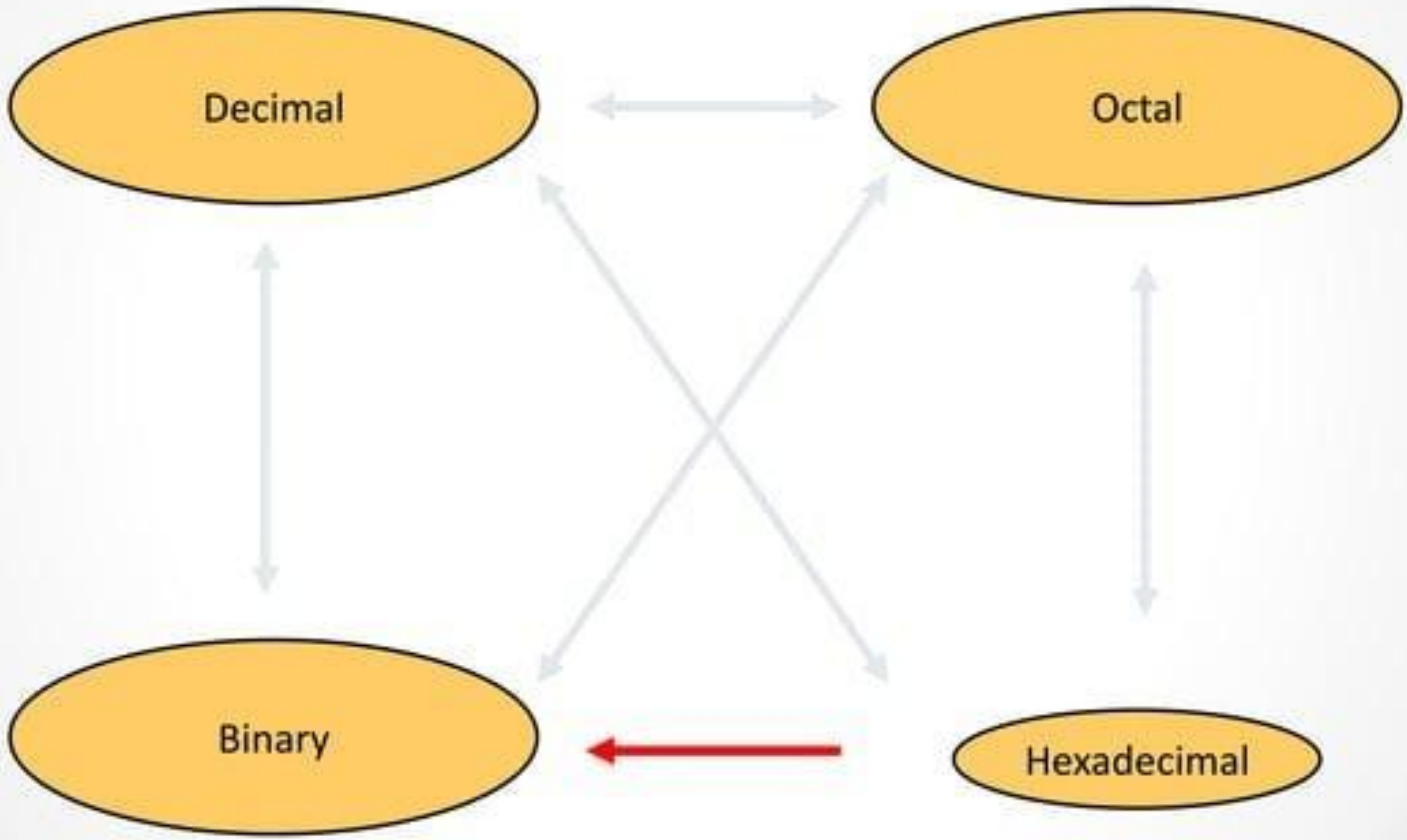
OR

$$\boxed{(364)_8 = (11110100)_2}$$

Exercise

- Convert following Octal Numbers in to its equivalent Binary Number:
    1. $(3006.05)_8 = (?)_2$
    2. $(273.56)_8 = (?)_2$
    3. $(6534.04)_8 = (?)_2$

Conversion from Hex Number to Binary Number



Conversion of Hexadecimal Number into Binary Number

- ✓ To get the binary equivalent of the given hex number we have to convert each hex digit into its equivalent 4 bit binary number

Example: Convert AFB2 hex number in to it's equivalent binary number.

Example: Convert AFB2 hex number in to it's equivalent binary number.

A	F	B	2

Example: Convert AFB2 hex number in to it's equivalent binary number.

A	F	B	2
↓	↓	↓	↓
1010	1111	1011	0010

Example: Convert AFB2 hex number in to it's equivalent binary number.

