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Narula Institute of Technology  
An Autonomous Institute under MAKAUT  
2023

END SEMESTER EXAMINATION - ODD 2023  
M(IT)302 - NUMERICAL METHODS AND STATISTICS

TIME ALLOTTED: 3Hours

FULL MARKS: 70

*Instructions to the candidate:*

*Figures to the right indicate full marks.*

*Draw neat sketches and diagram wherever is necessary.*

*Candidates are required to give their answers in their own words as far as practicable*

**Group A**

**(Multiple Choice Type Questions)**

**Answer any ten from the following, choosing the correct alternative of each question: 10×1=10**

1. The maximum likelihood estimate is a solution of the equation (1) CO1 BL1
  - a)  $\frac{\partial}{\partial \theta}(L(\theta))=0$
  - b)  $\frac{\partial}{\partial \theta}(L(\theta))=\theta$
  - c)  $\frac{\partial}{\partial \theta}(L(\theta))=\text{constant}$
  - d) None of these
  
2. The arithmetic mean of 20 data is calculated to be 89.40 . Later (1) CO1 BL1  
one data 78 is replaced by 87. Then the arithmetic mean of the data  
after replacement will be
  - a) 89
  - b) 85
  - c) 89.85
  - d) 85.89
  
3. In an iterative method, the amount of computation depends on the (1) CO1 BL1  
\_\_\_\_\_
  - a) Degree of accuracy
  - b) Rounding of errors
  - c) Ease of using the operators
  - d) Number of variables

4. Find the sum of the eigen values  $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$  (1) CO1 BL1
- a) 8  
b) 7  
c) 5  
d) 1
5. In the Fixed-Point Iteration method, what happens if the absolute value of  $g'(x)$  is greater than 1 for all  $x$  in the interval containing the fixed point? (1) CO1 BL1
- a) Convergence is guaranteed.  
b) The method may oscillate or diverge.  
c) The method may still converge, but more iteration is needed.  
d) It implies the fixed point is not unique.
6. In the Gauss-Seidel method, what is the condition for convergence? (1) CO1 BL1
- a) The matrix must be non-singular.  
b) The matrix must be symmetric.  
c) The matrix must be diagonally dominant or symmetric positive definite.  
d) Convergence is not guaranteed for Gauss-Seidel.
7. Newton-Raphson method is used to find the root of the equation  $x^2 - 2 = 0$ . (1) CO1 BL1  
If iterations are started from -1, then iterations will be
- a) converge to  $-\sqrt{2}$   
b) Converges to -1  
c) converge to  $\sqrt{2}$   
d) does not converge
8. If  $x$  be test statistic and  $(a,b)$  is region of acceptance corresponding to 3% level of significance then  $P(a \leq x \leq b)$  is (1) CO1 BL1
- a) 0.97  
b) 0.9  
c) 0.95  
d) 0.03
9. Let  $S^2$  be the sample variance of sample of size  $n$  drawn from a population (1) CO1 BL1  
with mean  $\mu$  and S.d.  $\sigma$  then  $E(S^2) =$

- a)  $(n/n-1)\sigma^2$
- b)  $(n-1/n)\sigma^2$
- c)  $(n+1/n)\sigma^2$
- d)  $(n-1/n)\sigma^2$

10. The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric \_\_\_\_\_ definite matrices. (1) CO1 BL1
- a) Positive
  - b) Negative
  - c) Zero
  - d) Equal
11. Which one of the following is an iterative technique for solving a system of simultaneous linear algebraic equations? (1) CO1 BL1
- a) Gauss-Seidel
  - b) Gauss-Elimination
  - c) Gauss-Jordan
  - d) LU-decomposition
12. Which one of the following is a statistic of  $n > 1$  dimension? (1) CO1 BL1
- a) mean of population
  - b) mean of sample
  - c) s.d. of population
  - d) random variable

### Group B

#### (Short Answer Type Questions)

(Answer any three of the following) 3x5=15

13. From a random sample of size 49 drawn from a normal population with standard deviation 2, find the 99% confidence interval of the population mean. Also find the interval if the mean of such a sample is 3. Given that  $\int_0^{2.58} \phi(z) dz = 0.495$ ,  $\phi(z)$  being the standard normal density function. (5) CO3 BL3
14. Use power method to find the largest eigen value of the matrix (5) CO3 BL3
- $$A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$$
15. Develop a recurrence formula for finding  $\sqrt{N}$  using Newton-Raphson method & hence compute  $\sqrt{27}$  to three decimal places. (5) CO4 BL4
16.  $T_1$  and  $T_2$  be two statistics with (5) CO2 BL2
- $$E(T_1) = 2\theta_1 + 3\theta_2 \text{ and } E(T_2) = \theta_1 + \theta_2,$$

find unbiased estimators of the parameters  $\theta_1, \theta_2$ .

17. Perform the five iterations of the bisection method to obtain a root of the equation  $f(x) = x^3 - x - 4 = 0$ . (5) CO4 BL3

**Group C**  
**(Long Answer Type Questions)**  
**(Answer any three of the following) 3x15=45**

18. (a) Let  $\{X_1, X_2, X_3, \dots, X_n\}$  be a random sample from a normal distribution with mean  $\mu$  and variance 1. (15) CO4 BL4

Show that  $\frac{1}{n} \sum_{i=1}^n X_i^2$  is an unbiased estimator of  $\mu^2 + 1$ . [6]

- (b) From the following data estimate the purchase when the sale 100.

Sales:	91	97	108	121	67	124	51	73	111	57
Purchases:	71	75	69	97	70	91	39	61	80	47

To get a purchase of 60 what would be the required sales?

[6]

- (c) Define Standard Error of a Statistic. State Central Limit theorem for Statistics. [3]

19. (a) Solve Equations  $x+y+z=3, 2x-y-z=3, x-y+z=9$  using Gauss Elimination method. [8] (15) CO4 BL4

- (b) The function  $f(x) = 4 \sin x - e^x$  has a zero in the interval  $[0, 0.5]$ . Find this zero correct to four significant digits using the secant method. [7]

20. (a) Solve Equations  $x+y+z=7, x+2y+2z=13, x+3y+z=13$  using Gauss Seidel method. [7] (15) CO3 BL3

- (b) Solve Equations  $x+y+z=3, 2x-y-z=3, x-y+z=9$  using LU-factorization method. [8]

21. (a) Find the root of the following equation  $e^{-x} - \sin x = 0$  by Regula-Falsi method correct to four significant digits. [7] (15) CO3 BL3

- (b) Find a root of  $x^4 - x^3 + 10x + 7 = 0$  correct to three decimal places between  $a = -2$  &  $b = -1$  by Newton Raphson method. [8]

22. (a) If samples are drawn from a Normal population, then the sample variance is a consistent estimator of the population variance. [5] (15) CO3 BL3

(b) The probability density function of a random variable is assumed to be of the form

$f(x) = cx^\alpha, 0 \leq x \leq 1$  for some constant  $c$ . If  $X_1, X_2, X_3, \dots, X_n$  is a random sample of size  $n$ , find the maximum likelihood estimate of  $\alpha$ .

[5]

(c)

The mean and standard deviation of marks of 70 students were found to be 65 and 5.2 respectively. Later it was found that the mark of one student was wrongly recorded as 85 instead of 58. Obtain the correct s.d.

[5]

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