Time-frequency selective channels labwork

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Lab 1 : Building a multipath time-variant channel simulator

In this labwork, we want to simulate a realistic multipath time-varying channel in the discrete-time domain. More precisely, we will implement a small-scale statistical model involving Rayleigh fading in a static (Sec. 1.2) or time-variant context (Sec. 1.3). Note that such simulation techniques are at the state-of-the-art to assess the behavior of mobile radio systems such as 5G cellular networks [Eur17].
A report on this work will be written (in French or in English) including the answers of the following questions as well as any interesting remark. Plots will be necessarily commented and provided with appropriate units, axes and legends.
The simulator will be realized using *Octave* or *Matlab*. The following instructions have to be written in a single Matlab script file (except if stated otherwise), for example `channel_simulator.m`.

Did you know... ? Few good practices to write good Matlab code...

- Each parameter of the simulation should be grouped at the beginning of the file.
- Please follow the notation given in [Isk08] and in this document, it will enable your teacher to help you more efficiently. For example, use `at` for $\tilde{\boldsymbol{a}}$, `U` for $\boldsymbol{U}$...
- Do not hard code values in the programs bodies; use suggestive constant names instead.
- Begin your programs with instructions `clearvars` and `close all` to empty your workspace.
- Do not copy pieces of source code from a PDF file, they might include invisible special characters and lead to syntax errors.
- Use instructions `help` and `doc` before using any new function (and asking to your teacher how it works).

1.1 : Preparatory work: reading a technical paper

To to develop (from scratch) our channel simulator, we will mainly refer to [Isk08] throughout this practice lab.
Here are few tricks to read a technical paper...

1. If possible, print the manuscript. You'll find useful to take notes and to underline portions of the paper.
2. First reading: just read the abstract, introduction and conclusion (10 minutes maximum). It will be sufficient to grab the context, contribution and the main results of the work. It also gives an insight of the relevance of the paper.
3. Second reading: focus on the system model and results. Do not hesitate to emphasize difficult steps to focus on later. It is the most time-consuming step since several derivations should be verified (few hours to few weeks...).
4. Beyond the paper: use cross-references and your favorite scientific search engine to enlighten unknown facts and obscure derivations.

In this labwork, the reader should focus on Section III. Examination of cross-references shall not be necessary. Do not worry about what I said with regard to the second reading: the following instructions will help you to complete the lab within a reasonable time.

1.2 : Static channel: interpolation and matrix based convolution

In this part, we will first focus on the simulation of a frequency selective only channel (time selectivity will be added in Sec. 1.3). It is thus a particular case where the columns of $\tilde{\boldsymbol{a}}$ are equal.

1.2.1 : Set general parameters of your simulation

First of all, I will suggest a set of "default parameters" to help you to verify each step. Of course, you will be encourage to change those values from time to time to qualify the behavior of the simulator.

```
% Time of arrival of each path [s]
tau=[0 0.2 0.5 1.6 2.3 5]*1e-6;
% Average gain of each path [SI]
sigma2a=10.^([-3 0 -2 -6 -8 -10]./10);
% Total number of paths
K=length(tau);

% Bandwidth of the transmitted signal [Hz]
B=19e6;
% Sampling period [s]
Ts=1/B;
% Transmitted block length (should be greater than N1+N2+1)
N=1000;

% Number of taps (N1+N2+1)
N1=10;
N2=ceil(tau(end)/Ts)+10;
```

1.2.2 : Generation the channel random coefficients

In this work, we will focus on Rayleigh fading which mean that the coefficients of $\tilde{\boldsymbol{a}}$ will follow a circular-symmetric Gaussian distribution. In other words, $[\tilde{\boldsymbol{a}}]_{k,n} \sim \mathcal{CN}(0, \sigma_a^2(k))$ where $\sigma_a^2(k)$ denotes the average path gain of the kth path (*e.g.*, the second path of the default parameters has $\sigma_a^2(2) = 0$ dB). Furthermore, as we restrict our analysis to static channels, we will form $\tilde{\boldsymbol{a}}$ as a replication of a single column denoted $\tilde{\boldsymbol{a}}_{\boldsymbol{c}} = [\tilde{a}_1, \ldots, \tilde{a}_K]^T$ taken as a realization of the Gaussian process.
Instructions:

1. create the column vector $\tilde{\boldsymbol{a}}_{\boldsymbol{c}}$ as a complex Gaussian realization, as explained above;
2. form the matrix $\tilde{\boldsymbol{a}}$ of size $(K \times N)$ from $\tilde{\boldsymbol{a}}_{\boldsymbol{c}}$.

