Advanced Theory of Communication

University of Tehran

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Homework 6

Due : 1403/2/25

Problem 1

In on-off keying of a carrier modulated signal, the two possible signals are

$$s_0(t) = 0, \quad s_1(t) = \sqrt{\frac{2\varepsilon_b}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b$$

The corresponding received signals are

$$r(t) = n(t), \quad 0 \leq t \leq T_b$$
$$r(t) = \sqrt{\frac{2\varepsilon_b}{T_b}} \cos(2\pi f_c t + \phi) + n(t), \quad 0 \leq t \leq T_b$$

where ϕ is the carrier phase and n(t) is AWGN.

1. Sketch a block diagram of the receiver (demodulator and detector) that employs non-coherent (envelope) detection.
2. Determine the PDFs for the two possible decision variables at the detector corresponding to the two possible received signals.
3. Derive the probability of error for the detector.

Problem 2

Assuming that it is desired to transmit information at the rate of R bits/s, determine the required transmission bandwidth of each of the following six communication systems, and arrange them in order of bandwidth efficiency, starting from the most bandwidth-efficient and ending at the least bandwidth-efficient.

1. Orthogonal BFSK
2. 8 PSK
3. QPSK
4. 64-QAM
5. BPSK
6. Orthogonal16-FSK

Problem 3

A binary communication system uses antipodal signals $s_1(t) = s(t)$ and $s_2(t) = -s(t)$ for transmission of two equiprobable messages m_1 and m_2. The block diagram of the communication system is given in Figure 1figure.caption.1.

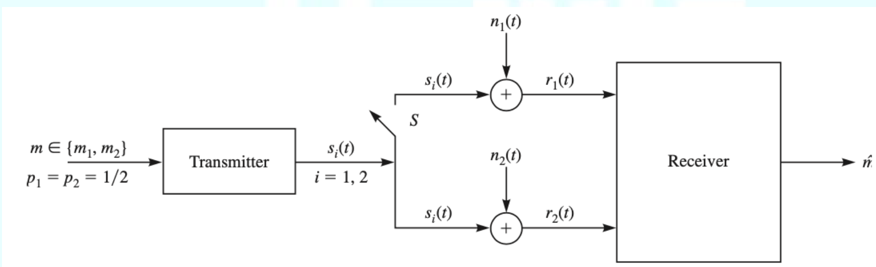

Figure 1

Message $s_i(t)$ is transmitted through two paths to a single receiver, and the receiver makes its decision based on the observation of both received signals $r_1(t)$ and $r_2(t)$. However, the upper channel is connected by a switch S which can either be closed or open. When the switch is open, $r_1(t) = n_1(t)$; i.e., the first channel provides only noise to the receiver. The switch is open or closed randomly with equal probability, but during the transmission it will not change position. Throughout this problem, it is assumed that the two noise processes are stationary, zero-mean, independent, white and Gaussian processes each with a power spectral density of $\frac{N_0}{2}$.

1. If the receiver does not know the position of the switch, determine the optimal decision rule.
2. Now assume that the receiver knows the position of the switch(the switch is still equally likely to be open or closed). What is the optimal decision rule in this case, and what is the resulting error probability?

Problem 4

A fading channel can be represented by the vector channel model $\mathbf{r} = a\mathbf{s}_m + \mathbf{n}$, where a is a random variable denoting the fading, whose density function is given by the Rayleigh distribution

$$p(a) = \begin{cases} \frac{2a}{\Omega} e^{-\frac{a^2}{\Omega}} & a \geq 0 \\ 0 & otherwise \end{cases}$$

1. Assuming that equiprobable signals, binary antipodal signaling, and coherent detection are employed, what is the structure of the optimal receiver?

2. Show that the bit error probability in this case can be written as

$$p_b = \frac{1}{2}(1 - \sqrt{\frac{E_b/N_0}{1 + E_b/N_0}})$$

   and for large SNR values we have

$$p_b \approx \frac{1}{4E_b/N_0}$$

3. Assuming an error probability of 10^{-5} is desirable, determine the required SNR per bit (in dB) if (i) the channel is nonfading and (ii) the channel is a fading channel. How much more power is required by the fading channel to achieve the same bit error probability?

4. Show that if binary orthogonal signaling and noncoherent detection are employed, we have

$$p_b = \frac{1}{2 + E_b/N_0}$$