Advanced Theory of Communication

University of Tehran

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Homework 5

Due : 1403/2/10

Problem 1

In the communication system shown in Figure 1., the receiver receives two signals r_1 and r_2, where r_2 is a "noisier" version of r_1. The two noises n_1 and n_2 are arbitrary not necessarily Gaussian, and not necessarily independent. Intuition would suggest that since r_2 is noisier than r_1, the optimal decision can be based only on r_1; in other words, r_2 is irrelevant. Is this true or false? If it is true, give a proof; if it is false, provide a counterexample and state under what conditions this can be true.

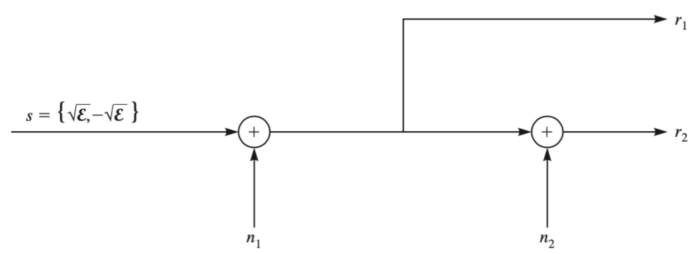

Figure 1

Problem 2

A binary digital communication system employs the signals:

$$
\begin{aligned}
s_0(t) &= 0 \quad 0 \le t \le T \\
s_1(t) &= A \quad 0 \le t \le T
\end{aligned}
$$

for transmitting the information. This is called on-off signaling.

a) Determine the optimum detector for an AWGN channel and the optimum threshold, assuming that the signals are equally probable.

b) Determine the probability of error as a function of the SNR. How does on-off signaling compare with antipodal signaling?

Problem 3

A communication system transmits one of the three messages m_1 , m_2 , and m_3 using signals $s_1(t)$, $s_2(t)$, and $s_3(t)$. The signal $s_3(t) = 0$, and $s_1(t)$ and $s_2(t)$ are shown in Figure 2. The channel is an additive white Gaussian noise channel with noise power spectral density equal to $\frac{N_0}{2}$.

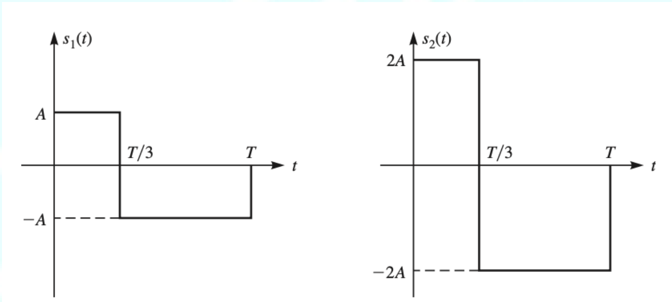

Figure 2

a) Determine an orthonormal basis for this signal set, and depict the signal constellation.

b) If the three messages are equiprobable, what are the optimal decision rules for this system? Show the optimal decision regions on the signal constellation you plotted in part 1.

c) If the signals are equiprobable, express the error probability of the optimal detector in terms of the average SNR per bit.

d) Assuming this system transmits 3000 symbols per second, what is the resulting transmission rate (in bits per second)?

Problem 4

Suppose that binary PSK is used for transmitting information over an AWGN with a power spectral density of $\frac{N_0}{2} = 10^{-10}$ W/HZ. The transmitted signal energy is $\mathcal{E}_b = \frac{1}{2}A^2T$, where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6} when the data rate is:

a) 10 kilobits/s

b) 15 megabits/s

c) 1 gigabits/s

Problem 5

Consider a signal detector with an input

$$r = \pm A + n \tag{1}$$

where $+A$ and $-A$ occur with equal probability and the noise variable n is characterized by the Laplacian PDF defined as

$$f(x) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{|x|\sqrt{2}}{\sigma}} \tag{2}$$

a) Determine the optimal decision rule and compute the optimal probability of error as a function of the parameters A and σ.

b) Determine the SNR required to achieve an error probability of 10^{-5}. How does the SNR compare with the result for a Gaussian PDF?

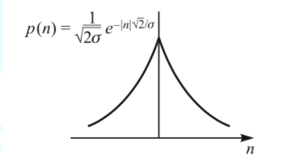

Figure 3

Problem 6

A binary communication system uses two equiprobable messages $s_1(t) = p(t)$ and $s_2(t) = -p(t)$. The channel noise is additive white Gaussian with power spectral density $\frac{N_0}{2}$ and signals have prior probabilities p and $1-p$. Assume that we have designed an optimal receiver for this channel, and let the error probability for the optimal receiver be P_e.

a) Find an expression for P_e.

b) If this receiver is used on an AWGN channel using the same signals but with the noise power spectral density $N_1 > N_0$, find the resulting error probability P_1 and explain how its value compares with P_e.

c) Let P_{e1} denote the error probability in part 2 when an optimal receiver is designed for the new noise power spectral density N_1. Find P_{e1} and compare it with P_1.