Tests:

- make sure that the mean power of each coefficient of $\tilde{\boldsymbol{a}}_{\boldsymbol{c}}$ is as expected (to do so, proceed to several hundreds of random experiments and compute the empirical mean power);
- check the size of $\tilde{\boldsymbol{a}}$.

Functions: `sqrt`, `randn`, `repmat`, `histfit`...

Question 1

What is the statistical distribution of a coefficient $|\tilde{a}_k|$, $k \in \{1, \dots, K\}$? Plot its histogram through several hundred of random experiments.

1.2.3 : Generation of the interpolation matrix

Instructions:

1. create the interpolation matrix $\boldsymbol{\alpha}$;
2. compute the interpolated coefficient $\tilde{\boldsymbol{g}}$.

Tests:

- check the size of $\boldsymbol{\alpha}$ and $\tilde{\boldsymbol{g}}$;
- try to set the delays $\boldsymbol{\tau}$ as integer multiples of the sampling period T_s.

Functions: `meshgrid`, `sinc`, `fft`, `fftshift`, `abs`, `log10`...

Question 2

Display (on the same graph) the infinite bandwidth impulse response (*i.e.*, a column of $\tilde{\boldsymbol{a}}$) along with the bandlimited impulse response (*i.e.*, a column of $\tilde{\boldsymbol{g}}$). Since those coefficients are complex, plot their modulus. How to choose N_1 and N_2 ?

Question 3

Compute the frequency response of the channel. To this extent, compute the DFT of a column of $\tilde{\boldsymbol{g}}$ (zero-pad the DFT by a factor 8 for a smooth display). Plot the power spectrum (in dB) between $-B/2$ and $B/2$.

Question 4

Discuss the impact of B, $\boldsymbol{\tau}$, $\boldsymbol{\sigma_a^2} = [\sigma_a^2(1), \dots, \sigma_a^2(K)]$ with regard to the channel's impulse and frequency responses.

1.2.4 : Using the channel with a QPSK signal

Instructions:

1. generate a sequence $\tilde{\boldsymbol{s}}$ of N QPSK symbols;
2. compute the signal matrix $\boldsymbol{U}$ by assuming $\boldsymbol{u} = \boldsymbol{0}$;
3. compute the output $\tilde{\boldsymbol{y}}$ of the channel.

Tests:

- check the constellation of $\tilde{\boldsymbol{s}}$;
- start by building a small scale matrix $\boldsymbol{U}$ with dummy a input and verify its first and last values with respect to (21);
- check that $\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{s}}$ if $\tilde{\boldsymbol{a}}_{\boldsymbol{c}} = [1, 0, \dots, 0]^T$ (*i.e.*, ideal channel).

Functions: `toeplitz`, `zeros`, `sum`, `round`, `rand`, `scatterplot`...

Question 5

What is the effect of the channel on the received constellation ? Discuss the impact of B, $\boldsymbol{\tau}$, $\boldsymbol{\sigma_a^2}$.

Question 6

Compare $\tilde{\boldsymbol{y}}$ with the ouput of `filter(gt(:,1),1,st)`. Use for the example the Euclidean distance as a comparison metric. Name the relation introduced by the channel between $\tilde{\boldsymbol{s}}$ and a column of $\tilde{\boldsymbol{g}}$.

Question 7

What is the excess bandwidth of the transmission filter assumed in this scenario ? In a more general framework, how should be sampled the transmitted signal ?

1.3 : Time-variant channel: white noise filtering

In this part, we will extend the previous simulator to the case of time-selective channels. In this case, the columns of $\tilde{\boldsymbol{a}}$ will evolve with respect to the specified Doppler spectrum. The proposed approach consist in generating a complex white Gaussian noise (WGN) to be filtered by the desired Doppler filter.

