

Plot the root locus plots for each of the systems below, in the order given. This will provide essential extra practice at creating the plots, and also help to illustrate the effect of different roots in the D and N polynomials.

As you prepare each plot, find the answers to the following usual questions:

- For what range of K (if any) is the system stable?
- For what range(s) of K (if any) is the system homogeneous response nonoscillatory?
- For what K does the system have the fastest settling time, and what is that settling time?
- Then, before creating the next plot, look at the change in the characteristic equation compared to the one you've just finished, and try to predict the effect it will have**

I generated each of the following plots using MATLAB; for example, here is how I generated the plot for number 3:

```
>> D=poly([-1 -3]);N=[1];rlocus(N,D);set(gca,'XGrid','on');set(gca,'YGrid','on');
```

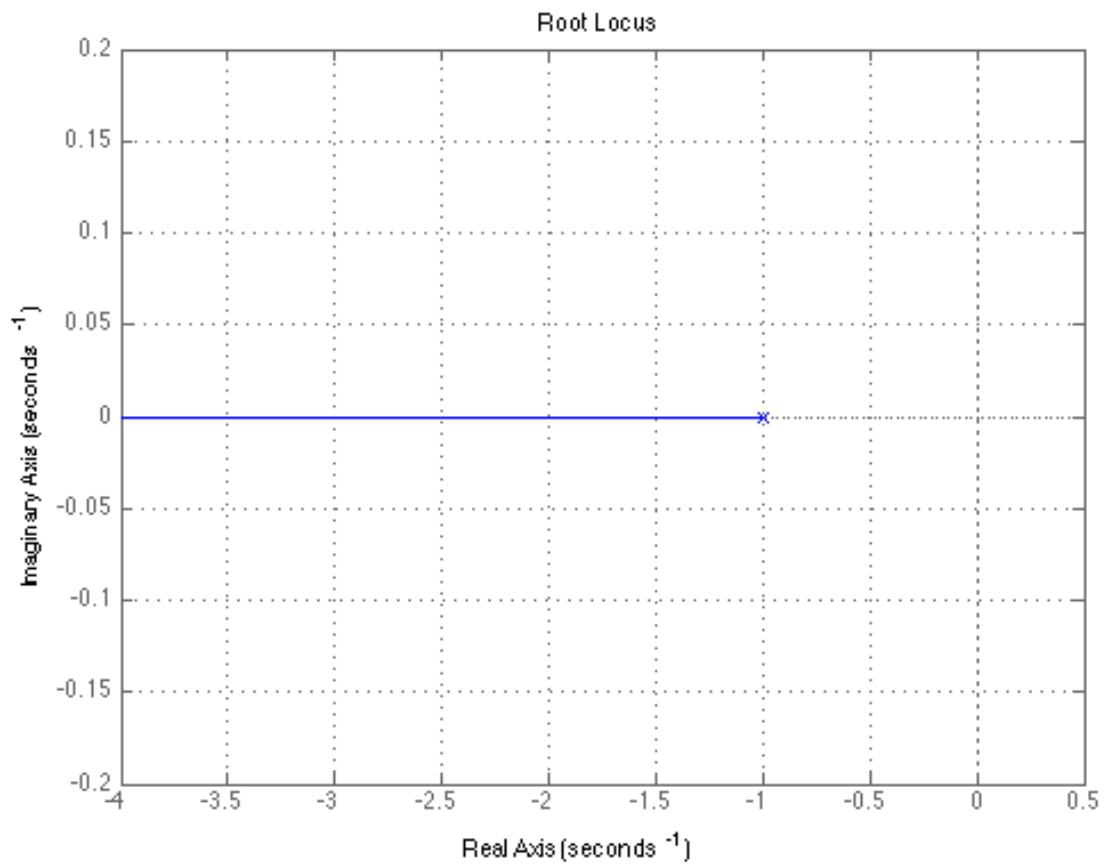
Then, in order to find any break-in or break-out points and the corresponding values of K , I used MATLAB such as:

```
>> BIBO=roots(conv(polyder(D),N))  
>> K=-polyval(D,BIBO)./polyval(N,BIBO)
```

These commands are described in somewhat more detail in the MATLAB guide attached to the 'Course Documents' section of the UBLearn website.

1.

