The general process for creating LTI ODE models for physical systems follows a common approach that is essentially the same regardless of what type of system is being modeled:

1. **Use standard *elements*.**
   (a) Each element represents a single physical property of the system type being modeled
   (b) Each element has a single mathematical equation associated with it that describes the *law* of that property
      i. Each element law (equation) relates two *time-dependent variables*
         A. one of these has the same value throughout the element, and is sometimes called the *through* variable
         B. the other one has different values through the element, and is sometimes called the *across* variable
      ii. Each element law contains a *constant*, which is a *physical parameter*
   (c) A table of the standard elements that we will use in MAE 340 is shown in Table (1). The *across* variable in each law is preceded by the symbol Δ, which indicates that the law is based on the difference in value of that variable from one side of the element to the other side.

Table 1: Standard Linear Model Elements

Type of System	Element	Physical Property	Mathematical Law, Physical Parameter
Translational	Spring	Linear Stiffness	Force = (spring constant k)*(Δdisplacement)
”	Damper	Linear Damping	Force = (damping coefficient c)*(Δvelocity)
”	Mass	Inertia	ΔForce = (mass m)*(acceleration)
Rotational	Spring	Rotational Stiffness	Moment = (spring constant k_θ)*(Δangular displacement)
”	Damper	Rotational Damping	Moment = (damping coefficient c_ω)*(Δangular velocity)
”	Mass	Mass Moment of Inertia	ΔMoment = (mass mom of inertia J)*(angular acceleration)
Electrical	Resistor	Electrical Resistance	ΔVoltage = (resistance R)*(current)
”	Inductor	Electrical Inductance	$\frac{d}{dt}$(Current) = $\frac{1}{(\text{Inductance } I)}$*($\Delta$voltage)
”	Capacitor	Electrical Capacitance	$\frac{d}{dt}$(ΔVoltage) = $\frac{1}{(\text{Capacitance } C)}$*(current)
Hydraulic	Resistor	Viscosity (Damping)	ΔPressure = (Resistance R_f)*(Volume flow rate)
”	Capacitor	Potential Energy	$\frac{d}{dt}$(Pressure) = $\frac{1}{(\text{Fluid Capacitance } C_f)}$*($\Delta$ Volume flow rate)
”	Inertor	Kinetic Energy	$\frac{d}{dt}$(Volume flow rate) = $\frac{1}{(\text{Fluid Inertance } I_f}$)*($\Delta$Pressure)
Thermal	Resistor	Insulation	(Heat flow rate) = $\frac{1}{(\text{Thermal resistance } R_h)}$*($\Delta$Temperature)
”	Capacitor	Thermal Energy	$\frac{d}{dt}$(Temp) = $\frac{1}{(\text{Thermal Capacitance } C_h)}$*($\Delta$Heat flow rate)

It is important to note that in order to limit the model to the form of an LTI ODE, the following approximations must be assumed:

(a) the type of behavior represented by an element is *spatially concentrated* in that element. For example, in any real electrical system, every component exhibits some resistance, so resistance is *distributed* (spread throughout) the electrical system. But for purposes of constructing an LTI ODE, it is necessary to model resistance by a discrete (not distributed) *resistor* element, which exists only at a distinct point and is not distributed throughout the system. A system may contain many elements of the same type (for example, an electrical system may contain numerous separate resistors), but each of these elements is a discrete point in the system configuration

(b) each element has only one type of elemental behavior. Different elements are used to model each type of behavior. A device may combine many different types of elemental behavior - for example, an electrical motor may have resistance, capacitance, and inductance - but the model of the device has these elemental behaviors separated into elements that each contain only one type of elemental behavior

(c) at points of connection between two elements, the appropriate local physical variables (for example, voltage and current in an electrical system) flow freely from one element to the next with the same numerical values. The values of these variables generally change within elements (according to the elemental laws), but they do not change in between elements.

2. Utilize *first principles* - i.e., basic laws of physics - to construct the LTI ODE's from the elemental laws and elemental connections. Each element has its own specific elemental law, and in addition, each combined system of elements must satisfy some overarching first principle of physics.

   A summary of the first principles used in each type of system, along with the typical state variables used in the models, is shown in Table (2).

