

A system's Bode plots are shown at left.

1. Estimate the system's transfer function:

The initial slope is +20 db/dec, so there is a zero at  $s = 0$ . From the two corners, there are poles at  $\omega = 1$  and  $\omega = 100$ . Thus, the TF is

$$TF = \frac{K * s}{(s + 1)(s + 100)} = \frac{K}{100} * \frac{s}{(s + 1)(s/100 + 1)}$$

To determine  $K$ , pick a convenient frequency; for example, at  $\omega = 1$ :

zero $s = 0$	$\rightarrow 0 \text{ db}$
poles $+ 1$	$\rightarrow -3 \text{ db}$
pole $\frac{s}{100} + 1$	$\rightarrow 0 \text{ db}$

Adding those terms, we see that the magnitude (without  $K/100$ ) is -3 db, which matches the plot at  $\omega = 1$ . Thus the magnitude of  $K/100$  must also be 0 db, i.e.,  $K = 100$ . The complete TF is

$$TF = \frac{100s}{(s + 1)(s + 100)}$$

- What kind of filter (if any) is this system? Bandpass
- What is this system's bandwidth (if any)?  $1 \leq \omega \leq 100$
- What is the response of this system to an input of  $100\sin(1000t)$ ?

At  $\omega = 1000$  we have -20 db, so the magnitude ratio is  $1/10$ . The phase is approximately  $-90^\circ$ , so the output is

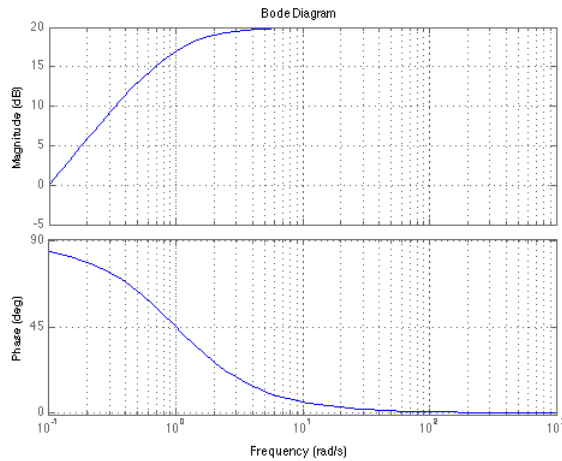
$$y(t) = 10\sin(1000t - 90^\circ) = 10\sin(1000t - \pi/2)$$

- Briefly but precisely describe how to convert the system into a low-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

We need to flatten the left end, so we may simply remove the zero. This creates a low-pass filter. In addition, we could remove one of the poles (but this isn't necessary).

- Briefly but precisely describe how to convert the system into a high-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

We need to flatten the right end, so remove either one of the two poles.



A system's Bode plots are shown at left.

1. Estimate the system's transfer function:

The initial slope is +20 dB/dec, so there is a zero at  $s = 0$ . From the corner, there is a pole at  $\omega = 1$ . Thus, the TF is

$$TF = \frac{K * s}{(s + 1)}$$

To determine  $K$ , pick a convenient frequency; for example, at  $\omega = 1$ :

$$\begin{array}{ll} \text{zero } s = 0 & \rightarrow 0 \text{ db} \\ \text{poles } + 1 & \rightarrow -3 \text{ db} \end{array}$$

(1)

Adding those terms, we see that the magnitude (without  $K$ ) is -3 db, but the plot at  $\omega = 1$  reads +17 db. Thus the magnitude of  $K$  must be +20 db, i.e.,  $K = 10$ . The complete TF is

$$TF = \frac{10s}{(s + 1)}$$

2. What kind of filter (if any) is this system? Highpass
3. What is this system's bandwidth (if any)?  $1 \leq \omega$
4. What is the response of this system to an input of  $100\sin(1000t)$ ?

At  $\omega = 1000$  we have +20 db, so the magnitude ratio is 10. The phase is approximately 0°, so the output is

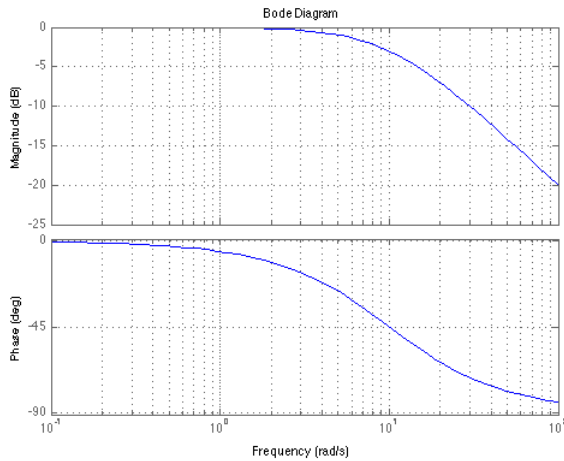
$$y(t) = 1000\sin(1000t)$$

5. Briefly but precisely describe how to convert the system into a low-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

We need to flatten the left end, so we may simply remove the zero. This creates a low-pass filter, since the pole now creates a corner followed by a -20 db/decade slope.

