MIMO Systems

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- MIMO information theory aspects
- Transmission over MIMO systems
  - Generalities
  - Tansmit diversity
  - Spatial Multiplexing
- Applications

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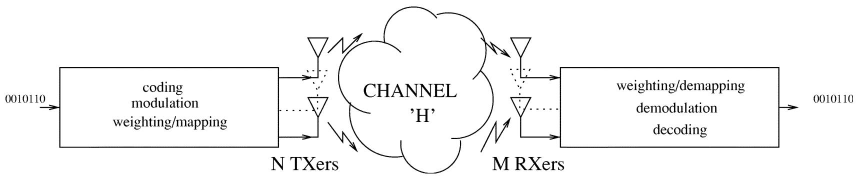
Principles of MIMO systems



- Multiple antennas at receiver AND emitter
- TIME dimension is complemented with SPATIAL dimension
- Objectives:
  - ➢Improve BER performances (diversity)
  - ➢Increase data rate (multiplexing gain)

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Recall : conventionnal smart antennas

- **Spatial diversity** (essentially receiver side)
  - ➢ Diversity order : number of decorrelated spatial branches available at the receiver
- **Beamforming** is essentially at the emitter side

  ⇒SNR and C/I improvment

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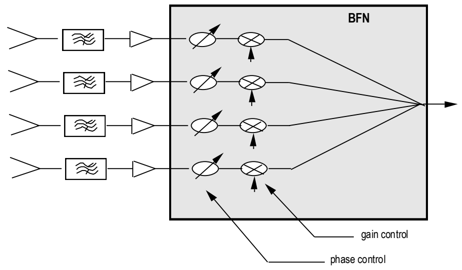
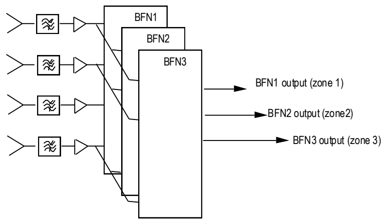
Beamforming Networks (1)

ANALOG IMPLEMENTATION



BFN principle



Example with 3 BFN's

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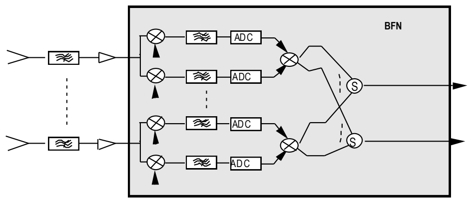
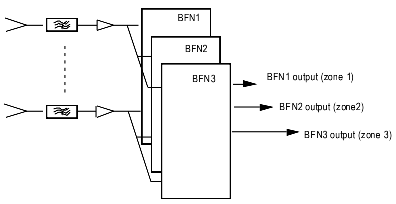
Beamforming Networks (2)

DIGITAL IMPLEMENTATION



BFN principle



Example with 3 BFN's

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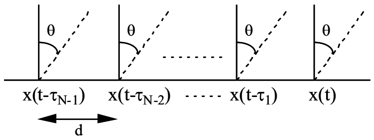
Beamforming Networks (3)



$$x(t-\tau) = \mathrm{Re}\left\{x_e(t-\tau_i)e^{j\omega_0 t}e^{j\Delta\Phi_i}\right\}$$
$$\cong \mathrm{Re}\left\{x_e(t)e^{j\omega_0 t}e^{j\Delta\Phi_i}\right\} if\ B << f_0$$
$$\tau_i(\theta) = i\frac{d}{c}\sin\theta, \Delta\Phi_i(\theta) = -2\pi\frac{d}{\lambda}i\sin\theta$$

After down-conversion and I-Q mixing, we have N complex signals :

<u>BFN output :</u> $$y(t) = \sum_{i=0}^{N-1} w_i y_i(t)$$

BFN performs SPATIAL FILTERING of the input signal.

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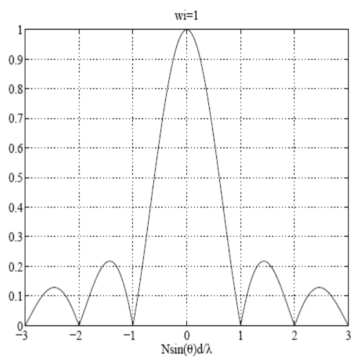
Beamforming Networks (4)

- <u>Example 1:</u>

wi=1

y(t)=xe(t)•F(q) with $$|F(\theta)| = \left|\frac{\sin\left(\frac{\pi}{\lambda}Nd\sin\theta\right)}{\sin\left(\frac{\pi}{\lambda}d\sin\theta\right)}\right|$$