Table 2: First Principles and State Variables

Type of System	State Variables	First Principles
Translational	Positions Velocities	$\Sigma F = ma$
Rotational	Angles Angular Velocities	$\Sigma M = J\alpha$
Electrical	(Δvoltage) in capacitors Current through inductors	Kircchoff Loop Law Kircchoff Node Law
Hydraulic	Pressure in capacitors Volume flow rate through inertors	Pressure equiv to Kircchoff Loop Law Flow equiv to Kircchoff Node Law
Thermal	Temperature of capacitors	$\Sigma Q_{net} = C_h \dot{T}_C$

STATE VARIABLES

- For Translational systems, we define the states = positions and velocities of each mass

  $\Rightarrow$ This is equivalent to our generic approach for LTI ODE $\rightarrow$ state space

- For Rotational systems, we define the states = angular positions and angular velocities of each rotational inertia

  $\Rightarrow$ This is equivalent to our generic approach for LTI ODE $\rightarrow$ state space

- For Electrical systems, it's easier to simply define the states as:
  (a) The voltage across each capacitor
  (b) The current through each inductor

  $\Rightarrow$ This approach yields first-order state equations directly

- For Hydraulic systems, it's easier to simply define the states as:
  (a) The pressure in each capacitor
  (b) The flow rate in each inertance

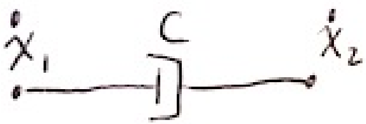
  $\Rightarrow$ This approach yields first-order state equations directly

- For Thermal systems, we define states as the temperature of each capacitor

  $\Rightarrow$ This approach yields first-order state equations directly

Translational System Elements

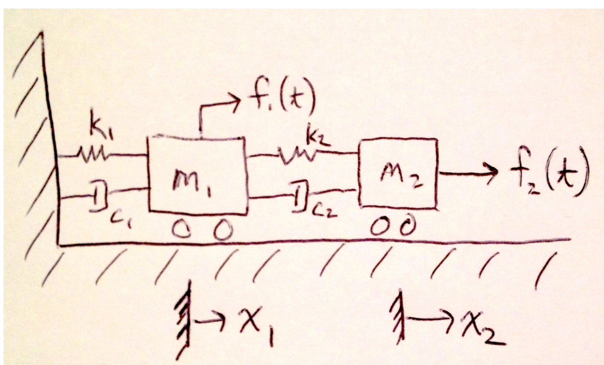
Spring: $F = k(x_2 - x_1)$
(tension positive)
F = through variable $\Rightarrow F_1 = F_2$
x = across variable $\Rightarrow x_1 \neq x_2$
k = spring constant = physical parameter



Damper $F = c(\dot{x}_2 - \dot{x}_1)$
(tension positive)
F = through variable $\Rightarrow F_1 = F_2$
$\dot{x}$ = across variable $\Rightarrow \dot{x}_1 \neq \dot{x}_2$
c = damping coefficient = physical parameter

Mass $F_2 - F_1 = ma$
a= through variable $\Rightarrow a_1 = a_2$
F = across variable $\Rightarrow F_1 \neq F_2$
m = mass = physical parameter

Example of a translational system: A two mass-spring-damper system



The element diagram is shown in the figure. Using the first principle for translational systems, $F = ma$, applied to each of the two masses:

$$\sum F_{m_1} = m_1\ddot{x}_1 = f_1(t) - k_1x_1 - c_1\dot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1)$$

$$\sum F_{m_2} = m_2\ddot{x}_2 = f_2(t) - k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1)$$

We convert this system to state space by defining

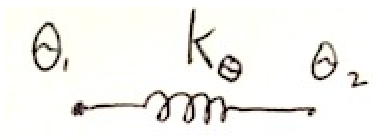
$$\vec{z}(t) \equiv \begin{pmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{pmatrix} \quad , \quad \vec{u}(t) \equiv \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

Thus, we see that the states are indeed positions and velocities of the two masses. The state equation is:

$$\dot{\vec{z}}(t) \equiv A\vec{z} + B\vec{u} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{k_2}{m_2} & -\frac{c_2}{m_2} \end{pmatrix} \vec{z}(t) + \begin{pmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{pmatrix} \vec{u}(t)$$

Using our previous techniques, an output equation can be constructed once outputs are defined.

