# Advanced Theory of Communication

University of Tehran

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Homework 8

Due : 1403/3/17

**Note**: This problem must be simulated in the **MATLAB** environment.

## Problem 1 Simulation Problem

Suppose we have bandpass BPSK modulation with central frequency $f_0$ and equivalent lowpass pulse shape $g(t)$.

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t \leq T \\ 0 & otherwise \end{cases}$$

The signal is passed through the channel with a lowpass impulse response $c(t)$.

$$c(t) = \begin{cases} \sqrt{\frac{3}{2T}}(1 - \frac{t}{2T}) & 0 \leq t \leq 2T \\ 0 & otherwise \end{cases}$$

After adding white noise with density $\frac{N_0}{2}$ to the received signal, the low pass equivalent signal is passed through the match filter $h^*(-t)$ (where $h(t) = g(t) * c(t)$) , and output of filter $y(t)$ is sampled at $t = nT$ denoted by $y_n = y(nT)$. Assume $T = 1$.

a. Derive the discrete model of this system shown in the Figure 1

b. Generate the sequence $y_n$ in Figure 1 using the discrete model described in part a. Create an $i.i.d$ sequence with equal probability and random variable $I_n \in \{\pm 1\}$ and a suitable Gaussian noise.

   In Figure 1, the $y_n$ sequence enters an equalizer and its output will be used for Symbol-by-Symbol detection. Answer the following questions.

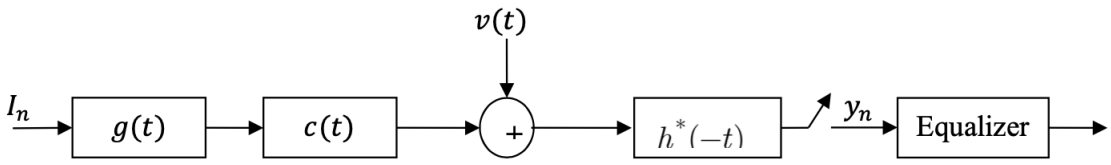


Figure 1

c. Derive the **ZF** equalizer with the number of coefficients equal to $5, 9, 13$. Determine the exact probability of error. By simulation, determine the error probability for SNRs between $0dB$ and $15dB$. Compare the your results with theoretical results by plotting the probability of error ($SNR = \frac{\mathbb{E}\{|I_n|^2\}}{2N_0}$). Also plot the SINR at the output of the equalizer versus SNR.

d. Derive the **MMSE** equalizer with the number of coefficients equal to $5, 9, 13$. Determine the exact probability of error. By simulation, determine the error probability for SNRs between $0dB$ and $15dB$. Compare the results with the theory by plotting the probability of error ($SNR = \frac{\mathbb{E}\{|I_n|^2\}}{2N_0}$). Also plot the SINR at the output of the equalizer versus SNR.

e. Derive the **DFE** equalizer with the number of coefficients equal to $5, 9, 13$. Determine the exact probability of error. By simulation, determine the error probability for SNRs between $0dB$ and $15dB$. Compare the results with the theory by plotting the probability of error ($SNR = \frac{\mathbb{E}\{|I_n|^2\}}{2N_0}$). Also plot the SINR at the output of the equalizer versus SNR. Assume that the number of Feedforward coefficients is one more than the number of Feedback coefficients, i.e, $K_1 = K_2$.

f. Compare the results with the optimal viterbi receiver from the previous homework.