Find the complete solution $x(t)$ and plot it to twice the settling time. For full credit, you must clearly label your axes including numerical values, and sketch a reasonably accurate plot.

$$\ddot{x}(t) + 2\dot{x}(t) + 2x(t) = 4 \quad , \; x(0) = 0 \; , \; \dot{x}(0) = 0$$

Solution:
The particular solution is a constant, $x_p = 2$. The general form of the homogeneous solution is:

$$\lambda^2 + 2\lambda + 2 = 0 \quad \Rightarrow \quad \lambda_{1,2} = -1 \pm i \quad \Rightarrow \quad x_h(t) = Ae^{-t} sin(t + \phi)$$

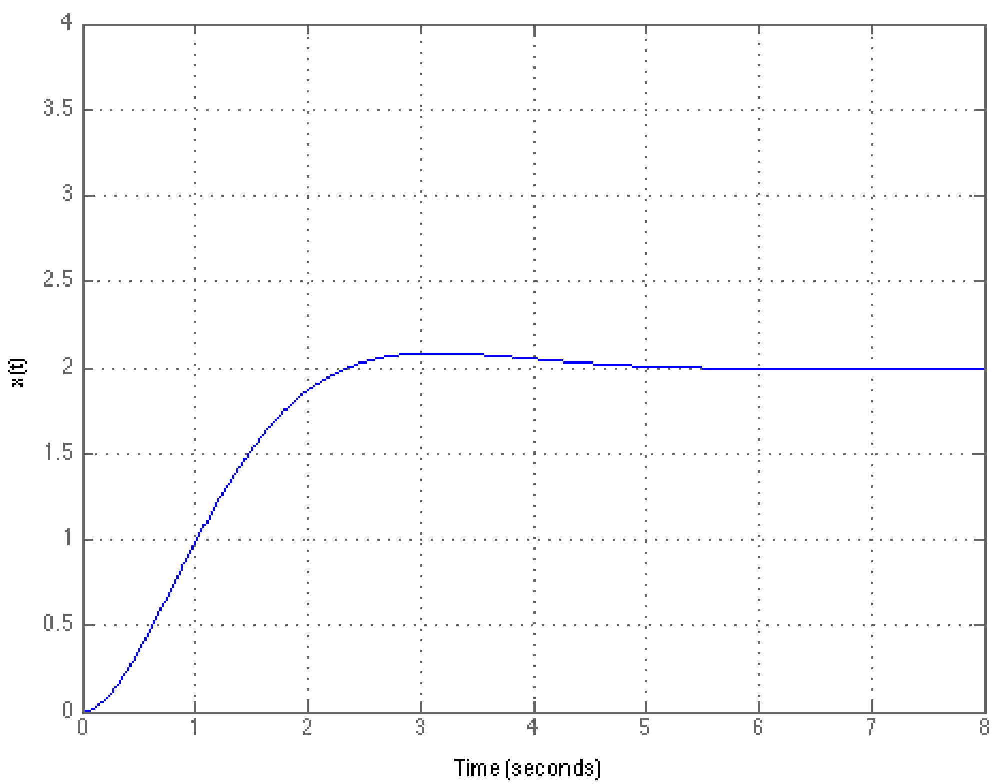
Now apply the initial conditions to determine A, ϕ :

$$x(0) = 0 = Asin\phi + 2 \qquad\qquad Asin\phi = -2$$
$$\dot{x}(0) = 0 = -Asin\phi + Acos\phi \qquad\qquad Acos\phi = -2$$
$$\Rightarrow \phi = \pi/4 \; , \; A = -2.83$$

The complete solution is thus

$$\underline{x(t) = -2.83e^{-t} sin(t + \pi/4) + 2}$$

The settling time is 4 seconds, so we plot out to 8 seconds. The solution begins at $x = 0$ with $\dot{x} = 0$, then settles at $x = 2$. Since the homogeneous solution is underdamped, we have some overshoot. The first crossing of $x = 2$ occurs at $sin(t + \pi/4) = 0$, i.e., $t = 3\pi/4$. The period of oscillation is 2π, which is much longer than the settling time, and the damping ratio is relatively high, $\zeta = 0.707$, so the solution settles to $x = 2$ with little oscillation. The following graph was generated by MATLAB, so your sketch may not match exactly, but it should be reasonably close.