Rotational System Elements

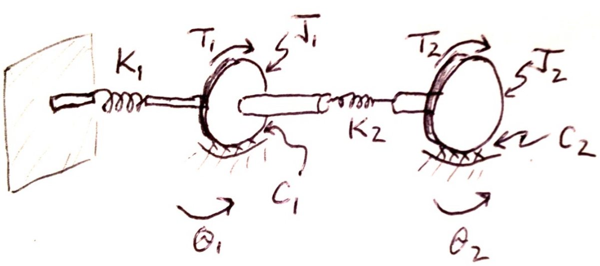


Rotational Spring $M = k_\theta(\theta_2 - \theta_1)$
M = through variable $\Rightarrow M_1 = M_2$
θ = across variable $\Rightarrow \theta_1 \neq \theta_2$
k_θ = spring constant = physical parameter

Rotational Damper $M = c_\omega(\dot{\theta}_2 - \dot{\theta}_1) \equiv c_\omega(\omega_2 - \omega_1)$
M = through variable $\Rightarrow M_1 = M_2$
ω = across variable $\Rightarrow \omega_1 \neq \omega_2$
c_ω = damping coefficient = physical parameter

Inertia $M_2 - M_1 = J\alpha$
α= through variable $\Rightarrow \alpha_1 = \alpha_2$
M = across variable $\Rightarrow M_1 \neq M_2$
J = mass moment of inertia = physical parameter

Example of a rotational system: A two shaft (inertia-spring-damper) system



The element diagram is shown in the figure. Using the first principle for rotational systems, $M = J\alpha$, applied to each of the two inertias:

$$\sum M_{J_1} = J_1\ddot{\theta}_1 = -T_1(t) - K_1\theta_1 - C_1\dot{\theta}_1 + K_2(\theta_2 - \theta_1) + C_2(\dot{\theta}_2 - \dot{\theta}_1)$$

$$\sum M_{J_2} = J_2\ddot{\theta}_2 = -T_2(t) - K_2(\theta_2 - \theta_1) - C_2(\dot{\theta}_2 - \dot{\theta}_1)$$

We convert this system to state space by defining

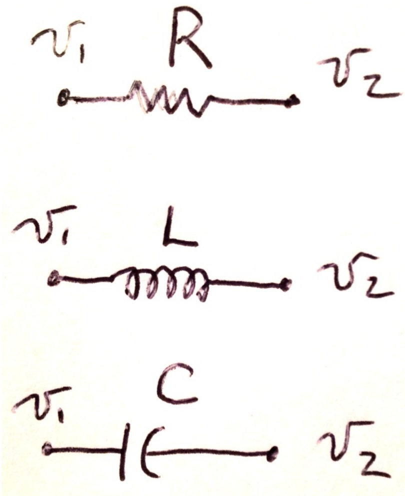
$$\vec{z}(t) \equiv \begin{pmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{pmatrix} \quad , \quad \vec{u}(t) \equiv \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$$

Thus, the states are angular positions and angular velocities of the two inertias. The state equation is:

$$\dot{\vec{z}}(t) \equiv A\vec{z} + B\vec{u} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1+K_2}{J_1} & -\frac{C_1+C_2}{J_1} & \frac{K_2}{J_1} & \frac{C_2}{J_1} \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{J_2} & \frac{C_2}{J_2} & -\frac{K_2}{J_2} & -\frac{C_2}{J_2} \end{pmatrix} \vec{z}(t) + \begin{pmatrix} 0 & 0 \\ -\frac{1}{J_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J_2} \end{pmatrix} \vec{u}(t)$$

Using our previous techniques, an output equation can be constructed once outputs are defined.

Electrical System Elements

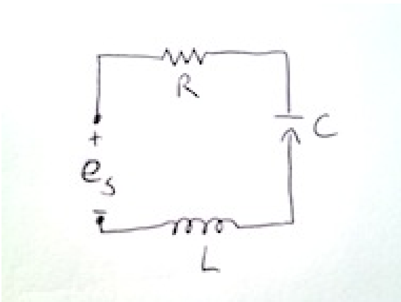


Resistor $v_1 - v_2 = e_R = Ri_R$
i_R = current = through variable $\Rightarrow i_1 = i_2$
v = voltage = across variable $\Rightarrow v_1 \neq \ v_2$
R = electrical resistance = physical parameter