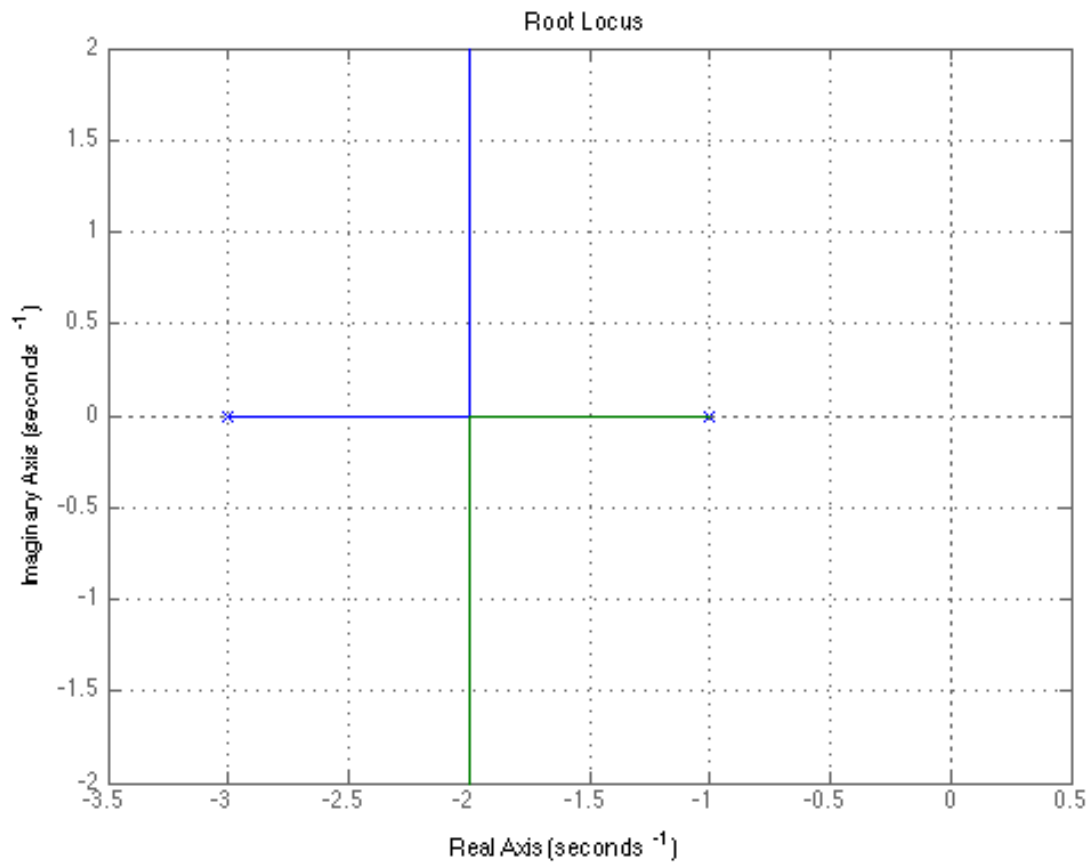
$$\lambda + 1 + K = 0$$



- a) The system is stable for all $0 \leq K \leq \infty$
- b) The response is non-oscillatory for all $0 \leq K \leq \infty$
- c) The settling time $\rightarrow 0$ as $K \rightarrow \infty$

2.

