

Code Generation & Optimization for Deep-Learning Computations via Multi-Dimensional Homomorphisms

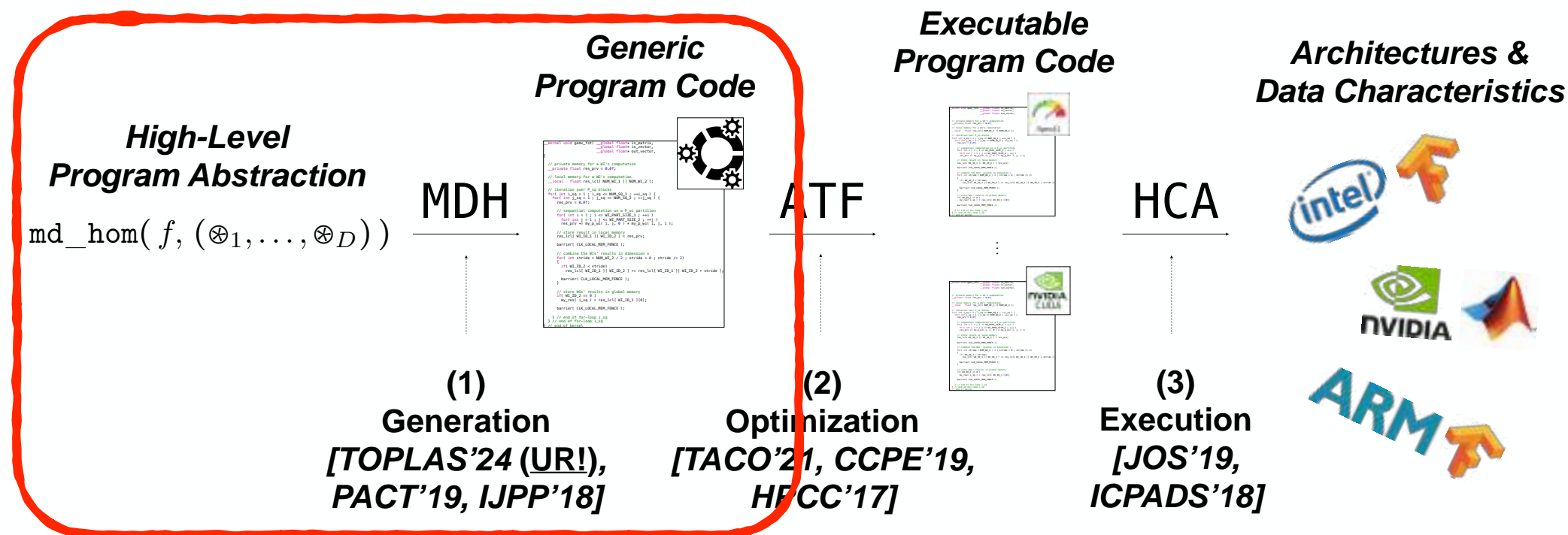
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Who are we?



We are the developers of the **MDH+ATF+HCA** approaches:



Richard Schulze

Focus Today

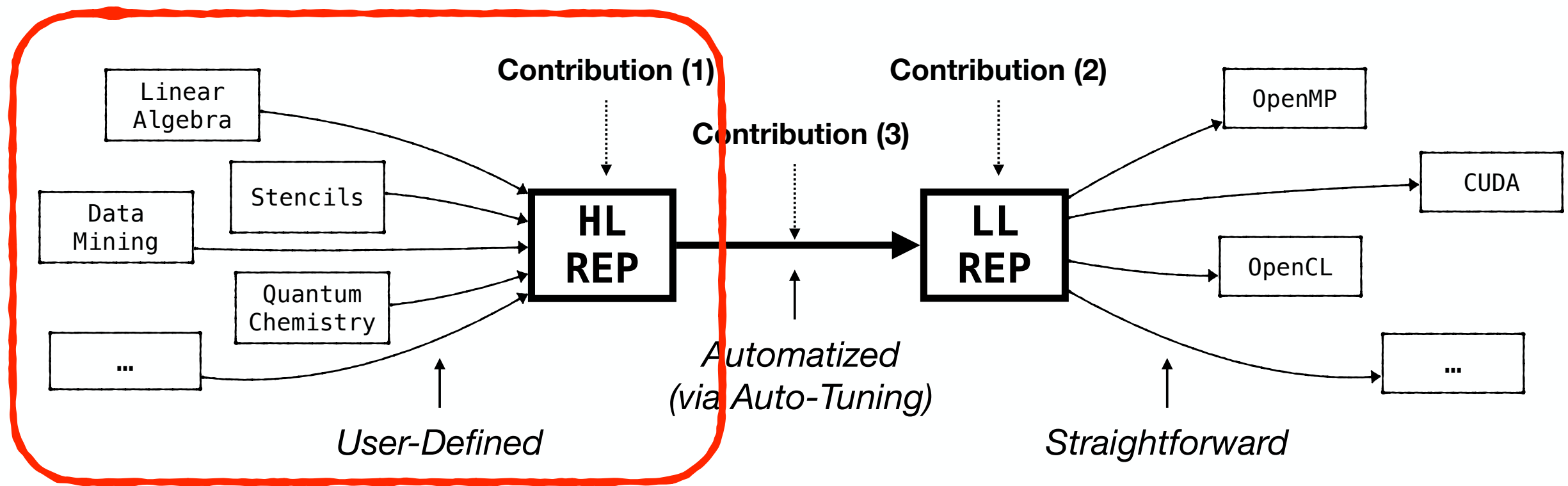


Ari Rasch

A holistic approach to code generation (MDH) & optimization (ATF) & execution (HCA):

- (1) MDH (Multi-Dimensional Homomorphisms): How to generate automatically optimizable (auto-tunable) code?
- (2) ATF (Auto-Tuning Framework): How to optimize (auto-tune) code?
- (3) HCA (Host Code Abstraction): How to execute code on (distr.) multi-dev. systems?

The MDH Approach



Focus Today

The MDH approach [1] (formally) introduces:

- (1) High-Level Program Representation for conveniently expressing data-parallel computations, agnostic from hardware and optimization details
- (2) Low-Level Program Representation that expresses device- and data-optimized de- and re-composition strategies of computations & straightforwardly transformable to executable program code
- (3) Lowering Process that *fully automatically* lowers a high-level MDH program to a device- and data-optimized low-level MDH program (based on auto-tuning [2])

[1] "(De/Re)-Composition of Data-Parallel Computations via Multi-Dimensional Homomorphisms" (*under review at ACM TOPLAS*)

[2] "Efficient Auto-Tuning of Parallel Programs with Interdependent Tuning Parameters via Auto-Tuning Framework (ATF)", *TACO'21*

The MDH High-Level Representation

Goals:

1. **Uniform:**

should be able to express any kind of data-parallel computation, but without relying on computation-specific building blocks, extensions, etc.