6. Briefly but precisely describe how to convert the system into a band-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

Add a pole anywhere but  $s = 0$



A system's Bode plots are shown at left.

1. Estimate the system's transfer function:

The initial slope is 0, followed by a corner at  $\omega = 10$ , followed by a slope of -20 dB/dec, so there is a pole at  $s = -10$ . Thus, the TF is

$$TF = \frac{K}{(s + 10)} = \frac{K}{10} * \frac{1}{s/10 + 1}$$

To determine  $K$ , pick a convenient frequency; for example, at  $\omega = 1$ , the term  $1/(s/10 + 1)$  has 0 db. Since the plot shows 0 db, we also need  $K/10$  to have 0 db, so  $K = 10$ :

$$TF = \frac{10}{s + 10}$$

2. What kind of filter (if any) is this system? Lowpass
3. What is this system's bandwidth (if any)?  $0 \leq \omega \leq 10$
4. What is the response of this system to an input of  $100\sin(1000t)$ ?

At  $\omega = 100$  we have -20 db and a slope of -20 db that continues to higher frequencies, so at  $\omega = 1000$  the plot would show -40 db. Thus, the magnitude ratio is 1/100. The phase is approximately -90°, so the output is

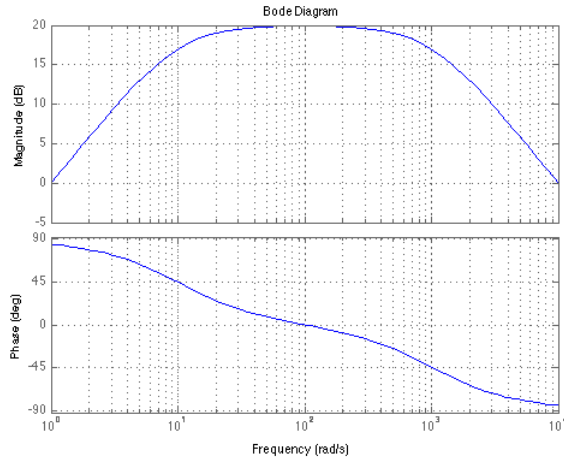
$$y(t) = \sin(1000t - 90^\circ) = \sin(1000t - \pi/2)$$

5. Briefly but precisely describe how to convert the system into a high-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

Add a zero to the left of  $\omega = 10$

6. Briefly but precisely describe how to convert the system into a band-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

Add a zero somewhere to the left of  $\omega = 10$ , then add a second pole anywhere to the right of that zero.



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1. Estimate the system's transfer function:

The initial slope is +20 db/dec, so there is a zero at  $s = 0$ . From the two corners, there are poles at  $\omega = 10$  and  $\omega = 1000$ . Thus, the TF is

$$TF = \frac{K * s}{(s + 10)(s + 1000)} = \frac{K}{1000} * \frac{s}{(s + 10)(s/1000 + 1)}$$

To determine  $K$ , pick a convenient frequency; for example, at  $\omega = 1$ :

zero $s = 0$	$\rightarrow 0 \text{ db}$
pole $(s + 10)$	$\rightarrow 0 \text{ db}$
pole $\frac{s}{1000} + 1$	$\rightarrow 0 \text{ db}$

Adding those terms, we see that the magnitude (without  $K/1000$ ) is 0 db, which matches the plot at  $\omega = 1$ . Thus the magnitude of  $K/1000$  must also be 0 db, i.e.,  $K = 1000$ . The complete TF is

$$TF = \frac{1000s}{(s + 10)(s + 1000)}$$

2. What kind of filter (if any) is this system? Bandpass
3. What is this system's bandwidth (if any)?  $10 \leq \omega \leq 1000$
4. What is the response of this system to an input of  $100\sin(1000t)$ ?

At  $\omega = 1000$  we have 17 db, so the magnitude ratio is 7.08. The phase is  $-45^\circ$ , so the output is

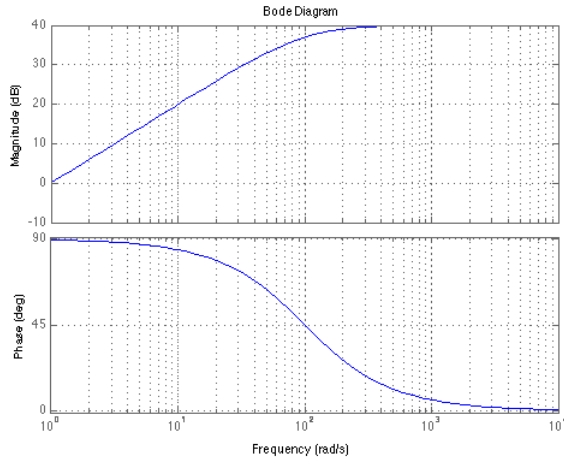
$$y(t) = 708\sin(1000t - 45^\circ) = 708\sin(1000t - \pi/4)$$

5. Briefly but precisely describe how to convert the system into a low-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

We need to flatten the left end, so we may simply remove the zero. This creates a low-pass filter. In addition, we could remove one of the poles (but this isn't necessary).