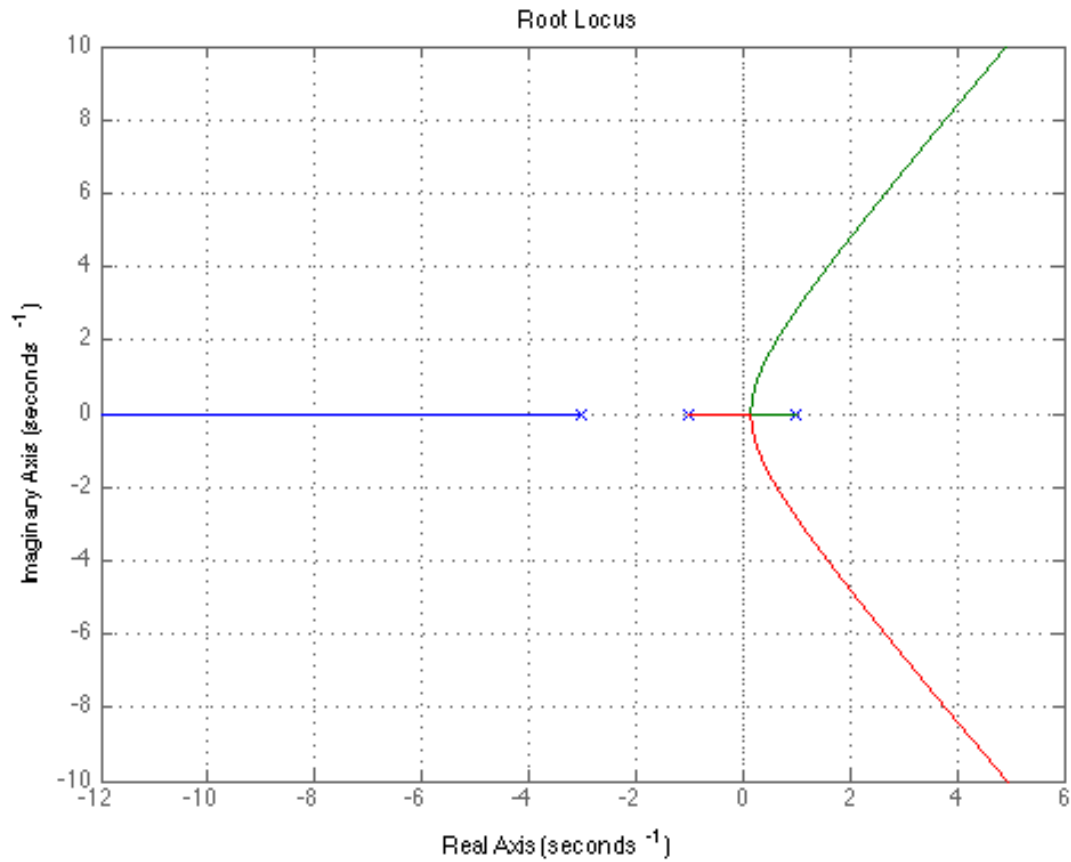
$$(\lambda + 1)(\lambda + 3) + K = 0$$



- a) The system is stable for all $0 \leq K \leq \infty$
- b) The response is non-oscillatory for $0 \leq K \leq 1$ (since $K = 1$ at the break-out point)
- c) The settling time is 2s for the value $K = 1$

3.

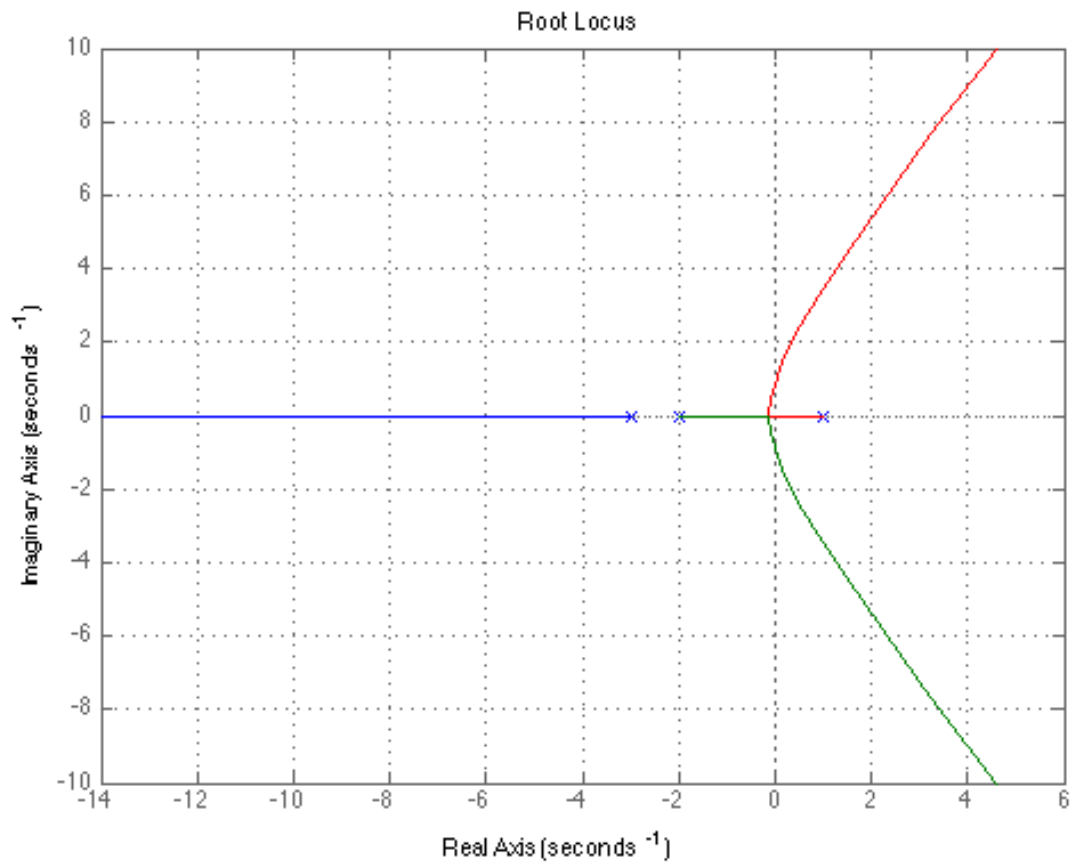
$$(\lambda - 1)(\lambda + 1)(\lambda + 3) + K = 0$$



