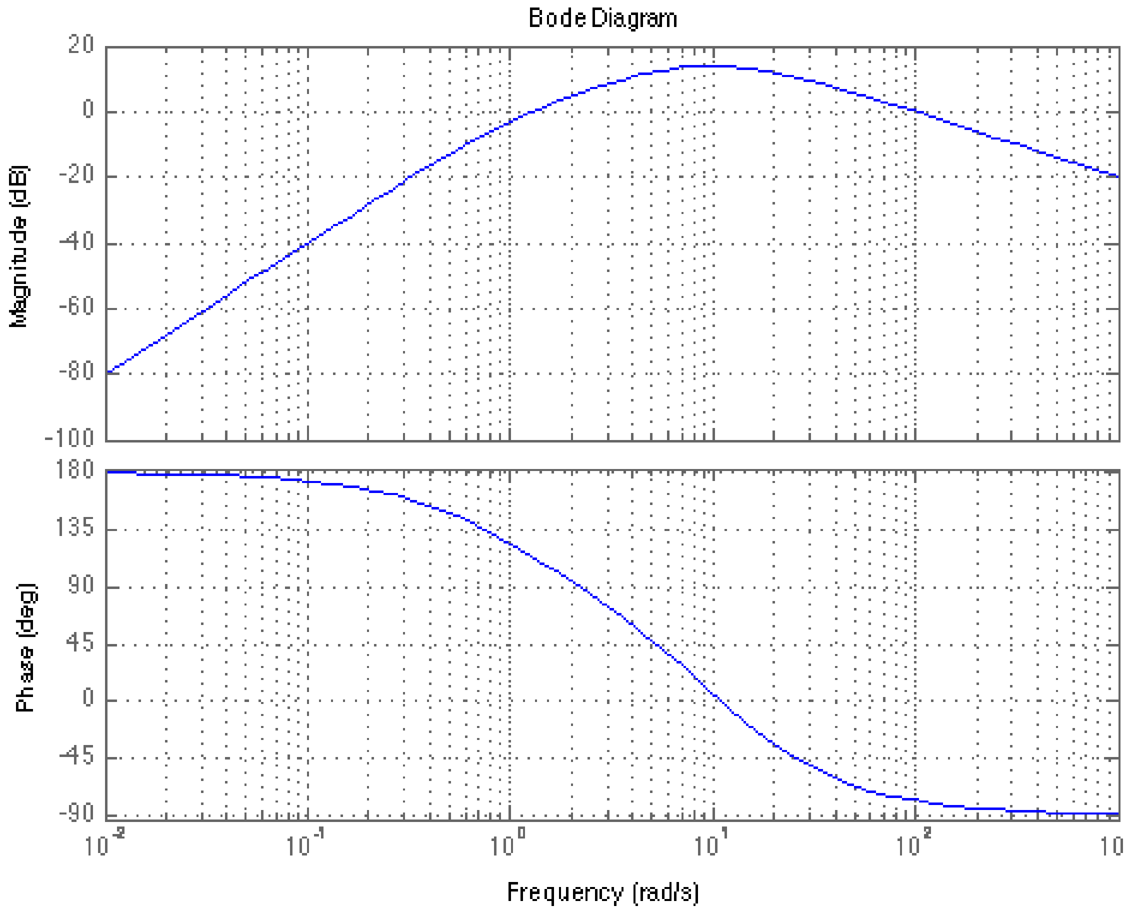


1. (5 pts each): Use the Bode plots to answer the following 10 questions (see BACK OF PAGE too!)

NOTE: You MUST clearly put a box around your answers, or receive NO CREDIT! NO CREDIT!

1. If the input magnitude is fixed at 3 (constant across all input frequencies), what input frequency produces the largest output? What is the magnitude of that output?

   $\boxed{\text{Peak magnitude in plot occurs at } \omega = 10\ rad/s}$

   The peak magnitude is 14 db, so $\boxed{y_{max} = 3 * 10^{14/20} = 15}$

2. Are there any poles or zeros equal to 0? If yes, how many of each?

   From both the mag and phase plots, clearly there are $\boxed{\text{two zeros at } 0}$

3. Assuming there is no pole/zero cancellation, what is the order of the system?

   From both mag and phase, we see that there are 3 poles, so the $\boxed{\text{system is } 3rd\text{-order}}$

4. For what range of frequencies, if any, does the system behave like an amplifier?

   $\boxed{\text{From approximately } 1.3 \leq \omega \leq 100\ rad/s}$

5. At what frequency, or frequencies, if any, will an input phase of 75^o result in an output phase of 120^o?

   We need phase angle of $120 - 75 = 45^o$, so $\boxed{\omega \approx 5.3\ rad/s}$

6. What is the difference between the number of poles and the number of zeros?

   From both mag and phase, we see 3 poles and 2 zeros, so the $\boxed{\text{difference} = 1 \text{ more pole than zero}}$

7. If the input is $u(t) = 400sin(0.1t) + 5sin(100t)$, what is the steady-state output?

   For $\omega = 0.1$, we have mag = -40 db and phase $\approx 170^o$

   For $\omega = 100$, we have mag = 0 db and phase $\approx -75^o$

   Therefore, the output is $\boxed{y = 4sin(0.1t + 170^o) + 5sin(100t - 75^o)}$

8. What kind of filter is this system (if any)?

   $\boxed{\text{Bandpass}}$

9. What is the system bandwidth (if any)?

   At the scale of the plot, it's hard to read exactly, but $\boxed{\text{BW}\approx 4.3 \leq \omega \leq 25 \text{ rad/s}}$