Find the complete solution $x(t)$ and plot it to twice the settling time. For full credit, you must clearly label your axes including numerical values, and sketch a reasonably accurate plot.

$$\ddot{x}(t) + 4\dot{x}(t) + 8x(t) = 8 \quad , \ x(0) = 0 \ , \ \dot{x}(0) = 0$$

Solution:
The particular solution is a constant, $x_p = 1$. The general form of the homogeneous solution is:

$$\lambda^2 + 4\lambda + 8 = 0 \quad \Rightarrow \quad \lambda_{1,2} = -2 \pm 2i \quad \Rightarrow \quad x_h(t) = Ae^{-2t}sin(2t + \phi)$$

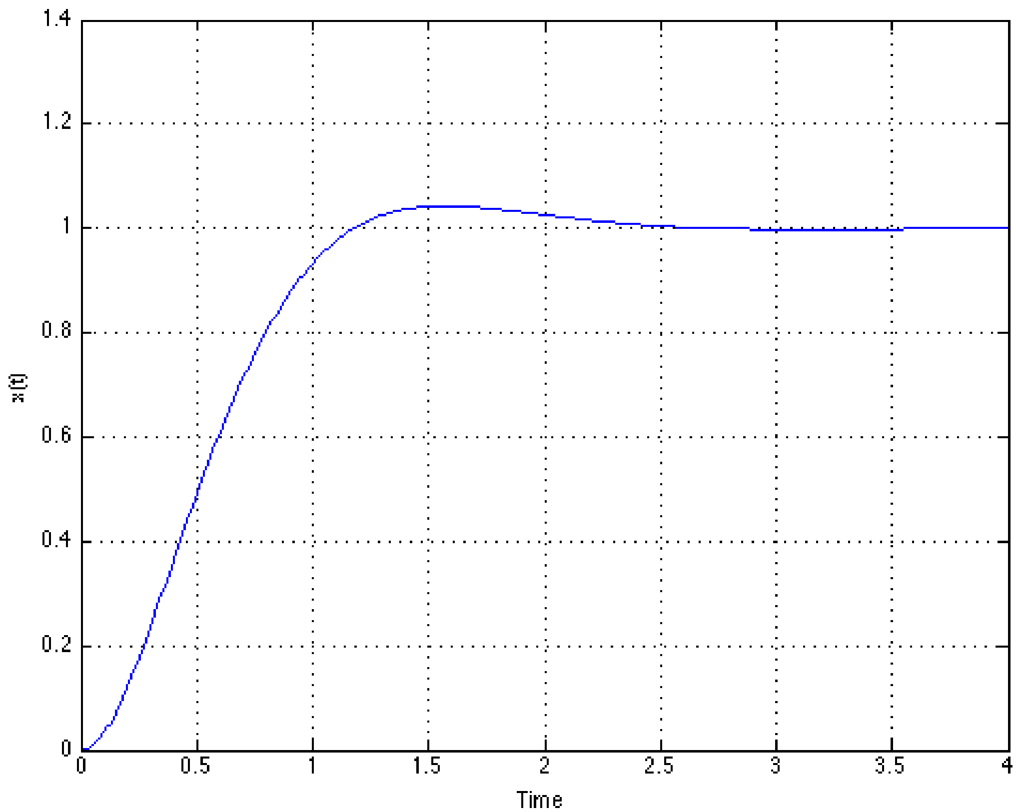
Now apply the initial conditions to determine A, ϕ :

$$\begin{aligned} x(0) = 0 &= Asin\phi + 1 & \qquad Asin\phi &= -1 \\ \dot{x}(0) = 0 &= -2Asin\phi + 2Acos\phi & \qquad Acos\phi &= -1 \\ \Rightarrow \phi &= \pi/4 \ , \ A = -1.414 \end{aligned}$$

The complete solution is thus

$$\underline{x(t) = -1.414e^{-2t}sin(2t + \pi/4) + 1}$$

The settling time is 2 seconds, so we plot out to 4 seconds. The solution begins at $x = 0$ with $\dot{x} = 0$, then settles at $x = 1$. Since the homogeneous solution is underdamped, we have some overshoot. The first crossing of $x = 1$ occurs at $sin(2t + \pi/4) = 0$, i.e., $t = 3\pi/8$. The period of oscillation is π, which is much longer than the settling time, and the damping ratio is relatively high, $\zeta = 0.707$, so the solution settles to $x = 1$ with little oscillation. The following graph was generated by MATLAB, so your sketch may not match exactly, but it should be reasonably close.



Find the complete solution $x(t)$ and plot it to twice the settling time. For full credit, you must clearly label your axes including numerical values, and sketch a reasonably accurate plot.

$$\ddot{x}(t) + 2\dot{x}(t) + 2x(t) = 8 \quad , \; x(0) = 4 \; , \; \dot{x}(0) = 0$$

Solution:
The particular solution is a constant, $x_p = 4$. The general form of the homogeneous solution is:

$$\lambda^2 + 2\lambda + 2 = 0 \quad \Rightarrow \quad \lambda_{1,2} = -1 \pm i \quad \Rightarrow \quad x_h(t) = Ae^{-t} sin(t + \phi)$$

