Introducción a la Inteligencia Artificial

Facultad de Ingeniería

Universidad de Buenos Aires

Índice

Clase 7

1. Motivación
    a. Aprendizaje No supervisado
    b. Aplicaciones
2. Gaussian Mixture Models
    a. Formulación
    b. Aplicaciones
3. Expectation Maximization

Aprendizaje no supervisado

Machine Learning Supervisado	Machine Learning no Supervisado
Proceso aleatorio $\bar{X}, y$	Proceso aleatorio $\bar{X}$
¿$f_{y/\bar{x}}(y\|\bar{x})$? ⟶ Bayes y M.V.	¿$f_{\bar{x}}(\bar{x})$? ⟶ Bayes y M.V.
Inferencias, predicciones	Clusterización, Reducción Dimensionalidad

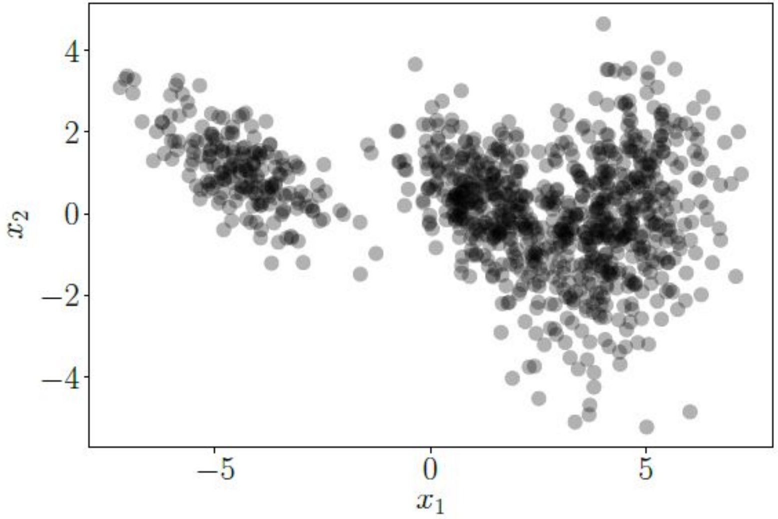
Gaussian Mixture Models

Aplicaciones Generales

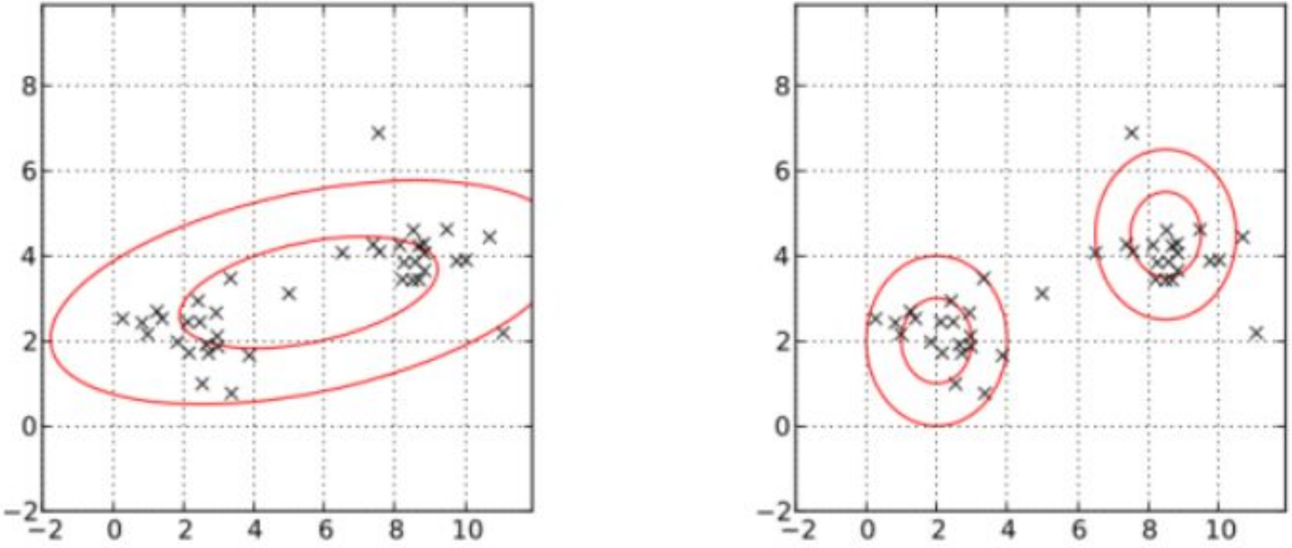
- Data Mining
- Pattern Recognition
- Statistical Analysis

