Colorful k-Center Algorithms

Ari Jordan

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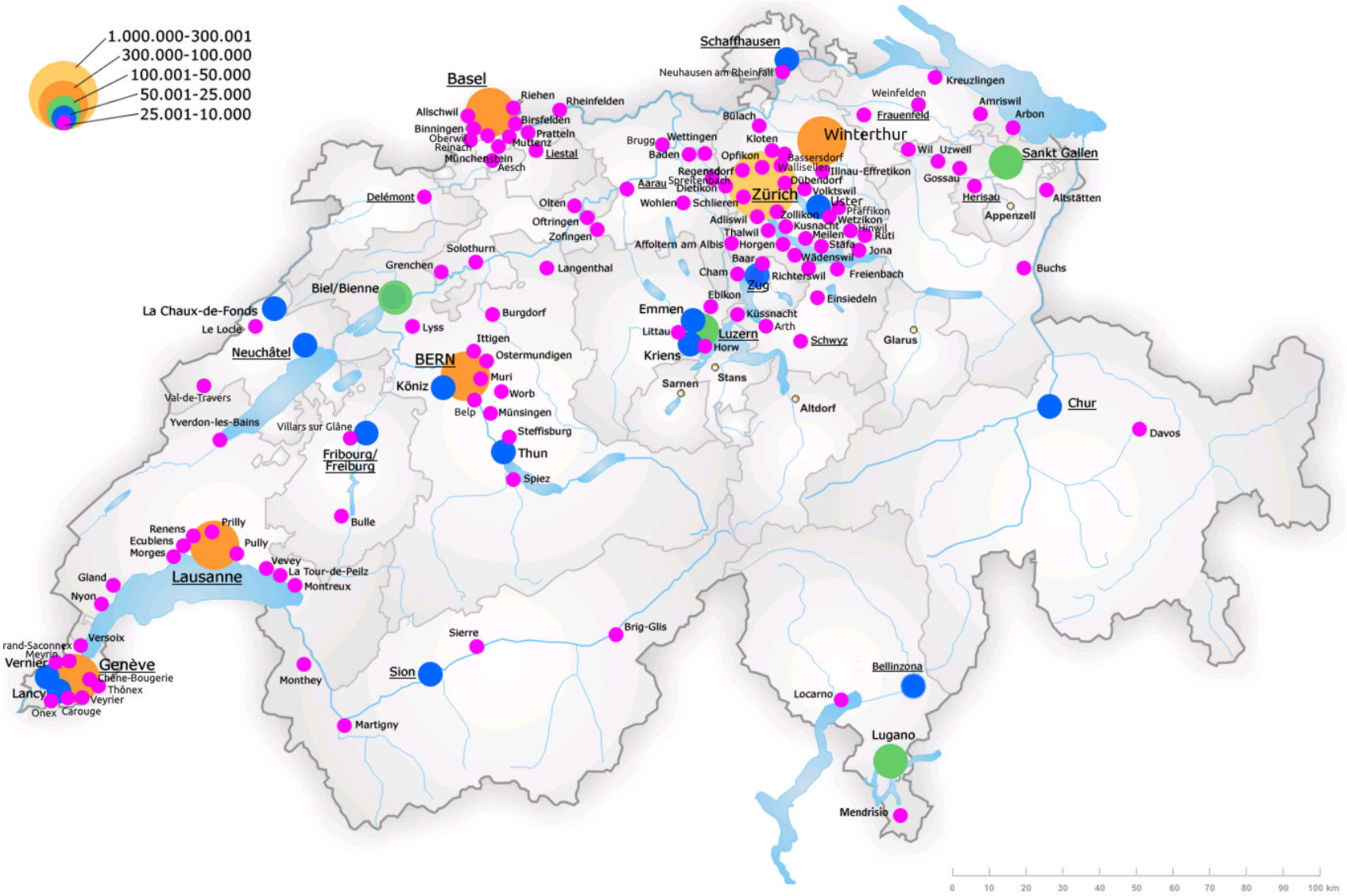
Motivation

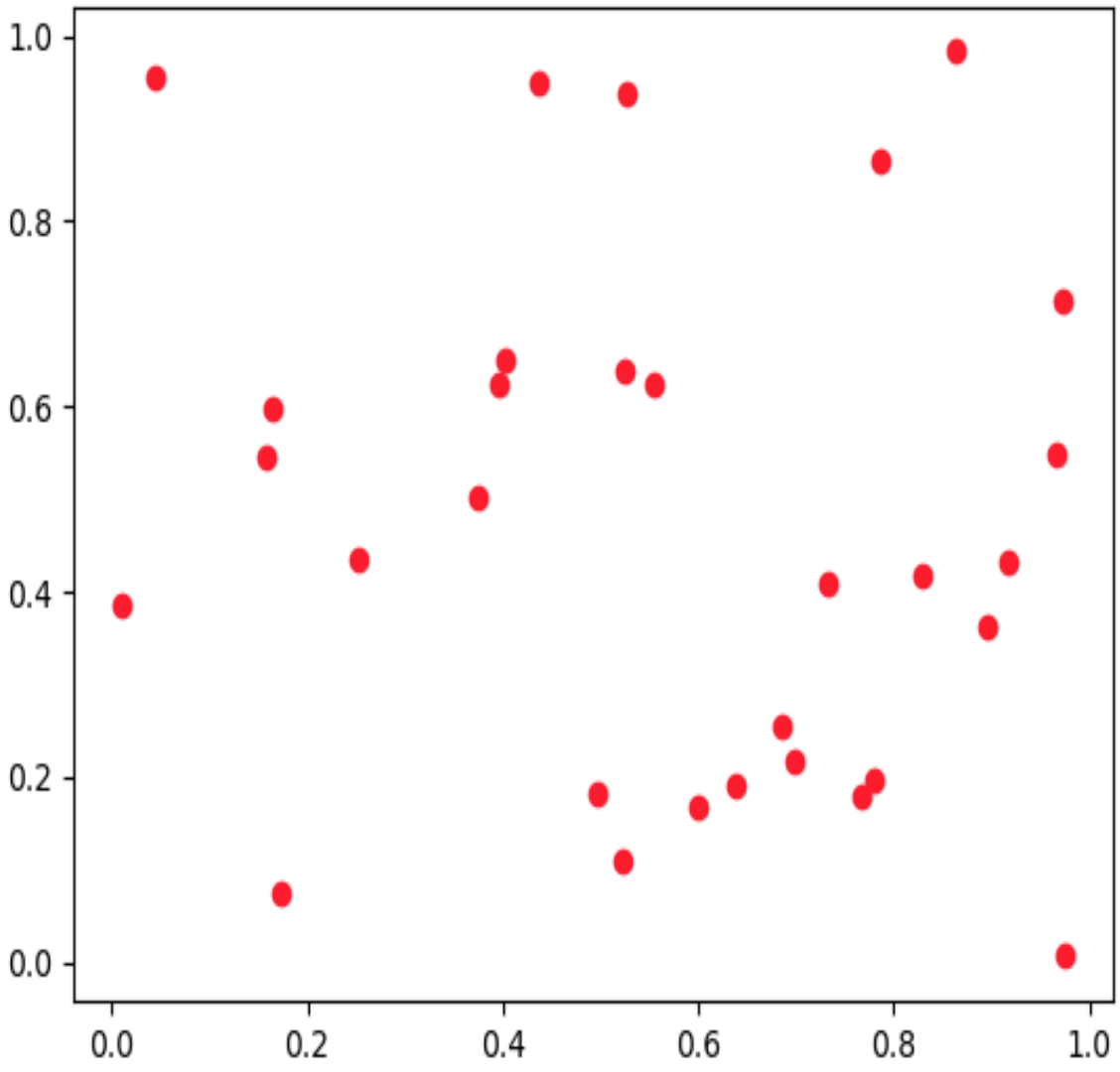

1.000.000-300.001
300.000-100.000
100.001-50.000
50.001-25.000
25.001-10.000
Schaffhausen
Neuhausen am Rheinfall
Basel
Riehen
Rheinfelden
Allschwil
Birsfelden
Binningen
Oberwil
Pratteln
Reinach
Muttenz
Münchenstein
Liestal
Aesch
Bülach
Kloten
Brugg
Wettingen
Baden
Opfikon
Winterthur
Bassersdorf
Wallisellen
Illnau-Effretikon
Regensdorf
Dübendorf
Aarau
Spreitenbach
Dietikon
Zürich
Volketswil
Uster
Wohlen
Schlieren
Adliswil
Zollikon
Pfäffikon
Wetzikon
Kusnacht
Hinwil
Thalwil
Meilen
Rüti
Affoltern am Albis
Horgen
Stäfa
Jona
Baar
Wädenswil
Cham
Richterswil
Freienbach
Zug
Ebikon
Einsiedeln
Emmen
Küssnacht
Arth
Littau
Luzern
Horw
Schwyz
Kriens
Stans
Sarnen
Altdorf
Glarus
Weinfelden
Kreuzlingen
Frauenfeld
Amriswil
Arbon
Wil
Uzwil
Sankt Gallen
Gossau
Herisau
Appenzell
Altstätten
Buchs
Chur
Davos
Delémont
Olten
Oftringen
Zofingen
Solothurn
Grenchen
Langenthal
Biel/Bienne
La Chaux-de-Fonds
Le Locle
Burgdorf
Lyss
Neuchâtel
Ittigen
Ostermundigen
BERN
Muri
Köniz
Worb
Belp
Münsingen
Val-de-Travers
Yverdon-les-Bains
Villars sur Glâne
Steffisburg
Thun
Fribourg/
Freiburg
Spiez
Bulle
Renens
Prilly
Ecublens
Pully
Morges
Vevey
La Tour-de-Peilz
Lausanne
Montreux
Gland
Nyon
Versoix
Genève
Chêne-Bougerie
Thônex
Veyrier
Onex
Carouge
Vernier
Lancy
Monthey
Sierre
Brig-Glis
Sion
Martigny
Bellinzona
Locarno
Lugano
Mendrisio
0
10
20
30
40
50
60
70
80
90
100 km