1.3.1 : Set general parameters of your simulation

Again, the following "default parameters" are proposed to help you to verify your result. For the sake of simplicity (and at the expense of an unnecessary computational complexity), we will avoid the interpolation operation proposed in [Isk08, Sec. III.C.3] by directly sampling the WGN at the input sampling rate B.

```
% Maximum Doppler frequency [Hz]
fd=100e4;
% Doppler filter sampling frequency and period
fs=B;
% Doppler filter sampling period
ts=1/fs;
% Doppler filter length
M=1000;
```

1.3.2 : White noise generation

Instructions:

1. define $N_p = M + N$, the number of WGN samples to be generated for each path;
2. generate a matrix $\boldsymbol{x}$ of size $(K \times N_p)$ filled with (centered and reduced) Gaussian samples.

Tests:

- make sure that the mean power of each coefficient of $\boldsymbol{x}$ is 1 (to do so, proceed to several hundreds of random experiments and compute the empirical mean power);
- check the size of $\boldsymbol{x}$.

Functions: `randn`, `sqrt`...

1.3.3 : Doppler filter specification

Instructions:

1. generate the Doppler filter impulse response $\boldsymbol{h}$ (of length M) using (27);
2. normalize the Doppler filter impulse response as in (50).

Tests:

- make sure that $\boldsymbol{h}$ is properly normalized.
- check if $\boldsymbol{h}$ has no discontinuities.
- check if the truncation of the impulse response to M coefficients is appropriate.

Functions: `gamma`, `besselj`, `abs`, `norm`...

1.3.4 : WGN filtering and power scaling

Instructions:

1. perform a row-wise filtering of the WGN $\boldsymbol{x}$ by $\boldsymbol{h}$ to obtain $\tilde{\boldsymbol{a}}$: `at=filter(h,1,x,[],2);`
2. discard the first M columns of $\tilde{\boldsymbol{a}}$ (correspond to the transient phase of the filtering operation);
3. perform a power scaling of each row of $\tilde{\boldsymbol{a}}$ as in (60).

Tests:

- check the size of $\tilde{\boldsymbol{a}}$.

Functions: `filter`...

Question 8
What is the theoretical power of the signal at the output of the Doppler filter (before power scaling) ? Verify this value by simulation: focus on a single coefficient of $\tilde{\boldsymbol{a}}$ and compute its mean power over several hundreds of random experiments. Same question after power scaling.

Question 9
Estimate the power spectral density of a row of $\tilde{\boldsymbol{a}}$ using the periodogram. Comment the obtained result.

Question 10
Plot a row of $\tilde{\boldsymbol{a}}$ (modulus) as a function of time. Discuss the influence of f_d...

1.3.5 : Using the channel with a QPSK signal

Instructions:
As in the previous part...

1. generate a sequence $\tilde{\boldsymbol{s}}$ of N QPSK symbols;
2. compute the signal matrix $\boldsymbol{U}$ by assuming $\boldsymbol{u} = \boldsymbol{0}$;
3. compute the output $\tilde{\boldsymbol{y}}$ of the channel.

Functions: `toeplitz`, `zeros`, `sum`, `round`, `rand`, `scatterplot`...

Question 11
Express the relation between f_d and the maximum radial velocity $v_{\max}$ between the transmitter and the receiver.

Question 12
Modify $\boldsymbol{\tau}$ and $\boldsymbol{\sigma_a^2}$ to consider a single path channel. What is the effect of such a channel on the received constellation ? Discuss the impact of f_d or equivalently $v_{\max}$.

References

[Eur17] European Telecommunications Standards Institute. 5g; study on channel model for frequencies from 0.5 to 100 GHz (3GPP TR 38.901 version 14.0.0 release 14), etsi tr 138 901, May 2017. Online: http://www.etsi.org/deliver/etsi_tr/138900_138999/138901/14.00.00_60/tr_138901v140000p.pdf.

[Isk08] C.D. Iskander. A MATLAB-based object-oriented approach to multipath fading channel simulation. Technical report, Hi-Tek Multisystems, 2008. Online: http://dsp.magdy.me/uploads/1/9/6/6/19664349/channelmodelingwhitepaper.pdf.