Aplicaciones Específicas

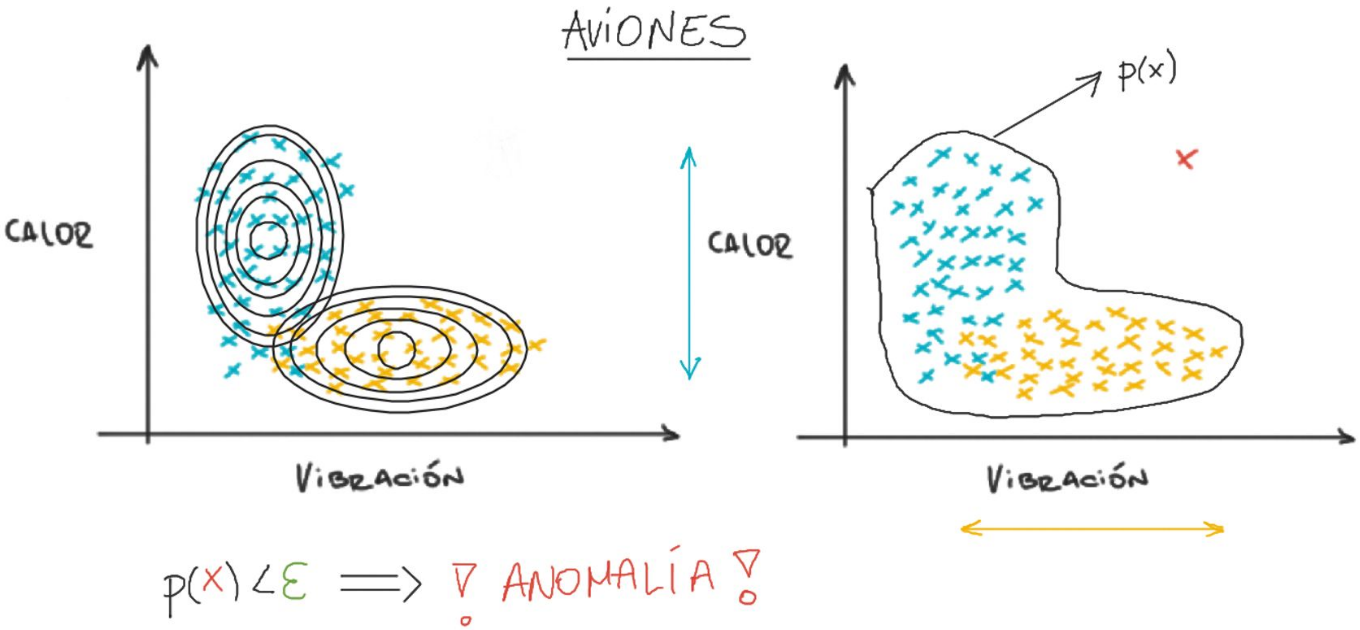
- Density Estimation
- Clustering
- Anomaly Detection
- Object Tracking
- Speech Feature Extraction



