Advanced Theory Of Communication

University of Tehran

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Homework 4

Due : 1403/1/31

Problem 1

Show that 16 - QAM can be represented as a superposition of two four-phase constant envelope signals where each component is amplified separately before summing, i.e.,

$$s(t) = G(A_n Cos(2\pi f_0 t) + B_n Sin(2\pi f_0 t)) + (C_n Cos(2\pi f_0 t) + D_n Sin(2\pi f_0 t))$$

where A_n , B_n, C_n, and D_n are statistically independent binary sequences with elements from the set $\{+1, -1\}$ and G is the amplifier gain. Thus, show that the resulting signal is equivalent to

$$s(t) = I_n Cos(2\pi f_0 t) + Q_n Sin(2\pi f_0 t)$$

and determine I_n and Q_n in terms of A_n, B_n, C_n, and D_n.

Problem 2

Consider a four-phase PSK signal represented by the equivalent lowpass signal

$$u(t) = \sum I_n g(t - nT)$$

where In takes on one of the four possible values $\sqrt{\frac{1}{2}}(\pm 1 \pm j)$ with equal probability. The sequence of information symbols { In } is statistically independent

a) Determine and sketch the power density spectrum of u(t) when

$$g(t) = \begin{cases} A & 0 < t < T \\ 0 & otherwise \end{cases}$$

b) Repeat Part 1 when

$$g(t) = \begin{cases} A\sin(\frac{\pi t}{T}) & 0 < t < T \\ 0 & otherwise \end{cases}$$

c) Compare the spectra obtained in Parts 1 and 2 in terms of the 3-dB bandwidth and the bandwidth to the first spectral zero.

Problem 3

A quadrature partial-response signal (QPRS) is generated by two separate partial-response signals of the type described in Problem 3.14 placed in phase quadrature. Hence, the QPRS is represented as

$$s(t) = Re[v(t)e^{2\pi f_0 t}]$$

where

$$v(t) = v_c(t) + jv_s(t) = \sum B_n g(t - nT) + j \sum C_n g(t - nT)$$

and $B_n = I_n + I_{n-1}$ and $C_n = J_n + J_{n-1}$. The sequences $\{B_n\}$ and $\{C_n\}$ are independent, and $I_n = \pm 1, J_n = \pm 1$ with equal probability.

a) Sketch the signal space diagram for the QPRS signal, and determine the probability of occurrence of each symbol.

b) Determine the autocorrelations and power spectral density of $v_c(t), v_s(t)$, and $v(t)$.

c) Sketch the Markov chain model, and indicate the transition probabilities for the QPRS.

Problem 4

The information sequence $\{a_n\}_{n=-\infty}^{\infty}$ is a sequence of iid random variables, each taking values +1 and -1 with equal probability. This sequence is to be transmitted at baseband by a biphase coding scheme, described by

$$s(t) = \sum a_n g(t - nT)$$

where $g(t)$ is shown in Figure 1

a) Find the power spectral density of $s(t)$.

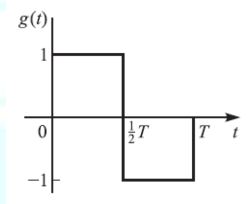

Figure 1:

b) Assume that it is desirable to have a zero in the power spectrum at $f = 1/T$. To this end, we use a precoding scheme by introducing$b_n = a_n + ka_{n-1}$, where k is some constant, and then transmit the $\{b_n\}$ sequence using the same $g(t)$. Is it possible to choose k to produce a frequency null at$f = 1/T$? If yes, what are the appropriate values and the resulting power spectrum?

c) Now assume we want to have zeros at all multiples of $f_0 = 1/4T$. Is it possible to have these zeros with an appropriate choice of k in the previous part? If not, then what kind of precoding do you suggest to achieve the desired result?

Problem 5

The two signal waveforms for binary FSK signal transmission with discontinuous phase are

$$s_0(t) = \sqrt{\frac{2E_b}{T_b}} cos[2\pi(f_c - \frac{\Delta f}{2})t + \theta_0] \qquad 0 \le t < T$$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} cos[2\pi(f_c - \frac{\Delta f}{2})t + \theta_1] \qquad 0 \le t < T$$

where $\Delta f = 1/T \ll f_c$, and θ_0 and θ_1 are independent uniformly distributed random variables on the interval $(0, 2\pi)$. The signals $s_0(t)$ and $s_1(t)$ are equally probable.

a) Determine the power spectral density of the FSK signal.

b) Show that the power spectral density decays as $1/f^2$ for$f \gg f_c$.

Problem 6

The elements of the sequence $\{I_n\}_{n=-\infty}^{\infty}$ are independent binary random variables taking values of ± 1 with equal probability. This data sequence is used to modulate the basic pulse $u(t)$ shown in Figure 2. The modulated signal is

$$X(t) = \sum I_n u(t - nT)$$

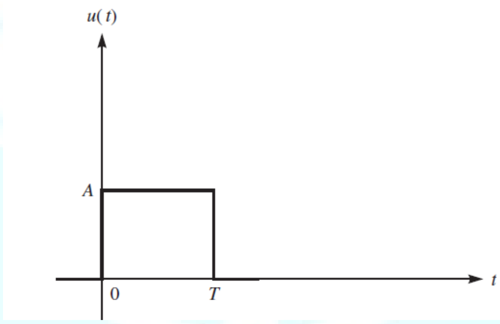

Figure 2:

a) Find the power spectral density of X(t).

b) If $u_1(t)$, shown in Figure 3, were used instead of $u(t)$, how would the power spectrum in part 1 change?