2. **Minimalistic:**

should rely on less building blocks to keep language small and simple

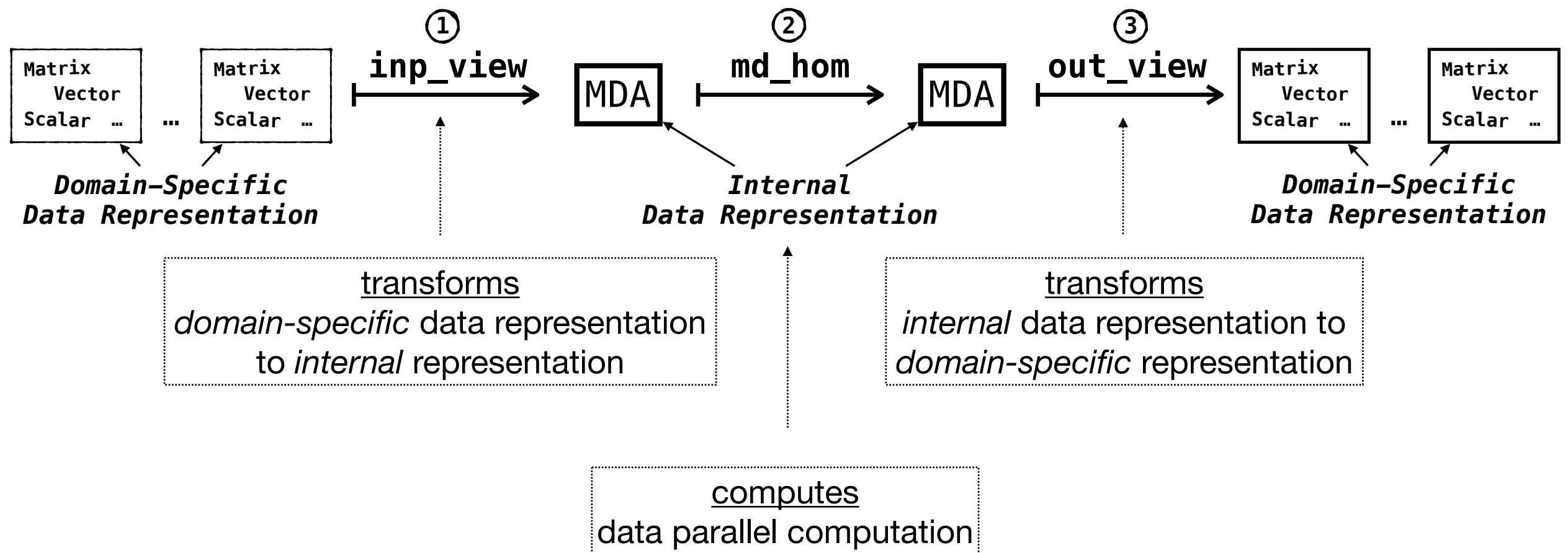
3. **Structured:**

avoiding compositions and nestings of building blocks as much as possible, thereby further contributing to usability and simplicity of our language

**While still capturing all information
relevant for generating high performing program code,
in a hardware- and data-agnostic manner**

The MDH High-Level Representation

Overview:



Our high-level representation expresses any data-parallel computation
— *agnostic from hardware and optimization details* —
using exactly three higher-order functions only

The MDH High-Level Representation

The MDH's high-level program representation illustrated:

```
MatVec<T ∈ TYPE | I, K ∈ ℕ> := out_view<T>( w: (i, k) ↦ (i) ) ∘  
                                md_hom<I, K>( *, (‡, +) ) ∘  
                                inp_view<T, T>( M: (i, k) ↦ (i, k) , v: (i, k) ↦ (k) )
```

MDH High-Level Representation¹ for MatVec

What is happening here:

- `inp_view` captures the accesses to input data
- `md_hom` expresses the data-parallel computation
- `out_view` captures the accesses to output data

¹We can generate such MDH expressions also automatically from straightforward (annotated) C code [IMPACT'19]

The MDH High-Level Representation

md_hom	f	$\otimes_1, \dots, \otimes_D$
Fill	id	++, ..., ++
ExpandDims<0>	id	++, ..., ++
ExpandDims<1>	id	++, ..., ++
ExpandDims<0,1>	id	++, ..., ++
⋮	⋮	⋮
Transpose< σ >	id	++, ..., ++
Exp	exp	++, ..., ++
Mul	*	++, ..., ++
BiasAdd<NHWC>	+	++, ++, ++, ++
BiasAdd<NCHW>	+	++, ++, ++, ++
Range	$(s, d, i) \mapsto (s + d * i)$	++