- a) The system is never stable for any $0 \leq K \leq \infty$
- b) Since the system is never stable, we don't bother with this
- c) No settling time for an unstable system

4.

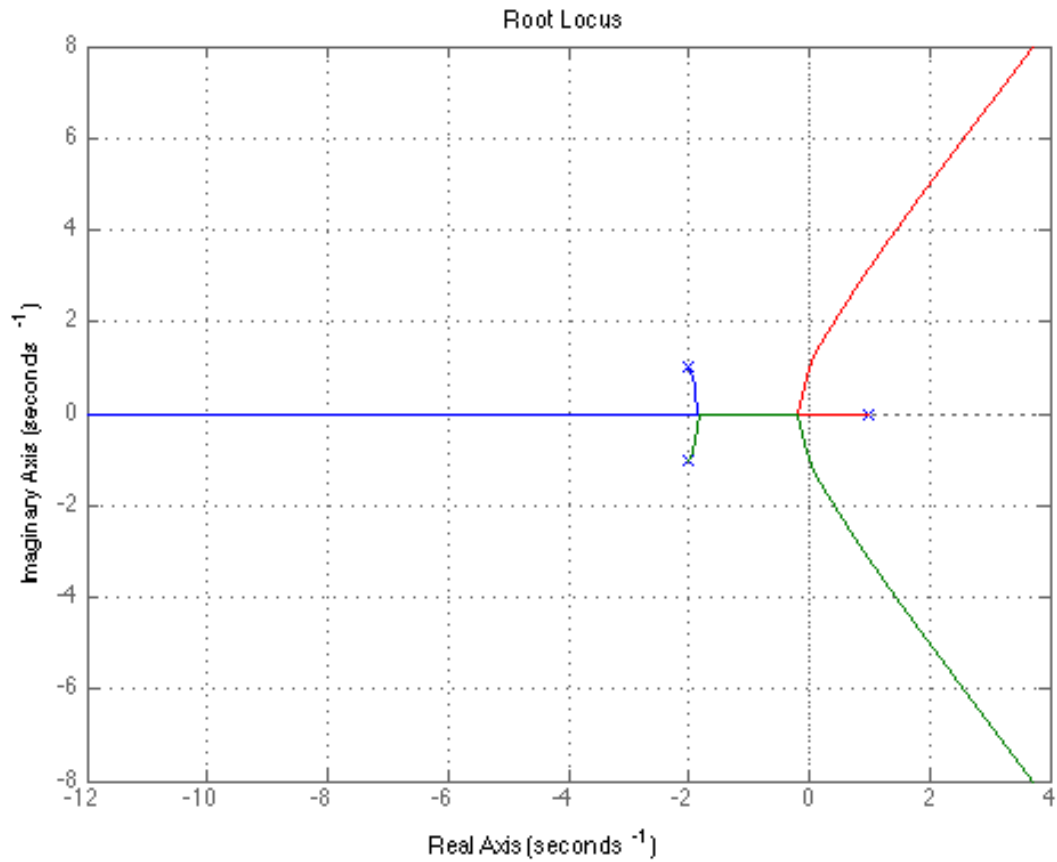
$$(\lambda - 1)(\lambda + 2)(\lambda + 3) + K = 0$$



- a) The system is stable for all $6 < K < 10$ (found from *Im* axis crossings)
- b) The response is non-oscillatory for $0 \leq K \leq 6.0646$ (since $K = 6.0646$ at the break-out point)
- c) The settling time is 30.4s for the value $K = 6.0646$

5.