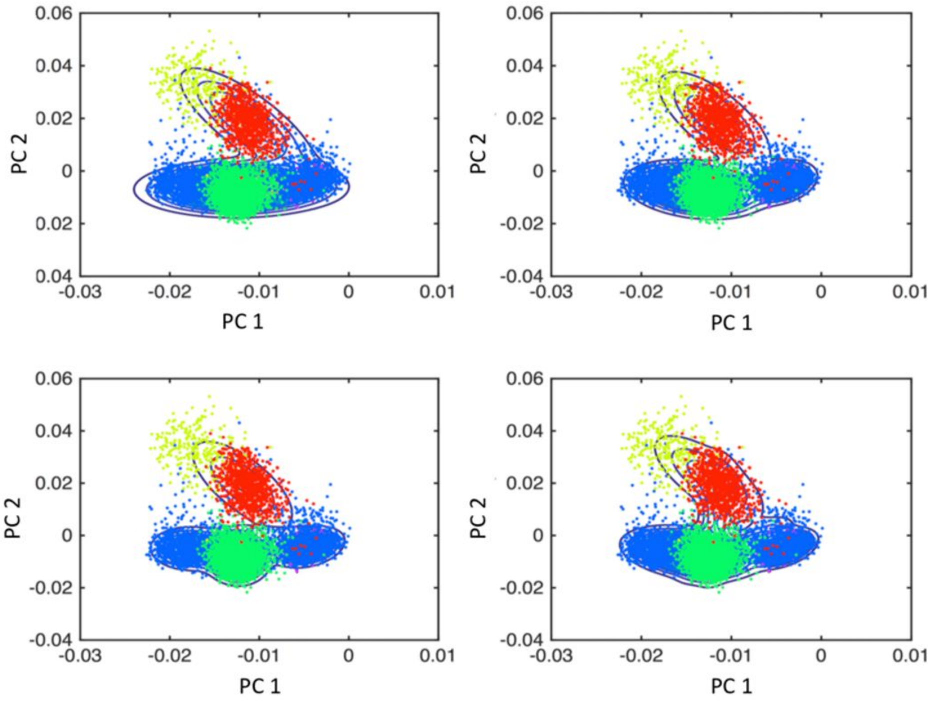
Gaussian Mixture Models



Gaussian Mixture Models

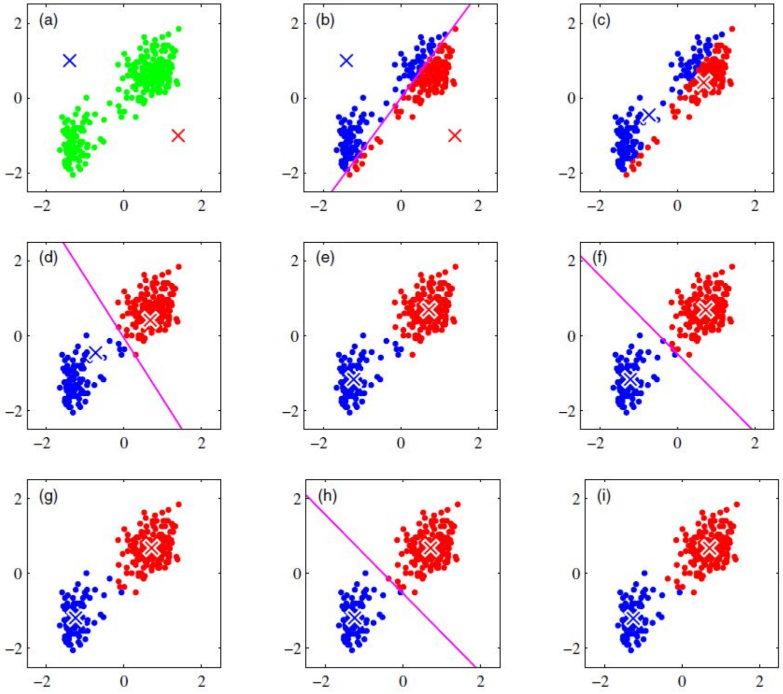


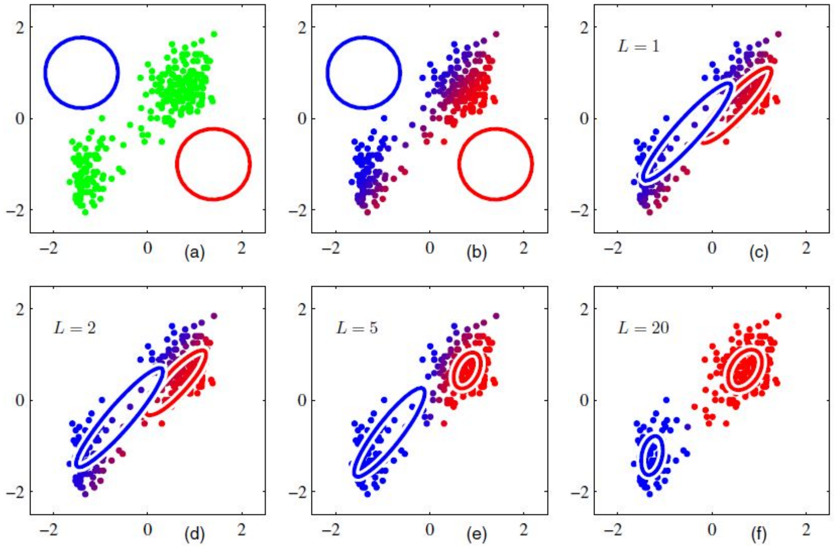
Gaussian Mixture Models



"MAPPING NEURONAL CELL TYPES USING INTEGRATIVE MULTI-SPECIES MODELING OF HUMAN AND MOUSE SINGLE CELL RNA SEQUENCING"