CC-Based Operators
(computations specification)

md_hom	f	$\otimes_1, \dots, \otimes_D$
MatMul<F,F>	*	++, ++, +
MatMul<F,T>	*	++, ++, +
MatMul<T,F>	*	++, ++, +
MatMul<T,T>	*	++, ++, +
BatchMatMul<F,F>	*	++, ..., ++, +
⋮	⋮	⋮
BiasAddGrad<NHWC>	id	+, ++, ++, ++
BiasAddGrad<NCHW>	id	+, ++, ++, ++
CheckNumerics	$(x) \mapsto (x == \text{NaN})$	\vee, \dots, \vee
Sum<0><F>	id	+, ++, ++, ..., ++
Sum<0><T>	id	+, ++, ++, ..., ++
Sum<1><F>	id	++, +, ++, ..., ++
Sum<0,1><F>	id	+, ++, ++, ..., ++
⋮	⋮	⋮
Prod<0><F>	id	*, ++, ++, ..., ++
⋮	⋮	⋮
All<0><F>	id	&&, ++, ++, ..., ++
⋮	⋮	⋮

CT-Based Operators
(computations specification)

Views	inp_view		out_view
	I_1	I_2	O
Fill	$(i_1, \dots, i_D) \mapsto ()$	/	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
ExpandDims<0>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (0, i_1, i_2, \dots, i_D)$
ExpandDims<0>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_1, 0, i_2, \dots, i_D)$
ExpandDims<0,1>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (0, 0, i_1, \dots, i_D)$
⋮	⋮	⋮	⋮
Transpose< σ >	$(i_1, \dots, i_D) \mapsto (\sigma(i_1), \dots, \sigma(i_D))$	/	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
Exp	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
Mul	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_{k-1}, i_{k+1}, \dots, i_D)$	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$
⋮	⋮	⋮	⋮
BiasAdd<NHWC>	$(n, h, w, c) \mapsto (n, h, w, c)$	$(n, h, w, c) \mapsto (c)$	$(n, h, w, c) \mapsto (n, h, w, c)$
BiasAdd<NCHW>	$(n, c, h, w) \mapsto (n, c, h, w)$	$(n, c, h, w) \mapsto (c)$	$(n, c, h, w) \mapsto (n, c, h, w)$
Range	$(i) \mapsto ()$	$(i) \mapsto ()$	$(i) \mapsto (i)$

CC-Based Operators
(data-access specification)

Views	inp_view		out_view
	I_1	I_2	O
MatMul<F,F>	$(i, j, k) \mapsto (i, k)$	$(i, j, k) \mapsto (k, j)$	$(i, j, k) \mapsto (i, j)$
MatMul<F,T>	$(i, j, k) \mapsto (i, k)$	$(i, j, k) \mapsto (j, k)$	$(i, j, k) \mapsto (i, j)$
MatMul<T,F>	$(i, j, k) \mapsto (k, i)$	$(i, j, k) \mapsto (k, j)$	$(i, j, k) \mapsto (i, j)$
MatMul<T,T>	$(i, j, k) \mapsto (k, i)$	$(i, j, k) \mapsto (j, k)$	$(i, j, k) \mapsto (i, j)$
BatchMatMul<F,F>	$(b_1, \dots, i, j, k) \mapsto (b_1, \dots, i, k)$	$(b_1, \dots, i, j, k) \mapsto (b_1, \dots, k, j)$	$(b_1, \dots, i, j, k) \mapsto (b_1, \dots, i, j)$
⋮	⋮	⋮	⋮
BiasAddGrad<NHWC>	$(n, h, w, c) \mapsto (n, h, w, c)$	/	$(n, h, w, c) \mapsto (n, h, w)$
BiasAddGrad<NCHW>	$(n, c, h, w) \mapsto (n, c, h, w)$	/	$(n, c, h, w) \mapsto (n, h, w)$
CheckNumerics	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto ()$
Sum<0><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_2, \dots, i_D)$
Sum<0><T>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (0, i_2, \dots, i_D)$
Sum<1><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_1, i_3, \dots, i_D)$
Sum<0,1><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_3, \dots, i_D)$
⋮	⋮	⋮	⋮
Prod<0><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_2, \dots, i_D)$
⋮	⋮	⋮	⋮
All<0><F>	$(i_1, \dots, i_D) \mapsto (i_1, \dots, i_D)$	/	$(i_1, \dots, i_D) \mapsto (i_2, \dots, i_D)$
⋮	⋮	⋮	⋮