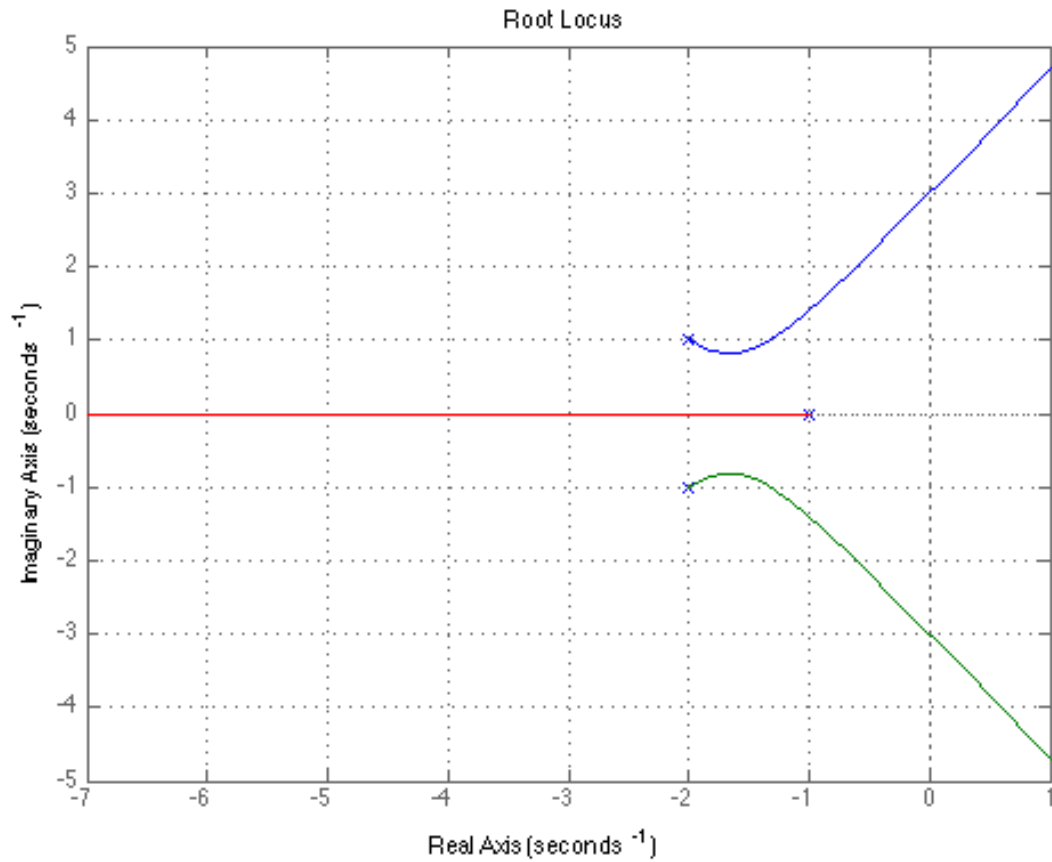
$$(\lambda - 1)(\lambda + 2 + i)(\lambda + 2 - i) + K = 0$$



- The system is stable for all $5 < K < 8$ (found from Im axis crossings)
- The response is non-oscillatory for $2.91 \leq K \leq 5.09$
(the values between the break-in point and the break-out point)
- The fastest settling occurs at the break-out point and is 21.8s for the value $K = 5.09$

6.

