Parametric Regions – 2017

Examples in 2D

Here are two examples together with *Mathematica* code that pictures them.

1. Butterfly Wings

   A. Parametric Equations

   $$x = r\sin(u)$$
   $$y = r\sin(2u)$$
   $$0 \le u \le 2\pi \text{ and } 0 \le r \le 1.$$

   B. *Mathematica* Code

   $$\text{ParametricPlot}[\{r * Sin[u], r * Sin[2u]\}, \{u, 0, 2 * Pi\}, \{r, 0, 1\}].$$

2. A Cardioid Cutout

   A. Parametric Equations

   $$x = r(1 - \cos(u))\sin(u)$$
   $$y = r(1 - \cos(u))\cos(u)$$
   $$0 \le u \le 2\pi \text{ and } .5 \le r \le 1.$$

   B. *Mathematica* Code

   $$\text{ParametricPlot}[\{r*(1-Cos[u])*Sin[u], r*(1-Cos[u])*Cos[u]\}, \{u, 0, 2*Pi\}, \{r, .5, 1\}].$$

Examples in 3D

Here are two 3D examples, but for this, *Mathematica* doesn't do well. I've inserted some *Mathematica* code that views a bit of the interior of these solids to give you the idea, but it does not view the entire solid.

1. Top Half of an Elliptical Torus

   A. Parametric Equations

   $$x = (4 + .5(3 + r\cos(v))\sin(u)$$
   $$y = (4 + .5(3 + r\cos(v))\cos(u)$$
   $$z = 4 + r\sin(v)$$
   $$0 \le u \le 2\pi,\ 0 \le v \le \pi \text{ and } 0 \le r \le 1.$$

   B. *Mathematica* Code for a Partial Image

   ParametricPlot3D[{{4 + .5 * (3 + Cos[v]) * Sin[u], 4 + .5 * (3 + Cos[v]) * Cos[u], 4 + Sin[v]},
   {4 + .5 * (3 + .5 * Cos[v]) * Sin[u], 4 + .5 * (3 + .5 * Cos[v]) * Cos[u], 4 + .5 * Sin[v]}},
   {u, 0, 2 * Pi}, {v, 0, Pi}].

2. A Spiral Tube

   A. Parametric Equations

   $$x_1 = r\cos(t) - s\sin(t)$$
   $$x_2 = r\sin(t) + s\cos(t)$$
   $$x_3 = t/3$$
   $$0 \le r \le 1,\ 0 \le s \le 1 \text{ and } -4\pi \le t \le 4\pi.$$

   B. *Mathematica* Code for a Partial Image

   ParametricPlot3D[{{r * Cos[t] - Sin[t], r * Sin[t] + Cos[t], t/3},
   {r * Cos[t] - .5 * Sin[t], r * Sin[t] + .5 * Cos[t], t/3},
   {r * Cos[t] - .2 * Sin[t], r * Sin[t] + .2 * Cos[t], t/3}},
   {r, 0, 1}, {t, -4 * Pi, 4 * Pi}].