CT-Based Operators
(data-access specification)

Our high-level representation is capable of expressing important DL operators

Experimental Results for DL Operators

Deep Learning	NVIDIA Ampere GPU									
	ResNet-50				VGG-16				MobileNet	
	Training		Inference		Training		Inference		Training	Inference
	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MCC
TVM+Ansor	1.00	1.26	1.05	2.22	0.93	1.42	0.88	1.14	0.94	1.00
PPCG	3456.16	8.26	-	7.89	1661.14	7.06	5.77	5.08	2254.67	7.55
PPCG+ATF	3.28	2.58	13.76	5.44	4.26	3.92	9.46	3.73	3.31	10.71
cuDNN	0.92	-	1.85	-	1.22	-	1.94	-	1.81	2.14
cuBLAS	-	1.58	-	2.67	-	0.93	-	1.04	-	-
cuBLASEx	-	1.47	-	2.56	-	0.92	-	1.02	-	-
cuBLASLt	-	1.26	-	1.22	-	0.91	-	1.01	-	-

Deep Learning	Intel Skylake CPU									
	ResNet-50				VGG-16				MobileNet	
	Training		Inference		Training		Inference		Training	Inference
	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MCC
TVM+Ansor	1.53	1.05	1.14	1.20	1.97	1.14	2.38	1.27	3.01	1.40
Pluto	355.81	49.57	364.43	13.93	130.80	93.21	186.25	36.30	152.14	75.37
Pluto+ATF	13.08	19.70	170.69	6.57	3.11	6.29	53.61	8.29	3.50	25.41
oneDNN	0.39	-	5.07	-	1.22	-	9.01	-	1.05	4.20
oneMKL	-	0.44	-	1.09	-	0.88	-	0.53	-	-
oneMKL (JIT)	-	6.43	-	8.33	-	27.09	-	9.78	-	-

Deep Learning	NVIDIA Volta GPU									
	ResNet-50				VGG-16				MobileNet	
	Training		Inference		Training		Inference		Training	Inference
	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MCC
TVM+Ansor	0.75	1.21	0.72	1.79	1.00	1.11	1.06	1.00	1.00	1.00
PPCG	1976.38	5.88	-	5.64	994.16	3.41	8.21	2.51	1411.92	7.26
PPCG+ATF	3.43	3.54	3.42	4.93	3.85	3.15	8.13	2.05	3.49	3.56
cuDNN	1.21	-	1.29	-	2.80	-	3.50	-	2.32	3.14
cuBLAS	-	1.33	-	1.14	-	1.09	-	1.04	-	-
cuBLASEx	-	1.21	-	1.07	-	1.04	-	1.03	-	-
cuBLASLt	-	1.00	-	1.07	-	1.04	-	1.02	-	-