Problem 7

The four signals shown in Figure 4. are used for communication of four equiprobable messages over an AWGN channel. The noise power spectral density is $\frac{N_0}{2}$.

a) Find an orthonormal basis,with lowest possible N,for representation of the signals.

b) Plot the constellation,and using the constellation, find the energy in each signal.What is the average signal energy and what is $E_{b,avg}$?

c) On the constellation that you have plotted, determine the optimal decision regions for each signal, and determine which signal is more probable to be received in error.

d) Now analytically (i.e., not geometrically) determine the shape of the decision region for signal $s_1(t)$, i.e., D_1, and compare it with your result in part 3.

Problem 8

In a binary communication system two equiprobable messages $\mathbf{s}_1 = (1, 1)$ and $\mathbf{s}_2 = (-1, -1)$ are used. The received signal is $\mathbf{r} = \mathbf{s} + \mathbf{n}$, where $\mathbf{n} = (n_1, n_2)$. It is assumed that n_1 and n_2 are independent and each is distributed according to:

$$f(n) = \frac{1}{2}e^{-|n|}$$

Determine and plot the decision regions D_1 and D_2 in this communication scheme and derive the probability of error for this system.

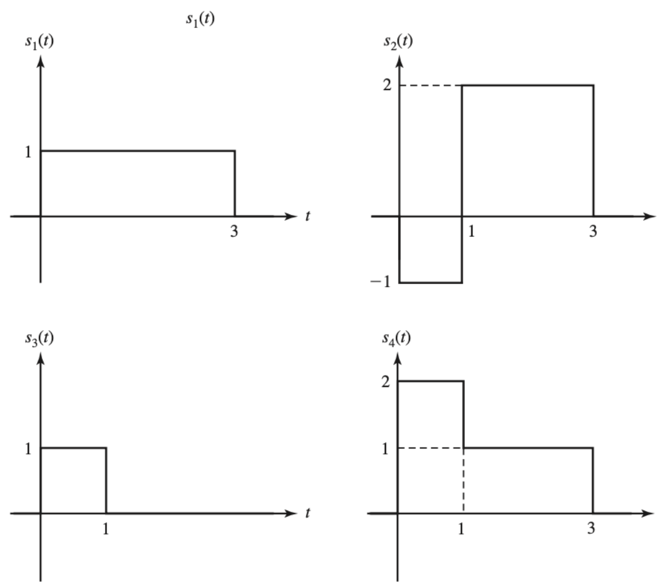

Figure 4

Problem 9

A digital communication system with two equiprobable messages uses the following signals:

$$s_1(t) = \begin{cases} 1 & 0 \le t < 1 \\ 2 & 1 \le t < 2 \\ 0 & otherwise \end{cases}$$

$$s_2(t) = \begin{cases} 1 & 0 \le t < 1 \\ -2 & 1 \le t < 2 \\ 0 & otherwise \end{cases}$$

a) Assuming that the channel is AWGN with noise power spectral density $\frac{N_0}{2}$,determine the error probability of the optimal receiver and express it in terms of $\frac{E_b}{N_0}$. By how many decibels does this system underperform a binary antipodal signaling system?

b) Assume that we are using the two-path channel shown in Figure 5figure.caption.5 . in which we receive both $r_1(t)$ and $r_2(t)$ at the receiver. Both $n_1(t)$ and $n_2(t)$ are independent white Gaussian processes each with power spectral density $\frac{N_0}{2}$. The receiver observes both $r_1(t)$ and $r_2(t)$ and makes its decision based on this observation. Determine the structure of the optimal receiver and the error probability in this case.

c) Now assume that $r_1(t) = As_m(t) + n_1(t)$ and $r_2(t) = s_m(t) + n_2(t)$, where m is the transmitted message and A is a random variable uniformly distributed over the interval $[0, 1]$. Assuming that the receiver knows the value of A, what is his optimal decision rule? What is the error probability in this case? (Note: This last question, regarding the error probability, is asked from you, and you do not know the value of A.)

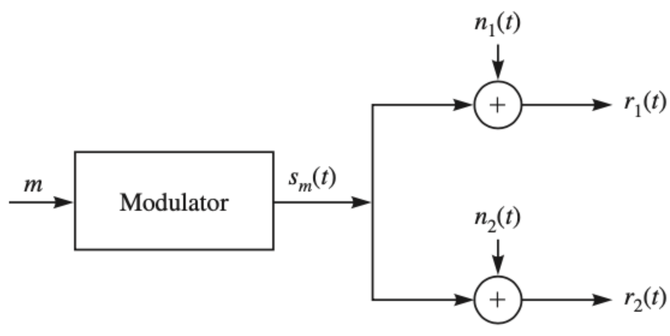

Figure 5