Gaussian Mixture Models





Formulación

$$p(\boldsymbol{x}) = \sum_{k=1}^{K} \pi_k p_k(\boldsymbol{x})$$

$$0 \leqslant \pi_k \leqslant 1\,, \quad \sum_{k=1}^{K} \pi_k = 1\,,$$

Mixture Models - General

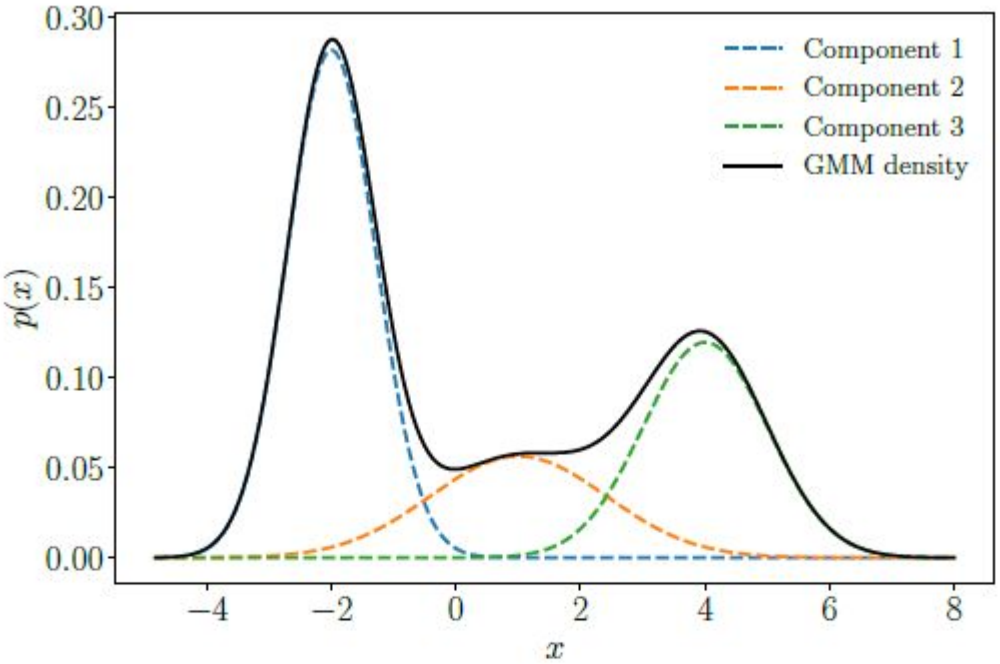
$$p(\boldsymbol{x} \,|\, \boldsymbol{\theta}) = \sum_{k=1}^{K} \pi_k \mathcal{N}\big(\boldsymbol{x} \,|\, \boldsymbol{\mu}_k,\ \boldsymbol{\Sigma}_k\big)$$

$$0 \leqslant \pi_k \leqslant 1\,, \quad \sum_{k=1}^{K} \pi_k = 1\,,$$

$$\boldsymbol{\theta} := \{\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k, \pi_k : \ k = 1, \ldots, K\}$$