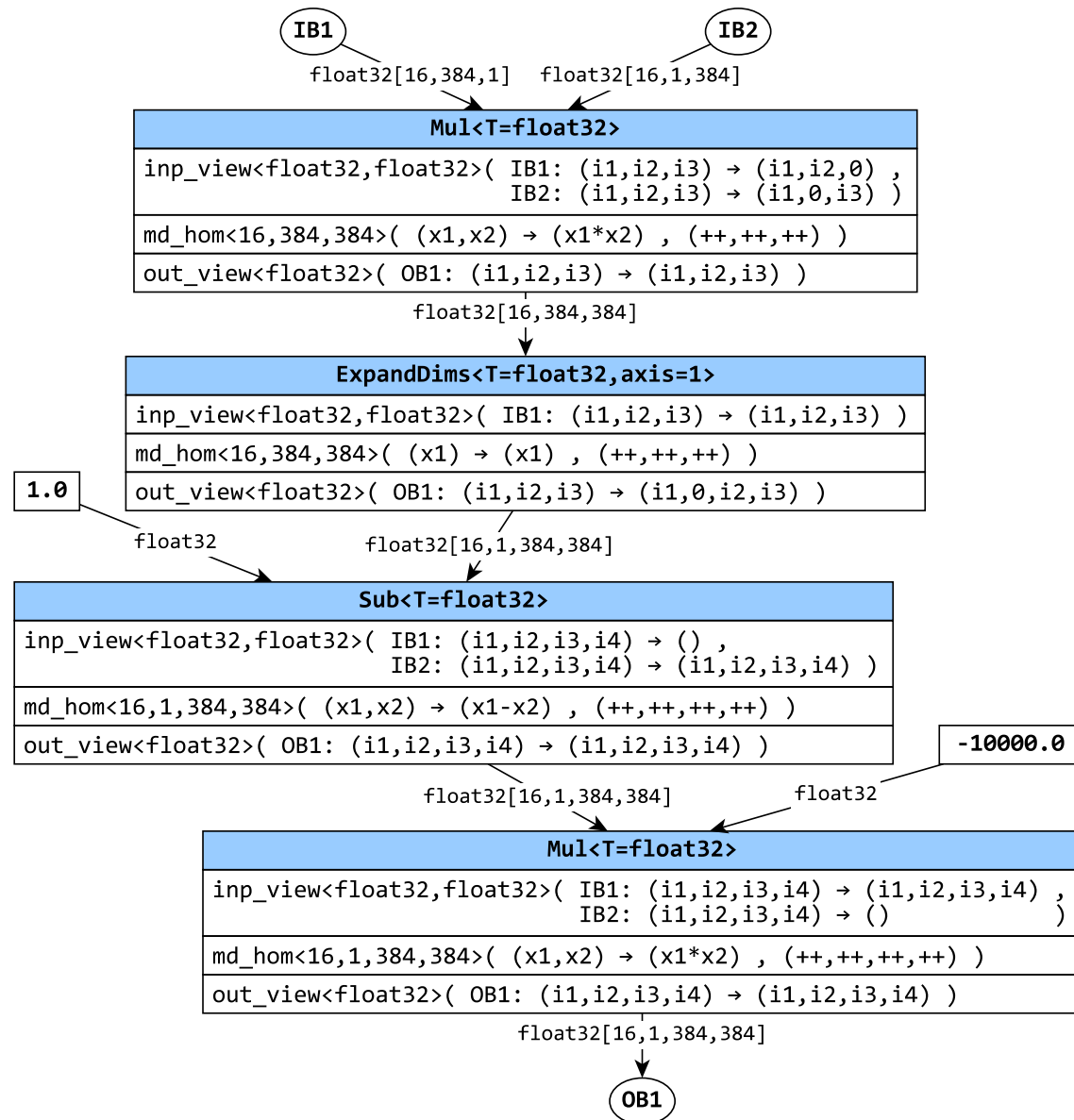
Deep Learning	Intel Broadwell CPU									
	ResNet-50				VGG-16				MobileNet	
	Training		Inference		Training		Inference		Training	Inference
	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MatMul	MCC	MCC
TVM+Ansor	1.53	1.60	1.29	1.53	1.32	1.00	1.27	1.02	2.42	1.92
Pluto	4349.20	40.41	137.21	15.96	1865.07	53.57	113.40	24.10	2255.00	53.85
Pluto+ATF	6.43	8.93	61.60	6.91	5.07	4.38	42.63	4.45	6.43	29.18
oneDNN	1.30	-	1.81	-	2.94	-	2.85	-	1.83	4.47
oneMKL	-	1.45	-	1.36	-	1.35	-	0.50	-	-
oneMKL (JIT)	-	19.78	-	9.77	-	50.58	-	10.70	-	-