Colorful k-Center

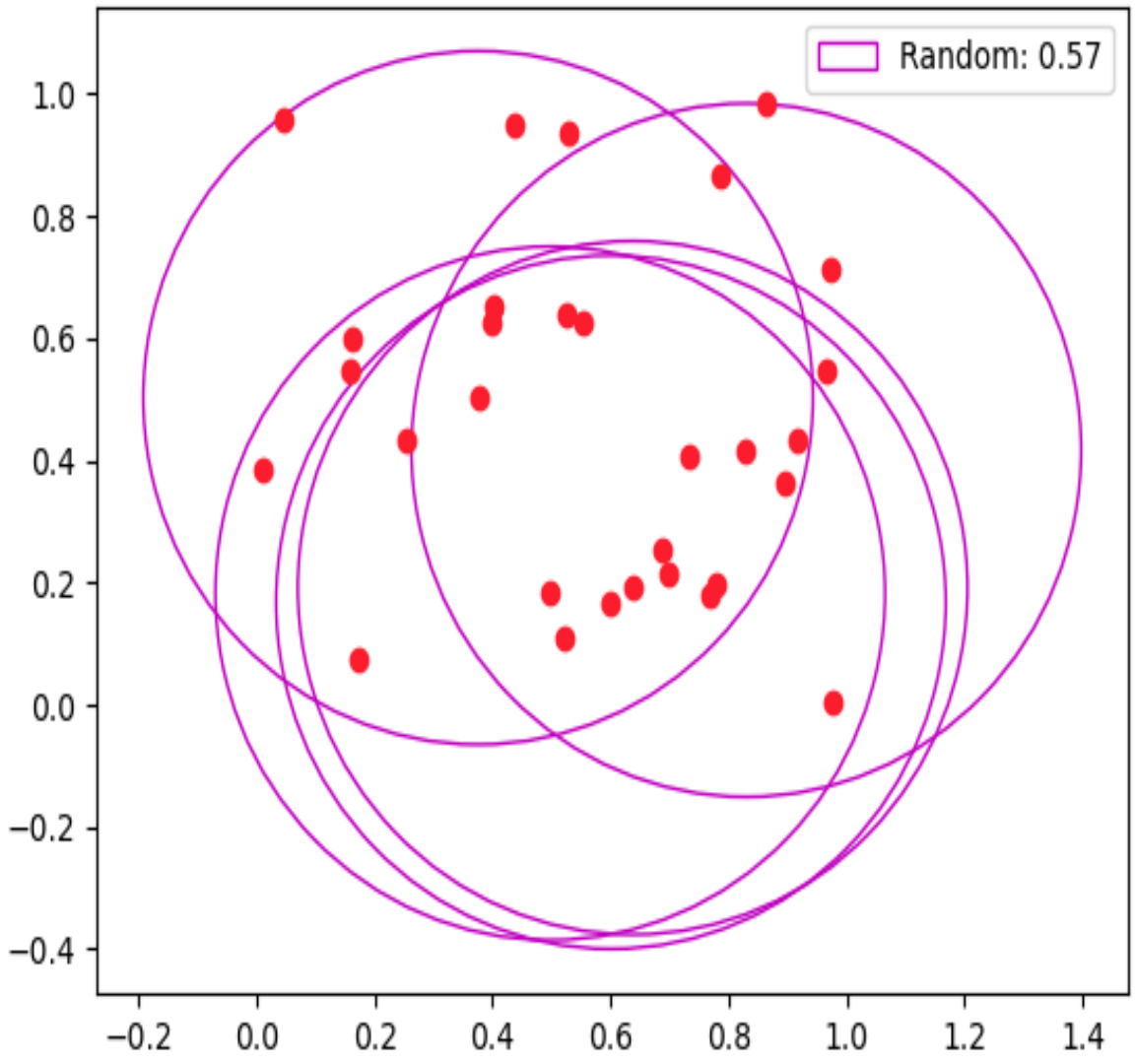
Input: P: n points with metric
$C_1, \ldots, C_\omega$: partition of P
$p_1, \ldots, p_\omega$: coverage requirements
Output: $F \subseteq P \mid |F| \leq k$: set of centers
Goal: Minimize radius r, such that

$$\left| \left(\bigcup_{k \in F} B(k, r) \right) \bigcap C_i \right| \geq p_i \;\; \forall i \in \{1, \ldots, \omega\}$$