A	F	B	2
↓	↓	↓	↓
1010	1111	1011	0010

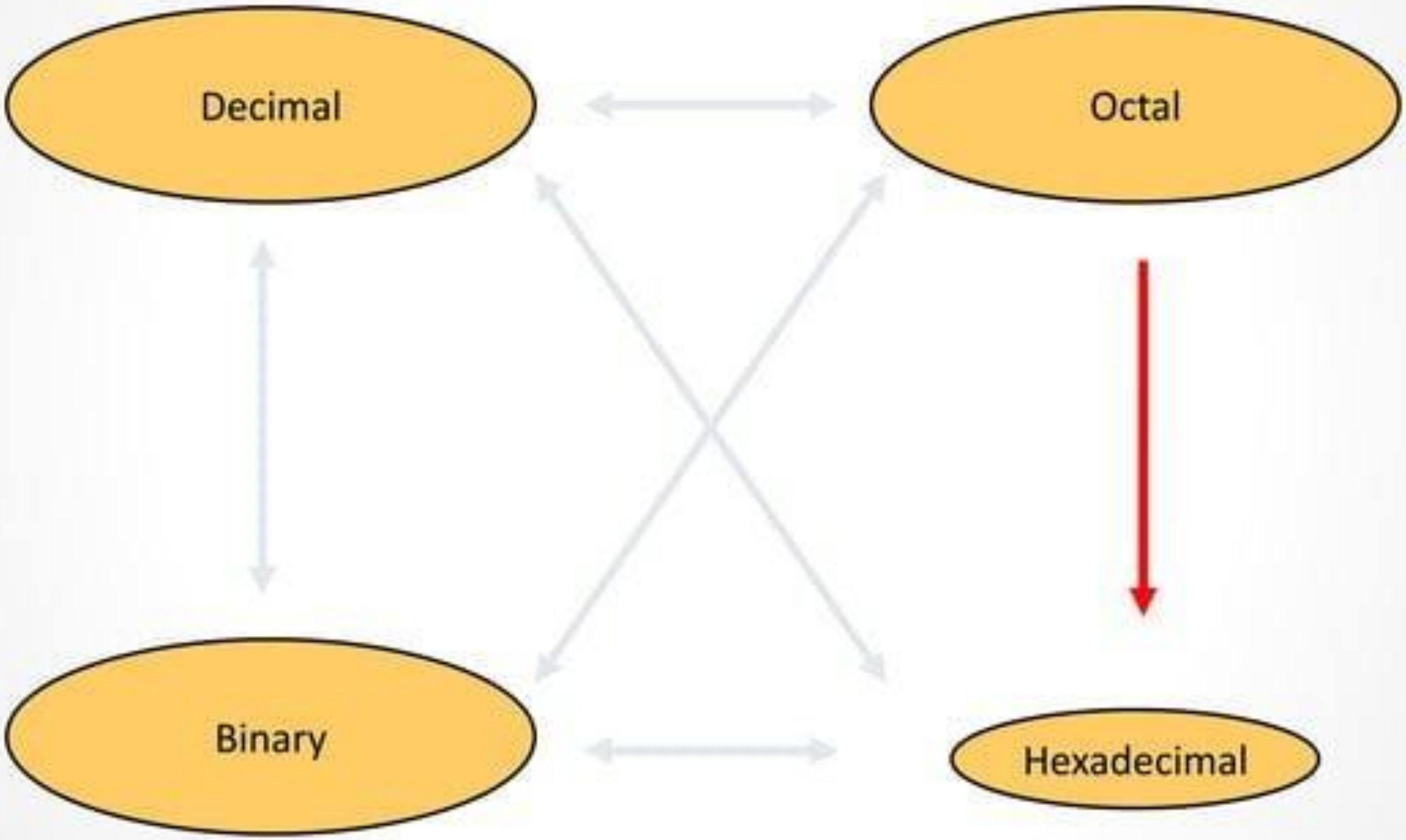
$$(AFB2)_{16} = (1010111110110010)_2$$

Exercise

- Convert following Hexadecimal Numbers in to its equivalent Binary Number:

    1. $(4056)_{16} = (\ ?\ )_2$
    2. $(6B7)_{16} = (\ ?\ )_2$
    3. $(8E47.AB)_{16} = (\ ?\ )_2$

Conversion from Octal Number to Hex Number



Conversion of Octal Number into Hexadecimal Number

✓ To get hex equivalent number of given octal number, first we have to convert octal number into its 3 bit binary equivalent and then convert binary number into its hex equivalent.

Example: Convert 364 octal number in to it’s equivalent hex number.

Example: Convert 364 octal number in to it's equivalent hex number.

3	6	4	Octal Number

Example: Convert 364 octal number in to it's equivalent hex number.

3	6	4	Octal Number
↓	↓	↓	
011	110	100	Binary Number

Example: Convert 364 octal number in to it's equivalent hex number.

3	6	4	Octal Number
↓	↓	↓	
011	110	100	Binary Number

011110100 Binary Number

Example: Convert 364 octal number in to it's equivalent hex number.

3	6	4	Octal Number
↓	↓	↓	
011	110	100	Binary Number

0	1111	0100	
011110100			Binary Number
↓	↓	↓	
0	F	4	Hex Number

Example: Convert 364 octal number in to it's equivalent hex number.

3	6	4	Octal Number
011	110	100	Binary Number

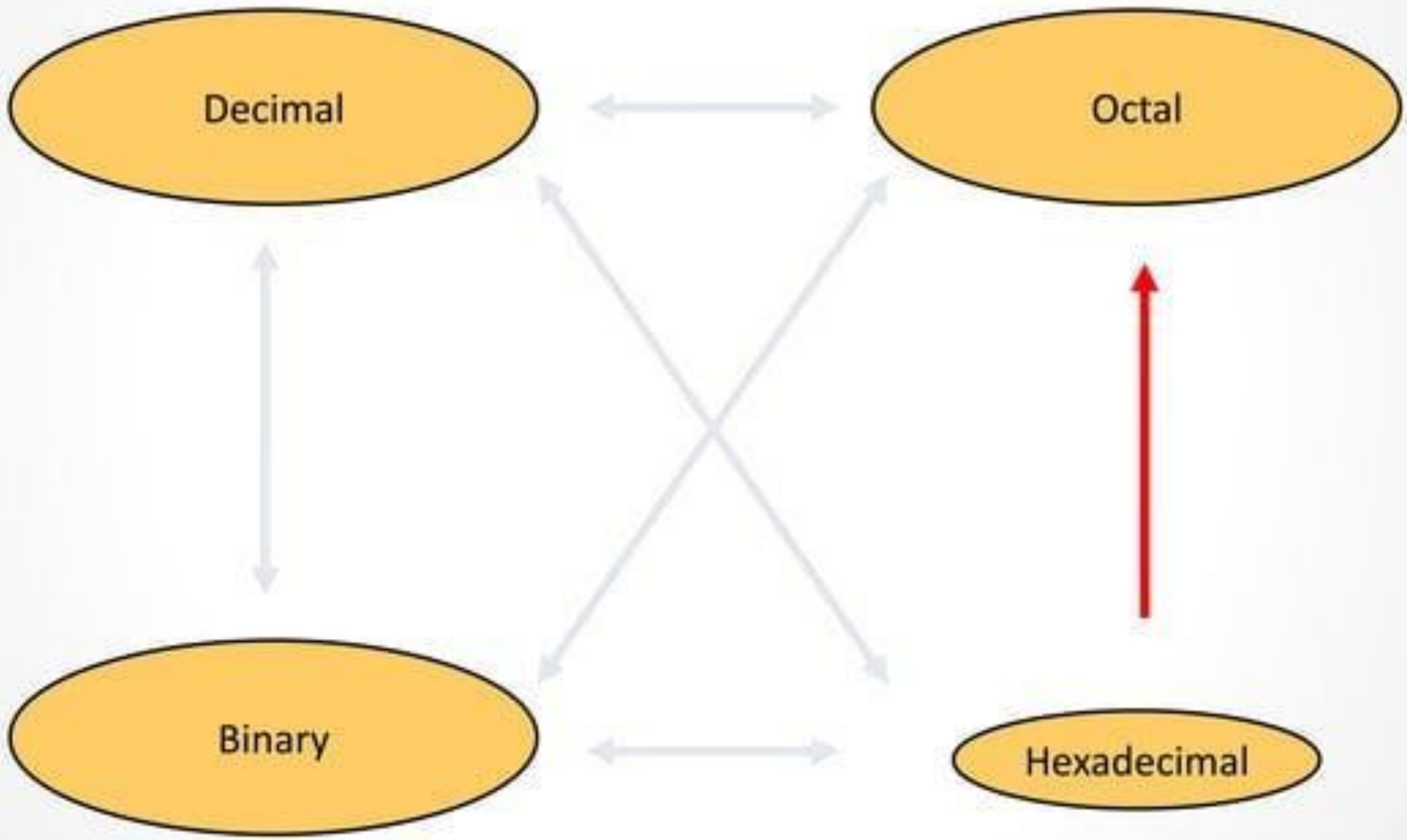
011110100	Binary Number
0 F 4	Hex Number

