

Matrix Algebra

A matrix is a collection of numbers ordered in rows and columns.

A matrix is a rectangular or square array of numbers or variables. We use uppercase bold letters to represent matrices.

Ex:- $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 6 \end{bmatrix}$ row

↓
col

2×3

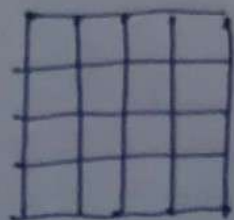
Matrices are called multi-dimensional, we have data being stored in different directions in a grid.

The dimensions of this matrix are "2x3" or "2 by 3", 2 rows and 3 columns

8, 4, 3, 2, 1, 6 → Each of these values Elements of the matrix. So our matrix is total of 6 Elements.

Matrices seem similar to tables and Spreadsheets

Ex:-



$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 6 \end{bmatrix}$$

However, the purpose of matrices not simply the store values but matrices are main characters in mathematical operations such as.

Addition, subtraction and multiplication of matrix.

A matrix can only contain numbers, symbols, or expressions.

$$A_{2 \times 3} = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3}$$

$$C = \begin{bmatrix} x-a & 5+b \\ f-e & z \end{bmatrix}_{2 \times 2}$$

Matrices can be of any size $m \times n$

$$A = \underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}_{n \text{ columns}} \left. \vphantom{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} \right\} m \text{ rows}$$

All numbers we know from algebra are referred to as scalars in linear algebra.

$[8] \rightarrow$ is a scalar.

Note: Scalars have 0 dimensions

A single real number is called a scalar. Thus,
 $[3]; [5]; [7]$ so on.

A vector is a matrix with a single row or column.

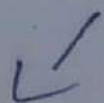
Ex:- $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $Z = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}_{3 \times 1}$

Elements in a vector are often identified by a single subscript.

Types of Vectors

1. Row Vector
2. Column Vector.

$$\begin{bmatrix} 6 & 3 & 2 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

What is the length of the vector?

Row vector length is $= 4$

Column vector length is $= 3$

Scalar - $[x]$ are of zero dimension

Vector - $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{m \times 1}$ one dimension

Matrix - $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ b_{11} & b_{12} & b_{13} \end{bmatrix}_{m \times n}$ - 2D dimension.

Two matrices or two vectors are equal if they are of the same size and if the elements in corresponding positions are equal:

For Ex:-

$$\begin{bmatrix} 1 & 6 & 7 \\ 4 & 2 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 7 \\ 4 & 2 & 9 \end{bmatrix}$$

Matrix
Equality

$$\begin{bmatrix} 5 & 2 & -9 \\ 6 & 3 & 1 \end{bmatrix}$$

$$\neq \begin{bmatrix} 4 & 2 & -9 \\ 6 & 3 & 1 \end{bmatrix}$$

→ value changed

Transpose Matrix

If we change the rows and columns of a matrix A , the resulting matrix is known as Transpose of A and is denoted by A'

Ex:-

$$A = \begin{bmatrix} 8 & 7 \\ 9 & 1 \\ 6 & -2 \end{bmatrix}$$

$$A' = \begin{bmatrix} 8 & 9 & 6 \\ 7 & 1 & -2 \end{bmatrix}$$

3 by 2 turned into 2 by 3 matrix after initiation of transpose.

Ex:- Addition of Matrix

$$A = \begin{bmatrix} 7 & 1 & 4 \\ 8 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad C = ?$$

$$C = A + B \quad C = c_{ij} = a_{ij} + b_{ij}$$

$$C = \begin{bmatrix} 7+6 & 1+2 & 4+3 \\ 8+4 & 2+5 & 3+6 \end{bmatrix} = \begin{bmatrix} 13 & 3 & 7 \\ 12 & 7 & 9 \end{bmatrix}$$

Python Code

import numpy as np \rightarrow Import necessary libraries.

m = np.array([[3, 4, 6], [7, 8, 9]])

m₁ = np.array([[1, 2, 3], [4, 5, 6]])

Output:

m + m₁

Op :- array([[4, 6, 9], [11, 13, 15]])

Ex:- Subtraction of Matrix

$$A = \begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 7 & -5 \\ 3 & 8 \end{bmatrix}$$

$$C = A - B = \begin{bmatrix} 5-7 & 3+5 \\ -2-3 & 4-8 \end{bmatrix} = \begin{bmatrix} -2 & 8 \\ -5 & -4 \end{bmatrix}$$

Code Snippet

m₃ = np.array([[5, 3], [-2, 4]])

m₄ = np.array([[7, -5], [3, 8]])

O/P: array([[-2, 8], [-5, -4]])