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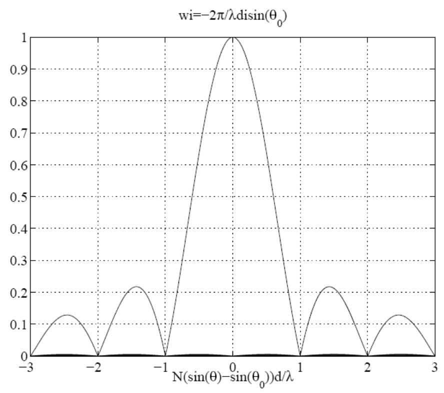
Beamforming Networks (5)

- **Example 2:**

$$w_i = \exp\left(\frac{2\pi}{\lambda} d i \sin\theta_0\right) \qquad |F(\theta)| = \left| \frac{\sin\left(\frac{\pi}{\lambda} N d[\sin\theta - \sin\theta_0]\right)}{\left(\frac{\pi}{\lambda} d[\sin\theta - \sin\theta_0]\right)} \right|$$

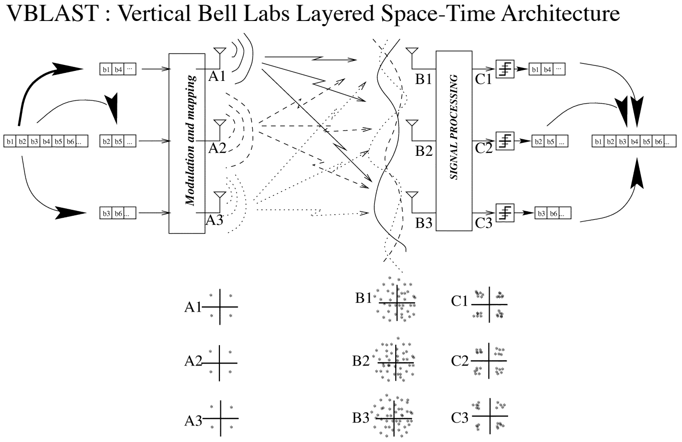


Differences between smart antennas and MIMO systems (1)

•Data are transmitted over a **matrix** rather than a vector (for linear beamformer)

Ex : VBLAST (1999) Spatial Multiplexing

VBLAST : Vertical Bell Labs Layered Space-Time Architecture



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Differences between smart antennas and MIMO systems (2)

VBLAST :

- Spectral efficiency is increased by a factor 3
- Linear system of 3 equations
- Channel matrix estimation is performed via pilot symbols
- Separation is assumed only if the channels are sufficiently independant (analogy with signatures in CDMA)
- Flat fading is assumed (if not, MIMO-MC-CDMA is a possible solution)

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Differences between smart antennas and MIMO systems (3)

- MIMO exploits multipaths
- N TX antennas, M RX antennas
- Conventionnal smart antenna (beamforming) are better suited to LOS or close to LOS conditions.
- Conventionnal smart antenna (diversity) are less efficient than MIMO systems

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Element of information theory (1)

- SISO systems

$$C = \log_2\left(1+\rho|h|^2\right) \text{b.s}^{-1}.Hz^{-1}$$

h: normalized gain of the channel

ρ : SNR at any RX antenna

- SIMO systems

$$C = \log_2\left(1+\rho\sum_{i=1}^{M}|h_i|^2\right) \text{b.s}^{-1}.Hz^{-1}$$

C increases logarithmically with M

Array gain

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Element of information theory (2)

- MISO (channel unknown at emitter)

$$C = \log_2\left(1+\frac{\rho}{N}\sum_{i=1}^{M}|h_i|^2\right) \text{b.s}^{-1}.Hz^{-1}$$

(fixed total transmitted power from N transmitters)

No array gain

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Element of information theory (3)

- MIMO channel : N TX antennas, M RX antennas

$$C_{EP} = \log_2\left(\det\left[I_M + \frac{\rho}{N} HH^*\right]\right) \text{b.s}^{-1}.Hz^{-1}$$

H : MxN channel matrix

EP : equal power uncorrelated sources

C grows **LINEARLY** with m=min(M,N) when the eigenvalues do not decay rapidly which is the case in practice (antennas not too closed)

Element of information theory (4)

C_{EP} is a RANDOM variable

- Ergodic capacity : $E(C_{EP})$
- Outage capacity (usually evaluated by simulation)

  Ex: $C_{0.1}$: capacity value supported 90% of the time

  $C_{0.01}$: capacity value supported 99% of the time

Element of information theory (5)

General formula :

$$r = Hs + n$$

$$C = \log_2\left(\det\left[I_M + HQH^*\right]\right)\text{b.s}^{-1}.Hz^{-1}$$

$$Q = \text{cov}(s) \text{ with } trace(Q) \le \rho$$

For equal power uncorrelated sources : $Q=(\rho/N)I_N$=> $C=C_{EP}$

Questions:

- What is the impact of Q on the capacity ?
- What is the impact of the channel on the capacity ?