Gaussian Mixture Models

Formulación

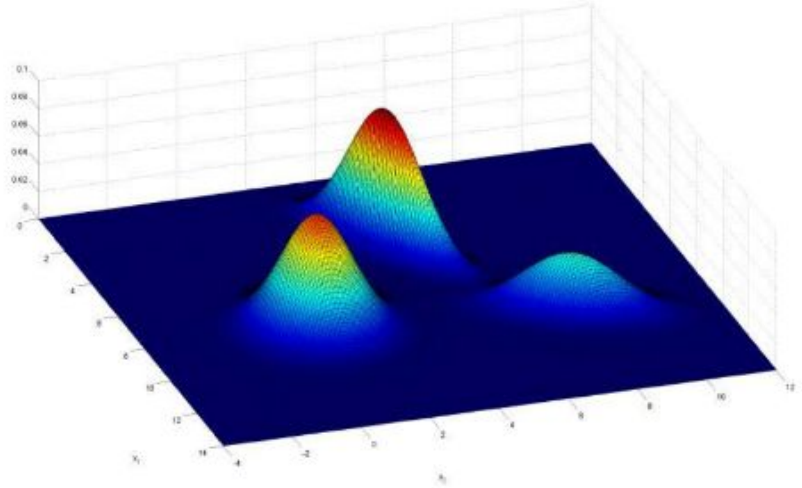


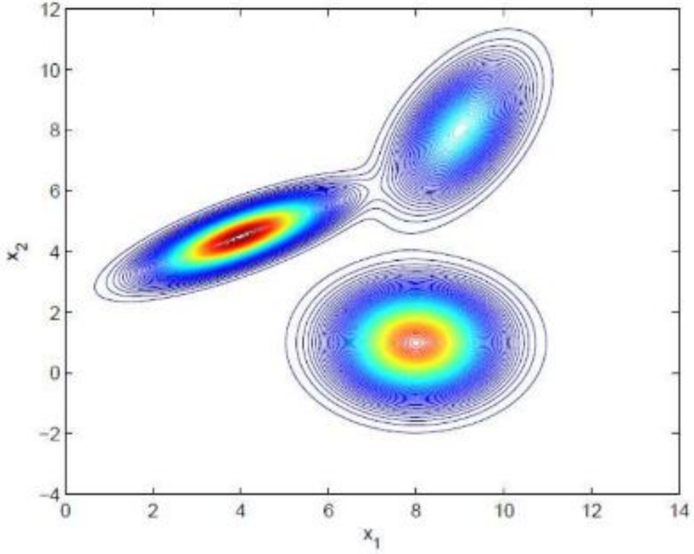
$$p(x \mid \boldsymbol{\theta}) = 0.5\mathcal{N}\left(x \mid -2, \tfrac{1}{2}\right) + 0.2\mathcal{N}\left(x \mid 1, 2\right) + 0.3\mathcal{N}\left(x \mid 4, 1\right)$$

Gaussian Mixture Models

Formulación

$$p(x) = \underbrace{0.3}_{\pi_1} \mathcal{N}\left(x \mid \underbrace{\begin{pmatrix} 4 \\ 4.5 \end{pmatrix}}_{\mu_1}, \underbrace{\begin{pmatrix} 1.2 & 0.6 \\ 0.6 & 0.5 \end{pmatrix}}_{\Sigma_1}\right) + \underbrace{0.5}_{\pi_2} \mathcal{N}\left(x \mid \underbrace{\begin{pmatrix} 8 \\ 1 \end{pmatrix}}_{\mu_2}, \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\Sigma_2}\right) + \underbrace{0.2}_{\pi_3} \mathcal{N}\left(x \mid \underbrace{\begin{pmatrix} 9 \\ 8 \end{pmatrix}}_{\mu_3}, \underbrace{\begin{pmatrix} 0.6 & 0.5 \\ 0.5 & 1.5 \end{pmatrix}}_{\Sigma_3}\right)$$

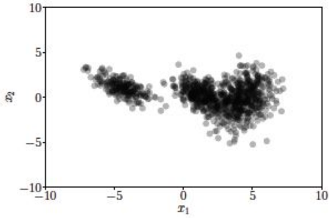




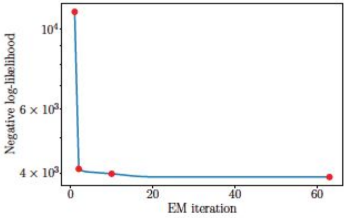
(1) GMM - EJERCICIOS DE APLICACIÓN

(1) GMM - TEORÍA

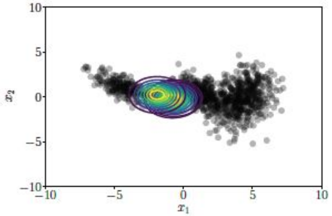
Gaussian Mixture Models - Teoría



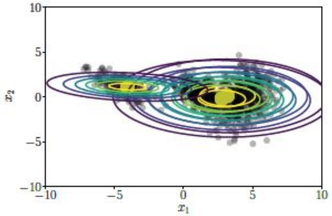
(a) Dataset.



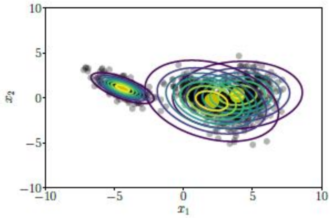
(b) Negative log-likelihood.



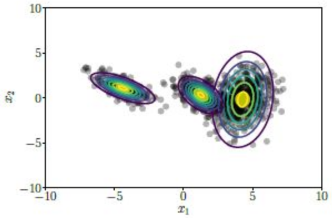
(c) EM initialization.