$$(\lambda + 1)(\lambda + 2 + i)(\lambda + 2 - i) + K = 0$$



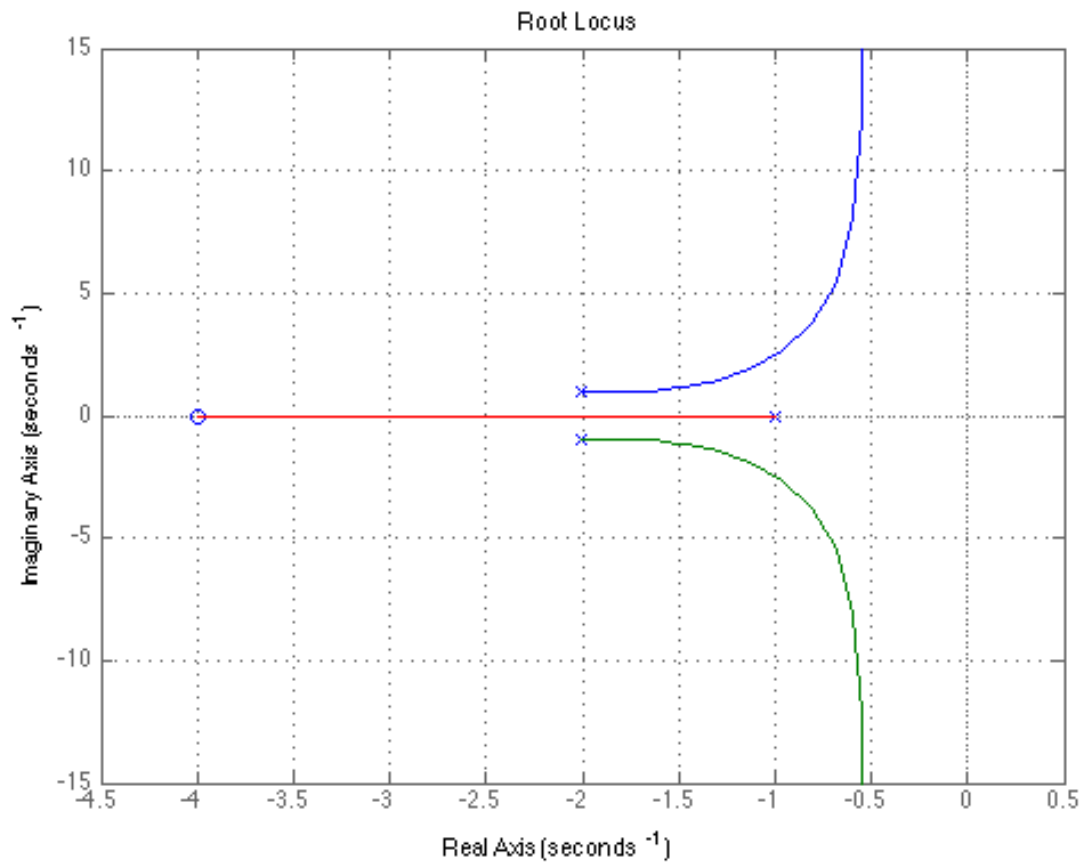
- The system is stable for all $0 \leq K < 40$ (found from *Im* axis crossings)
- The response is oscillatory for all $0 \leq K \leq \infty$
- The fastest settling time occurs when the three branches line up. Multiplying roots, we find the original polynomial for the system as

$$\lambda^3 + 5\lambda^2 + 9\lambda + 5 + K = 0$$

Using Rule 9, we find the three roots are aligned for $\lambda = -5/3$, so we have fastest settling time = 2.4 s at the value $K = 0.741$

7.

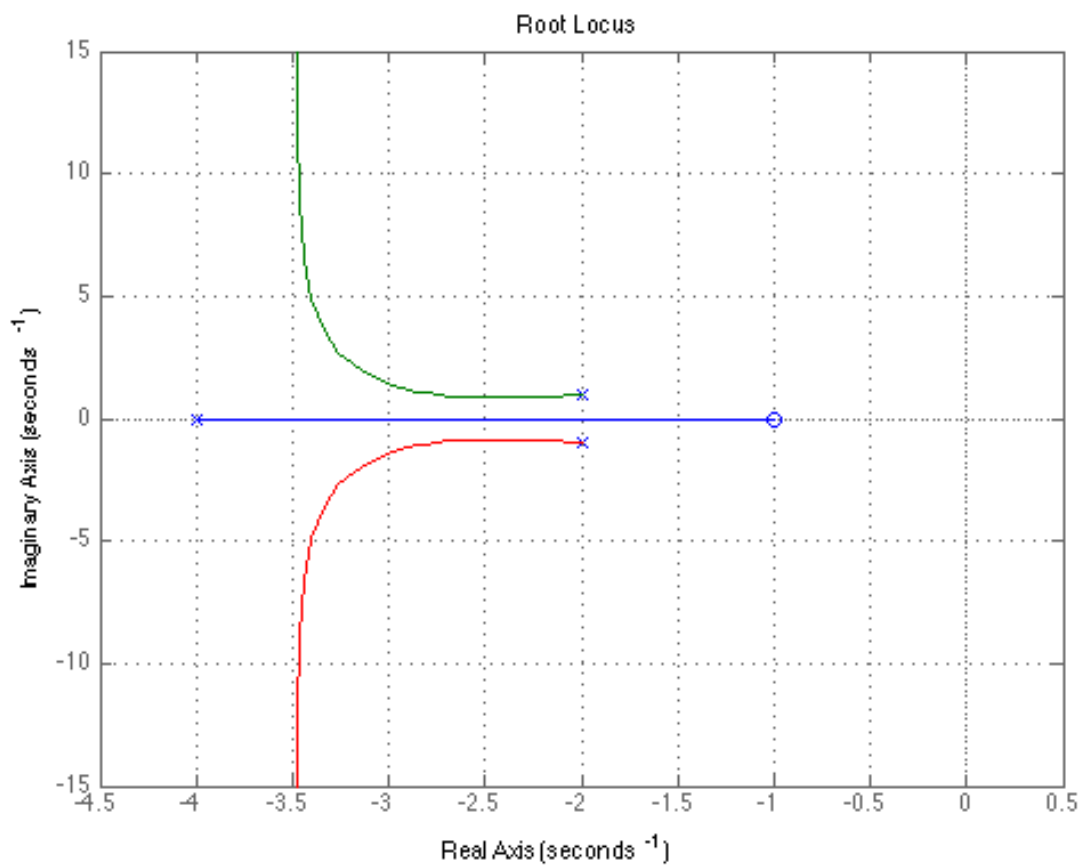
$$(\lambda + 1)(\lambda + 2 + i)(\lambda + 2 - i) + K(\lambda + 4) = 0$$