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Element of information theory (6)

- Impact of Q on the capacity

  Two cases :

  - Channel unknown at the receiver
  - Channel known at the receiver

Case 1 : channel unknown at the receiver

$Q=(\rho/N)I_N$ is shown to be optimal (equal power between antennas)

Case 2 : channel known at the receiver

$\Rightarrow$ **Waterfilling** solutions

Q is optimized subject to trace(Q)≤ρ

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Element of information theory (7)

- Impact of channel H on capacity

$$a)C_{EP} = \sum_{i=1}^{m} \log_2\left(1+\frac{\rho}{N}\lambda_i\right)$$

b) SVD of H : H=UDV* with U,V : unitary matrix, D : diagonal matrix (m=min(M,N))

$$D = diag(\sqrt{\lambda_1},\sqrt{\lambda_2},\ldots,\sqrt{\lambda_m},0,\ldots,0)$$

$$r = Hs + n \Rightarrow \tilde{r} = D\tilde{s} + \tilde{n}\ (\tilde{r} = U^{*}r, \tilde{s} = V^{*}s, \tilde{n} = V^{*}n)$$

MIMO channel ⇔ m parallel SISO channels with signal power λ_i

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MIMO systems

- Two types of transmission schemes:
  - Diversity maximization
  - Data rate maximization

Diversity maximization : ST-TC (space time trellis codes)
ST-BC (space time bock codes)

Data rate maximization : Spatial multiplexing

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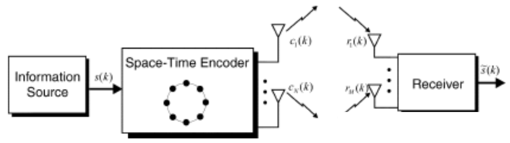
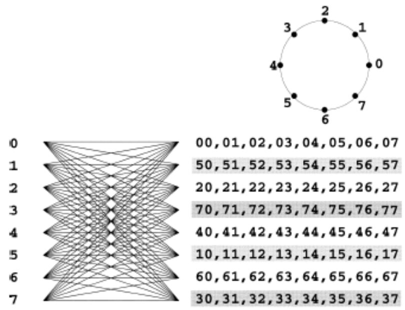
Diversity maximization (1)

Space-Time Trellis Code (STTC)

Example with Spectral efficiency equal to 3b/symbol



Input:	0	1	5	7	6	4
Tx 1:	0	0	5	1	3	6
Tx 2:	0	1	5	7	6	4



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Diversity maximization (2)

Space-Time Trellis Code (STTC)

-no loss of spectral efficiency

- Diversity gain ≤NM

-coding gain

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Diversity maximization (3)

Space Time Block codes :

- Simpler processing (linear processing) than STTC
- Diversity gain
- No coding gain

Example : Alamouti scheme, 1998 , 2x1 STBC

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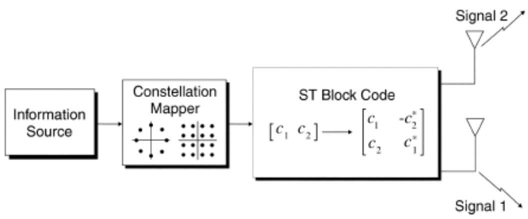
Diversity maximization (4)

Exercise : Alamouti scheme :



h_1 : channel TX #1 – RX , h_2 : channel TX #2 – RX

Hyp: h_i constant over 2 consecutive symbols

The received signals over two consecutive periods are noted r_1 and r_2. The associated thermal noise is noted n_1 and n_2 .

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Diversity maximization (5)

1) Express r_1 and r_2 in function of $c_1, c_2, n_1, n_2, h_1, h_2$
2) Let
$$r = [r_1 \quad r_2^*]^T \quad c = [c_1 \quad c_2]^T \quad n = [n_1 \quad n_2^*]^T$$

Show that r can be expressed as : r=Hc+n where H is a matrix to be defined.

3) Show that $H^*H=\alpha I_2$ where α is to be defined (I_2 : 2x2 identity matrix) and * means transpose conjugate.