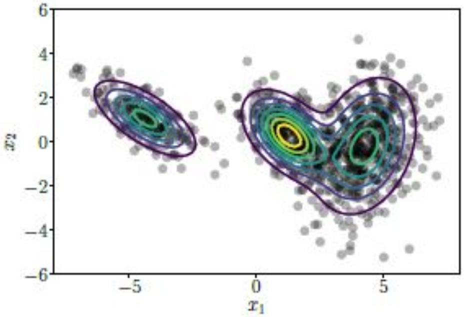
(d) EM after one iteration.



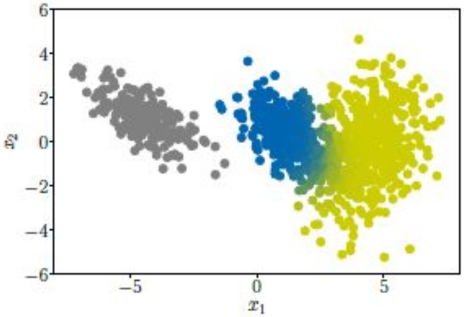
(e) EM after 10 iterations.



(f) EM after 62 iterations.

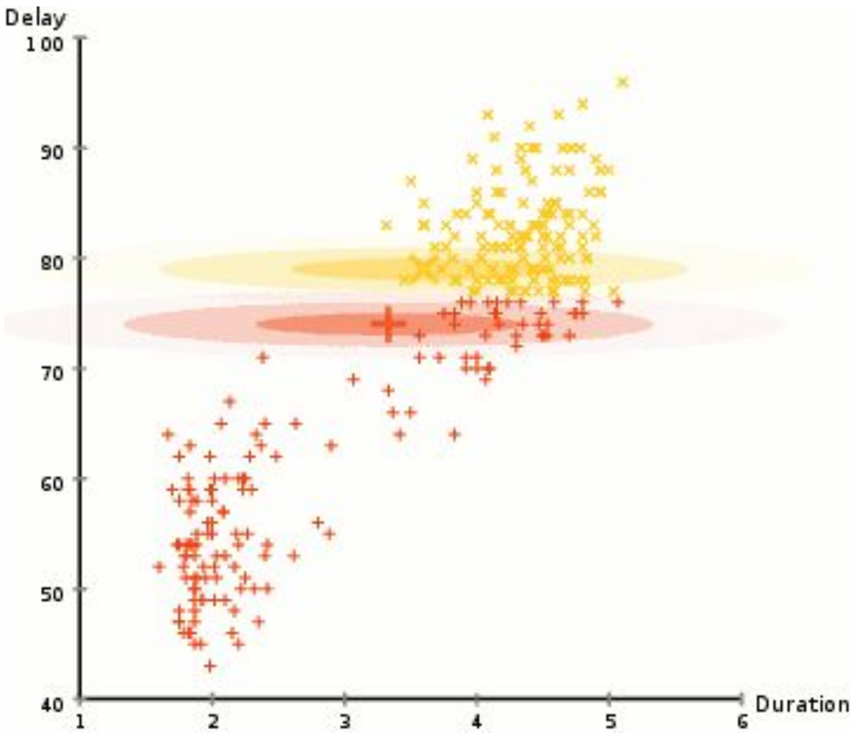


(a) GMM fit after 62 iterations.

