# Climate Modelling

## The Energy Balance way

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# A slight modification

The equation of the slightly modified model would be:

$$R\frac{dT}{dt} = Q(1-\alpha) - (A + BT)$$

which can also be rewritten as:

$$\frac{dT}{dt} + \frac{B}{R}T = \frac{Q}{R}(1-\alpha) - \frac{A}{R}$$

Let's take $k = \frac{Q}{R}(1-\alpha) - \frac{A}{R}$.
This turns our equation to:

$$\frac{dT}{dt} + \frac{B}{R}T = k$$

The integrating factor for this equation is $e^{\int \frac{B}{R}dt} = e^{\frac{B}{R}t}$.
Using this integration factor gives us:

$$T(t)e^{\frac{B}{R}t} = k\frac{R}{B}e^{\frac{B}{R}t} + c$$

which can be written as:

$$T(t) = k\frac{R}{B} + ce^{-\frac{B}{R}t}$$

or, if we replace $k = \frac{Q}{R}(1-\alpha) - \frac{A}{R}$, then

$$T(t) = \frac{Q(1-\alpha) - A}{B} + ce^{-\frac{B}{R}t}$$

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# Whatever happened to equilibirum?

The slightly modified model yields:

$$T(t) = \frac{Q(1-\alpha) - A}{B} + ce^{-\frac{B}{R}t}$$

which, at equilibrium, ends up being:

$$T^* = \frac{Q(1-\alpha) - A}{B}$$

which, by the way, is also the asymptotic value of the temperature. From Graves et al. [1993], we have the best current estimates of $A$ and $B$. $A = 202W/m^2$ and $B = 1.90W/m^2K$.

Now, if we take the values of Q and $\alpha$ from previous slides, then:

$$T^* = \frac{343x(1-0.3) - 202}{1.9}$$

which then yields $T^* = 19.7^oC$ or $290.7K$.

This is pretty close to the actual average of $15.4^oC$.

Makes sense? Yeah. This model attempts to take into consideration the role our atmosphere plays in determining surface temperature.

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# Nullclines and change of variables

Given the change of variables $w = (u_0 + v_0)/2$ and $z = u_0 - v_0$:

$R\dot{w} = \frac{1}{2}(R\dot{u_0} + R\dot{v_0})$

$R\dot{w} = \frac{1}{2}(Q(1-\alpha_1)-A-(B+C)u_0+C\bar{T}+Q(1-\alpha_2)-A-(B+C)v_0+C\bar{T})$

$R\dot{w} = Q(1-\alpha_0) - A - \frac{1}{2}(B+C)(u_0+v_0) + C\bar{T}$

$R\dot{w} = Q(1-\alpha_0) - A - (B+C)w + C\bar{T}$

where $\alpha_0 = \frac{(\alpha_1+\alpha_2)}{2}$

Similarly...

$R\dot{z} = R\dot{u_0} - R\dot{v_0}$

$R\dot{z} = Q(1-\alpha_1)-A-(B+C)u_0+C\bar{T}-Q(1-\alpha_2)+A+(B+C)v_0-C\bar{T}$

$R\dot{z} = Q(\alpha_2-\alpha_1) - (B+C)(u_0-v_0) = Q(\alpha_2-\alpha_1) - (B+C)z$

# Nullclines and change of variables

$\bar{T} = w + (\eta - \frac{1}{2})\frac{Q(\alpha_2-\alpha_1)}{B+C} + P_2(\eta)(\frac{Qs_2(1-\alpha_1)}{B+C} - \frac{Qs_2(1-\alpha_2)}{B+C})$
$= w + (\eta - \frac{1}{2})\frac{Q(\alpha_2-\alpha_1)}{B+C} + P_2(\eta)(\frac{Qs_2(\alpha_2-\alpha_1)}{B+C})$
$= w + (\eta - \frac{1}{2} + s_2 P_2(\eta))\frac{Q(\alpha_2-\alpha_1)}{B+C}$
Plugging $\bar{T}$ into the expression for $\dot{w}$:
$\dot{w} = \frac{1}{R}(Q(1-\alpha_0) - A - Bw + \frac{CQ(\alpha_2-\alpha_1)}{B+C}(\eta - 1/2 + s_2 P_2(\eta)))$
which can be written as $\dot{w} = -\frac{B}{R}(w - F(\eta))$.
Here $F(\eta)$ is the cubic polynomial:
$F(\eta) = \frac{1}{B}[Q(1-\alpha_0) - A + \frac{CQ(\alpha_2-\alpha_1)}{B+C}(n - \frac{1}{2} + s_2 P_2(\eta))]$

# Nullclines and change of variables

We now consider a dynamic equation for the ice line by adding the equation:

$\dot{\eta} = \epsilon(T(t,\eta) - T_c)$

where $T(t,\eta) = w + \frac{u_2+v_2}{2}p_2(\eta)$.

Now, using $u_{2eq} = \frac{Qs_2(1-\alpha_1)}{B+C}$ and $v_{2eq} = \frac{Qs_2(1-\alpha_2)}{B+C}$, we get:

$T(t,\eta) = w + \frac{1}{2}(\frac{Qs_2(1-\alpha_1)}{B+C} + \frac{Qs_2(1-\alpha_2)}{B+C})p_2(\eta)$

$T(t,\eta) = w + \frac{Qs_2(1-\alpha_0)}{B+C}p_2(\eta)$ where $\alpha_0 = \frac{1}{2}(\alpha_1 + \alpha_2)$

This means that $\dot{\eta}$ depends on both $w$ and $\eta$.

$\dot{\eta} = \epsilon(w + \frac{Qs_2(1-\alpha_0)}{B+C}p_2(\eta) - T_c) = \epsilon(w - G(\eta))$,

where $G(\eta) = -\frac{Q}{B+C}s_2(1-\alpha_0)p_2(\eta) + T_c$