$$(364)_8 = (F4)_{16}$$

Exercise

- Convert following Octal Numbers in to its equivalent Hex Number:
  1. $(3006.05)_8 = (?)_{16}$
  2. $(273.56)_8 = (?)_{16}$
  3. $(6534.04)_8 = (?)_{16}$

Conversion from Hex Number to Octal Number



Conversion of Hexadecimal Number into Octal Number

✓ To get octal equivalent number of given hex number, first we have to convert hex number into its 4 bit binary equivalent and then convert binary number into its octal equivalent.

Example: Convert 4CA hex number in to it's equivalent octal number.

Example: Convert 4CA hex number in to it's equivalent octal number.

4	C	A	Hex Number

Example: Convert 4CA hex number in to it's equivalent octal number.

4	C	A	Hex Number
↓	↓	↓	
0100	1100	1010	Binary Number

Example: Convert 4CA hex number in to it's equivalent octal number.

4	C	A	Hex Number
↓	↓	↓	
0100	1100	1010	Binary Number

010011001010 Binary Number

Example: Convert 4CA hex number in to it's equivalent octal number.

4	C	A	Hex Number
↓	↓	↓	
0100	1100	1010	Binary Number

010011001010 Binary Number

2 3 1 2 Octal Number

Example: Convert 4CA hex number in to it's equivalent octal number.

4	C	A	Hex Number
0100	1100	1010	Binary Number

010011001010 Binary Number

2 3 1 2 Octal Number

$$(4CA)_{16} = (2312)_8$$

Exercise

- Convert following Hexadecimal Numbers in to its equivalent Octal Number:
    1. $(4056)_{16} = (\ ?\ )_8$
    2. $(6B7)_{16} = (\ ?\ )_8$
    3. $(8E47.AB)_{16} = (\ ?\ )_8$

Chapter I – Number System

- **Introduction to digital signal, Advantages of Digital System over analog systems)**Number Systems: Different types of number systems(Binary, Octal, Hexadecimal), Conversion of number systems,
  - ✓ **Binary arithmetic: Addition, Subtraction, Multiplication, Division.**
  - ✓ Subtraction using 1's complement and 2's complement
- **Codes**
  - ✓ Codes -BCD, Gray Code, Excess-3, ASCII code
  - ✓ BCD addition, BCD subtraction using 9's and 10' complement
- **(Numericals based on above topic)**.

Binary Addition

➢ Following are the four most basic cases for binary addition

0	+	0	=	0
0	+	1	=	1
1	+	0	=	1
1	+	1	=	10 i.e. 0 with carry 1

Binary Addition

Example: Perform $(10111)_2 + (11001)_2$

Binary Addition

Example: Perform $(10111)_2 + (11001)_2$

$$
\begin{array}{cccccc}
 & & & & \color{red}{1} & \\
 & 1 & 0 & 1 & 1 & 1 \\
+ & 1 & 1 & 0 & 0 & 1 \\
\hline
 & & & & & 0
\end{array}
$$

Binary Addition

Example: Perform $(10111)_2 + (11001)_2$

$$
\begin{array}{cccccc}
 & & & 1 & 1 & \\
 & 1 & 0 & 1 & 1 & 1 \\
+ & 1 & 1 & 0 & 0 & 1 \\
\hline
 & & & & 0 & 0
\end{array}
$$

Binary Addition

Example: Perform $(10111)_2 + (11001)_2$

$$\begin{array}{cccccc} & & 1 & 1 & & \\ & 1 & 0 & 1 & 1 & 1 \\ + & 1 & 1 & 0 & 0 & 1 \\ \hline & & & 0 & 0 & 0 \end{array}$$

Binary Addition

Example: Perform $(10111)_2 + (11001)_2$

$$
\begin{array}{cccccc}
 & 1 & 1 & & & \\
 & 1 & 0 & 1 & 1 & 1 \\
+ & 1 & 1 & 0 & 0 & 1 \\
\hline
 & & 0 & 0 & 0 & 0
\end{array}
$$

Binary Addition

Example: Perform $(10111)_2 + (11001)_2$

$$
\begin{array}{cccccc}
 & \color{red}{1} & & & & \\
 & 1 & 0 & 1 & 1 & 1 \\
+ & 1 & 1 & 0 & 0 & 1 \\
\hline
1 & 1 & 0 & 0 & 0 & 0
\end{array}
$$

Binary Addition

Example: Perform $(10111)_2 + (11001)_2$

		1	1	1	1	
		1	0	1	1	1
+		1	1	0	0	1
	1	1	0	0	0	0

$$(10111)_2 + (11001)_2 = (110000)_2$$

Binary Addition

Example: Perform $(1101.101)_2 + (111.011)_2$

Binary Addition

Example: Perform $(1101.101)_2 + (111.011)_2$

$$\begin{array}{cccccccccc}
 & & 1 & 1 & 1 & 1 & & 1 & 1 & \\
 & & 1 & 1 & 0 & 1 & . & 1 & 0 & 1 \\
+ & & & 1 & 1 & 1 & . & 0 & 1 & 1 \\
\hline
 & 1 & 0 & 1 & 0 & 1 & . & 0 & 0 & 0
\end{array}$$

$$(1101.101)_2 + (111.011)_2 = (10101.000)_2$$

Exercise

- Perform Binary Addition of following:

1. $(11011)_2+(1101)_2$
2. $(1011)_2+(1101)_2+(1001)_2+(1111)_2$
3. $(1010.11)_2+(1101.10)_2+(1001.11)_2+(1111.11)_2$
4. $(10111.101)_2+(110111.01)_2$

Binary Subtraction

- Following are the four most basic cases for binary subtraction

	Subtraction	Borrow
0 - 0 =	0	0
0 - 1 =	1	1
1 - 0 =	1	0
1 - 1 =	0	0

Binary Subtraction

Example: Perform $(1010.010)_2 - (111.111)_2$

Binary Subtraction

Example: Perform $(1010.010)_2 - (111.111)_2$

$$
\begin{array}{cccccccccc}
 & & 1 & & 1 & & 1 & 10 & \\
 & 0 & 10 & 0 & 10 & & 10 & 0 & 10 \\
 & 1 & 0 & 1 & 0 & . & 0 & 1 & 0 \\
- & & 1 & 1 & 1 & . & 1 & 1 & 1 \\
\hline
 & 0 & 0 & 1 & 0 & . & 0 & 1 & 1
\end{array}
$$

$$(1010.010)_2 + (111.111)_2 = (0010.011)_2$$

Exercise

- Perform Binary Subtraction of following:

    1. $(1011)_2 - (101)_2$
    2. $(1100.10)_2 - (111.01)_2$
    3. $(10110)_2 - (1011)_2$
    4. $(10001.01)_2 - (1111.11)_2$

Binary Multiplication

- Following are the four most basic cases for binary multiplication

0	X	0	=	0
0	X	1	=	0
1	X	0	=	0
1	X	1	=	1

Binary Multiplication

Example: Perform $(1001)_2 + (1000)_2$

Binary Multiplication

Example: Perform $(1001)_2 + (1000)_2$

				1	0	0	1
×				1	0	0	0
				0	0	0	0
			0	0	0	0	x
+		0	0	0	0	x	x
	1	0	0	1	x	x	x
	1	0	0	1	0	0	0

$$(1001)_2 + (1000)_2 = (1001000)_2$$

Exercise

- Perform Binary Multiplication of following:

1. $(1101)_2 \text{ X } (101)_2$
2. $(1101.11)_2 \text{ X } (101.1)_2$
3. $(11001)_2 \text{ X } (10)_2$
4. $(10110)_2 \text{ X } (10.1)_2$

Binary Division

Example: Perform $(110110)_2 / (101)_2$

Binary Division

Example: Perform $(110110)_2 / (101)_2$

```
        1 0 1 0
      ___________
101  |  1 1 0 1 1 0
       -1 0 1
       -------
        0 0 1 1
           -0 0
           -------
            1 1 1
           -1 0 1
           -------
            0 1 0 0
             -0 0 0
             -------
              1 0 0
```

Exercise

- Perform Binary Division of following:

1. $(1010)_2$ by $(11)_2$
2. $(11110)_2$ by $(101)_2$
3. $(11011)_2$ by $(10.1)_2$
4. $(110111.1)_2$ by $(101)_2$

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- (Numericals based on above topic).

1’s Complement

- The 1’s complement of a number is obtained by simply complementing each bit of the number that is by changing all 0’s to 1’s and all 1’s to 0’s.
- This system is called as 1’s complement because the number can be subtracted from 1 to obtain result

1's Complement

Example: Obtain 1's complement of the 1010

$$
\begin{array}{r}
1\ 1\ 1\ 1 \\
-\ \ 1\ 0\ 1\ 0 \\
\hline
0\ 1\ 0\ 1
\end{array}
$$

1's complement of the 1010 is 0101

1's Complement

Sr. No.	Binary Number	1's Complement
1	1101 0101	0010 1010
2	1001	0110
3	1011 1111	0100 0000
4	1101 1010 0001	0010 0101 1110
5	1110 0111 0101	0001 1000 1010
6	1011 0100 1001	0100 1011 0110
7	1100 0011 0010	0011 1100 1101
8	0001 0010 1000	1110 1101 0111

Subtraction Using 1's Complement

- In 1's complement subtraction, add the 1's complement of subtrahend to the minuend.
- If there is carry out, bring the carry around and add it to LSB.
- Look at the sing bit (MSB), if this is 0, the result is positive and is in its true binary form.
- If the MSB is 1(whether there is a carry or no carry at all), the result is negative & is in its 1's complement form. So take 1's complement to obtain result.

Subtraction using 1's Complement

Example: Perform using 1′ complement $(9)_{10} - (4)_{10}$

Subtraction using 1's Complement

Example: Perform using 1' complement $(9)_{10} - (4)_{10}$

Step 1: Take 1' complement of $(4)_{10} = (0100)_2$

$$= 1011$$

Step 2: Add 9 with 1' complement of 4

$$
\begin{array}{cccccc}
 & & 1 & 0 & 0 & 1 \\
+ & & 1 & 0 & 1 & 1 \\
\hline
\text{final carry} \rightarrow & \boxed{1} & 0 & 1 & 0 & 0
\end{array}
$$

$\underline{0\ 1\ 0\ 0}$ Result

Step 3:If carry is generated add final carry to the result

Example Continue

$$\begin{array}{rccccc} & & 1 & 0 & 0 & 1 \\ + & & 1 & 0 & 1 & 1 \\ \hline & 1 & 0 & 1 & 0 & 0 \\ & & & & & 1 \\ \hline & & 0 & 1 & 0 & 1 \end{array}$$

final carry

Result

Final Result

When the final carry is produced the answer is positive and is in its true binary form

Exercise

- Perform Binary Subtraction using 1's Complement method
  1. $(52)_{10} - (17)_{10}$
  2. $(46)_{10} - (84)_{10}$
  3. $(63.75)_{10} - (17.5)_{10}$
  4. $(73.5)_{10} - (112.75)_{10}$

2's Complement

✓ The 2's complement of a number is obtained by adding 1 to the 1's complement of that number

2’s Complement

Example: Obtain 2’s complement of the 1010

2’s Complement

Example: Obtain 2’s complement of the 1010

$$
\begin{array}{rcccccl}
 & 1 & 1 & 1 & 1 & & \\
- & 1 & 0 & 1 & 0 & & \\
\hline
 & 0 & 1 & 0 & 1 & & \text{---1's complement} \\
 & & & & 1 & & \\
\hline
 & 0 & 1 & 1 & 0 & & \text{----2's complement}
\end{array}
$$

2’s complement of the 1010 is 0110

2's Complement

Sr. No.	Binary Number	1's Complement	2's Complement
1	1101 0101	0010 1010	0010 1011
2	1001	0110	0111
3	1011 1111	0100 0000	0100 0001
4	1101 1010 0001	0010 0101 1110	0010 0101 1111
5	1110 0111 0101	0001 1000 1010	0001 1000 1011

Subtraction Using 2's Complement

- In 2's complement subtraction, add the 2's complement of subtrahend to the minuend.
- If carry is generated then the result is positive and in its true form.
- If the carry is not produced, then the result is negative and in its 2's complement form.