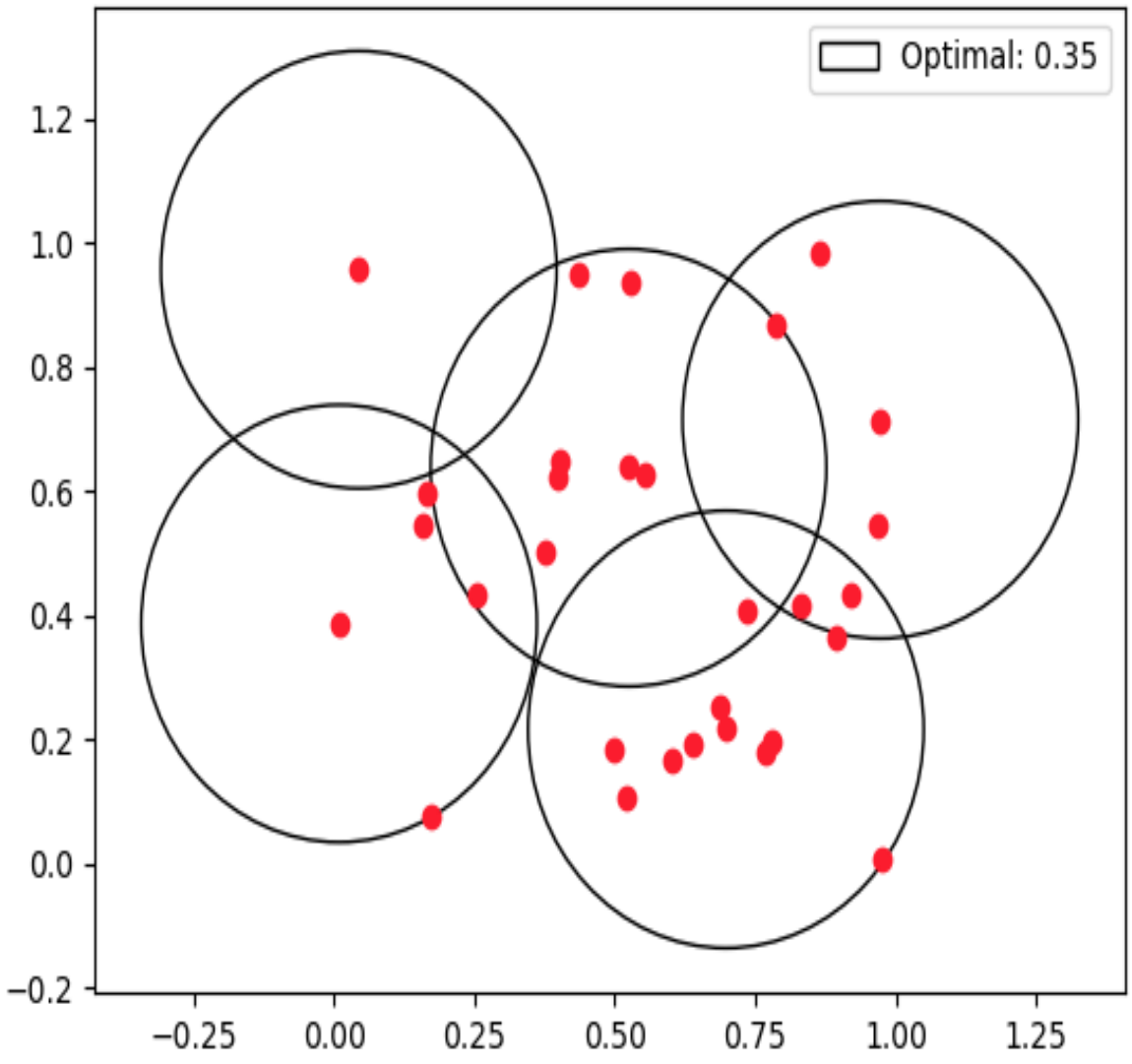
Instance



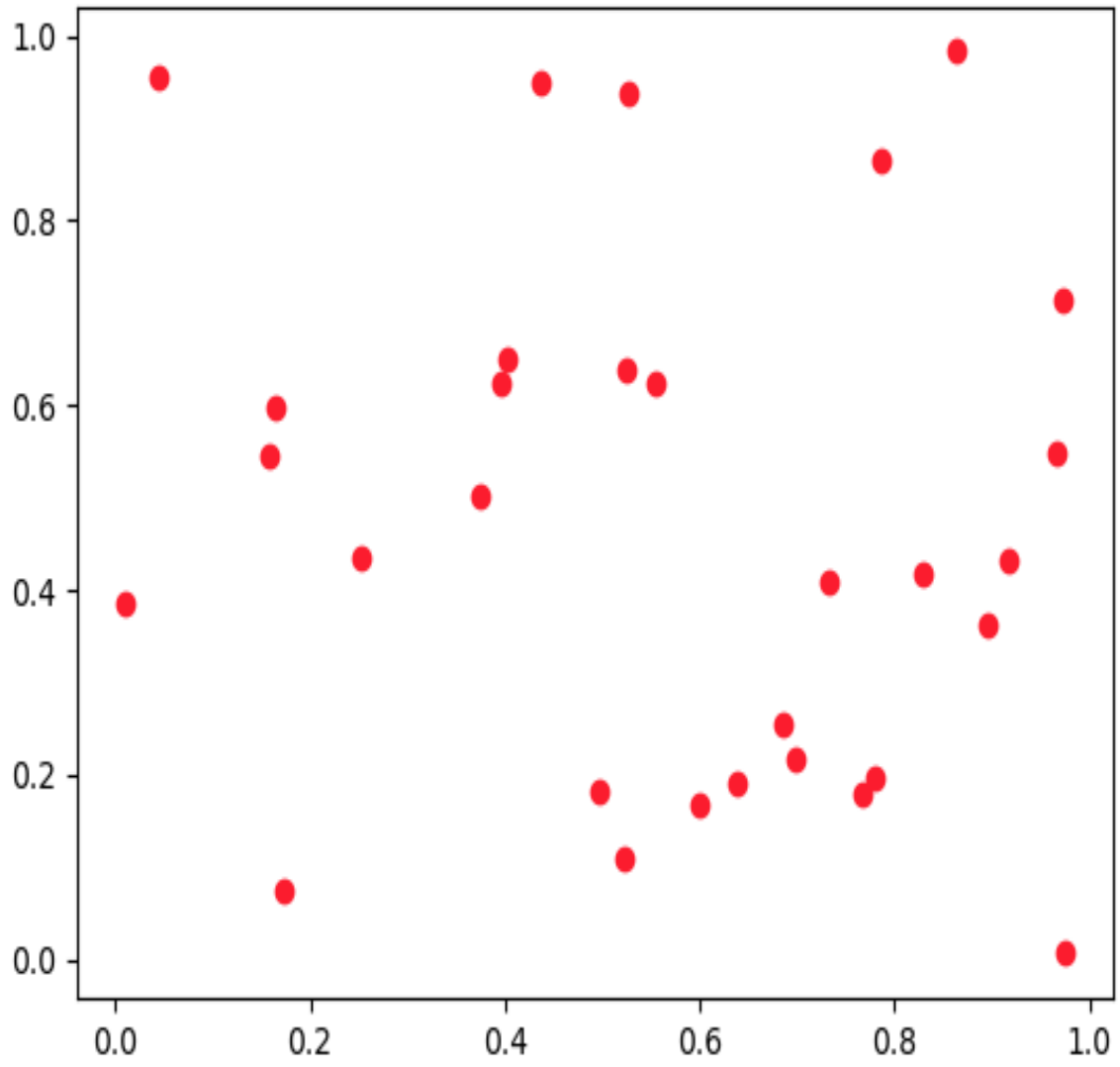
Random



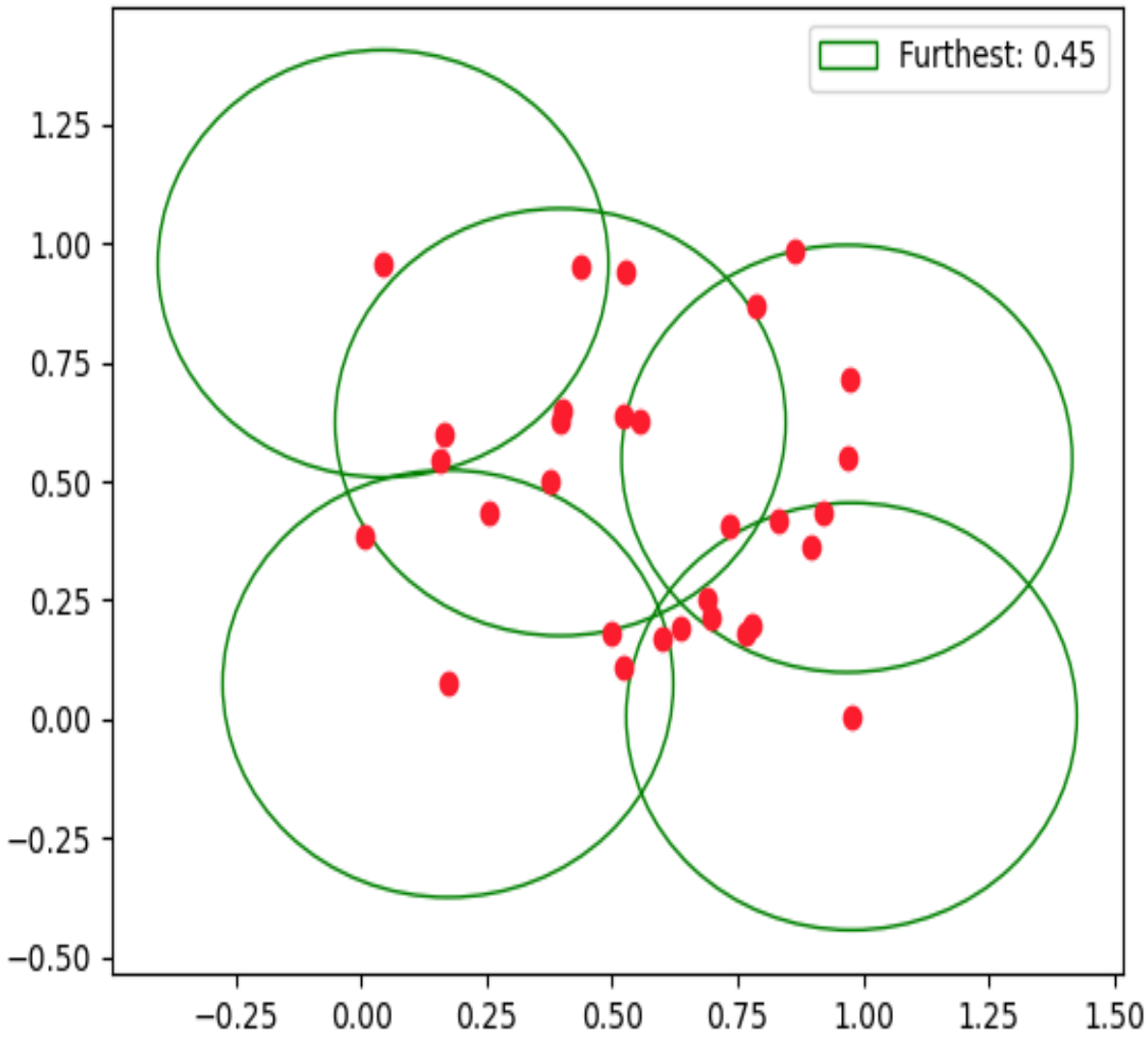
Optimal



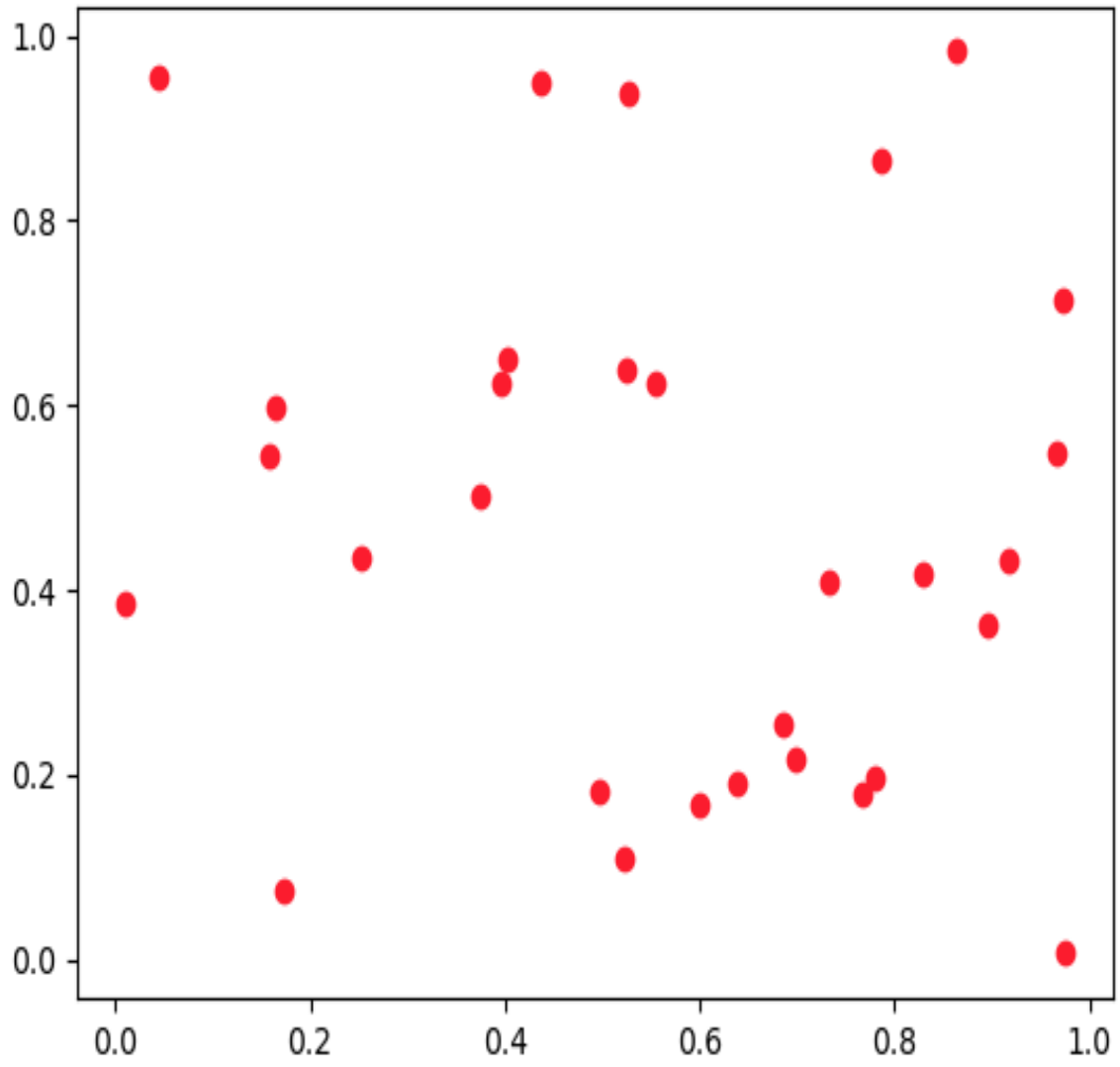
Furthest



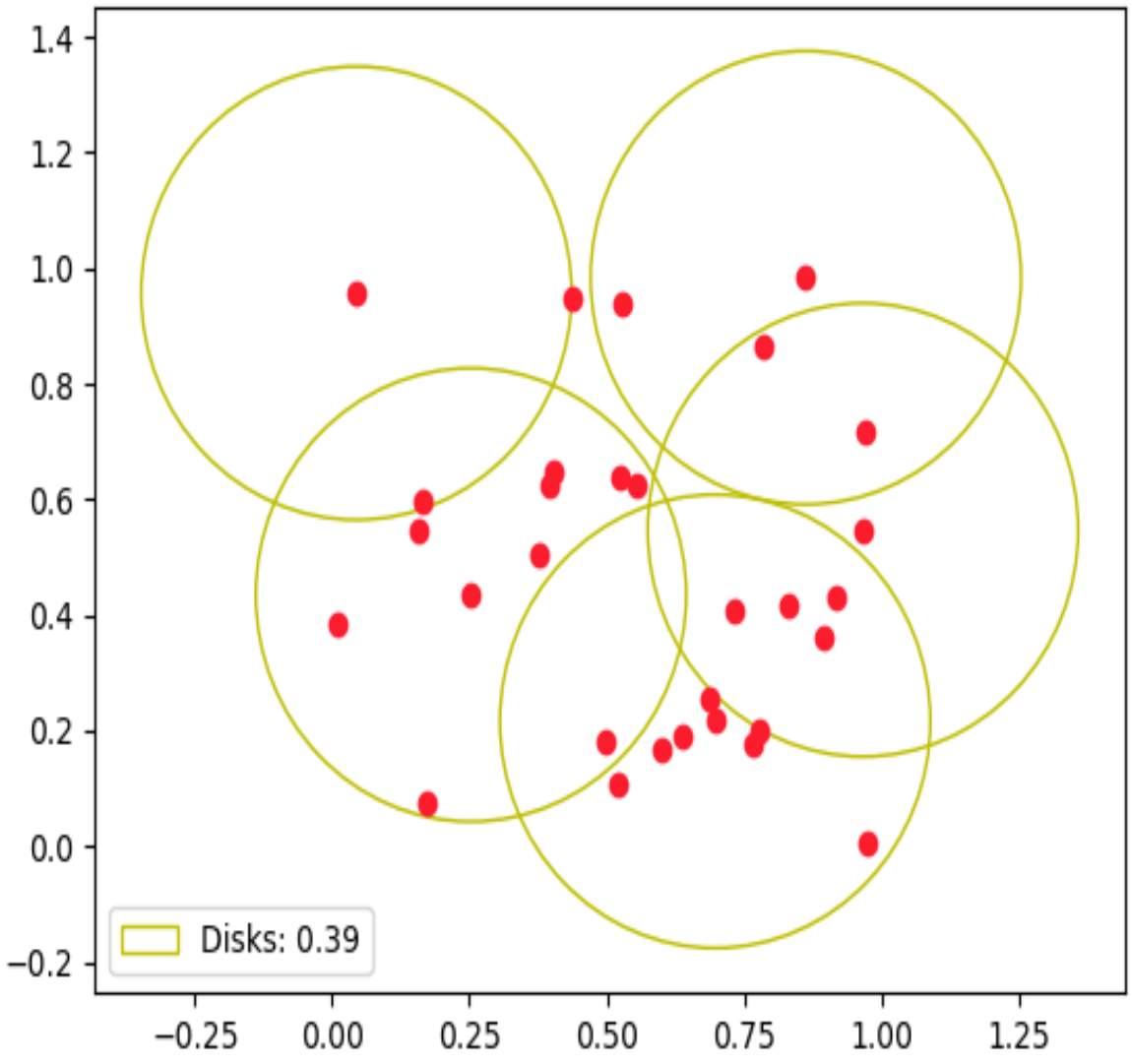
Furthest


Furthest: 0.45