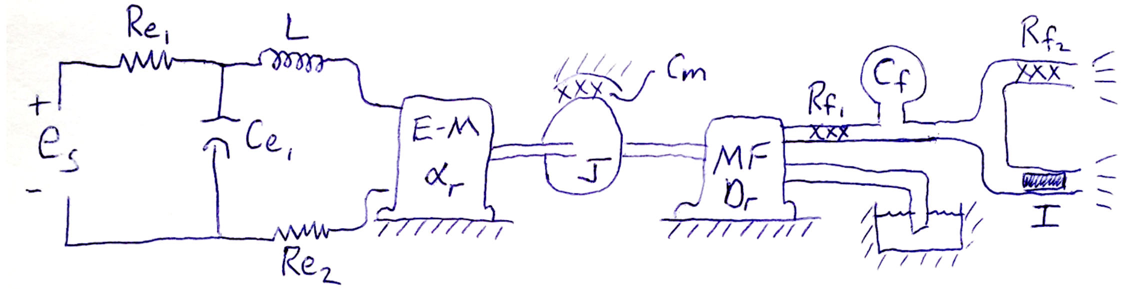
10. What is the approximate transfer function for the system?

    Looking at both mag and phase plots, there appear to be two zeros at zero, followed by a pole at 1 and a pair of poles at 10:

    $$TF = \frac{Ks^2}{(s+1)(s+10)^2} = \frac{K}{100}\frac{s^2}{(s+1)(s/10+1)^2}$$

    To find K, check the magnitude at any frequency. I choose $\omega = 1$ for convenience. The plot shows that the magnitude there is (-3 db). Adding up the individual contributions of the factors in the TF, we have 0 db from the two zeros; (-3 db) from the pole factor (s+1); and 0 db from the pole factors (s/10+1). Therefore, we need 0 db from the pole factor $K/100$, so therefore $K = 100$.

    $$\boxed{TF = \frac{100s^2}{(s+1)(s+10)^2}}$$



2. (50 points) Find a complete state-space model (A, B, C, and D matrices, plus $\underline{z}, \underline{u}, \underline{y}$ vectors) for the system shown.

The only input is the electric voltage source e_s

The two outputs are
(i) the flow into the capacitance C_f, and
(ii) the current through the resistor R_{e1}

For the transducers, $i_{EM} = \alpha_r T_{EM}$; $\dot{\theta} = \alpha_r e_{EM}$; $T_{MF} = D_r P_{MF}$; $Q_{MF} = D_r \dot{\theta}$

Begin with the usual sub-system states:

$$\underline{z}(t) = \begin{pmatrix} e_{Ce1} \\ i_L \\ \dot{\theta} \\ P_{Cf} \\ Q_I \end{pmatrix} \quad , \quad \underline{u} = (e_s) \quad , \quad \underline{y} = \begin{pmatrix} Q_{Cf} \\ i_{Re1} \end{pmatrix}$$

Now proceed to find the state-space model:

$$\dot{e}_{Ce1} = \frac{1}{C_{e1}} i_{Ce1} = \frac{1}{C_{e1}}(i_{Re1} - i_L) = \frac{1}{C_{e1}}(\frac{e_s - e_{Ce1}}{R_{e1}} - i_L)$$

$$\dot{i}_L = \frac{1}{L} e_L = \frac{1}{L}(e_{Ce1} - e_{EM} - e_{Re2}) = \frac{1}{L}(e_{Ce1} - \frac{\dot{\theta}}{\alpha_r} - R_{e2} i_L)$$

$$\begin{aligned} \ddot{\theta} &= \frac{1}{J}(T_{EM} - C_m \dot{\theta} - T_{MF}) = \frac{1}{J}(\frac{i_{EM}}{\alpha_r} - C_m \dot{\theta} - D_r P_{MF}) \\ &= \frac{1}{J}(\frac{i_L}{\alpha_r} - C_m \dot{\theta} - D_r(P_{Cf} + P_{Rf1})) = \frac{1}{J}(\frac{i_L}{\alpha_r} - C_m \dot{\theta} - D_r(P_{Cf} + R_{f1} Q_{MF})) \\ &= \frac{1}{J}(\frac{i_L}{\alpha_r} - C_m \dot{\theta} - D_r(P_{Cf} + R_{f1} D_r \dot{\theta})) \end{aligned}$$

$$\dot{P}_{Cf} = \frac{1}{C_f} Q_{Cf} = \frac{1}{C_f}(Q_{MF} - Q_I - Q_{Rf2}) = \frac{1}{C_f}(D_r \dot{\theta} - Q_I - \frac{P_{Cf}}{R_{f2}})$$

$$\dot{Q}_I = \frac{1}{I} P_I = \frac{1}{I} P_{Cf}$$

Thus,

$$A = \begin{pmatrix} -\frac{1}{C_{e1}R_{e1}} & -\frac{1}{C_{e1}} & 0 & 0 & 0 \\ \frac{1}{L} & -\frac{R_{e2}}{L} & -\frac{1}{L\alpha_r} & 0 & 0 \\ 0 & \frac{1}{J\alpha_r} & -\frac{C_m+D_r^2R_{f1}}{J} & -\frac{D_r}{J} & 0 \\ 0 & 0 & \frac{D_r}{C_f} & -\frac{1}{C_fR_{f2}} & -\frac{1}{C_f} \\ 0 & 0 & 0 & \frac{1}{I} & 0 \end{pmatrix} \quad , \quad B = \begin{pmatrix} \frac{1}{C_{e1}R_{e1}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & D_r & -\frac{1}{R_{f2}} & -1 \\ -\frac{1}{R_{e1}} & 0 & 0 & 0 & 0 \end{pmatrix} \quad , \quad D = \begin{pmatrix} 0 \\ \frac{1}{R_{e1}} \end{pmatrix}$$