4)Let: $\tilde{r} = H^* r \ \ with\ \ \tilde{r} = [\tilde{r}_1 \quad \tilde{r}_2]$

Sow that : $\tilde{r} = \alpha c + \tilde{n}$ with $\tilde{n} = H^* n$ where $\tilde{n}$ is a complex gaussian process with zero mean and covariance $\alpha N_0 I_2$.

5) Deduce the decoding rule for c_1 and c_2. What can you say about the decoding complexity ?

6)What is the diversity order of the system?

7)Draw a block diagram of the receiver.

8)What is the transmission rate (=transmited rate/information rate)?

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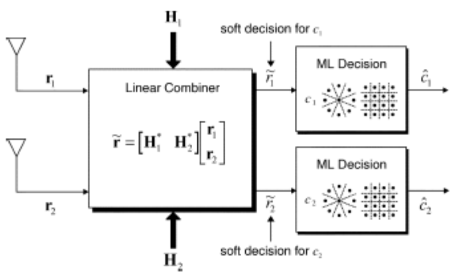
Diversity maximization (6)

<u>Extensions of Alamouti scheme:</u>

2x2 STBC



Show that diversity order=4 and transmission rate=1, power gain : 0 dB

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Diversity maximization (7)

STBC for N > 2 TX antennas

- No existing solutions for complex constellations and a transmission rate =1
- For transmission rate <1 STBC can be found.

  Example : 4 TX antennas, 1 RX antennas => 4x1 STBC

$$C = \begin{bmatrix} c_1 & -c_2 & -c_3 & -c_4 & c_1^* & -c_2^* & -c_3^* & -c_4^* \\ c_2 & c_1 & c_4 & -c_3 & c_2^* & c_1^* & c_4^* & -c_3^* \\ c_3 & -c_4 & c_1 & c_2 & c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & c_3 & -c_2 & c_1 & c_4^* & c_3^* & -c_2^* & c_1^* \end{bmatrix}$$

Diversity order : 4 , Transmission rate : ½, power gain : 3dB

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Diversity maximization (8)

- Channel estimation has to be performed
  - ➢ Use of pilot symbols
- STBC in frequency selective channels
  - ➢ Use of equalization methods (difficult)
  - ➢ Use of MIMO-OFDM or MIMO-MC-CDMA schemes

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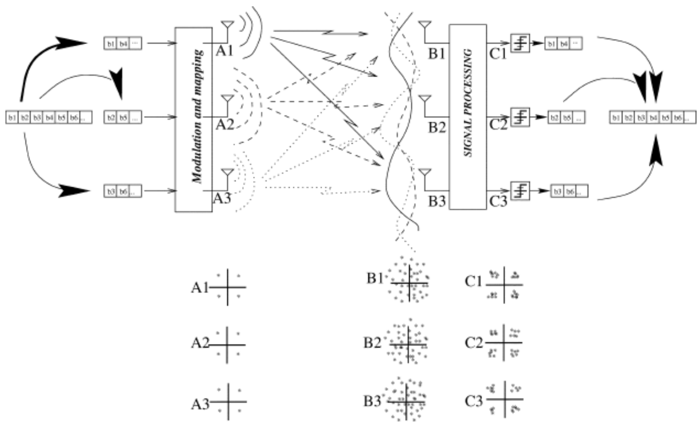
Data rate maximization (1)

- Independant bit streams are sent over different antennas

Example : 3x3 VBLAST



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Data rate maximization (2)

- Generalization : NxM MIMO system

  Transmitted data block C : NxL

  Received data block Y : MxL

  $$Y=HC+N$$

- MIMO receiver (estimation of C)
  - Linear receivers (ZF, MMSE)
  - Non linear receiver (ML, SIC,…)

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Data rate maximization (3)

Ex1 : ZF for 3x3 MIMO system

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + noise = H \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + noise$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = H^{-1} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

Ex1 : ZF for MxN MIMO system

$$\begin{bmatrix} r_1 \\ r_2 \\ \dots \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \dots \\ h_{21} & h_{22} & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \end{bmatrix} + noise = H \begin{bmatrix} x_1 \\ x_2 \\ \dots \end{bmatrix} + noise$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \dots \end{bmatrix} = H^{\#} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \text{ with } \mathrm{H}^{\#} \text{ pseudo-inverse } H^{\#} = (H^{*}H)^{-1}H^{*}$$

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Applications

Standarts:

- LTE :

MIMO+OFDM : date rate 10 times higher than HSPA in urban environment.

- WIFI : 802.11n
- WIMAX : 802.16e
- 4G : MIMO-OFDM
- 5G : massive MIMO

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