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50

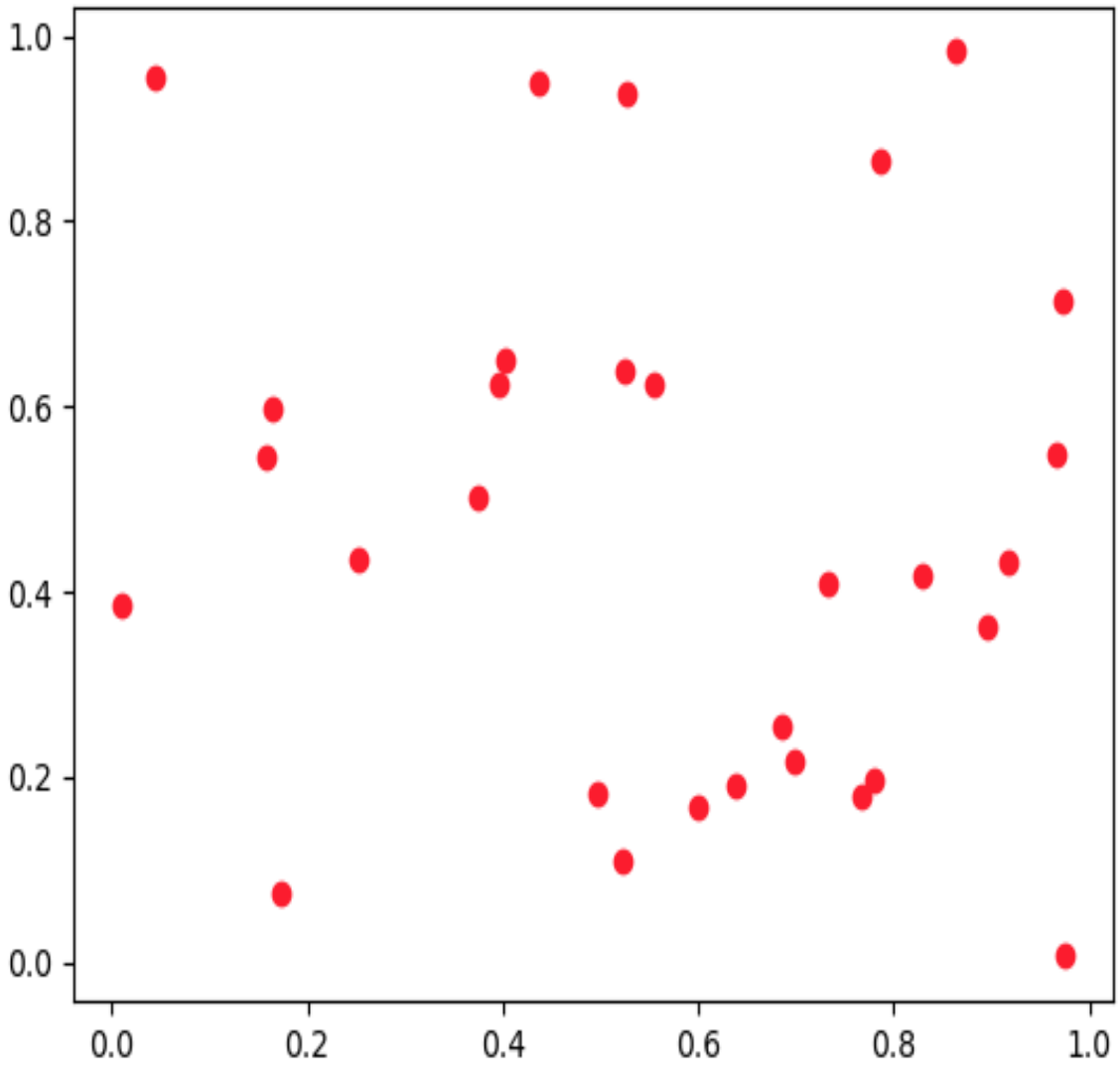
Disks



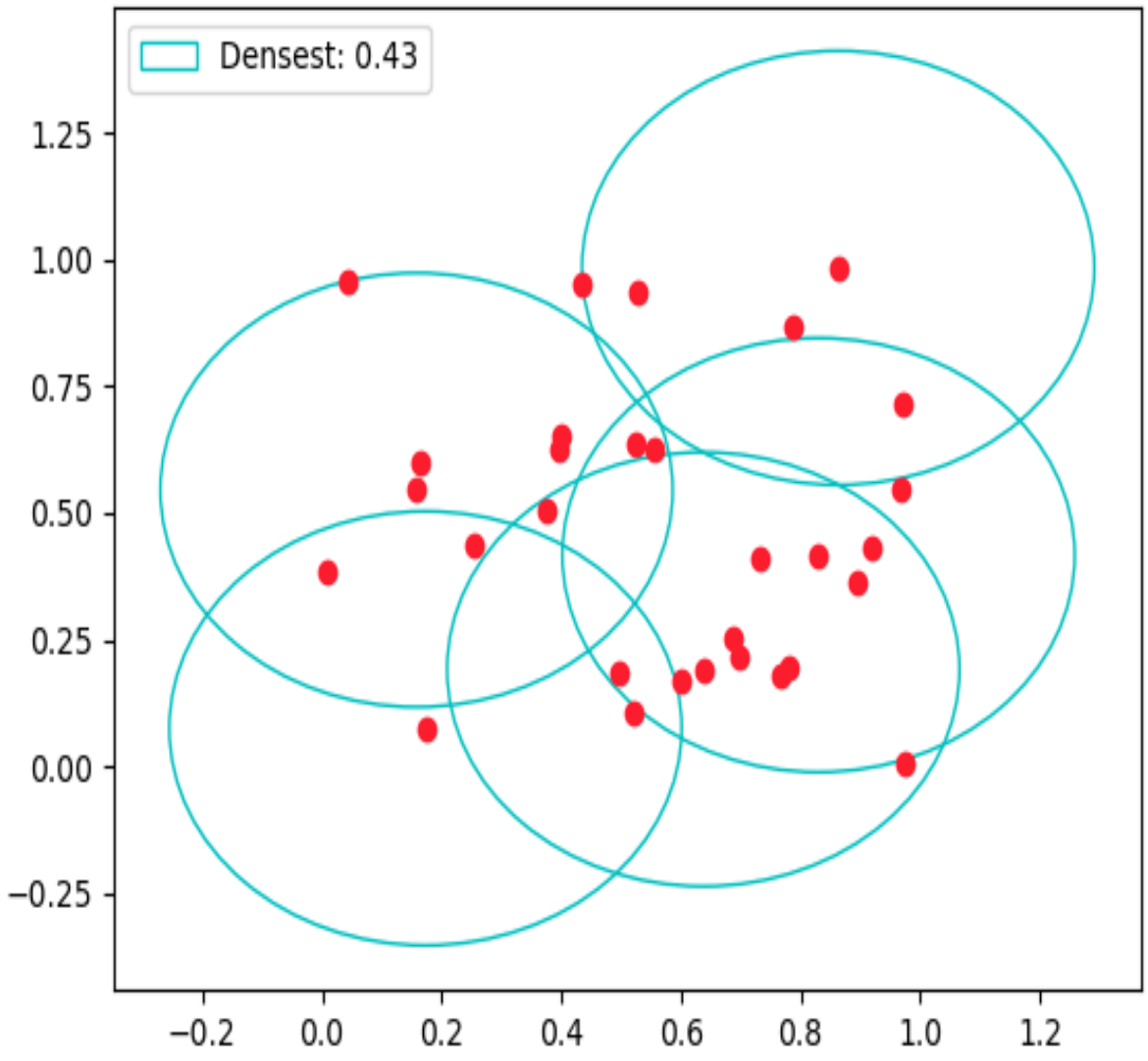
Disks



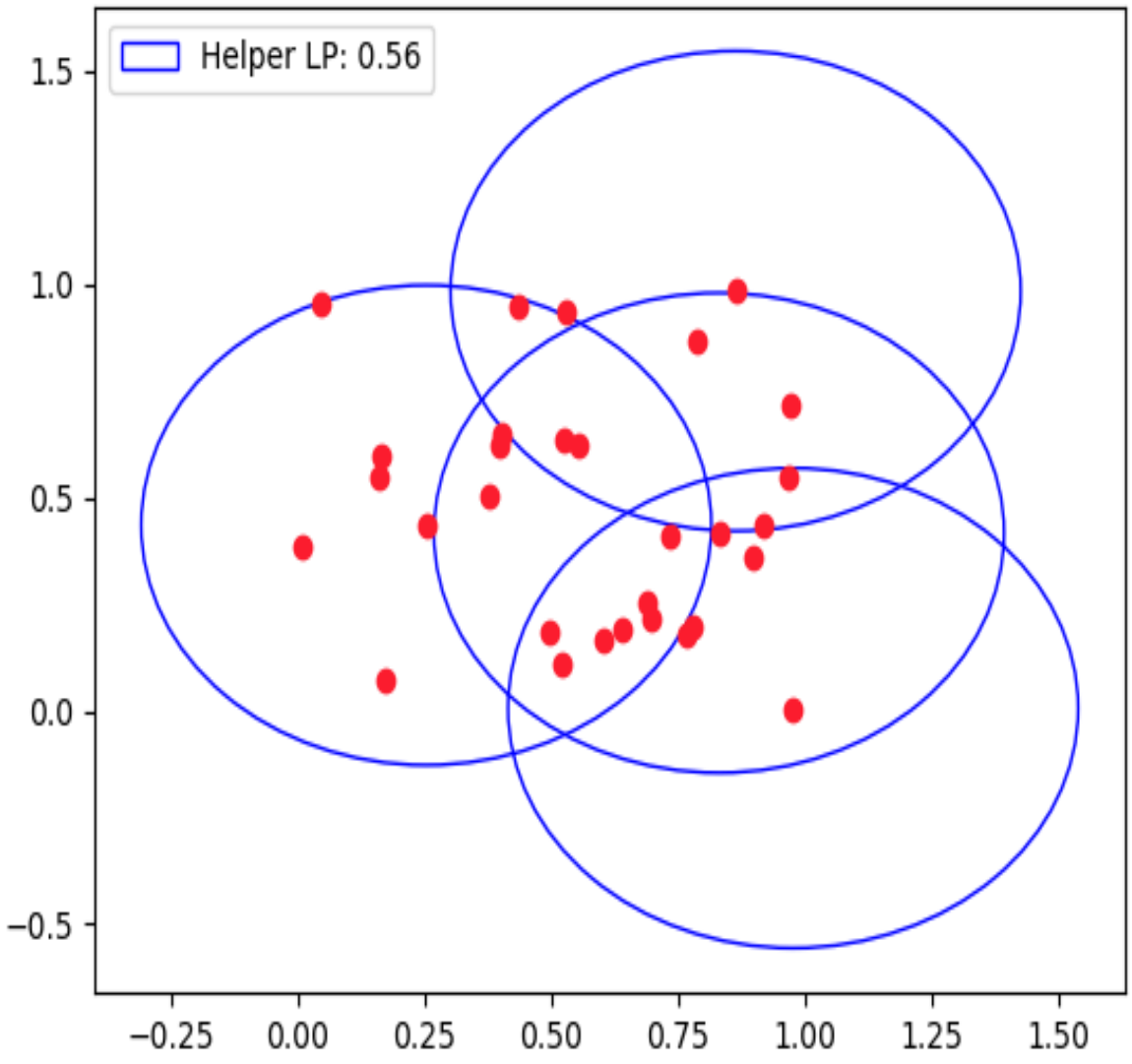
Densest



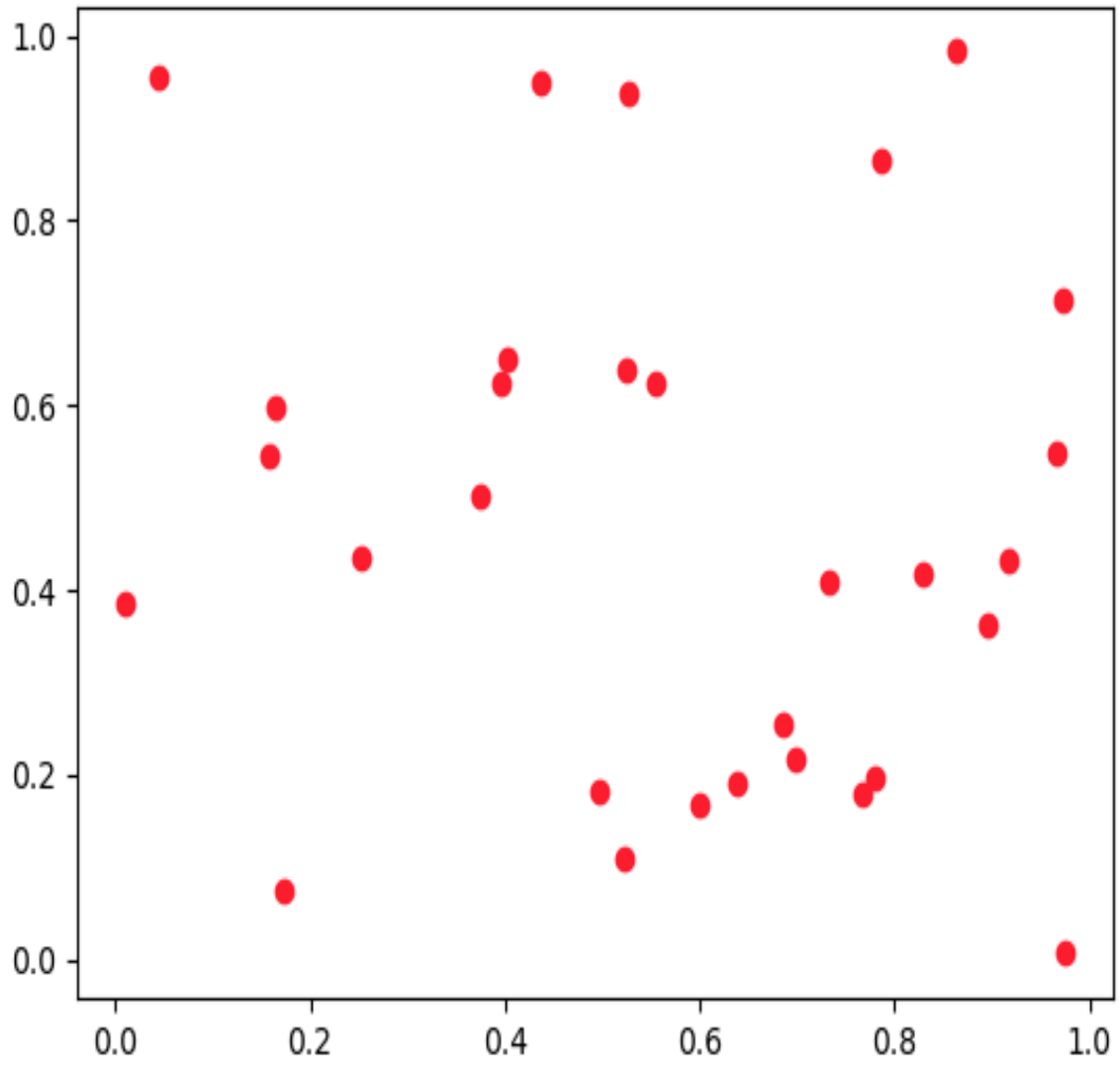
Densest



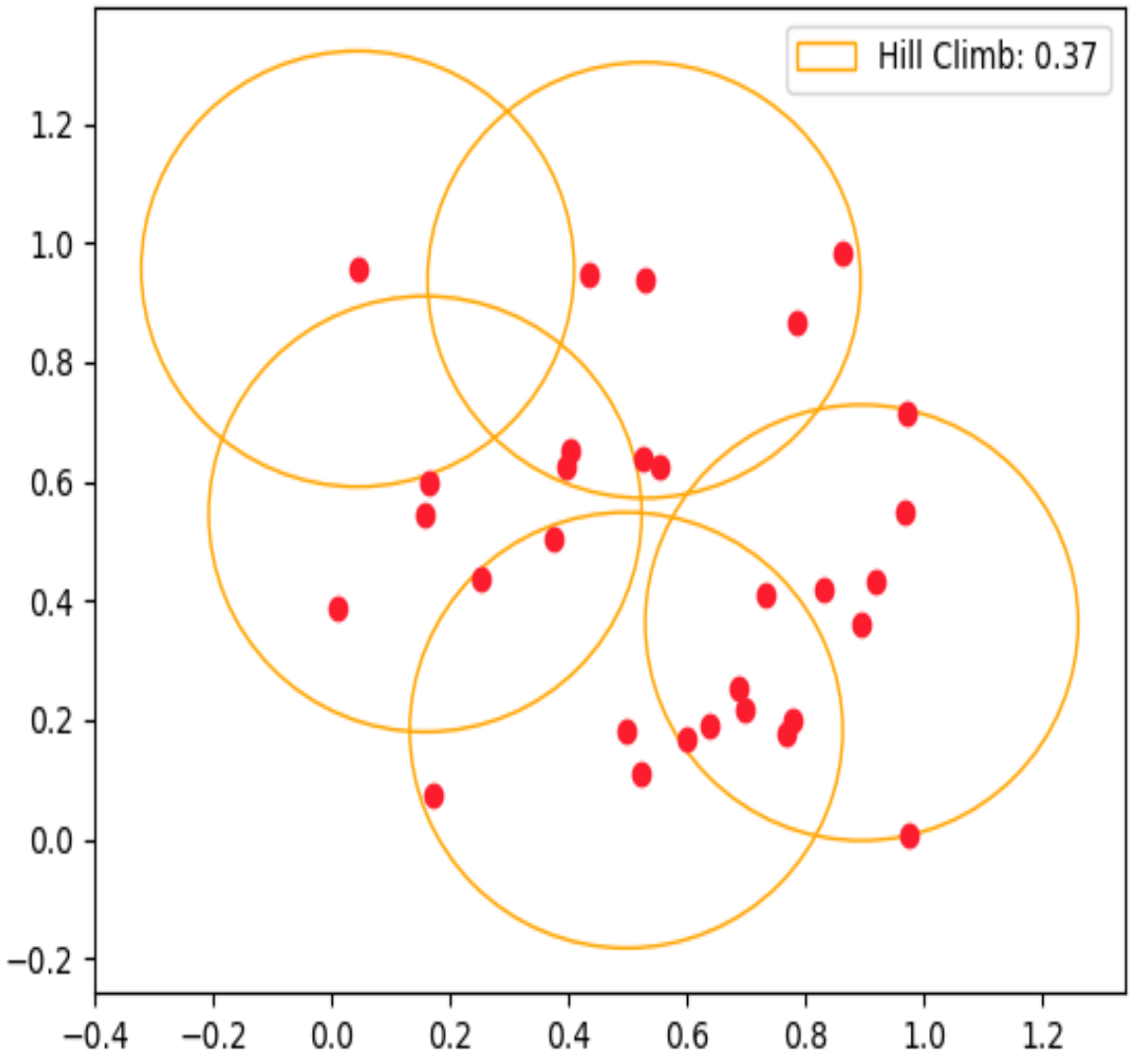
