Knapsack Problem

Truck – 10t capacity

What is the optimum cargo combination of the following items?

- Item 1: $5 (3t)
- Item 2: $7 (4t)
- Item 3: $8 (5t)

Knapsack Problem

First, define output function *f(i,w)*

⇔

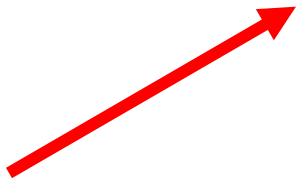
Function *f* is the optimum value for a combination of items 1 to i with a cumulative weight of w or less.

- Item 1: $value_1$=\$5 ; w_1=3t
- Item 2: $value_2$ =\$7 ; w_2=4t
- Item 3: $value_3$ =\$8 ; w_3=5t

Knapsack Problem

Output function f(i,w)

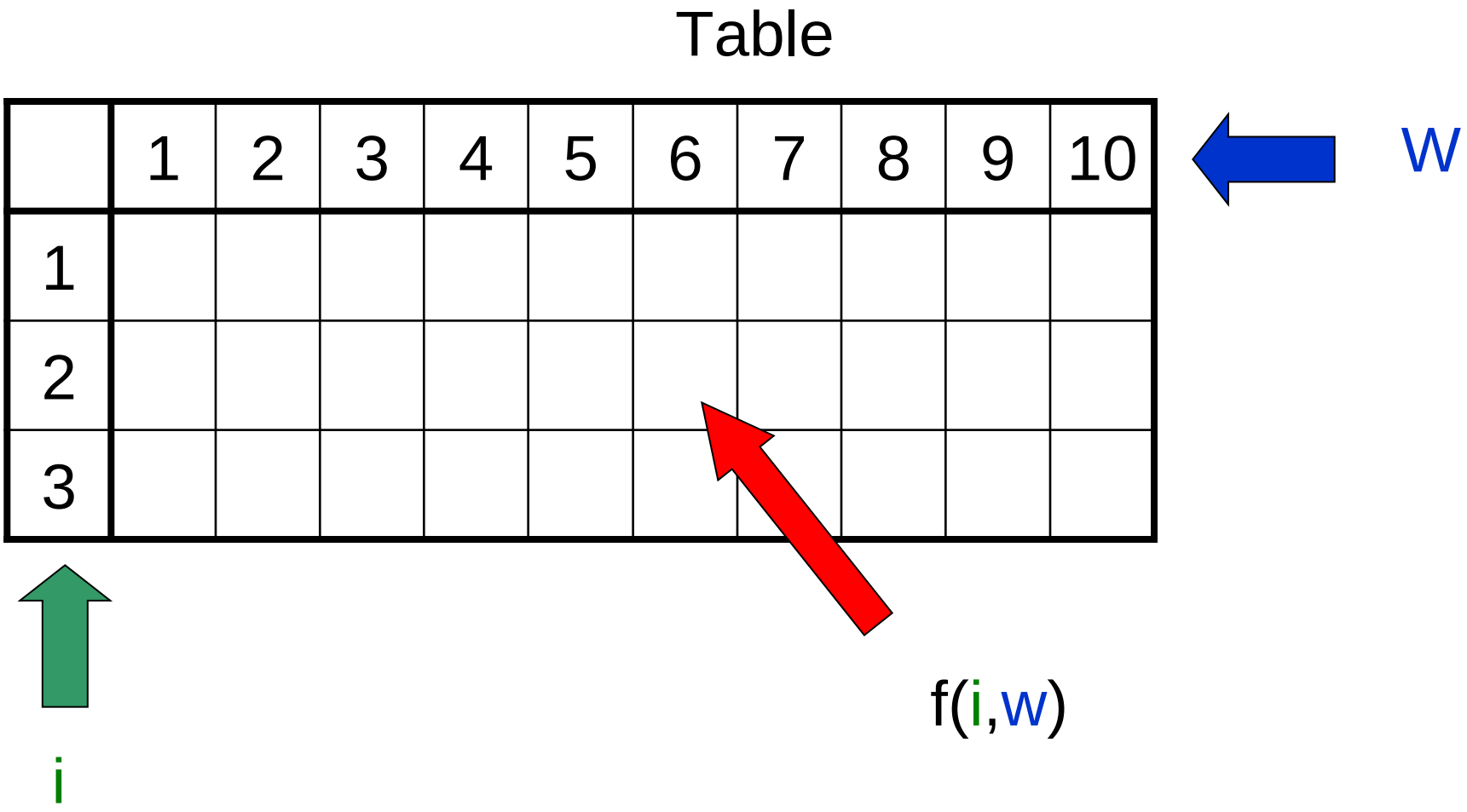
$$f(i,w)=Max[\ value_i + f(i,w-w_i)\ ;\ f(i-1,w)]$$