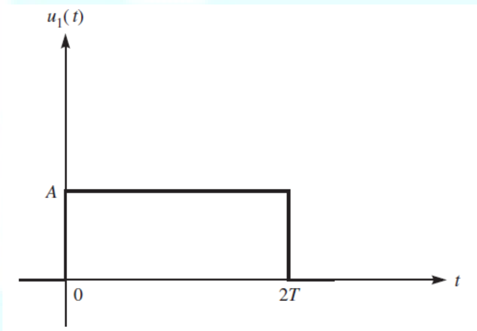

Figure 3:

c) In part 2, assume we want to have a null in the spectrum at $f = \frac{1}{3T}$. This is done by a precoding of the form $b_n = I_n + \alpha I_{n-1}$. Find the value of α that provides the desired null.

d) Is it possible to employ a precoding of the form $b_n = I_n + \sum_{i=1}^{N} \alpha_i I_{n-i}$ for some finite N such that the final power spectrum will be identical to zero for $\frac{1}{3T} \leq |f| \leq \frac{1}{2T}$? If yes, how? If no, why? (Hint: Use properties of analytic functions.)

Problem 7

A binary memoryless source generates the equiprobable outputs $\{a_k\}_{k=-\infty}^{\infty}$ which take values in $\{0, 1\}$. The source is modulated by mapping each sequence of length 3 of the source outputs into one of the eight possible $\{\alpha_i, \theta_i\}_{i=1}^{8}$ pairs and generating the modulated sequence

$$u(t) = \sum \alpha_n g(t - nT) Cos(2\pi f_0 t + \theta_n)$$

where

$$g(t) = \begin{cases} 2t/T & 0 < t < T/2 \\ 2 - 2t/T & T/2 < t < T \\ 0 & otherwise \end{cases}$$

a) Find the power spectral density of $s(t)$ in terms of $\alpha^2 = \sum_{i=1}^{8} |\alpha_i|^2$ and $\beta = \sum_{i=1}^{8} \alpha_i e^{j\theta_i}$

b) For the special case of $\alpha_{odd} = a, \alpha_{even} = b$, and $\theta_i = (i-1)\pi/4$, determine the power spectral density of $s(t)$.

c) Show that for $a = b$, case 2 reduces to a standard 8-PSK signaling scheme, and determine the power spectrum in this case.

d) If a precoding of the form$b_n = a_n \oplus a_{n-1}$ (where $\oplus$ denotes the binary addition) were applied to the source outputs prior to modulation, how would the results in parts 1, 2, and 3 change?

Problem 8

An information source generates the ternary sequence.$\{I_n\}_{n=-\infty}^{\infty}$ Each In can take one of the three possible values 2, 0, and -2 with probabilities 1/4, 1/2, and 1/4, respectively. The source outputs are assumed to be independent. The source outputs are used to generate the lowpass signal.

$$c(t) = \sum I_n g(t - nT)$$

a) Determine the power spectral density of the process v(t), assuming g(t) is the signal shown in Figure 4.

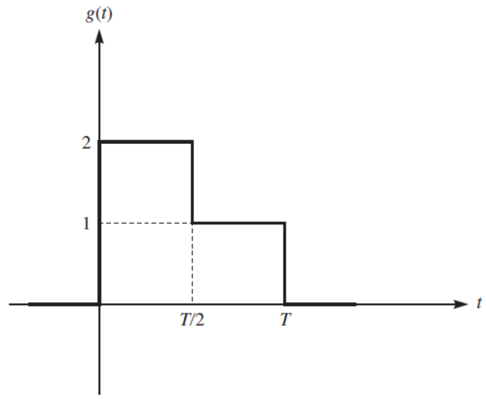

Figure 4:

b) Repeat Part 1 when

$$w(t) = \sum J_n g(t - nT) \quad (1)$$

where $J_n = I_{n-1} + I_n + I_{n+1}$.

Problem 9

The information sequence $\{a_n\}$ is an iid sequence taking the values $-1, 2,$ and 0 with probabilities $1/4, 1/4, and 1/2$. This information sequence is used to generate the baseband signal

$$v(t) = \sum a_n Sinc(\frac{t - nT}{T})$$

a) Determine the power spectral density of v(t).

b) Define the sequence $\{b_n\}$ as $b_n = a_n + a_{n-1} - a_{n-2}$ and generate the baseband signal

$$u(t) = \sum b_n Sinc(\frac{t - nT}{T})$$

Determine the power spectral density of u(t). What are the possible values for the b_n sequence?

c) Determine the power spectral density of u(t). What are the possible values for the b_n sequence?

$$W(t) = \sum c_n Sinc(\frac{t - nT}{T})$$

where $c_n = a_n + ja_{n-1}$. Determine the power spectral density of w(t).(Hint: You can use the relation $\sum_{m=-\infty}^{\infty} e^{-j2\pi fmT} = \frac{1}{T}\sum_{m=-\infty}^{\infty} \delta(f - m/T)$.)

Problem 10

$\{a_n\}_{n=-\infty}^{\infty}$ is a sequence of iid random variables each taking 0 or 1 with equal probability.

a) The sequence bn is defined as $b_n = a_{n-1} \oplus a_n$ where $\oplus$ denotes binary addition (EXCLUSIVE-OR). Determine the autocorrelation function for the sequence bn and the power spectral density of the PAM signal

$$v(t) = \sum b_n g(t - nT)$$

where

$$g(t) = \begin{cases} 1 & 0 < t < T \\ 0 & otherwise \end{cases}$$

b) Compare the result in part 1 with the result when $b_n = a_{n-1} + a_n$.

Problem 11

Sketch the phase tree, the state trellis, and the state diagram for partial-response CPM with $h = 1/2$ and

$$g(t) = \begin{cases} 1/4T & 0 < t < 2T \\ 0 & otherwise \end{cases}$$