**We achieve encouraging experimental results
for DL Operators [1]**

Excursion: MDH High-Level Optimization

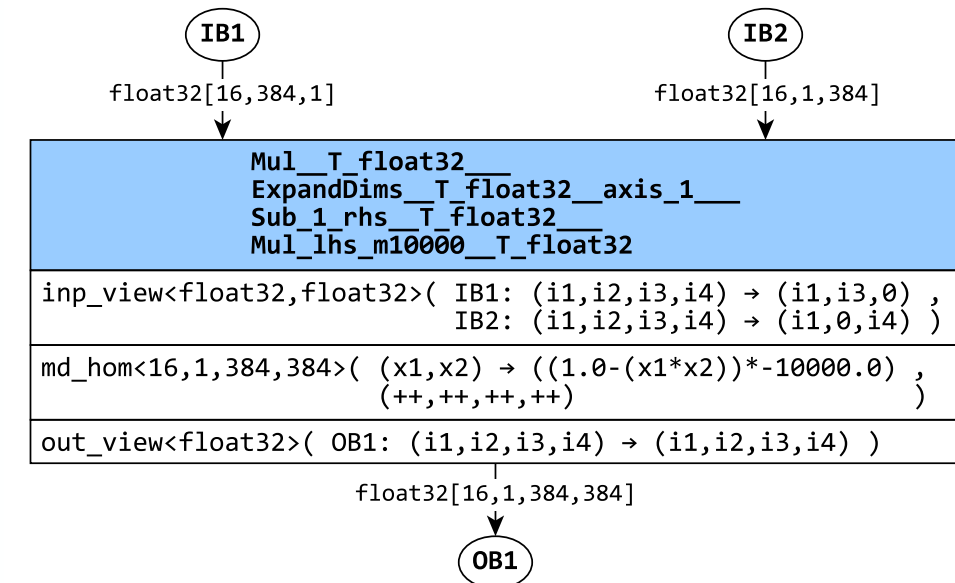
WIP
Results

An optimization is considered **High Level** iff it operates on **MDH's High-Level Representation**:



b) MDH subgraph (naive)

MDH HL-Opt



c) MDH subgraph (fused)

We exploit the uniform MDH representation to analyze and fuse the DL graph

Experimental Results for DL Graphs

**WIP
Results**

NVIDIA Ampere GPU				
	Number of Operators	Operators Occurring in Subgraph	Runtime Share	Speedup over TF
				MDH TC
1.	13	(Sub,1),(Mul,5),(AddV2,4),(RealDiv,1),(Sqrt,1),(Square,1)	25.96%	65.93 45.72
2.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	11.40%	30.99 30.50
3.	41	(BiasAddGrad,1),(AddN,1),(Mul,17),(TanhGrad,2),(Pow,4),(AddV2,4),(Tanh,3),(BiasAdd,9),(Sub,1)	3.79%	1.41 0.05
4.	3	(Mul,1),(Reshape,1),(AddV2,1)	3.60%	23.79 14.72
5.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	2.87%	6.79 6.73
6.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	2.70%	6.36 6.24
7.	2	(BiasAddGrad,1),(Reshape,1),(Transpose,1)	2.64%	9.86 0.57
8.	5	(BiasAddGrad,1),(Mul,2),(Reshape,1),(Cast,1),(GreaterEqual,1)	2.45%	4.55 1.60
9.	13	(Sub,1),(Mul,5),(AddV2,4),(RealDiv,1),(Sqrt,1),(Square,1)	2.40%	39.57 36.93
10.	9	(AddV2,2),(Mul,3),(Cast,1),(BiasAdd,1),(GreaterEqual,1),(Reshape,1)	1.47%	1.63 1.60
Total Speedup over TF:				2.29 0.79