*Carry is always to be discarded

Subtraction Using 2's Complement

Example: Perform using 2' complement $(9)_{10} - (4)_{10}$

Subtraction Using 2's Complement

Example: Perform using 2' complement $(9)_{10} - (4)_{10}$

Step 1: Take 2' complement of $(4)_{10} = (0100)_2$

$$= 1011 + 1 = 1100$$

Step 2: Add 9 with 2' complement of 4

$$\begin{array}{cccccc}
 & & 1 & 0 & 0 & 1 \\
+ & & 1 & 1 & 0 & 0 \\
\hline
 & 1 & 0 & 1 & 0 & 1
\end{array}$$

final carry Discard

Final Result

If Carry is generated, discard carry. The result is positive and its true binary form

Exercise

- Perform Binary Subtraction using 2's Complement method

    1. $(46)_{10} - (19)_{10}$
    2. $(27)_{10} - (75)_{10}$
    3. $(125.3)_{10} - (46.7)_{10}$
    4. $(36.75)_{10} - (89.5)_{10}$

Chapter I – Number System

- **Introduction to digital signal, Advantages of Digital System over analog systems**
  - ✓ Number Systems: Different types of number systems(Binary, Octal, Hexadecimal), Conversion of number systems,
  - ✓ Binary arithmetic: Addition, Subtraction, Multiplication, Division.
  - ✓ Subtraction using 1's complement and 2's complement
- **Codes**
  - ✓ **Codes -BCD,** Gray Code, Excess-3, ASCII code
  - ✓ BCD addition, BCD subtraction using 9's and 10' complement
- **(Numericals based on above topic)**.

BCD or 8421 Code

- The smallest BCD number is (0000) and the largest is (1001). The next number to 9 will be 10 which is expressed as (0001 0000) in BCD.

- There are six illegal combinations 1010, 1011, 1100, 1101, 1110 and 1111 in this code i.e. they are not part of the 8421 BCD code

Decimal to BCD Conversion

Sr. No.	Decimal Number	BCD Code
1	8	1000
2	47	0100 0111
3	345	0011 0100 0101
4	99	1001 1001
5	10	0001 0000

Exercise

- Convert following Decimal Numbers into BCD

1. $(286)_{10}$
2. $(807)_{10}$
3. $(429.5)_{10}$
4. $(158.7)_{10}$

Chapter I – Number System

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- (Numericals based on above topic).

BCD Addition

- The BCD addition is performed by individually adding the corresponding digits of the decimal number expressed in 4 bit binary groups starting from LSD.

- If there is no carry & the sum term is not an illegal code, no correction is needed.

BCD Addition

- If there is a carry out of one group to the next group or if the sum term is an illegal code then 6 i.e. 0110 is added to the sum term of that group and resulting carry is added to the next group.

- This is done to skip the six illegal states.

BCD Addition

Addition of two BCD numbers

Sum≤9, Carry=0	Sum≤9, Carry=1	Sum>9, Carry=0
Answer is correct. No correction required.	Add 6 to the sum term to get the correct answer	Add 6 to the sum term to get the correct answer

BCD Addition

Example: Perform in BCD $(57)_{10} + (26)_{10}$

BCD Addition

Example: Perform in BCD $(57)_{10} + (26)_{10}$

$$
\begin{array}{rr}
 & 57 \\
+ & 26 \\
\hline
 & 83
\end{array}
\qquad
\begin{array}{rccccccccc}
 & & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\
+ & & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
\hline
 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1
\end{array}
$$

Final Carry 0	Valid BCD Code	Invalid BCD Code

Thus we have to add 0110 in illegal BCD code

Example Continue

Add 0110 in only invalid code

	0	1	1	1	1	1	0	1
+	0	0	0	0	0	1	1	0
	1	0	0	0	0	0	1	1

$$(57)_{10} + (26)_{10} = (83)_{10}$$

Exercise

- Perform BCD Addition

1. $(275)_{10} + (493)_{10}$
2. $(109)_{10} + (778)_{10}$
3. $(88.7)_{10} + (265.8)_{10}$
4. $(204.6)_{10} + (185.56)_{10}$

BCD Subtraction

- The BCD subtraction is performed by subtracting the digits of each 4 bit group of the subtrahend from the corresponding 4 bit group of the minuend in binary starting from the LSD.

BCD Subtraction

- If there is no borrow from the next higher group then no correction is required .

- If there is a borrow from the next group, then 0110 is subtracted from the difference term of this group.

BCD Subtraction

Example: Perform in BCD $(38)_{10} - (15)_{10}$

BCD Subtraction

Example: Perform in BCD $(38)_{10} - (15)_{10}$

$$
\begin{array}{rr}
 & 38 \\
- & 15 \\
\hline
 & 23
\end{array}
\qquad
\begin{array}{rccccccccc}
 & & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
- & & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
\hline
 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1
\end{array}
$$

Final Borrow 0 | Valid BCD Code | Valid BCD Code

No borrow, hence difference is correct

Exercise

- Perform BCD Subtraction
  1. $(920)_{10} - (356)_{10}$
  2. $(79)_{10} - (27)_{10}$
  3. $(476.7)_{10} - (258.9)_{10}$
  4. $(634.6)_{10} - (328.7)_{10}$

9's Complement

- The nine's complement of a BCD number can be obtained by subtracting each digit of BCD from 9.

Ex. Nine's complement of 168

$$
\begin{array}{rccc}
 & 9 & 9 & 9 \\
- & 1 & 6 & 8 \\
\hline
 & 8 & 3 & 1
\end{array}
$$

9's complement of 168 is 831

BCD Subtraction using 9'sComplement

- Obtain 9's complement of subtrahend
- Add minuend with 9's complement of subtrahend
- If a carry is generated then add it to the sum to obtain the final result.
- If a carry is not produced then the result is negative hence take the 9's complement of the result.