- The system is stable for all $0 \leq K \leq \infty$
- The response is oscillatory for all $0 \leq K \leq \infty$
- The fastest settling occurs when the three branches have equal real parts; using guide 9, this occurs at $\sigma = -5/3$ so the fastest settling time is 2.4s for the value $K = 0.3175$

8.

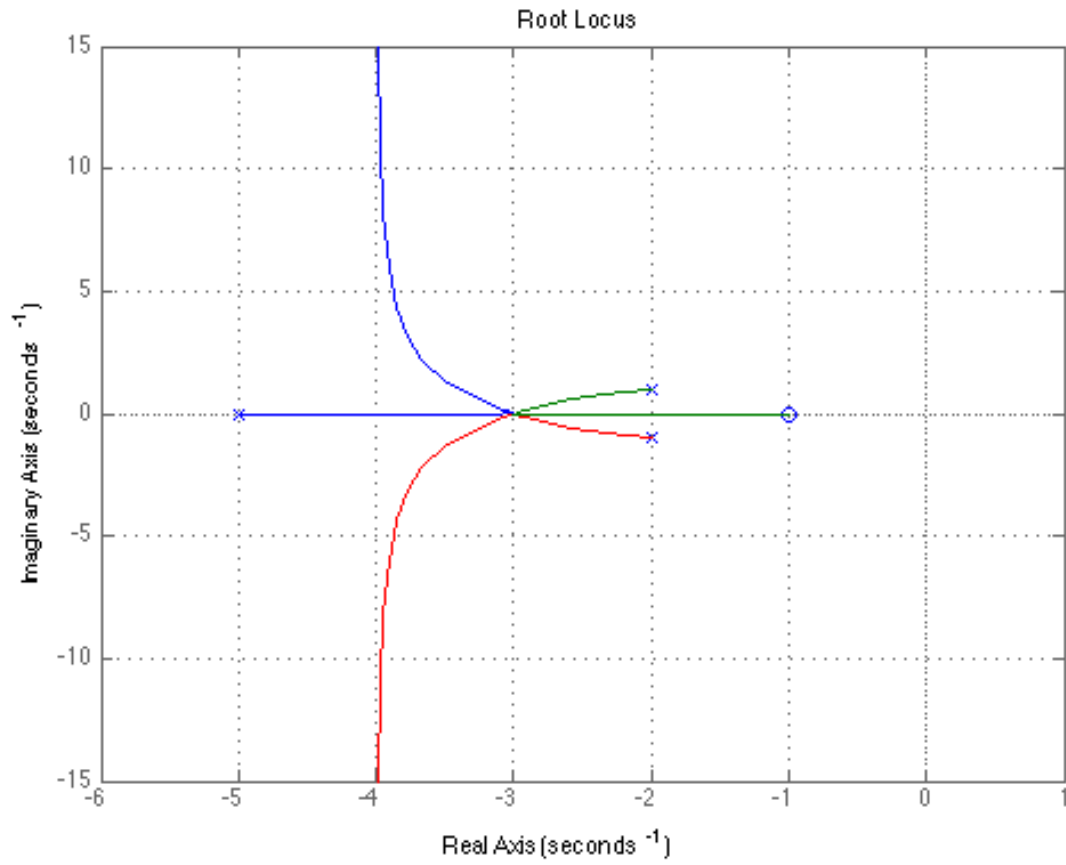
$$(\lambda + 4)(\lambda + 2 + i)(\lambda + 2 - i) + K(\lambda + 1) = 0$$



- The system is stable for all $0 \leq K \leq \infty$
- The response is oscillatory for all $0 \leq K \leq \infty$
- The fastest settling occurs when the three branches have equal real parts; using guide 9, this occurs at $\sigma = -8/3$ so the fastest settling time is 1.5s for the value $K = 1.156$

9.