Inductor $\dot{i}_L = \frac{1}{L}(v_1 - v_2) = \frac{1}{L}e_L$
i_L = current = through variable $\Rightarrow i_1 = i_2$
v = voltage = across variable $\Rightarrow v_1 \neq \ v_2$
I = electrical inductance = physical parameter

Capacitor $\dot{(v_1 - v_2)} = \dot{e}_C = \frac{1}{C}i_C$
i_L = current = through variable $\Rightarrow i_1 = i_2$
v = voltage = across variable $\Rightarrow v_1 \neq \ v_2$
C = electrical capacitance = physical parameter

Example: State Space Modeling of an Electrical System



Electrical System Example 1

Find a state-space model for the electrical circuit shown, where e_s is a voltage source (note: a voltage source is assumed to provide the same voltage regardless of the current; a battery is normally considered to be a voltage source). The output of this system is the voltage change across the resistor.

Solution

From Table (2), the states in electrical systems are voltages across capacitors, and currents through inductors. Thus, we may write the state vector, input vector, and output vector as follows:

$$\vec{z}(t) \equiv \begin{pmatrix} e_C \\ i_L \end{pmatrix} \quad , \quad \vec{u} = (e_s) \quad , \quad \vec{y} = (e_R)$$

To obtain the state-space equations, begin by writing the elemental laws for the capacitor and the inductor, since these already contain the 1^{st}-order derivatives of the states:

$$\dot{\vec{z}}(t) = \begin{pmatrix} \dot{e}_C \\ \dot{i}_L \end{pmatrix} = \begin{pmatrix} \frac{1}{C} i_C \\ \frac{1}{L} e_L \end{pmatrix}$$

We need to obtain

$$\dot{\vec{z}} = A\vec{z} + b\vec{u}$$

but the right hand sides contain terms (e_L, i_C) that are neither states nor inputs. Therefore, we must find substitutions for e_L and i_C in terms of states and/or inputs. We use the first principles and perhaps other elemental equations (for any resistors) to substitute on the right-hand sides until this is accomplished. For example:

$$\text{From the Kircchoff node law } \rightarrow i_C = i_L \quad \Rightarrow \underline{\dot{e}_C = \frac{1}{C} i_L} \quad \text{RHS is in terms of a state}$$

Similarly,

$$\text{From the Kircchoff loop law } \rightarrow e_L = e_S - e_R - e_C \quad \Rightarrow \underline{\dot{i}_L = \frac{1}{L}(e_S - e_R - e_C)} \quad \text{RHS still contains } e_R$$

Since e_R is neither a state nor an input, we must find another substitution. Since we have not yet used the element law for a resistor, it will probably be helpful:

$$e_R = R i_R$$

Now we may again use the Kircchoff node law to get

$$i_R = i_C = i_L$$

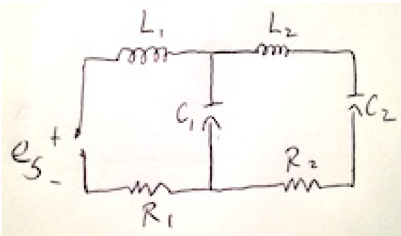
and, since i_L is a state, we may substitute $e_R = R i_R = R i_L$ to obtain

$$\dot{i}_L = \frac{1}{L}(e_S - R i_L - e_C)$$

The state model is

$$A = \begin{pmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{pmatrix} \quad , \quad B = \begin{pmatrix} o \\ \frac{1}{L} \end{pmatrix} \quad , \quad C = (0 \quad R) \quad , \quad D = (0)$$

Example: State Space Modeling of an Electrical System



Electrical System Example 2

Find a state-space model for the electrical circuit shown, where e_s is a voltage source. The outputs of this system are
(i) the voltage across the resistor R_2
(ii) the current through the capacitor C_1.