BCD Subtraction using 9's Complement

Example: Perform in BCD using 9's complement $(83)_{10} - (21)_{10}$

BCD Subtraction using 9's Complement

Example: Perform in BCD using 9's complement $(83)_{10} - (21)_{10}$

9's complement of 21 99-21=78

Add 83 with 9's complement of 21 i.e. 78

$$
\begin{array}{lcccccccc}
83 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
78 + & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
\hline
 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1
\end{array}
$$

Invalid BCD Code

Invalid BCD Code

Thus we have to add 0110 in illegal BCD code

```
      1 1 1 1     1 0 1 1
 +    0 1 1 0     0 1 1 0
            1
 ------------------------
 1    0 1 1 0   1 0 0 0 1
                        1
 ------------------------
      0 1 1 0     0 0 1 0
```

$$(83)_{10} - (21)_{10} = (62)_{10}$$

Exercise

- Perform BCD Subtraction using 9's Complement

1. $(274)_{10} - (86)_{10}$
2. $(93)_{10} - (615)_{10}$
3. $(574.6)_{10} - (279.7)_{10}$
4. $(376.3)_{10} - (765.6)_{10}$

10's Complement

➢ The 10's complement of a BCD number can be obtained by adding 1 in 9's complement.

Ex. 10's complement of 168

$$
\begin{array}{rccc}
 & 9 & 9 & 9 \\
- & 1 & 6 & 8 \\
\hline
 & 8 & 3 & 1 \\
+ & & & 1 \\
\hline
 & 8 & 3 & 2
\end{array}
$$

10's complement of 168 is 832

BCD Subtraction using 10’s Complement

- Obtain 10’s complement of subtrahend
- Add minuend with 10’s complement of subtrahend
- If a carry is generated, discard carry and result is positive and in its true form.
- If a carry is not produced then the result is negative hence take the 10’s complement of the result.

BCD Subtraction using 10's Complement

Example: Perform in BCD using 10's complement $(83)_{10} - (21)_{10}$

BCD Subtraction using 10's Complement

Example: Perform in BCD using 10's complement $(83)_{10} - (21)_{10}$

9's complement of 21 99-21=78

10's complement of 21 78+1=79

Add 83 with 10's complement of 21 i.e. 79

83		1	0	0	0	0	0	1	1
79	+	0	1	1	1	1	0	0	1
		1	1	1	1	1	1	0	0
		Invalid BCD Code				Invalid BCD Code			

Thus we have to add 0110 in illegal BCD code

Example: continue……

$$
\begin{array}{ccccccccccc}
 & & 1 & 1 & 1 & 1 & & 1 & 1 & 0 & 0 \\
+ & & 0 & 1 & 1 & 0 & & 0 & 1 & 1 & 0 \\
 & & & & & 1 & & & & & \\
\hline
 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
 & & 0 & 1 & 1 & 0 & & 0 & 0 & 1 & 0
\end{array}
$$

$$(83)_{10} - (21)_{10} = (62)_{10}$$

Exercise

- Perform BCD Subtraction using 10's Complement

  1. $(274)_{10} - (86)_{10}$
  2. $(93)_{10} - (615)_{10}$
  3. $(574.6)_{10} - (279.7)_{10}$
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- **(Numericals based on above topic)**.

Gray Code

- ✓ The gray code is non-weighted code.
- ✓ It is not suitable for arithmetic operations.
- ✓ It is a cyclic code because successive code words in this code differ in one bit position only i.e. unit distance code

Binary to Gray Code Conversion

- ✓ If an n bit binary number is represented by $B_n, B_{n-1}, \ldots\ldots B_1$ and its gray code equivalent by $G_n, G_{n-1}, \ldots\ldots G_1$ where and are the MSBs, then gray code bits are obtained from the binary code as follows;

			....	

*where the symbol represents Exclusive-OR operation

Binary to Gray Code Conversion

Example 1: Convert 1011 Binary Number into Gray Code

Binary to Gray Code Conversion

Example 1: Convert 1011 Binary Number into Gray Code

Binary Number	1	0	1	1

Example 1: Continue

Binary Number	1	0	1	1
Gray Code	1			

Binary Number	1 $\rightarrow \oplus \leftarrow$	0	1	1
Gray Code	1	1		

Example 1: Continue

Binary Number	1	0	1	1
Gray Code	1	1	1	

Binary Number	1	0	$1 \rightarrow \oplus \leftarrow 1$	
Gray Code	1	1	1	0

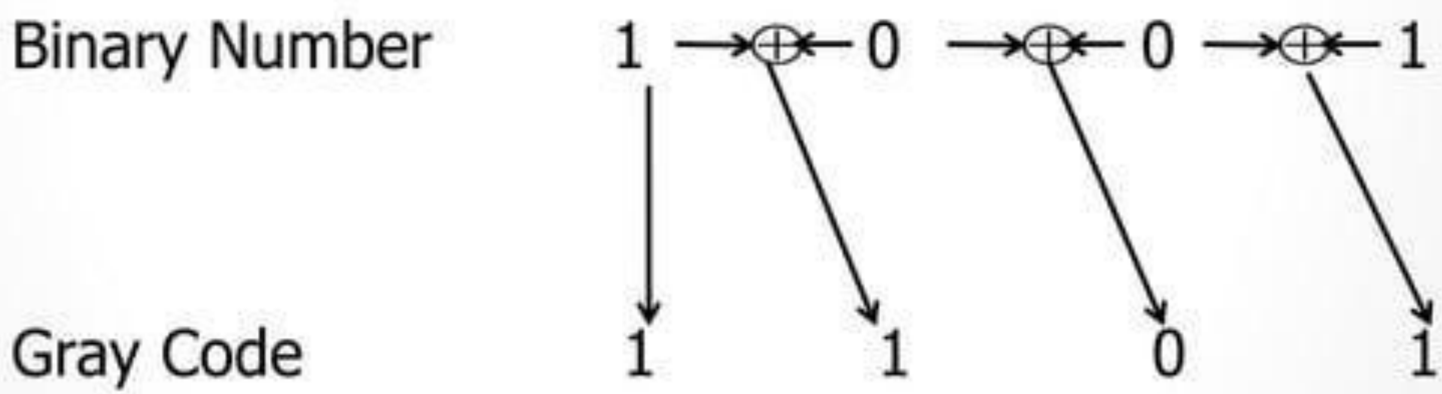
Binary Number	1	$\rightarrow \oplus \leftarrow$ 0	$\rightarrow \oplus \leftarrow$ 1	$\rightarrow \oplus \leftarrow$ 1
Gray Code	1	1	1	0