Intel Skylake CPU				
	Number of Operators	Operators Occurring in Subgraph	Runtime Share	Speedup over TF
				MDH
1.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	17.33%	571.14
2.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	7.20%	248.81
3.	9	(AddV2,2),(Mul,3),(Cast,1),(BiasAdd,1),(GreaterEqual,1),(Reshape,1)	6.94%	110.51
4.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	6.06%	199.32
5.	8	(Mul,4),(Cast,1),(Softmax_Div,1),(GreaterEqual,1),(AddV2,1)	5.45%	12.76
6.	11	(Mul,6),(Sub,1),(Softmax_Div,1),(AddV2,1),(Cast,1),(GreaterEqual,1)	5.20%	10.81
7.	41	(BiasAddGrad,1),(AddN,1),(Mul,17),(TanhGrad,2),(Pow,4),(AddV2,4),(Tanh,3),(BiasAdd,9),(Sub,1)	3.71%	2.82
8.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	1.50%	24.77
9.	2	(Transpose,1),(Reshape,1)	0.59%	12.08
10.	15	(Sub,1),(Mul,6),(AddV2,5),(RealDiv,1),(Sqrt,1),(Square,1)	0.42%	875.92
Total Speedup over TF:				2.11

**We achieve encouraging experimental results
also for DL Graphs with MDH**

Excursion: MDH in MLIR



MLIR is a compiler framework that offers a solid, uniform infrastructure for compiler developers to conveniently design and implement *Domain-Specific Languages (DSLs)* (a.k.a. *dialect* in MLIR terminology)



Implemented by Jens & Lars Hunloh
(University of Muenster, Germany)

```
func.func @main()
{
  %M = memref.alloc() : memref<128x64xf32>
  %v = memref.alloc() : memref<64xf32>

  %w = mdh.compute "mdh_matvec"
  {
    inp_view =
    [
      [ affine_map<( i,k ) -> ( i,k )> ],
      [ affine_map<( i,k ) -> ( k )> ]
    ],

    md_hom =
    {
      scalar_func = @mul,
      combine_ops = [ "cc", ["pw",@add] ]
    },

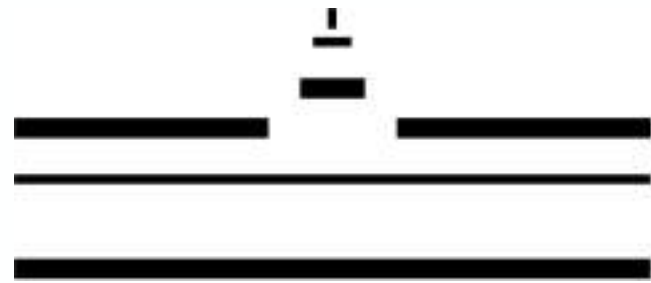
    out_view =
    [
      [ affine_map<( i,k ) -> ( i )> ]
    ]
  }
  {
    inp_types = [ f32, f32 ],
    mda_size = [ 128, 64 ],
    out_types = [ f32 ]
  }
  { (%A,%B) : ( memref<128x64xf32>, memref<64xf32> )
    -> memref<128xf32> }

  return
}
```

MatVec^{<T ∈ TYPE | I, K ∈ ℕ>} := out_view<T>(w:(i,k)↦(i)) ◦
md_hom<I, K>(*, (#, +)) ◦
inp_view<T, T>(M:(i,k)↦(i,k) , v:(i,k)↦(k))

MDH

WIP
Results



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**We look forward
to discussions
at the poster session**



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<https://mdh-lang.org>



<https://atf-tuner.org>



<https://hca-project.org>