Solution

The states in electrical systems are voltages across capacitors, and currents through inductors. Thus, we have a four-state system with a single input and two outputs:

$$\vec{z}(t) \equiv \begin{pmatrix} e_{C_1} \\ e_{C_2} \\ i_{L_1} \\ i_{L_2} \end{pmatrix} \quad , \quad \vec{u} = (e_s) \quad , \quad \vec{y} = \begin{pmatrix} e_{R_2} \\ i_{C_1} \end{pmatrix}$$

Begin by writing the elemental laws for the capacitors and the inductors, since these are 1^{st}-order ODE's in the states, and then use Kircchoff Laws and elemental laws for the resistors to substitute for any terms on the right-hand sides that are neither states nor inputs:

$$\begin{aligned}
\dot{e}_{C_1} &= \frac{1}{C_1} i_{C_1} = \frac{1}{C_1}(i_{L_1} - i_{L_2}) \\
\dot{e}_{C_2} &= \frac{1}{C_2} i_{C_2} = \frac{1}{C_2} i_{L_2} \\
\dot{i}_{L_1} &= \frac{1}{L_1} e_{L_1} = \frac{1}{L_1}(e_S - e_{C_1} - R_1 i_{L_1}) \\
\dot{i}_{L_2} &= \frac{1}{L_2} e_{L_2} = \frac{1}{L_2}(e_{C_1} - e_{C_2} - R_2 i_{L_2})
\end{aligned}$$

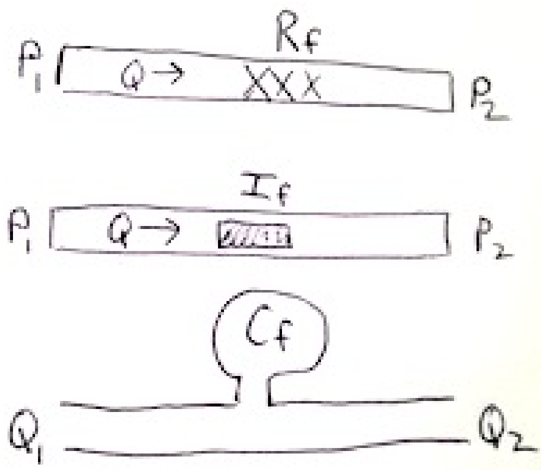
The outputs are

$$\begin{aligned}
y_1 &= e_{R_2} = R_2 i_{L_2} \\
y_2 &= i_{C_1} = (i_{L_1} - i_{L_2})
\end{aligned}$$

Substituting the above, the state model is

$$A = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} & -\frac{1}{C_1} \\ 0 & 0 & 0 & \frac{1}{C_2} \\ -\frac{1}{L_1} & 0 & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & -\frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{pmatrix} \quad , \quad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_1} \\ 0 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 0 & 0 & 0 & R_2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \quad , \quad D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Hydraulic System Elements



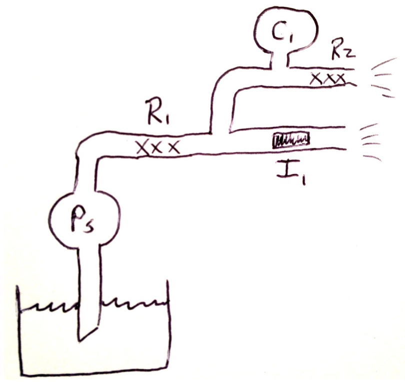
Hydraulic System Elements

Resistor $P_1 - P_2 = P_{R_f} = R_f Q_R$
Q_R = flow rate = through variable $\Rightarrow Q_1 = Q_2$
P = pressure = across variable $\Rightarrow P_1 \neq P_2$
R_f = hydraulic resistance = physical parameter

Inertor $\dot{Q}_I = \frac{1}{I_f}(P_1 - P_2) = \frac{1}{I_f} P_I$
Q_I = flow rate = through variable $\Rightarrow Q_1 = Q_2$
P_I = pressure = across variable $\Rightarrow P_1 \neq P_2$
I_f = fluid inertance = physical parameter

Capacitor $\dot{P}_{C_f} = \frac{1}{C_f}(Q_1 - Q_2) = Q_C$
P_C = pressure = through variable $\Rightarrow P_1 = P_2$
Q_C = flow rate = across variable $\Rightarrow Q_1 \neq Q_2$
C_f = fluid capacitance = physical parameter

Example: State Space Modeling of an Hydraulic System



Find a state-space model for the hydraulic system shown.

All pressure is measured relative to *ambient* pressure, denoted by an opening to the air around the system.

P_s is a pressure source (a pump), which is modeled as a constant addition to the pressure from the input side to the output side of the pump.

The output of this system is defined as the flow rate across the resistor R_2.