ONE Item i + optimum combination of weight $w-w_i$

NO Item i + optimum combination items 1 to i-1

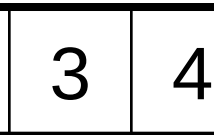
Knapsack Problem



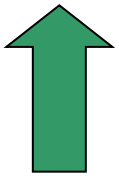
Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1			**Using only item 1**							
2										
3										



W

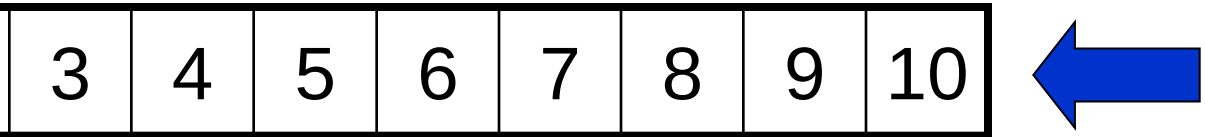


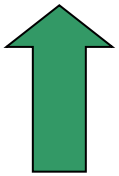
i

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1										
2		**Using only item 1 & 2**								
3										

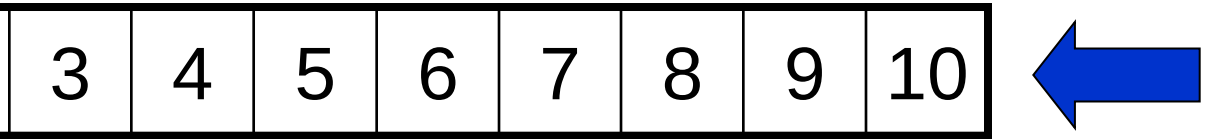


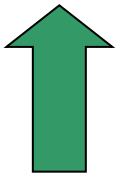


Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1										
2										
3			**Using items 1, 2 & 3**							





Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10				
2										
3										

W

3t / $5

Item 1 doesn't fit

(f(i, w-w_i) not valid!)

A single item 1 fits

$w_1 = 3$

Two item 1's fit

$2 * w_1 = 6$

Notice f(i-1, w) not valid for first row!

$f(i,w) = \max[\ value_i + f(i,w-w_i)\ ;\ f(i-1,w)\ ]$

Knapsack Problem

Table – filling in f(2, 5)

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7						
3										

3t / $5

4t / $7

+ $value_2$ (= 7)

$w - w_2$ =

5 – 4 = 1

$$f(i,w)=\text{Max}[\ value_i + f(i,w-w_i)\ ;\ f(i-1,w)\ ]$$

Knapsack Problem

Table – filling in f(2, 5)

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7					
3										

3t / $5

4t / $7

+ $value_2$ (= 7)

$w - w_2$ =

5 – 4 = 1

$$f(i,w)=\text{Max}[\ value_i + f(i,w-w_i)\ ;\ f(i-1,w)\ ]$$

Knapsack Problem

Table – filling in f(2, 6)

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7					
3										

3t / $5

4t / $7

+ $value_2$ (= 7)

$w - w_2$ =

6 – 4 = 2

$$f(i,w)=\text{Max}[\ value_i + f(i,w-w_i)\ ;\ f(i-1,w)\ ]$$

Knapsack Problem

Table – filling in f(2, 6)

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10				
3										

3t / $5

4t / $7

+ $value_2$ (= 7)

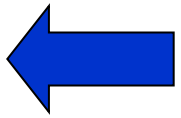
$w - w_2$ = 6 – 4 = 2

$$f(i,w)=\text{Max}[\ value_i + f(i,w-w_i)\ ;\ f(i-1,w)\ ]$$

Knapsack Problem (truck)

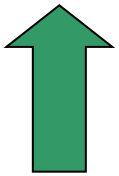
Table – fill row for first item

	1	2	3	4	5	6	7	8	9	10
1										



Weight

3t / $5



i

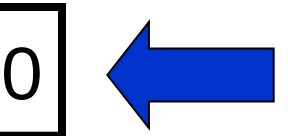
(item number)

(Fill in table values)

Knapsack Problem (truck)

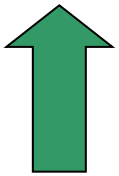
Table – fill row for second item

	1	2	3	4	5	6	7	8	9	10
1										
2										



3t / $5

4t / $7



i

(item number)

Knapsack Problem (truck)

Table – fill row for third item

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										

Weight

3t / $5

4t / $7

5t / $8

i

(item number)

Knapsack Problem

COMPLETED TABLE

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10	12	14	15	17
3	0	0	5	7	8	10	12	14	15	17

Knapsack Problem

Path – Which items were used?

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10	12	14	15	17
3	0	0	5	7	8	10	12	14	15	17

Optimal: 2 x Item 1 + 1 x Item 2