Helper LP



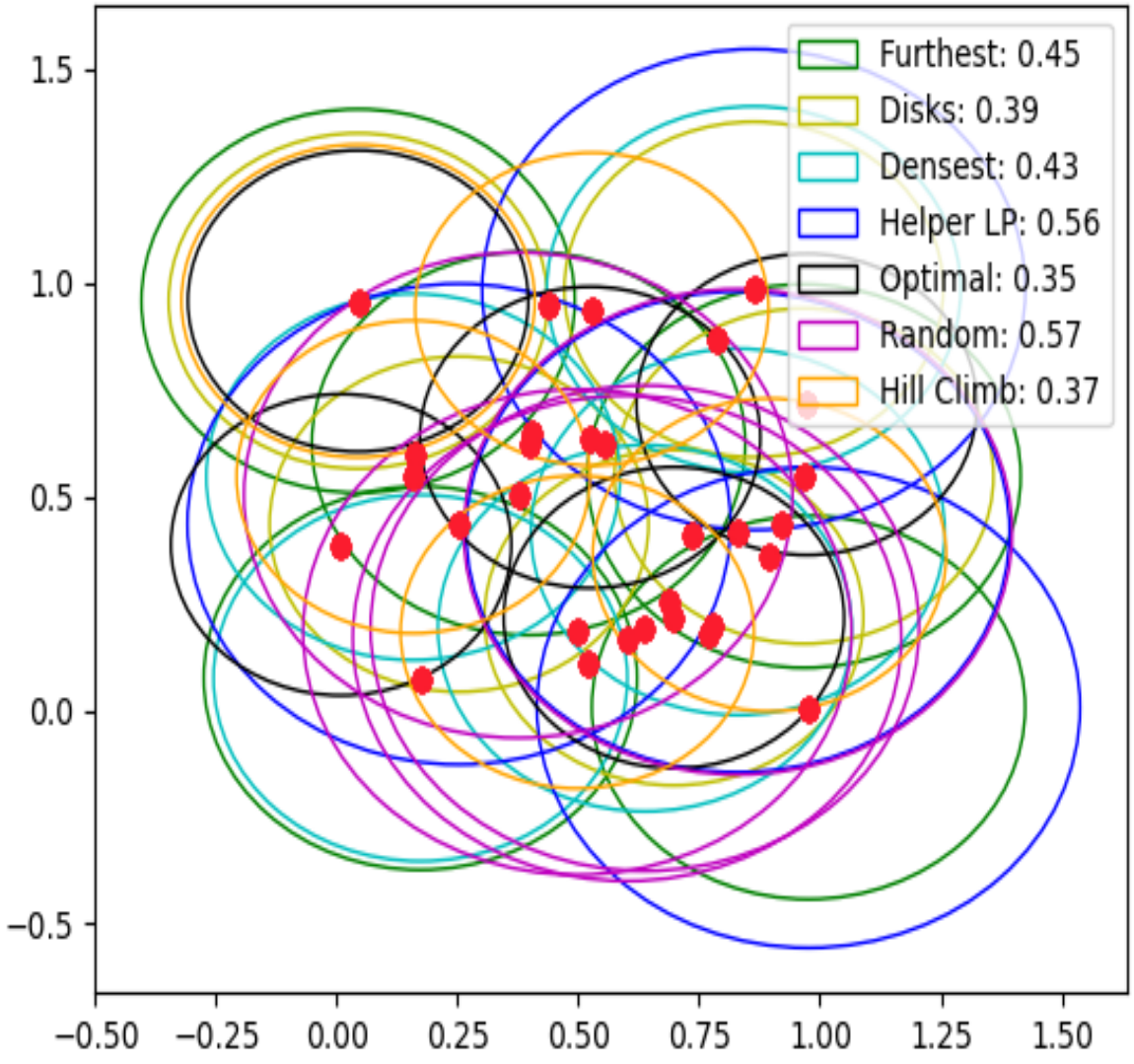
Hill Climbing



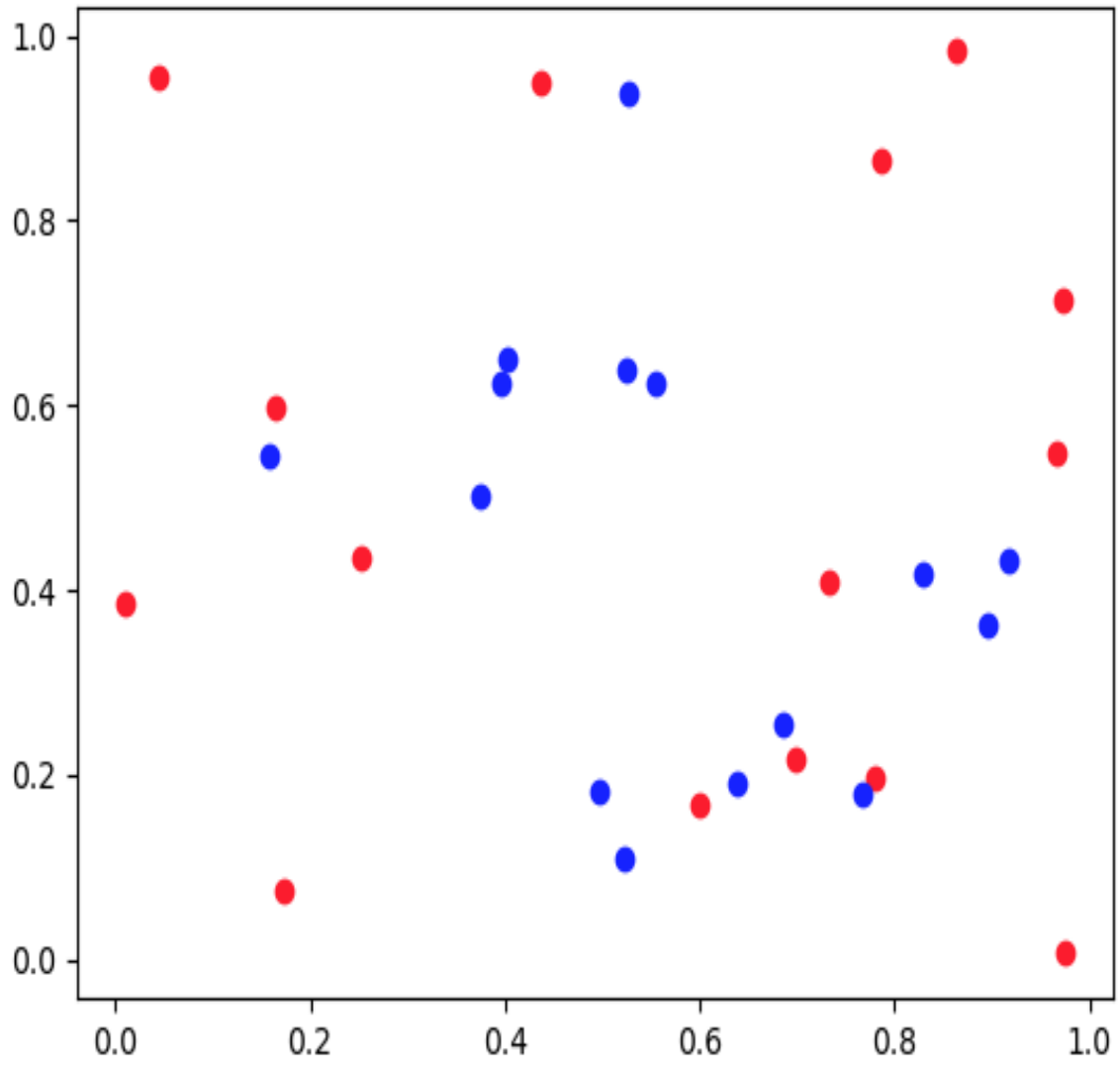
Hill Climbing



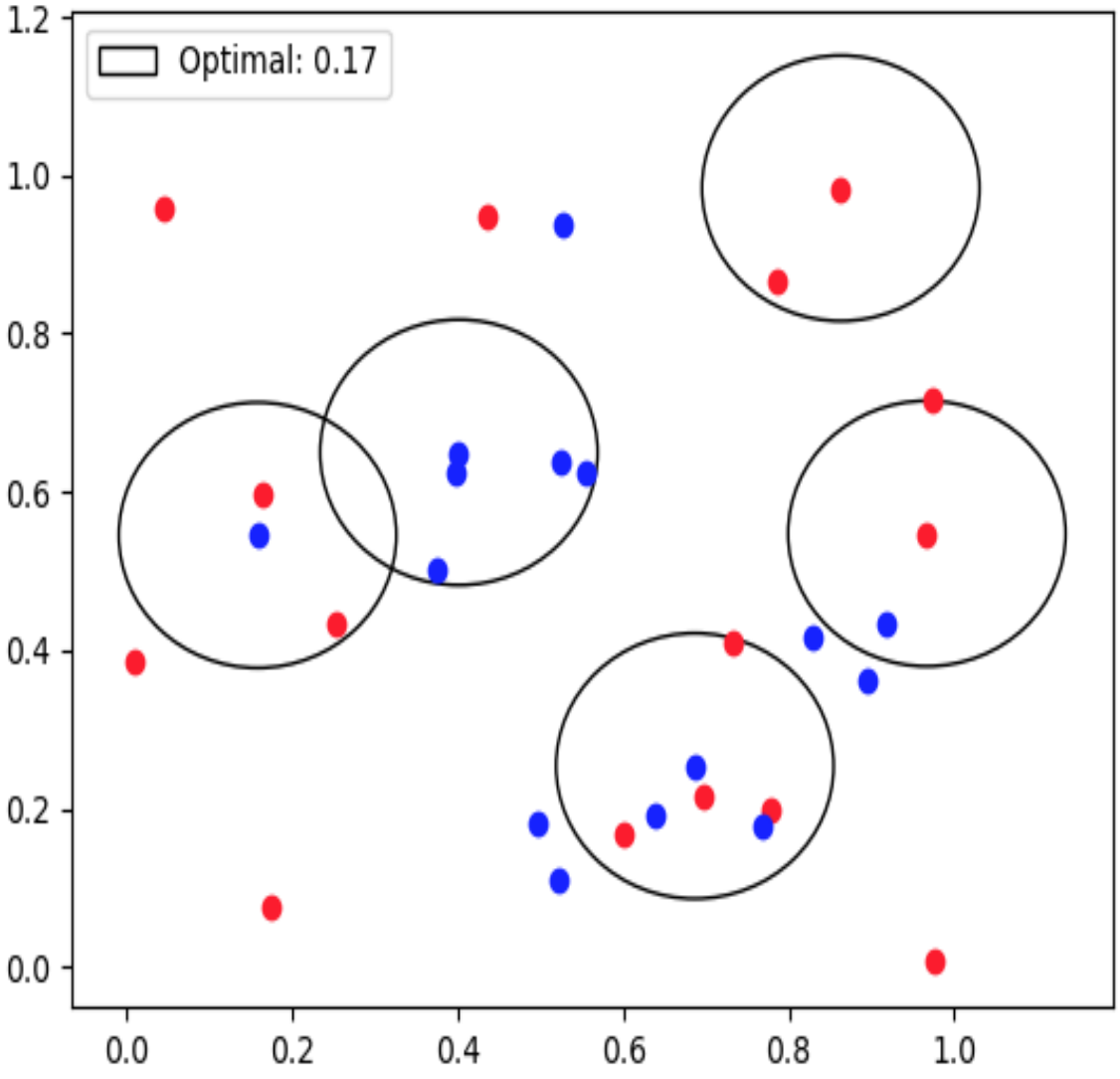
All Algorithms



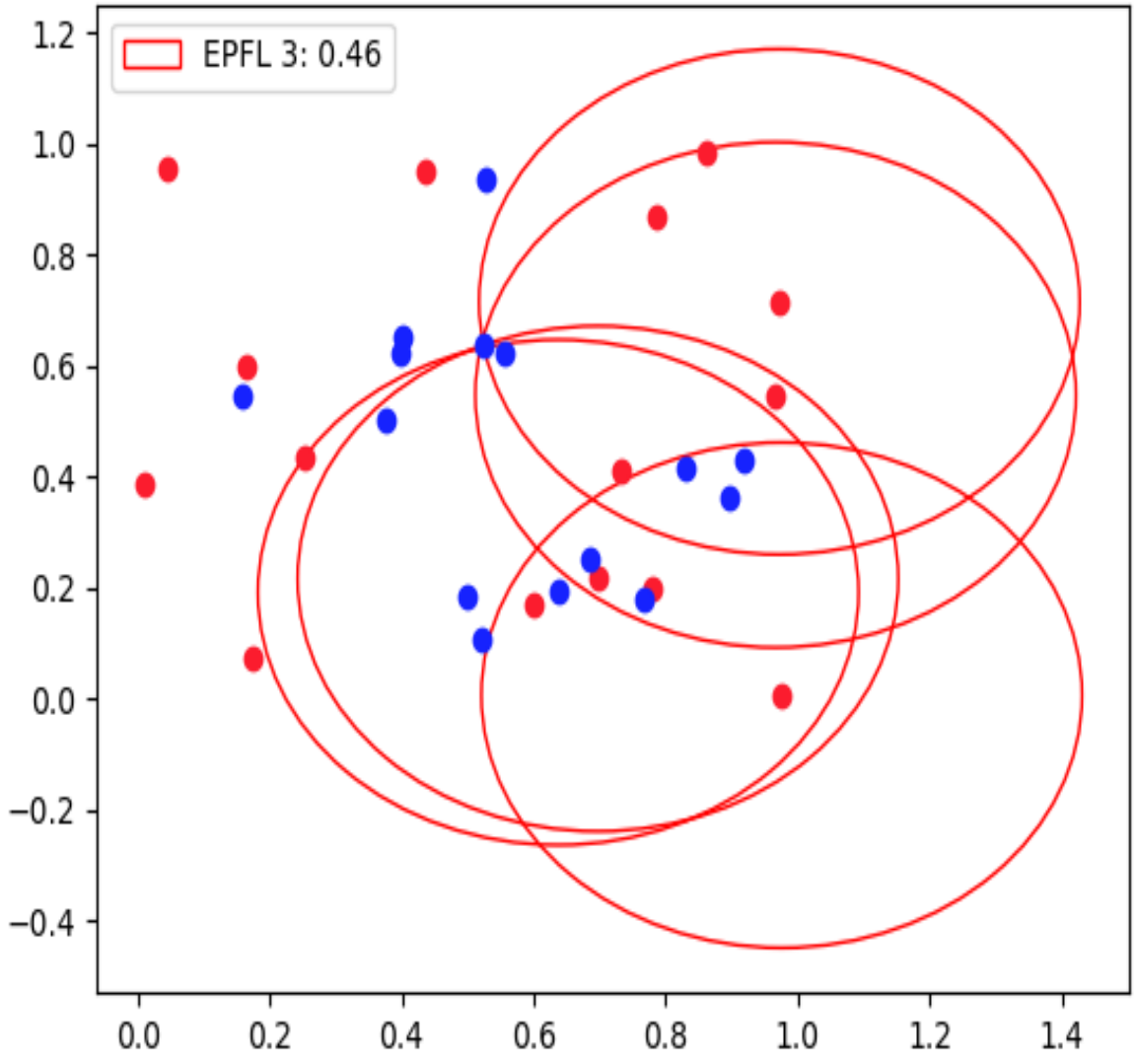
2 Colors



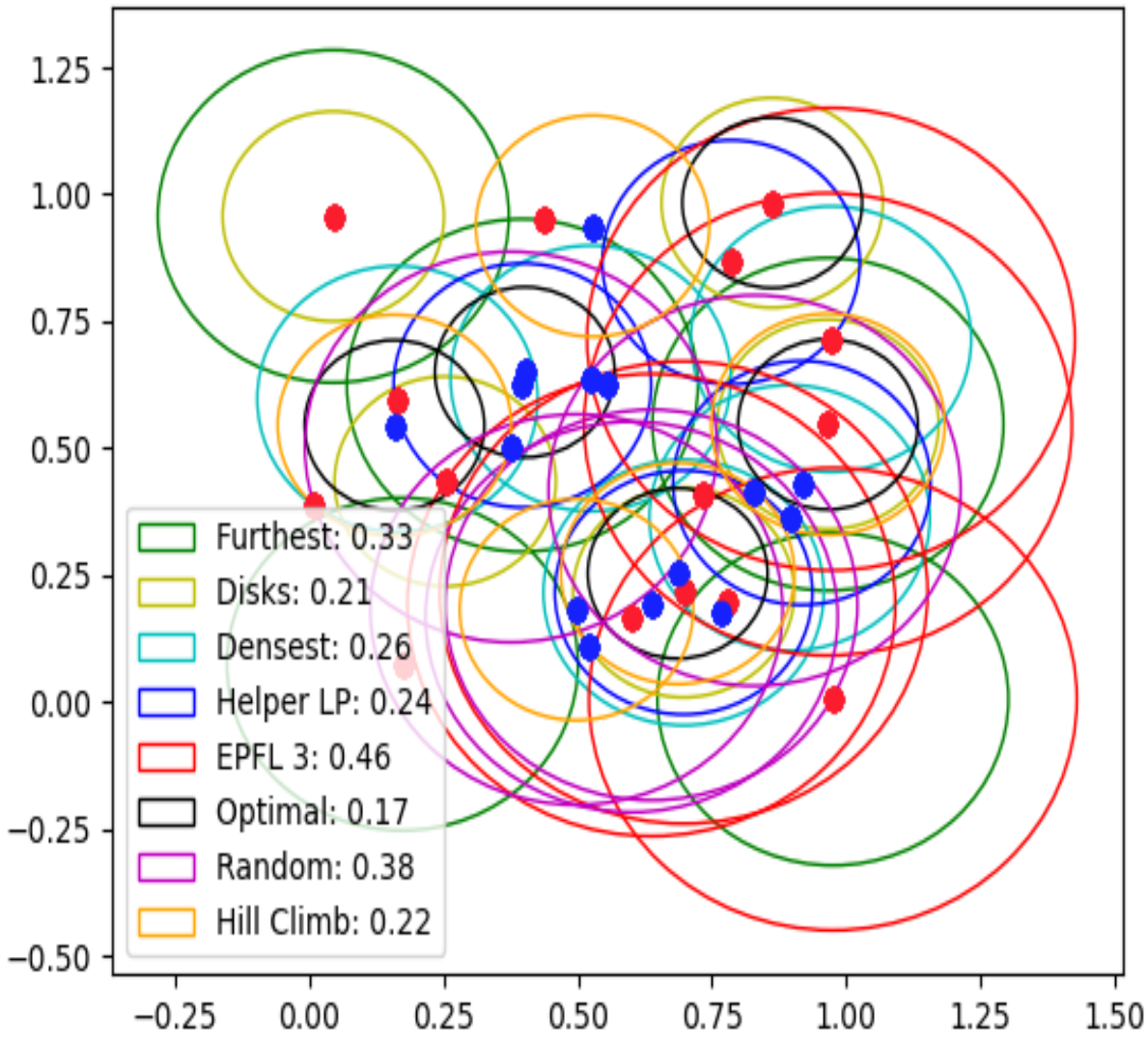