6. Briefly but precisely describe how to convert the system into a high-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

We need to flatten the right end, so remove either one of the two poles.



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The initial slope is +20 db/dec, so there is a zero at  $s = 0$ . From the corner, there is a pole at  $\omega = 100$ . Thus, the TF is

$$TF = \frac{K * s}{(s + 100)} = \frac{K}{100} \frac{s}{s/100 + 1}$$

To determine  $K$ , pick a convenient frequency; for example, at  $\omega = 1$ :

$$\begin{array}{ll} \text{zero } s = 0 & \rightarrow 0 \text{ db} \\ \text{poles } + 100 & \rightarrow 0 \text{ db} \end{array}$$

(2)

Adding those terms, we see that the magnitude (without  $K$ ) is 0 db, and the plot at  $\omega = 1$  reads 0 db. Thus the magnitude of  $K/100$  must be 0 db, i.e.,  $K = 100$ . The complete TF is

$$TF = \frac{100s}{(s + 100)}$$

2. What kind of filter (if any) is this system? Highpass
3. What is this system's bandwidth (if any)?  $100 \leq \omega$
4. What is the response of this system to an input of  $100\sin(1000t)$ ?

At  $\omega = 1000$  we have +40 db, so the magnitude ratio is 100. The phase is  $-45^\circ$ , so the output is

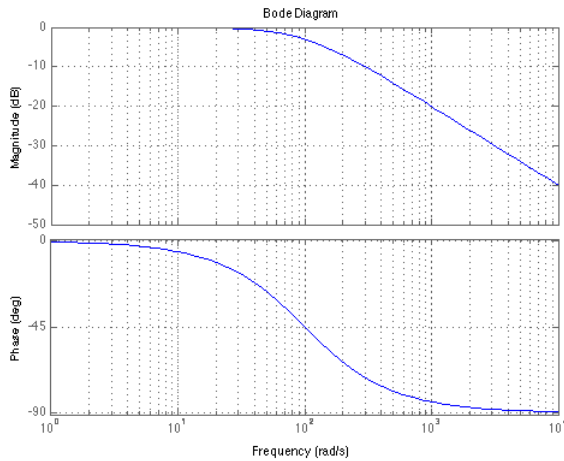
$$y(t) = 10,000\sin(1000t - 45^\circ) = 10,000\sin(1000t - \pi/4)$$

5. Briefly but precisely describe how to convert the system into a low-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

We need to flatten the left end, so we may simply remove the zero. This creates a low-pass filter, since the pole now creates a corner followed by a -20 db/decade slope.

6. Briefly but precisely describe how to convert the system into a band-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

Add a pole anywhere but 0 (i.e., a pole at any real number).



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1. Estimate the system's transfer function:

The initial slope is 0, followed by a corner at  $\omega = 100$ , followed by a slope of -20 db/dec, so there is a pole at  $s = -100$ . Thus, the TF is

$$TF = \frac{K}{(s + 100)} = \frac{K}{100} * \frac{1}{s/100 + 1}$$

To determine  $K$ , pick a convenient frequency; for example, at  $\omega = 1$ , the term  $1/(s/100 + 1)$  has 0 db. Since the plot shows 0 db, we also need  $K/100$  to have 0 db, so  $K = 100$ :

$$TF = \frac{100}{s + 100}$$

2. What kind of filter (if any) is this system? Lowpass
3. What is this system's bandwidth (if any)?  $0 \leq \omega \leq 100$
4. What is the response of this system to an input of  $100\sin(1000t)$ ?

At  $\omega = 100$  we have -20 db, so the magnitude ratio is  $1/100$ . The phase is approximately  $-90^\circ$ , so the output is

$$y(t) = \sin(1000t - 90^\circ) = \sin(1000t - \pi/2)$$

(the phase angle is actually a little less than  $90^\circ$ , say,  $85^\circ$ )

5. Briefly but precisely describe how to convert the system into a high-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

Add a zero to the left of  $\omega = 100$

6. Briefly but precisely describe how to convert the system into a band-pass filter by adding or subtracting poles or zeros, OR, explain why it is impossible to do so. Be specific about the numerical values of any poles(s) and/or zero(s) to be added (or subtracted) from the system's TF.

Add a zero somewhere to the left of  $\omega = 100$ , then add a second pole anywhere to the right of that zero.