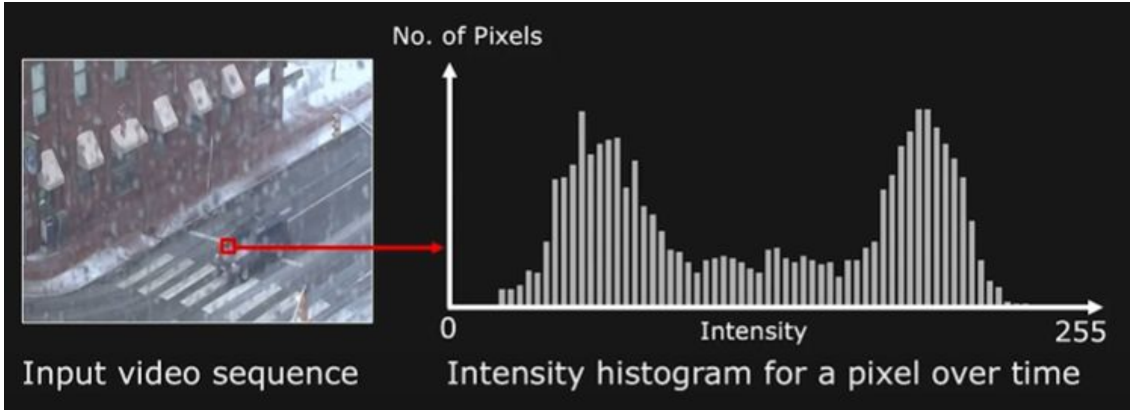


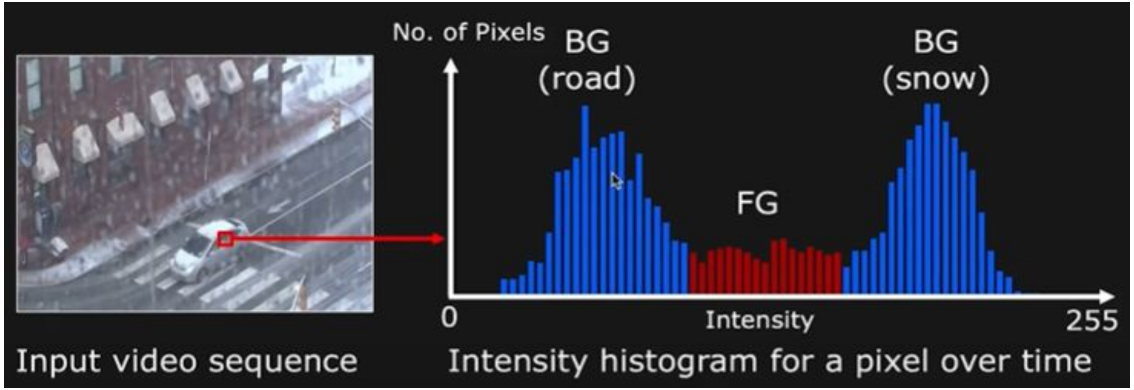
(b) Dataset colored according to the responsibilities of the mixture components.

Gaussian Mixture Models - Teoría

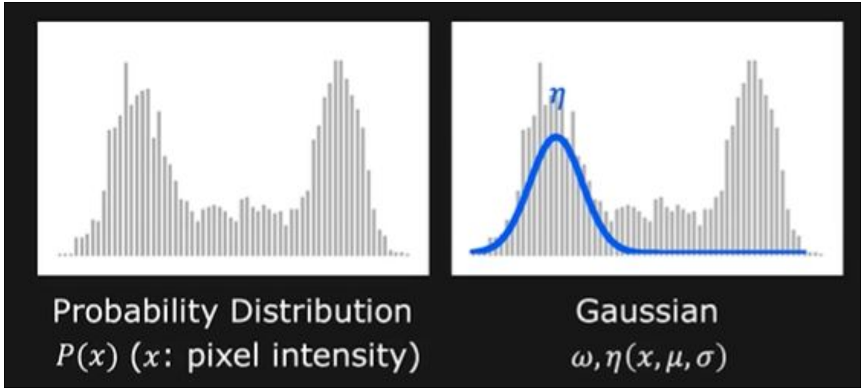


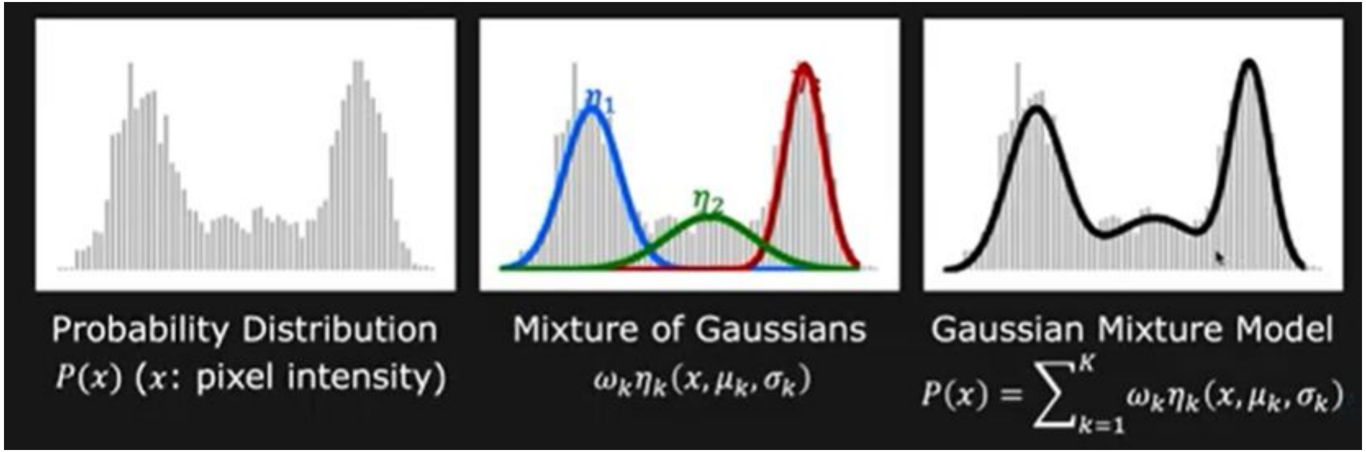
Gaussian Mixture Models - Object Tracking