Optimal



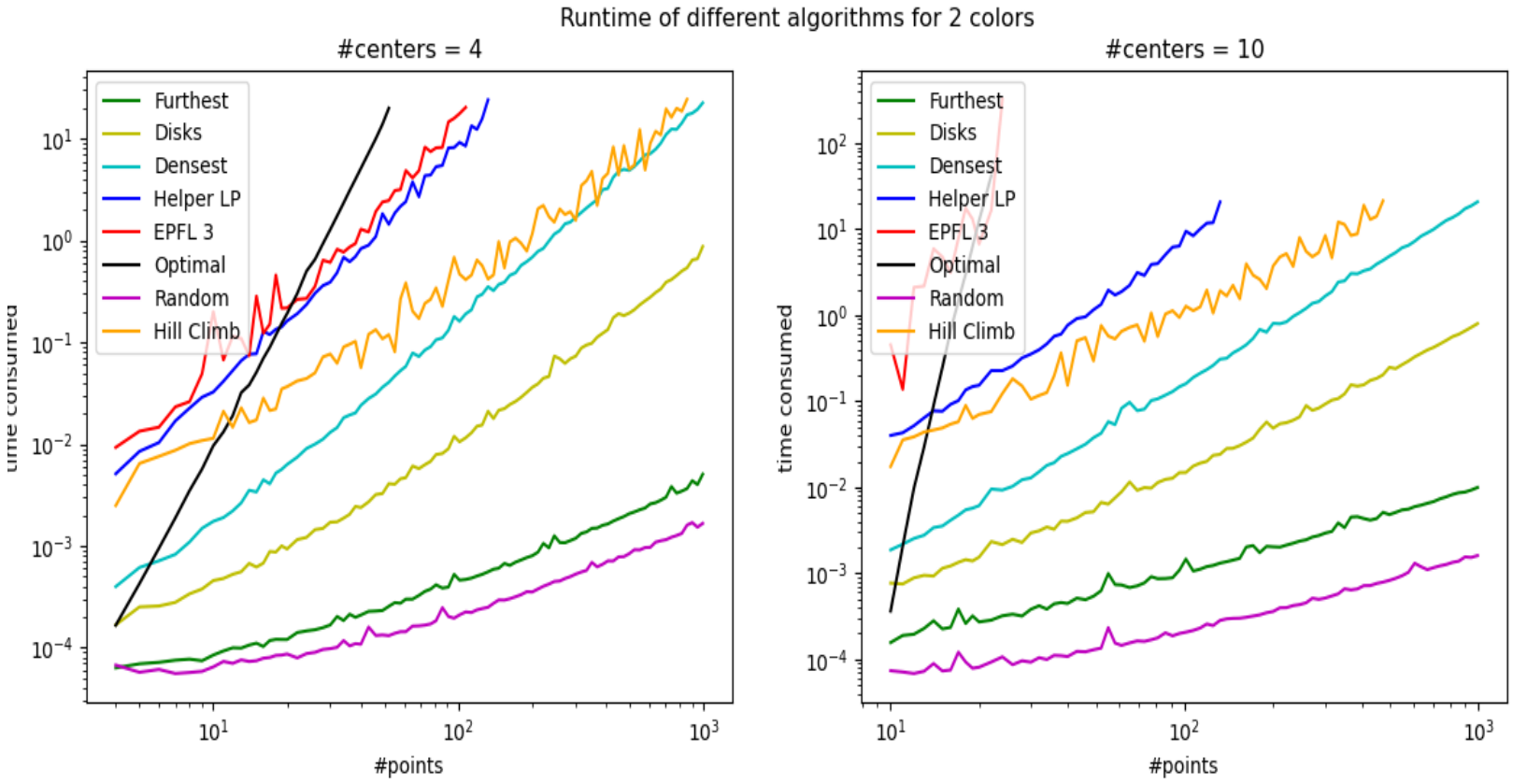
EPFL 3-Approximation



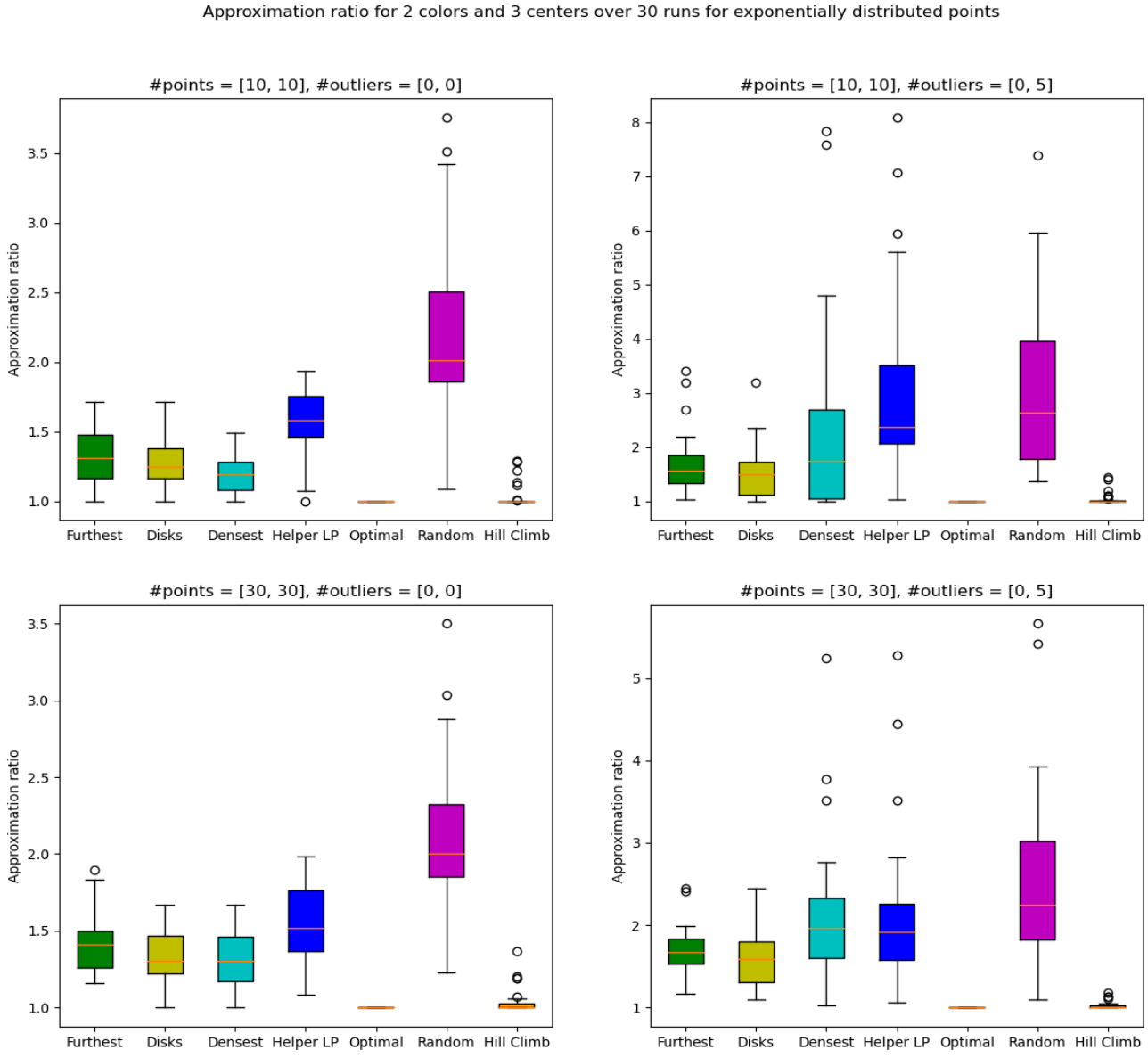
All Algorithms



Empirical Runtime



Empirical Approximation ratio



Conclusion

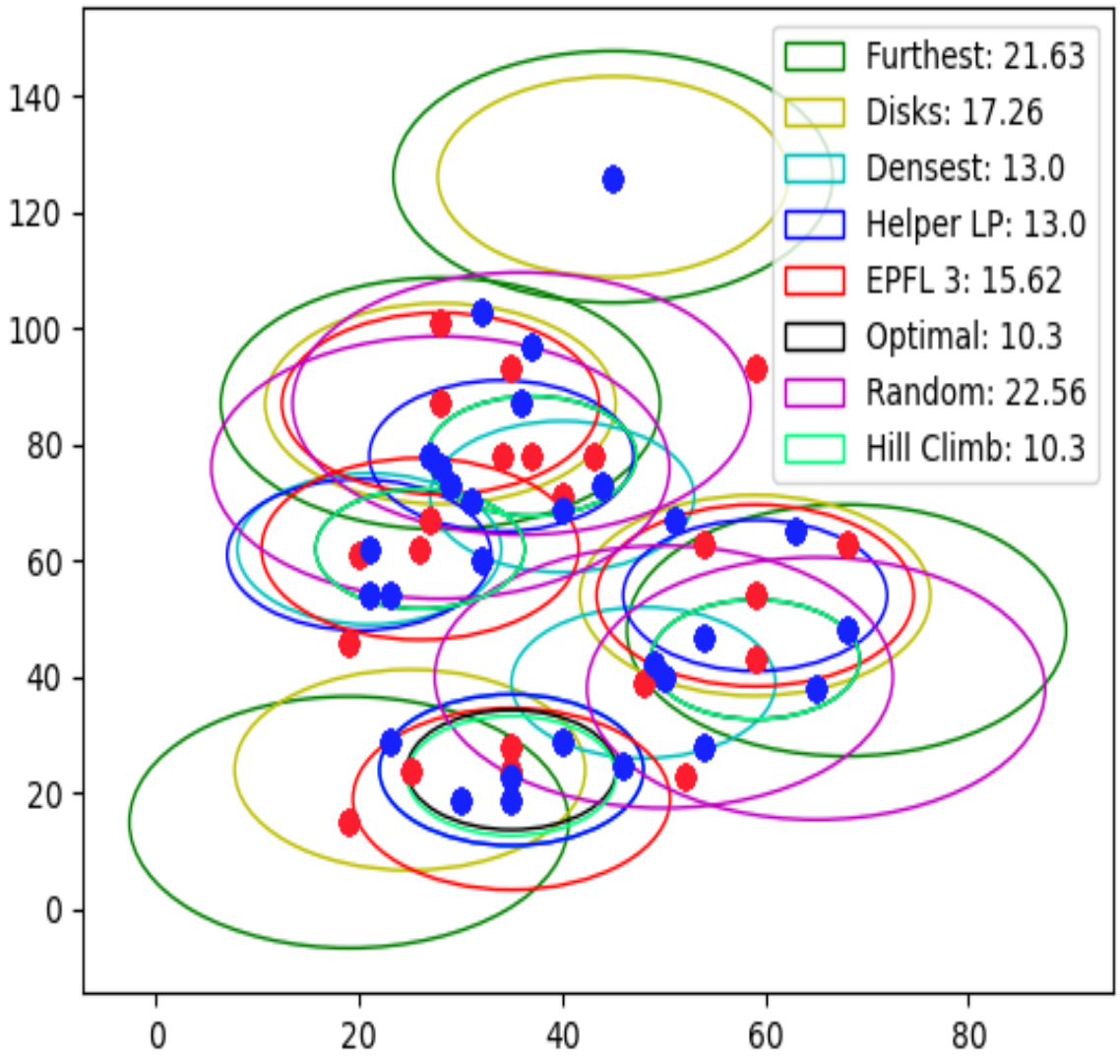
Takeaways

- EPFL 3-approximation has bad runtime