Solution

From Table (2), the states in hydraulic systems are the pressure in capacitors, and the flow through inertances. Thus, we may write the state vector, input vector, and output vector as follows:

$$\vec{z}(t) \equiv \begin{pmatrix} P_C \\ Q_I \end{pmatrix} \quad , \quad \vec{u} = (P_s) \quad , \quad \vec{y} = (Q_{R_2})$$

To obtain the state-space equations, begin by writing the elemental laws for the capacitor and the inertance, since these already contain the 1^{st}-order derivatives of the states:

$$\dot{\vec{z}}(t) = \begin{pmatrix} \dot{P}_C \\ \dot{Q}_I \end{pmatrix} = \begin{pmatrix} \frac{1}{C} Q_C \\ \frac{1}{I} P_I \end{pmatrix}$$

As always, we need to obtain

$$\dot{\vec{z}} = A\vec{z} + b\vec{u}$$

but here the right hand sides contain terms (Q_C, P_I) that are neither states nor inputs. Therefore, we must find substitutions for Q_C and P_I in terms of states and/or inputs. We use the first principles and perhaps other elemental equations (for any resistors) to substitute on the right-hand sides until this is accomplished. For example:

From the equivalent to the Kircchoff node law, the sum of flows at a node must be zero:

$$\rightarrow Q_C = (Q_{R_1} - Q_I) - Q_{R_2}$$

Neither Q_{R_1} nor Q_{R_2} are states or inputs, so we use the resistor laws to substitute for them:

$$Q_{R_1} = \frac{1}{R_1}\Delta P_{R_1} = \frac{1}{R_1}(P_s - P_C) \qquad \text{RHS is states and/or inputs}$$

$$Q_{R_2} = \frac{1}{R_2}\Delta P_{R_2} = \frac{1}{R_2}(P_C - 0) = \frac{1}{R_2}P_C \qquad \text{RHS is states and/or inputs}$$

In ΔP_{R_2}, we have used the fact that the open pipe leads to ambient pressure, so the pressure change across R_2 is simply P_C. Making these substitutions, the first state equation is

$$\Rightarrow \underline{\dot{P}_C = \frac{1}{C}Q_C = \frac{1}{C}\{\frac{1}{R_1}(P_s - P_C) - \frac{1}{R_2}P_C - Q_I\}}$$

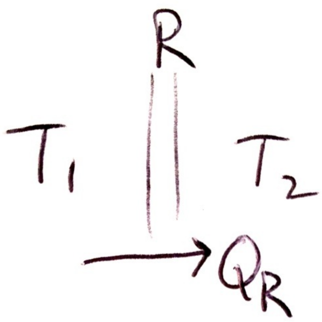
The substitution for P_I is trivial:

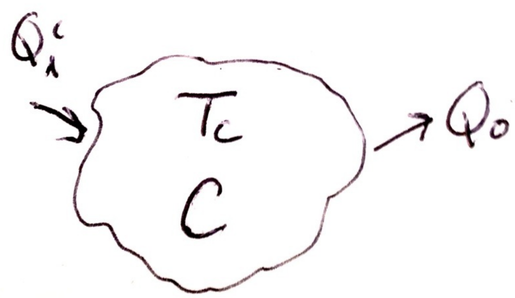
$$\underline{\dot{Q}_I = \frac{1}{I}P_I = \frac{1}{I}P_C}$$

where we note that the entire area of piping/tank enclosed by the elements R_1, R_2, and I_1 has the same pressure (P_C) because pressure only changes *across* an element. The state model is thus

$$\dot{\vec{z}}(t) = A\vec{z} + B\vec{u} = \begin{pmatrix} -\frac{1}{C}(\frac{1}{R_1} + \frac{1}{R_2}) & -\frac{1}{C} \\ \frac{1}{I} & 0 \end{pmatrix} \vec{z} + \begin{pmatrix} \frac{1}{CR_1} \\ 0 \end{pmatrix} \vec{u} \quad , \quad \vec{y}(t) = Q_{R_2} = (\frac{1}{R_2} \quad 0)\vec{z} + (0)\vec{u}$$