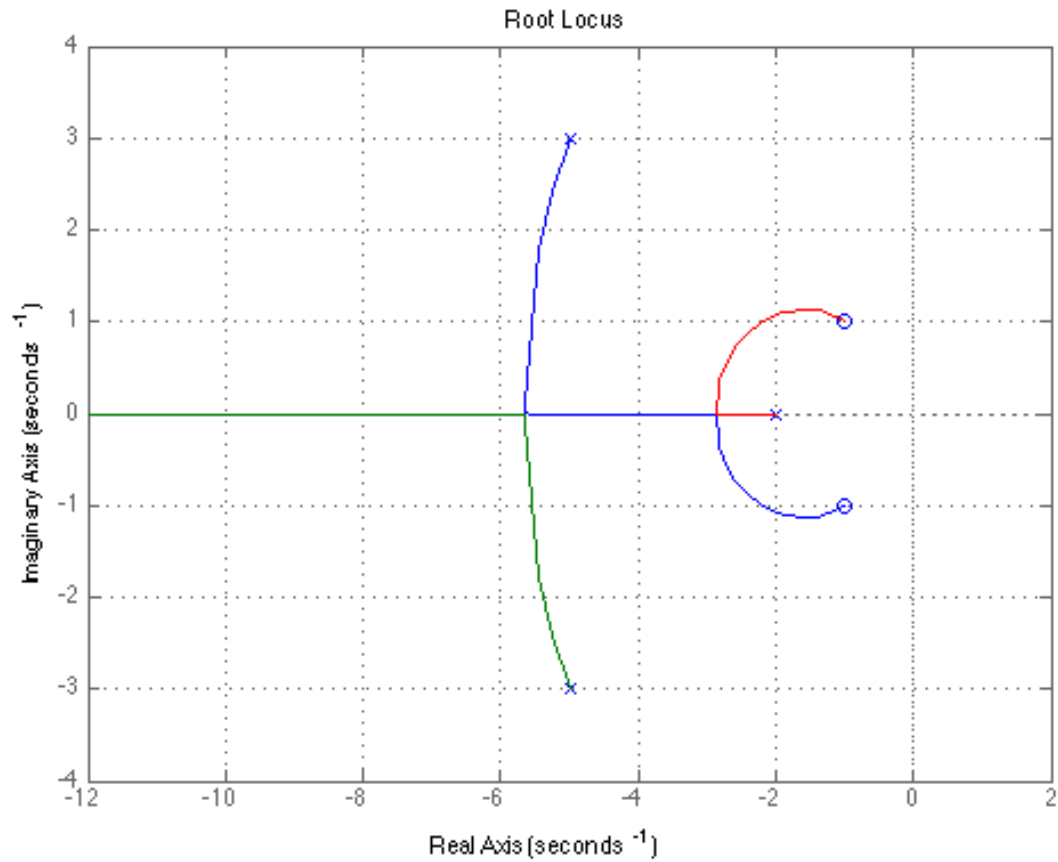
$$(\lambda + 5)(\lambda + 2 + i)(\lambda + 2 - i) + K(\lambda + 1) = 0$$



- The system is stable for all $0 \leq K \leq \infty$
- The response is oscillatory for all $0 \leq K \leq \infty$ **EXCEPT** for a triple real root at $\lambda = -3$ for $K = 2$
- The fastest settling time occurs at the triple real root at $\lambda = -3$ and is 1.333s , for $K = 2$

10.

$$(\lambda + 6)(\lambda + 2 + i)(\lambda + 2 - i) + K(\lambda + 1) = 0$$



- The system is stable for all $0 \leq K \leq \infty$
- The response is non-oscillatory for $2.888 < K < 3.338$, which are the values at the break-in and break-out points
- The fastest settling time is 1.516 s, and occurs at the break-in point for $K = 2.888$