Binary to Gray Code Conversion

Example 2: Convert 1001 Binary Number into Gray Code

Binary to Gray Code Conversion

Example 2: Convert 1001 Binary Number into Gray Code

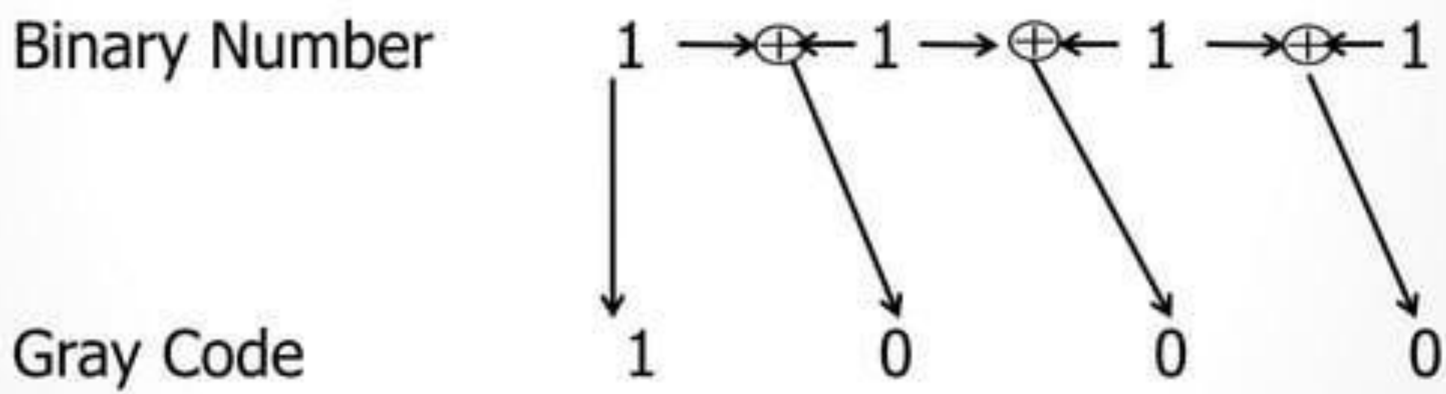


Binary to Gray Code Conversion

Example 3: Convert 1111 Binary Number into Gray Code

Binary to Gray Code Conversion

Example 3: Convert 1111 Binary Number into Gray Code



Binary to Gray Code Conversion

Example 4: Convert 1010 Binary Number into Gray Code

Binary to Gray Code Conversion

Example 4: Convert 1010 Binary Number into Gray Code

Binary Number	1	0	1	0
Gray Code	1	1	1	1

Binary and Corresponding Gray Codes

Decimal No.	Binary No.	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Exercise

- Convert following Binary Numbers into Gray Code

    1. $(1011)_2$
    2. $(110110010)_2$
    3. $(101010110101)_2$
    4. $(100001)_2$

Gray Code to Binary Conversion

- ✓ If an n bit gray code is represented by $G_n, G_{n-1}, \ldots\ldots G_1$ and its binary equivalent $B_n, B_{n-1}, \ldots\ldots B_1$ then binary bits are obtained from gray bits as follows;

$B_n = G_n$	$B_{n-1} = B_n \oplus G_{n-1}$	$B_{n-2} = B_{n-1} \oplus G_{n-2}$	…………	$B_1 = B_2 \oplus G_1$

*where the symbol represents Exclusive-OR operation

Gray Code to Binary Conversion

Example 1: Convert 1110 Gray code into Binary Number.

Gray Code to Binary Conversion

Example 1: Convert 1110 Gray code into Binary Number.

Gray Code	1	1	1	0

Example 1: Continue

Gray Code	1	1	1	0
Binary Number	1			

Example 1: Continue

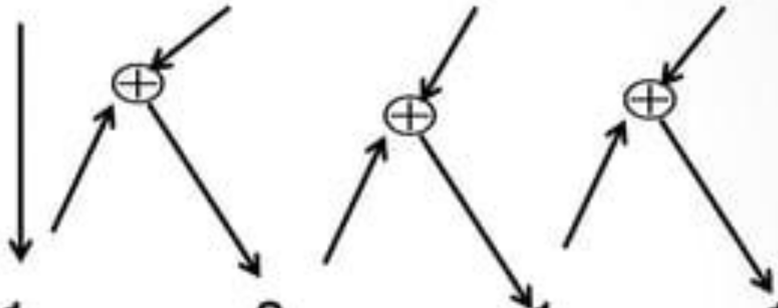
Gray Code	1	1	1	0
	$\oplus$			
Binary Number	1	0		

Example 1: Continue

Gray Code	1	1	1	0
			$\oplus$	
Binary Number	1	0	1	

Gray Code	1	1	1	0
				$\oplus$
Binary Number	1	0	1	1

Gray Code 1 1 1 0



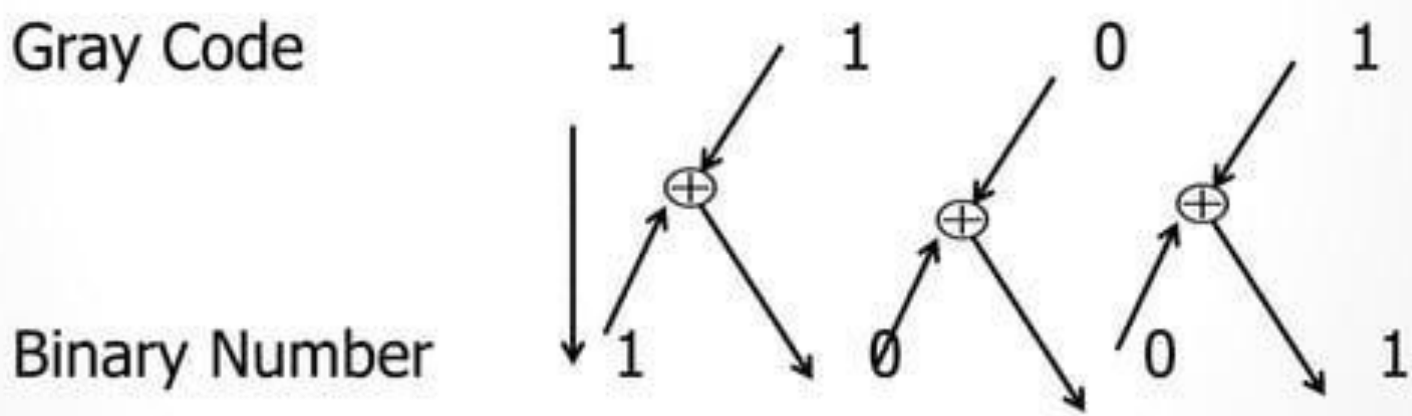
Binary Number 1 0 1 1

Gray Code to Binary Conversion

Example 2: Convert 1101 Gray code into Binary Number.