Thermal System Elements



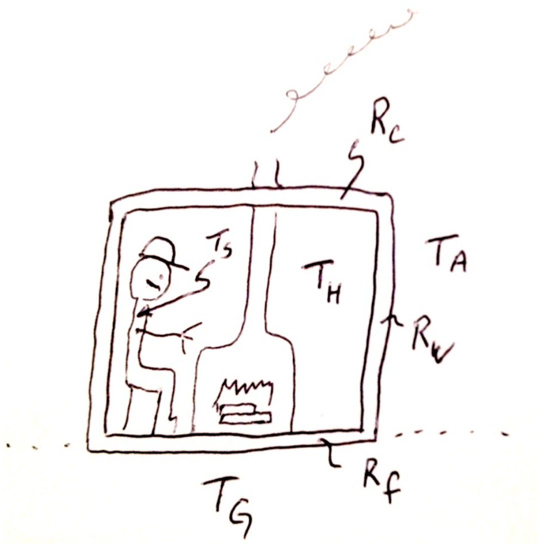


Thermal System Elements

Resistor $T_1 - T_2 = R_h Q_R$
Q_R = heat flow rate = through variable
$T_1 - T_2$ = temperature difference = across variable
R_h = thermal resistance = physical parameter

Capacitor $\dot{T_C} = \frac{1}{C_h}(Q_i - Q_o)$
T_C = temperature = constant throughout
$Q_i - Q_o$ = across variable = net heat flow (in - out)
C_h = thermal capacitance = physical parameter

Example: State Space Modeling of a Thermal System



An MAE 340 student sits in a house with a nice fire that is a heat source, Q_f. The house interior is modeled by a thermal capacitance with C_H and temperature T_H, and the student is modeled by a second thermal capacitance with C_S and temperature T_S. The warm-hearted student acts as a second source, Q_S. The temperature of the outside air is T_A, and the ground below is T_G. Both of these should be considered constant inputs to the model. The walls, ceiling, and floor all act like thermal resistors with R_W, R_F, and R_C, respectively. The student's clothing acts a thermal resistance R_S between T_S and T_H.

The output of this system is defined as (i) the net heat flow rate out of the house, and (ii) the temperature of the house T_H.

Solution

From Table (2), the states in thermal systems are the temperatures in capacitors. Thus, we may write the state vector, input vector, and output vector as follows:

$$\vec{z}(t) \equiv \begin{pmatrix} T_S \\ T_H \end{pmatrix} \quad , \quad \vec{u} = \begin{pmatrix} Q_f \\ Q_S \\ T_A \\ T_G \end{pmatrix} \quad , \quad \vec{y} = \begin{pmatrix} -Q_W - Q_C - Q_F \\ T_H \end{pmatrix}$$

To obtain the state-space equations, begin by writing the elemental laws for the capacitors:

$$\dot{\vec{z}}(t) = \begin{pmatrix} \dot{T}_S \\ \dot{T}_H \end{pmatrix} = \begin{pmatrix} \frac{1}{C_S}\{\frac{1}{R_S}(T_H - T_S) + Q_S\} \\ \frac{1}{C_H}\{Q_f + \frac{1}{R_S}(T_S - T_H) - \frac{1}{R_W}(T_H - T_A) - \frac{1}{R_C}(T_H - T_A) - \frac{1}{R_F}(T_H - T_G)\} \end{pmatrix}$$

Everything on the RHS is either a state or input term, so we may collect terms to find:

$$\dot{\vec{z}}(t) = A\vec{z} + B\vec{u} = \begin{pmatrix} -\frac{1}{C_S R_S} & \frac{1}{C_S R_S} \\ \frac{1}{C_H R_S} & -\frac{1}{C_H R_S} - \frac{1}{C_H R_W} - \frac{1}{C_H R_F} - \frac{1}{C_H R_C} \end{pmatrix} \vec{z} + \begin{pmatrix} 0 & \frac{1}{C_S} & 0 & 0 \\ \frac{1}{C_H} & 0 & \frac{1}{C_H}(\frac{1}{R_W} + \frac{1}{R_C}) & \frac{1}{C_H R_F} \end{pmatrix} \vec{u}$$

$$\vec{y}(t) = C\vec{z} + D\vec{u} = \begin{pmatrix} 0 & \frac{1}{R_W} + \frac{1}{R_C} + \frac{1}{R_F} \\ 0 & 1 \end{pmatrix} \vec{z} + \begin{pmatrix} 0 & 0 & (-\frac{1}{R_W} - \frac{1}{R_C}) & -\frac{1}{R_F} \\ 0 & 0 & 0 & 0 \end{pmatrix} \vec{u}$$