

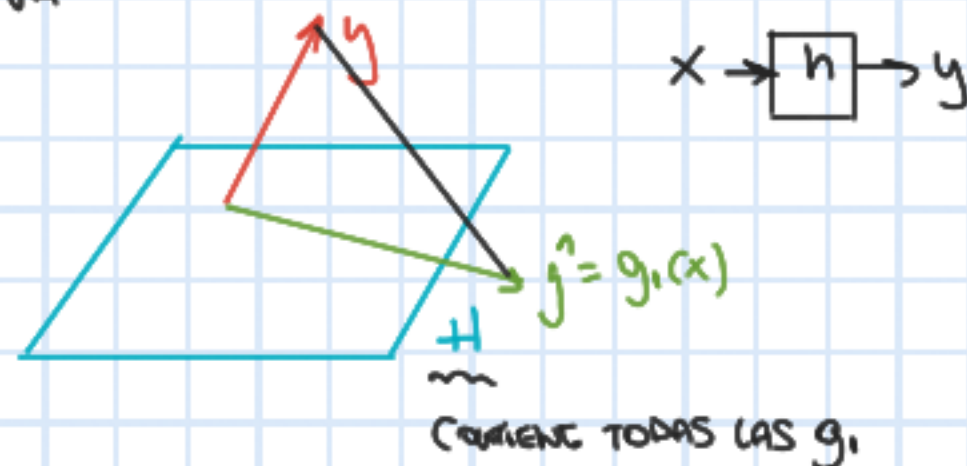
Teórico

$$X = [x_1, x_2, \dots, x_n]^T, y = [y_1, y_2, \dots, y_n]^T, f_{xy}(\bar{x}, \bar{y})$$

$$\hat{y} = g(\bar{x}) \rightarrow \text{Distancia}$$

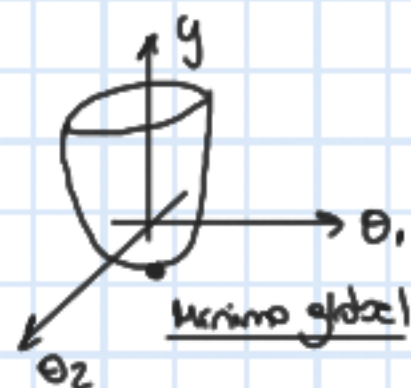
Distancia \rightarrow Producto Interno

$$\langle \bar{x}, \bar{y} \rangle = E[xy] = \iint_{\mathbb{R}^2} xy f_{xy}(x, y) dx dy$$



$$\min_{g_1(x)} \| \hat{y} - y \| \rightarrow \min_{g_1(x)} \sqrt{\langle \hat{y} - y, \hat{y} - y \rangle}$$

$$= \min_{g_1(x)} \sqrt{E[(\hat{y} - y)^2]} \rightarrow \min_{g_1(x)} E[(g_1(x) - y)^2]$$



$$g_1(x) = E(y/x)$$

Regresión Lineal

No conozco distribución conjunta

$$\begin{bmatrix} x_1 & x_2 & \dots & x_m & y \\ x_{1,1} & x_{1,2} & \dots & x_{1,m} & y_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{n,1} & x_{n,2} & \dots & x_{n,m} & y_n \end{bmatrix}$$

No tengo acceso al dataset completo

$$\hat{y} = g_2(x)$$

(1) R.L (2) MV (3) Bayesiano

$$(1) \ell(g_2(x), y) = (g_2(x) - y)^2 \text{ E.C.M.}$$

$$L = \frac{1}{n} \sum_{i=1}^n \ell(g_2(x_i), y_i) \quad \hat{y} = g_2(x) = \bar{w}^T x$$

$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} \in \mathbb{R}^m$$

$$X^{n \times m} \cdot W^{m \times 1} = Y^{n \times 1}$$

$$(1) \quad J_2 = \frac{1}{n} \sum_{i=1}^N \underbrace{(\omega^T x_i - y_i)^2}_{g_2(x)} \xrightarrow{\min} \underline{\nabla_{\omega} J_2 = 0}$$

$$\nabla_{\omega} \|x\bar{\omega} - y\|_2^2 = \frac{1}{n} \nabla_{\omega} (x\bar{\omega} - y)^T (x\bar{\omega} - y)$$

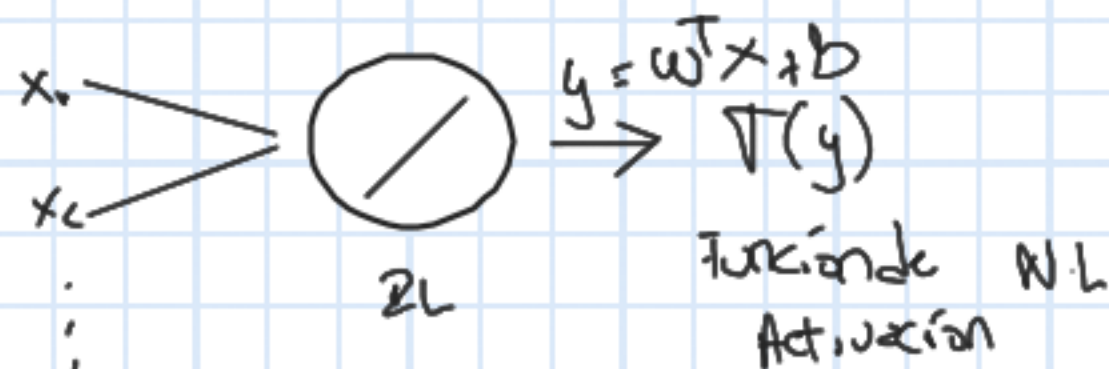
$$= \frac{1}{n} \nabla_{\omega} (\bar{\omega}^T x^T x \bar{\omega} - 2 \bar{\omega}^T x^T y + y^T y) = 0$$

$$2x^T x \bar{\omega} - 2x^T y = 0$$

$$\phi(x) = x^2$$

$$\bar{\omega} = (x^T x)^{-1} x^T y$$

$$\bar{\omega} = (\phi(x)^T \phi(x))^{-1} x^T y$$



$$\hat{y} = \omega^T x \quad \hat{y} = \omega^T x + b$$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad x_m = \begin{bmatrix} x_{1m} \\ \vdots \\ x_{nm} \end{bmatrix} \quad \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_m \\ b \end{bmatrix}$$