Gray Code to Binary Conversion

Example 2: Convert 1101 Gray code into Binary Number.

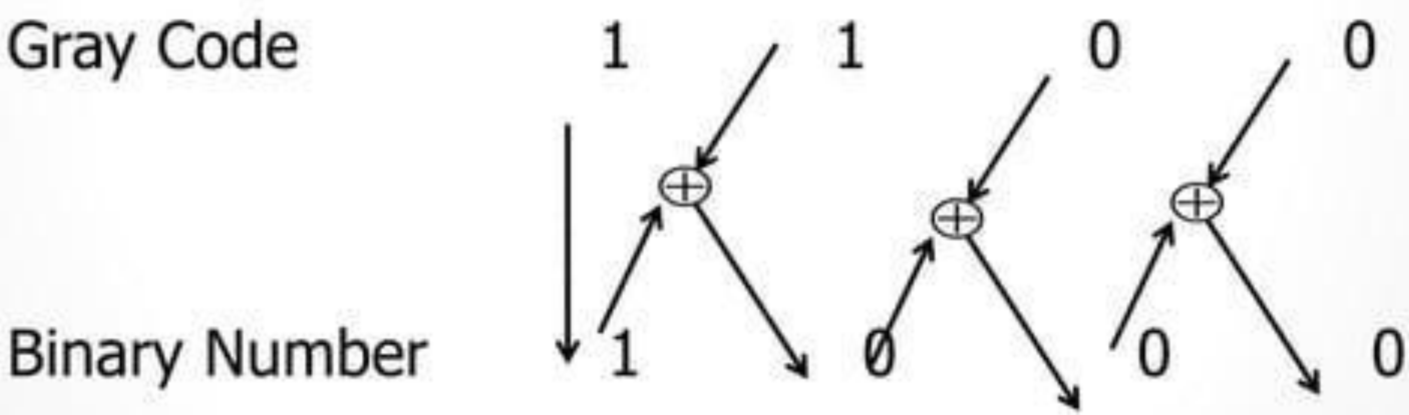


Gray Code to Binary Conversion

Example 3: Convert 1100 Gray code into Binary Number.

Gray Code to Binary Conversion

Example 3: Convert 1100 Gray code into Binary Number.



Exercise

- Convert following Gray Numbers into Binary Numbers

    1. $(1111)_{GRAY}$
    2. $(101110)_{GRAY}$
    3. $(100010110)_{GRAY}$
    4. $(11100111)_{GRAY}$

Excess-3 Code (XS-3)

- ✓ The Xs-3 is non-weighted BCD code.
- ✓ This code derives its name from the fact that each binary code word is the corresponding 8421 code word plus 0011.
- ✓ It is a sequential code & therefore can be used for arithmetic operations.
- ✓ It is a self complementing code

Excess-3 Code (XS-3)

Decimal No.	BCD Code	Excess-3 Code= BCD + Excess-3
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

Excess-3 Code (XS-3)

Example 1: Obtain Xs-3 Code for 428 Decimal

Excess-3 Code (XS-3)

Example 1: Obtain Xs-3 Code for 428 Decimal

	4	2	8
	0100	0010	1000
+	0011	0011	0011
	0111	0101	1011

Exercise

- Convert following Decimal Numbers into Excess-3 Code
  1. $(40)_{10}$
  2. $(88)_{10}$
  3. $(64)_{10}$
  4. $(23)_{10}$

ASCII Codes

- ✓ The **American Standard Code for Information Interchange** is a character-encoding scheme originally based on the English alphabet.
- ✓ ASCII codes represent text in computers, communications equipment, and other devices that use text.
- ✓ Most modern character-encoding schemes are based on ASCII, though they support many additional characters.

ASCII Codes

- ✓ ASCII developed from telegraphic codes. Its first commercial use was as a seven-bit tele-printer code promoted by Bell data services.
- ✓ Work on the ASCII standard began on October 6, 1960, with the first meeting of the American Standards Association's (ASA) X3.2 subcommittee.
- ✓ The first edition of the standard was published during 1963.

ASCII Codes

- ✓ ASCII includes definitions for 128 characters: 33 are non-printing control characters (many now obsolete) that affect how text and space is processed and 95 printable characters, including the space (which is considered an invisible graphic)

ASCII Codes

				B7	0	0	0	0	1	1	1	1
				B6	0	0	1	1	0	0	1	1
				B5	0	1	0	1	0	1	0	1
B4	B3	B2	B1		0	1	2	3	4	5	6	7
0	0	0	0	0	NUL	DLE	SP	0	@	P	‘	p
0	0	0	1	1	SOH	DC1	|	1	A	Q	a	q
0	0	1	0	2	STX	DC2	"	2	B	R	b	r
0	0	1	1	3	ETX	DC3	#	3	C	S	c	s
0	1	0	0	4	EOT	DC4	$	4	D	T	d	t
0	1	0	1	5	ENQ	NAK	%	5	E	U	e	u
0	1	1	0	6	ACK	SYN	&	6	F	V	f	v
0	1	1	1	7	BEL	ETB	’	7	G	W	g	w
1	0	0	0	8	BS	CAN	(	8	H	X	h	x
1	0	0	1	9	HT	EM	)	9	I	Y	i	y
1	0	1	0	10	LF	SUB	"	:	J	Z	j	z
1	0	1	1	11	VT	ESC	+	;	K	[	k	{
1	1	0	0	12	FF	FC	,	<	L	\	l	!
1	1	0	1	13	CR	GS		=	M	]	m	}
1	1	1	0	14	SO	RS	.	>	N	^	n	~

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- ✓ Digital Techniques by A. Anand Kumar

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- ✓ http://www.electronics-tutorials.ws/binary/bin1.html

Thank You