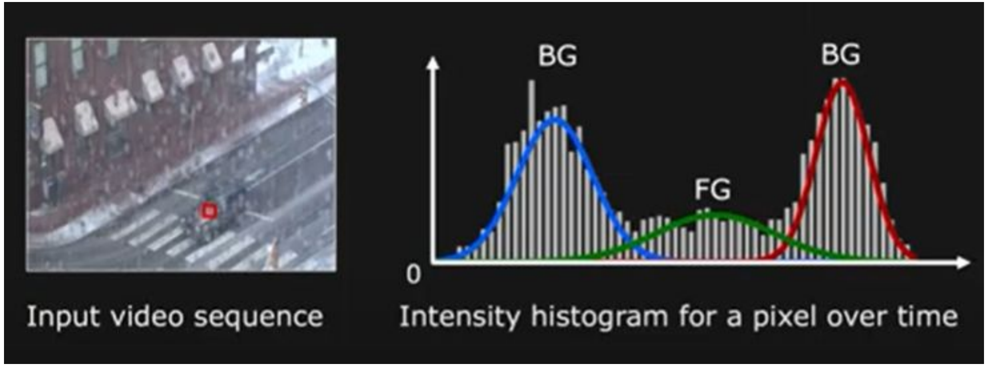


Gaussian Mixture Models - Object Tracking





Gaussian Mixture Models - Object Tracking



$\uparrow \frac{\omega}{\sigma}$ **Background**

$\downarrow \frac{\omega}{\sigma}$ **Foreground**

Gaussian Mixture Models - Object Tracking

$$\boxed{P(\mathbf{X}) \cong \sum_{k=1}^{K} \omega_k \eta_k(\mathbf{X}, \boldsymbol{\mu}_k, \Sigma_k)} \quad \text{such that} \quad \sum_{k=1}^{K} \omega_k = 1$$

where: $$\boxed{\eta(\mathbf{X}, \boldsymbol{\mu}, \Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\mathbf{X}-\boldsymbol{\mu})^T(\Sigma)^{-1}(\mathbf{X}-\boldsymbol{\mu})}}$$

Mean $\boldsymbol{\mu} = \begin{bmatrix} \mu_r \\ \mu_g \\ \mu_b \end{bmatrix}$ Covariance matrix $\Sigma = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$ (can be a full matrix)

Gaussian Mixture Models - Object Tracking

[Stauffer 1998]
[Bowden 2001]
[Zivkovic 2004-2006]

Algoritmo:

Para cada pixel:

1. Calcular el histograma **H** usando los primeros **N** frames.
2. Normalizar el histograma.
3. Modelar el histograma normalizado como un GMM con **K** componentes.
4. Para cada frame subsiguiente:

    a. *El píxel **X** corresponde al componente **k** del GMM para el cual ||**X-media_k**|| es mínimo y ||**X-media_k**||<**2.5*cov_k***

    b. *Clasificar el píxel como background o foreground usando la regla anterior.*

    c. *Actualizar el histograma **H** con el nuevo píxel.*

    d. *Si el nuevo histograma normalizado difiere en gran medida respecto del anterior, actualizar el histograma y hacer fit nuevamente al GMM.*

(1) EM - Teoría

Ejercicio integrador

1. Implementar el algoritmo de Gaussian Mixture Models en NumPy.
2. Aplicar el modelo a un dataset de elección.
3. Comparar los resultados con Scikit-Learn .

Bibliografía

- Mathematics for Machine Learning | Deisenroth, Faisal, Ong
- Pattern Recognition and Machine Learning | Bishop
- Gaussian Mixture Model | John McGonagle, Geoff Pilling, Andrei Dobre
- Expectation-Maximization Algorithms | Stanford CS229: Machine Learning
- First Principles of Computer Vision| Computer Science Department, School of Engineering and Applied Sciences, Columbia University