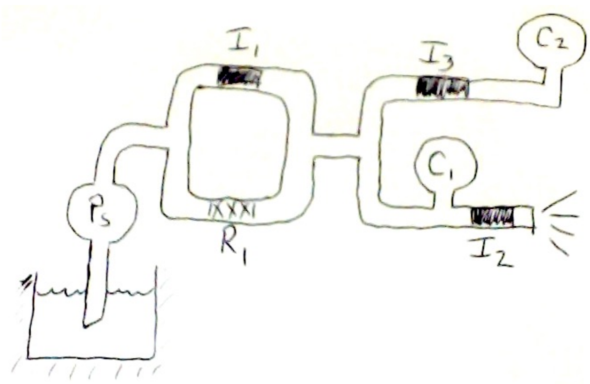


Find a complete state-space model (A, B, C, and D matrices, plus $\underline{z}, \underline{u}, \underline{y}$ vectors) for the system shown at left.

The input is the pump pressure source P_s

The two outputs are
(i) the pressure change across the inertance Q_1, and
(ii) the flow through the resistor R

Solution:
Begin by defining the state, input, and output vectors:

$$\underline{z}(t) = \begin{bmatrix} P_{C1} \\ P_{C2} \\ Q_{I1} \\ Q_{I2} \\ Q_{I3} \end{bmatrix} \qquad \underline{u}(t) = (P_s) \qquad \underline{y}(t) = \begin{bmatrix} P_{I1} \\ Q_{R1} \end{bmatrix}$$

Now proceed to find the state and output equations and fill in A, B, C, and D:

$$\dot{P}_{C1} = \frac{1}{C_1} Q_{C1} = \frac{1}{C_1}\left[Q_{I1} + Q_{R1} - Q_{I2} - Q_{I3}\right] = \frac{1}{C_1}\left[Q_{I1} + \frac{P_s - P_{C1}}{R_1} - Q_{I2} - Q_{I3}\right]$$
$$\dot{P}_{C2} = \frac{1}{C_2} Q_{C2} = \frac{1}{C_2} Q_{I3}$$
$$\dot{Q}_{I1} = \frac{1}{I_1} P_{I1} = \frac{1}{I_1}\left[P_s - P_{C1}\right]$$
$$\dot{Q}_{I2} = \frac{1}{I_2} P_{I2} = \frac{1}{I_2} P_{C1}$$
$$\dot{Q}_{I3} = \frac{1}{I_3} P_{I3} = \frac{1}{I_3}\left[P_{C1} - P_{C2}\right]$$

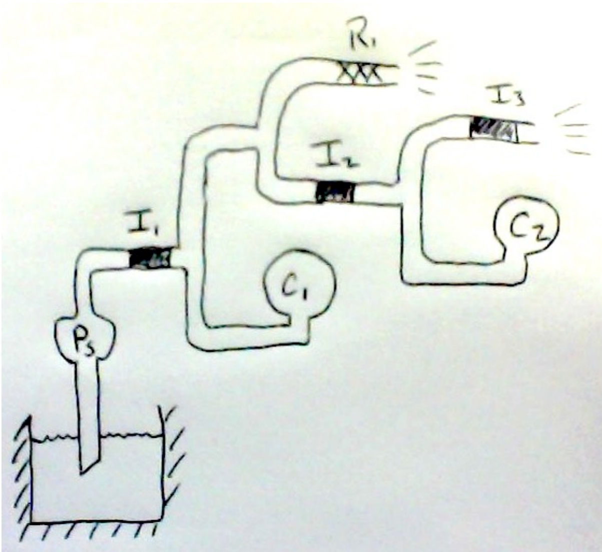
Thus,

$$A = \begin{bmatrix} -\frac{1}{C_1 R_1} & 0 & \frac{1}{C1} & -\frac{1}{C1} & -\frac{1}{C1} \\ 0 & 0 & 0 & 0 & \frac{1}{C2} \\ -\frac{1}{I1} & 0 & 0 & 0 & 0 \\ \frac{1}{I2} & 0 & 0 & 0 & 0 \\ \frac{1}{I3} & -\frac{1}{I3} & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{C_1 R_1} \\ 0 \\ \frac{1}{I1} \\ 0 \\ 0 \end{bmatrix}$$

The output equation is

$$\underline{y}(t) = \begin{bmatrix} P_{I1} \\ Q_{R1} \end{bmatrix} = \begin{bmatrix} P_s - P_{C1} \\ \frac{1}{R_1}(P_s - P_{C1}) \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ -\frac{1}{R1} & 0 & 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 1 \\ \frac{1}{R1} \end{bmatrix}$$



Find a complete state-space model (A, B, C, and D matrices, plus $\underline{z}, \underline{u}, \underline{y}$ vectors) for the system shown at left.

The input is the pump pressure source P_s

The two outputs are
(i) the flow into the capacitance C_1, and
(ii) the flow through the resistor R_1

Solution:
Begin by defining the state, input, and output vectors:

$$\underline{z}(t) = \begin{bmatrix} P_{C1} \\ P_{C2} \\ Q_{I1} \\ Q_{I2} \\ Q_{I3} \end{bmatrix} \qquad \underline{u}(t) = (P_s) \qquad \underline{y}(t) = \begin{bmatrix} Q_{C1} \\ Q_{R1} \end{bmatrix}$$

Now proceed to find the state and output equations and fill in A, B, C, and D:

$$\begin{aligned}
\dot{P}_{C1} &= \frac{1}{C_1} Q_{C1} = \frac{1}{C_1}\left[Q_{I1} - Q_{I2} - Q_{R1}\right] = \frac{1}{C_1}\left[Q_{I1} - Q_{I2} - \frac{P_{C1}}{R_1}\right] \\
\dot{P}_{C2} &= \frac{1}{C_2} Q_{C2} = \frac{1}{C_2}(Q_{I2} - Q_{I3}) \\
\dot{Q}_{I1} &= \frac{1}{I_1} P_{I1} = \frac{1}{I_1}(P_s - P_{C1}) \\
\dot{Q}_{I2} &= \frac{1}{I_2} P_{I2} = \frac{1}{I_2}(P_{C1} - P_{C2}) \\
\dot{Q}_{I3} &= \frac{1}{I_3} P_{I3} = \frac{1}{I_3} P_{C2}
\end{aligned}$$

Thus,

$$A = \begin{bmatrix} -\frac{1}{C_1 R_1} & 0 & \frac{1}{C_1} & -\frac{1}{C_1} & 0 \\ 0 & 0 & 0 & \frac{1}{C_2} & -\frac{1}{C_2} \\ -\frac{1}{I_1} & 0 & 0 & 0 & 0 \\ \frac{1}{I_2} & -\frac{1}{I_2} & 0 & 0 & 0 \\ 0 & \frac{1}{I_3} & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{I_1} \\ 0 \\ 0 \end{bmatrix}$$

The output equation is

$$\underline{y}(t) = \begin{bmatrix} Q_{C1} \\ Q_{R1} \end{bmatrix} = \begin{bmatrix} Q_{I1} - Q_{I2} - \frac{P_{C1}}{R_1} \\ \frac{P_{C1}}{R_1} \end{bmatrix}$$

$$C = \begin{bmatrix} -\frac{1}{R1} & 0 & 1 & -1 & 0 \\ \frac{1}{R1} & 0 & 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$