Advanced Theory of Communication

University of Tehran

Instructor: Dr. Ali Olfat — Spring 2024

Homework 9

Due : 1403/3/30

Problem 1

Suppose that the binary signal $\pm s_l(t)$ is transmitted over a fading channel and the received signal is:

$$r_l(t) = \pm a s_l(t) + z(t), \quad 0 \le t \le T$$

where $z(t)$ is zero-mean white Gaussian noise with autocorrelation function

$$R_{zz}(\tau) = 2N_0\delta(\tau)$$

The energy in the transmitted signal is $\mathcal{E} = \frac{1}{2}\int_0^T |s_l(t)|^2\,dt$. The channel gain a is specified by the probability density function

$$p(a) = 0.1\delta(a) + 0.9\delta(a-2)$$

(a) Determine the average probability of error P_b for the demodulator that employs a filter matched to $s_l(t)$.

(b) What value does P_b approach as $\frac{\mathcal{E}}{N_0}$ approaches infinity?

(c) Suppose that the same signal is transmitted on two statistically independently fading channels with gains a_1 and a_2, where

$$p(a_k) = 0.1\delta(a_k) + 0.9\delta(a_k-2), \quad \forall k = 1,2$$

The noises on the two channels are statistically independent and identically distributed. The demodulator employs a matched filter for each channel and simply adds the two filter outputs to form the decision variable. Determine the average P_b.

(d) For the case in (c) what value does P_b approach as $\frac{\mathcal{E}}{N_0}$ approaches infinity?

Problem 2

Consider the model for a binary communication system with diversity as shown in Figure 1. The channels have fixed attenuations and phase shifts. The $z_k(t)$ are complex- valued white Gaussian noise processes with zero-mean and autocorrelation functions

$$R_{zz}(t) = \mathbb{E}\left[z_k^*(t)z_k(t+\tau)\right] = 2N_{0k}\delta(\tau)$$

(Note that the spectral densities N_{0k} are all different.) Also, the noise processes $z_k(t)$ are mutually statistically independent. The β_k are complex-valued weighting factors to be determined. The decision variable from the combiner is

$$U = \Re\left(\sum_{k=1}^{L} \beta_k U_k\right) \gtrless_{-1}^{1} 0$$

1. Determine the PDF $p(u)$ when $+1$ is transmitted.
2. Determine the probability of error P_b as a function of the weightsβ_k.
3. Determine the values of β_k that minimize P_b.

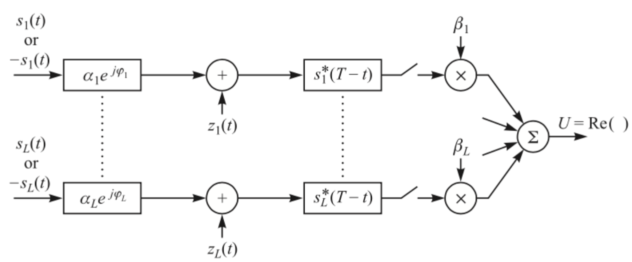

Figure 1

Problem 3

A binary sequence is transmitted via binary antipodal signaling over a Rayleigh fading channel with Lth-order diversity. When $s_l(t)$ is transmitted, the received equivalent low-pass signals are

$$r_k(t) = \alpha_k e^{j\phi_k} s_l(t) + z_k(t), \quad k = 1, 2, \cdots, L$$

The fading among the L subchannels is statistically independent. The additive noise terms $z_k(t)$ are zero-mean, statistically independent, and identically distributed white

Gaussian noise processes with autocorrelation function $R_{zz}(\tau) = 2N_0\delta(\tau)$. Assuming that recever knows only ϕ_k for $k = 1, ..., L$, each of the L signals is passed through a filter matched to $s_l(t)$ and the output is phase-corrected to yield

$$U_k = \Re\left[e^{-j\phi_k} \int_0^T r_k(t)s_l^*(t)\,dt\right], \quad k = 1, 2, \cdots, L$$

The U_k are combined by a linear combiner to form the decision variable

$$U = \sum_{k=1}^{L} U_k$$

(a) Determine the PDF of U conditional on fixed values for the α_k.

(b) If receiver decides on U, determine the expression for the probability of error when the α_k are statistically independent and identically distributed Rayleigh random variables.

(c) Determine the optimum detector when α_k and ϕ_k for $k = 1, ..., L$ are available at receiver and derive the expression for the probability of error when the α_k are statistically independent and identically distributed Rayleigh random variables.

Problem 4

A multipath fading channel has a multipath spread of $T_m = 1s$ and a Doppler spread $B_d = 0.01Hz$.The total channel bandwidth at bandpass available for signal transmission is $W = 5Hz$. To reduce the effects of intersymbol interference, the signal designer selects a pulse duration $T = 10s$.

(a) Determine the coherence band width and the coherence time.

(b) Is the channel frequency selective? Explain.

(c) Is the channel fading slowl or fast? Explain.

(d) Suppose that the channel is used to transmit binary data via (antipodal) coherently detected PSK in a frequency diversity mode. Explain how you would use the available channel bandwidth to obtain frequency diversity and determine how much diversity is available.

(e) For the case in(d),what is the approximate SNR required per diversity to achieve an error probability of 10^{-6}?