- The Furthest, Disks, Densest, and Helper LP algorithms give bad approximations for colorful k-center
- Hill climbing is a decent heuristic for colorful k-center

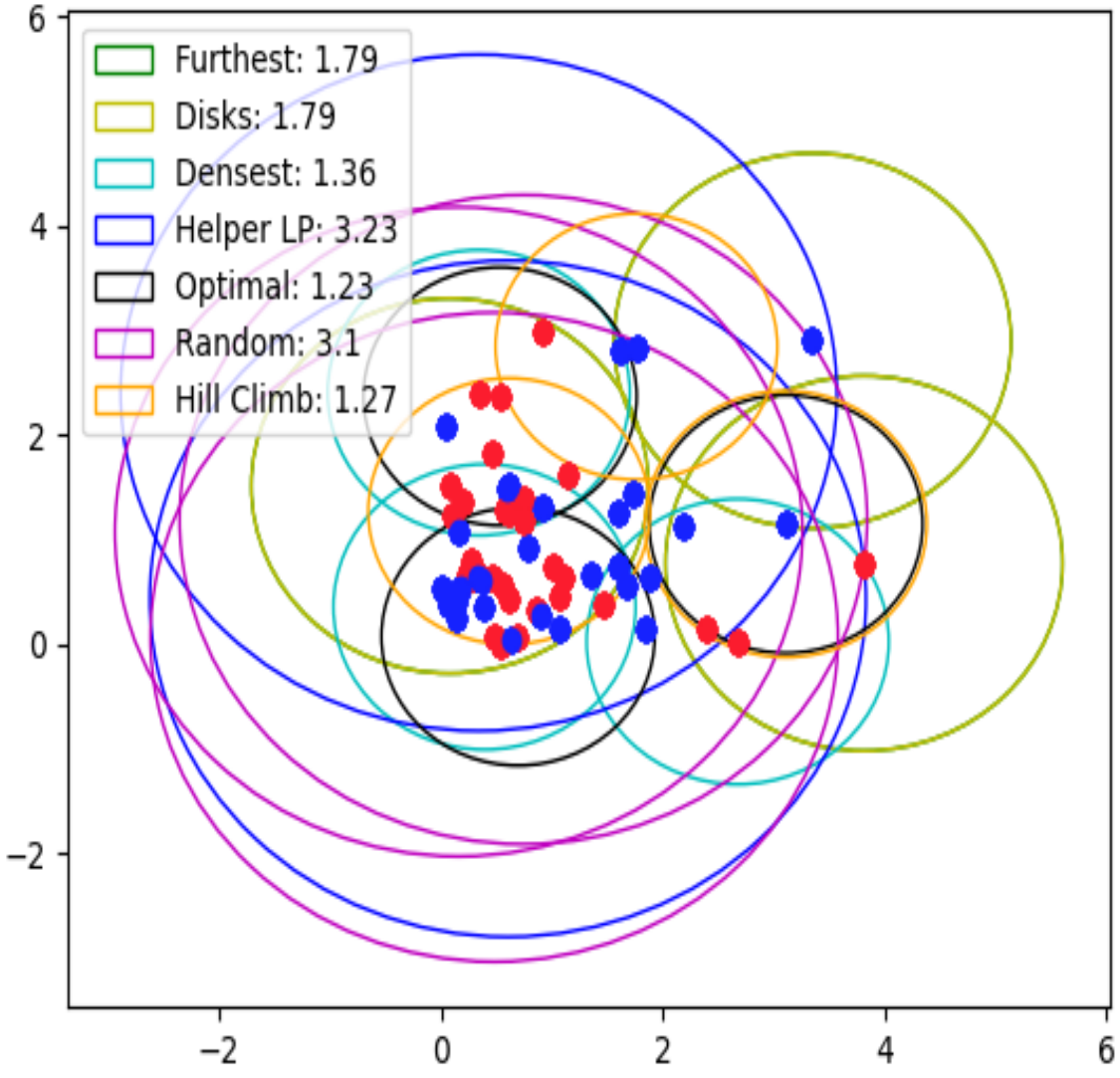
Open Questions

- Is there a 2-approximation for colorful k-center?
- Is there a 3-approximation for colorful k-center with a better runtime?

Mall Data


Furthest: 21.63
Disks: 17.26
Densest: 13.0
Helper LP: 13.0
EPFL 3: 15.62
Optimal: 10.3
Random: 22.56
Hill Climb: 10.3
140
120
100
80
60
40
20
0
0
20
40
60
80

Exponential Instance



Normal Instance with small p