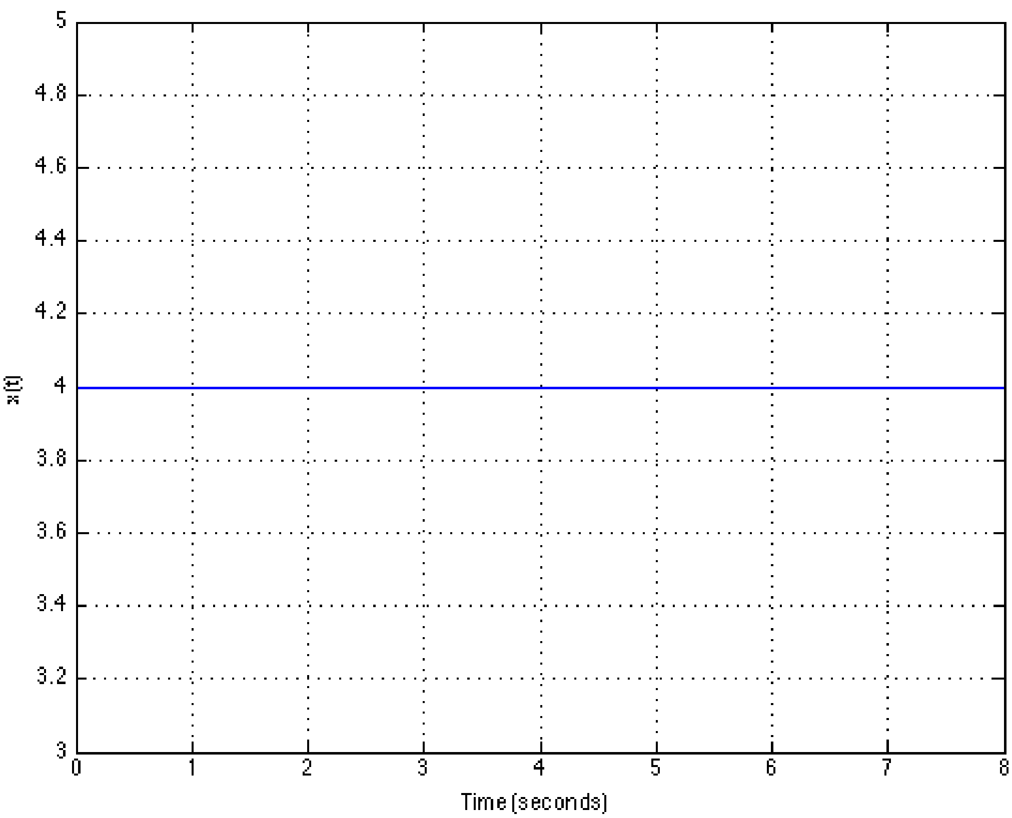
Now apply the initial conditions to determine A, ϕ :

$$\begin{aligned} x(0) &= 4 = Asin\phi + 4 & \qquad Asin\phi &= 0 \\ \dot{x}(0) &= 0 = -Asin\phi + Acos\phi & \qquad Acos\phi &= 0 \\ &\Rightarrow A = 0 \end{aligned}$$

The complete solution is thus

$$\underline{x(t) = 4}$$

The settling time is 4 seconds, so we plot out to 8 seconds. The solution is a straight line at $x = 4$



Find the complete solution $x(t)$ and plot it to twice the settling time. For full credit, you must clearly label your axes including numerical values, and sketch a reasonably accurate plot.

$$\ddot{x}(t) + 4\dot{x}(t) + 8x(t) = 8 \quad , \; x(0) = 2 \; , \; \dot{x}(0) = 0$$

Solution:
The particular solution is a constant, $x_p = 1$. The general form of the homogeneous solution is:

$$\lambda^2 + 4\lambda + 8 = 0 \quad \Rightarrow \quad \lambda_{1,2} = -2 \pm 2i \quad \Rightarrow \quad x_h(t) = Ae^{-2t} sin(2t + \phi)$$

Now apply the initial conditions to determine A, ϕ :

$$x(0) = 2 = Asin\phi + 1 \qquad\qquad Asin\phi = 1$$
$$\dot{x}(0) = 0 = -2Asin\phi + 2Acos\phi \qquad\qquad Acos\phi = 1$$
$$\Rightarrow \phi = \pi/4 \; , \; A = 1.414$$

The complete solution is thus

$$\underline{x(t) = 1.414e^{-2t} sin(2t + \pi/4) + 1}$$

The settling time is 2 seconds, so we plot out to 4 seconds. The solution begins at $x = 0$ with $\dot{x} = 0$, then settles at $x = 1$. Since the homogeneous solution is underdamped, we have some overshoot. The first crossing of $x = 1$ occurs at $sin(2t + \pi/4) = 0$, i.e., $t = 3\pi/8$. The period of oscillation is π, which is much longer than the settling time, and the damping ratio is relatively high, $\zeta = 0.707$, so the solution settles to $x = 1$ with little oscillation. The following graph was generated by MATLAB, so your sketch may not match exactly, but it should be reasonably close.