Advanced Theory Of Communications

University of Tehran

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Homework 1

Due : 1402/12/14

Problem 1

Prove the following properties of the Hilbert transform:

1. If $x(t) = x(-t)$ then $\hat{x}(t) = -\hat{x}(-t)$
2. If $x(t) = -x(-t)$ then $\hat{x}(t) = \hat{x}(-t)$
3. If $x(t) = \cos\omega_0 t$ then $\hat{x}(t) = \sin\omega_0 t$
4. If $x(t) = \sin\omega_0 t$ then $\hat{x}(t) = -\cos\omega_0 t$
5. $\hat{\hat{x}}(t) = -x(t)$
6. $\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} \hat{x}^2(t)\,dt$
7. $\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = 0$

Problem 2

Let $x(t)$ and $y(t)$ denote two bandpass signals, and let $x_l(t)$ and $y_l(t)$ denote their lowpass equivalents with respect to center frequency f_0.We know that in general $x_l(t)$ and $y_l(t)$ are complex signals.

a. Show that

$$\int_{-\infty}^{\infty} x(t)y(t)dt = \frac{1}{2}Re\left[\int_{-\infty}^{\infty} x_l(t)y_l^{\star}(t)dt\right]$$

b. Using the result of part (a) prove $\varepsilon_x = \frac{1}{2}\varepsilon_{x_l}$, i.e., the energy in a bandpass signal is one-half the energy in its lowpass equivalent.

Problem 3

Consider the four waveforms shown in Figure 1.

1. Determine the dimensionality of the waveforms and a set of basis functions.
2. Use the basis functions to represent the four waveforms by vectors $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3$ and $\mathbf{s}_4$.
3. Determine the minimum distance between any pair of vectors.

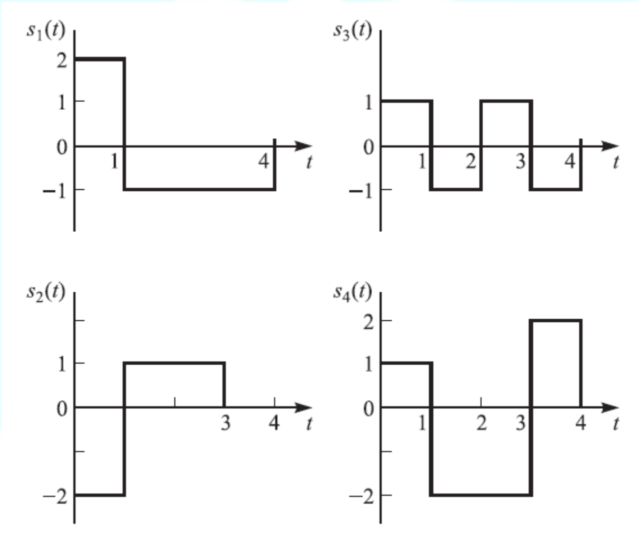

Figure 1:

Problem 4

Suppose $s(t)$ is real band-pass signal and let $s_l(t)$ denote its lowpass equivalent with respect to f_0 , i.e.,

$$s(t) = Re\{s_l(t)e^{j2\pi f_0 t}\}$$

We show Hilbert Transform of $s(t)$ with $\hat{s}(t)$. Express $\hat{s}(t)$ in terms of $s_l(t)$ in a simple form.

Problem 5

$x(t)$ is a bandpass signal with bandwidth W and Fourier Transform $X(f)$. Let $x_i(t)$ and $x_q(t)$ denote the inphase and quadrature components of $x(t)$ with respect to central frequency f_0. Determine the Fourier Transform of $x_i(t)$ and $x_q(t)$ and show that they are both lowpass signals.

Problem 6

Determine a set of orthonormal functions for the four signals shown in Figure 2 and then derive signal space representation of them.

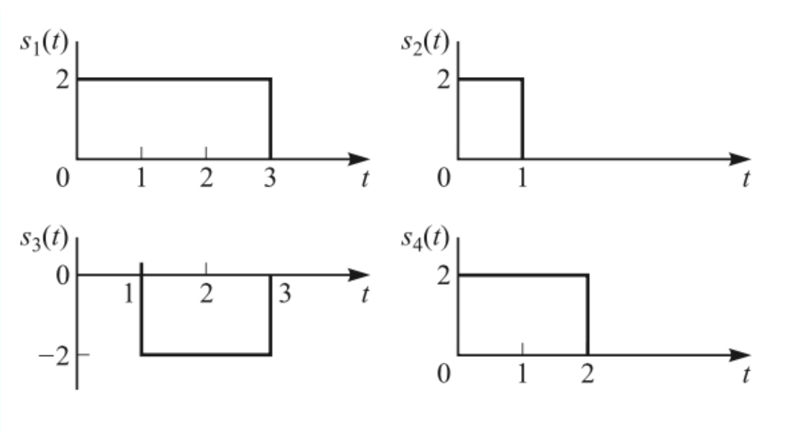

Figure 2:

Problem 7

Suppose $m(t)$ is a bandpass signal with bandwidth of W. We define $s_i(t); i = 1, 2, 3, 4$ as,

$$s_i(t) = m(t)\cos\left(2\pi f_0 t + \frac{(i-1)\pi}{4}\right)$$

where $f_0 \gg W$.

1. Determine a set of orthonormal bases function for the set of signals $s_i(t); i = 1, 2, 3, 4$. What is the dimensionality of these signals?

2. Derive $s_{l,i}(t)$ the lowpass equivqlent of $s_i(t); i = 1, 2, 3, 4$. Determine a set of orthonormal lowpass signals for representation of $s_{l,i}(t)$; $i = 1, 2, 3, 4$. What is the dimensionality of the lowpass signals?