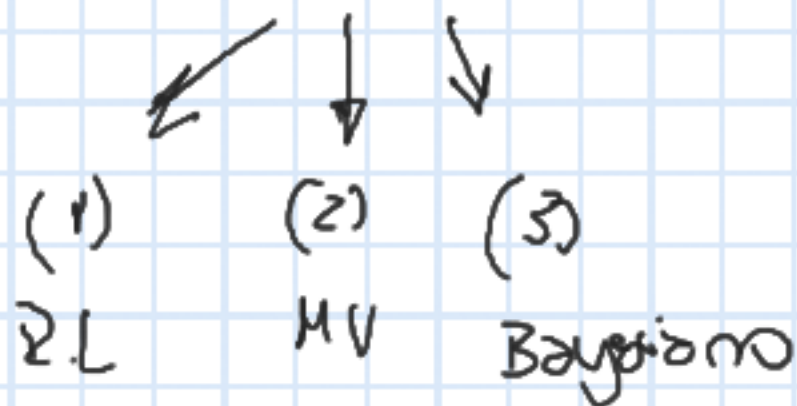


$$y = g_2(x)$$



Regresión  
Lineal -  
Máxima  
Verosimilitud

$x_1, x_2, x_3, \dots, x_n$  i.i.d

$$\mathcal{F} = \{f_\theta : \theta \in \Theta\} \rightarrow$$

$f_{y/x}$

$$\max_{\theta} P_{\theta}(\underbrace{x_1=x_1, x_2=x_2, \dots, x_n=x_n}_{\text{i.i.d}})$$

$$\max_{\theta} \prod_{i=1}^n P_{\theta}(x_i = x_i) \rightarrow \prod_{i=1}^n f_{\theta}(x_i)$$

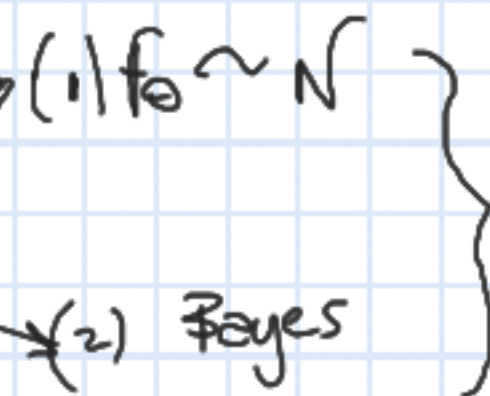
$$\theta_{MV} = \max_{\theta} \prod_{i=1}^n f_{\theta}(x_i)$$

$$\arg \max_{\theta} \prod_{i=1}^n f_{\theta}(y/x)$$

$$(1) f_{\theta}(y_i/x_i) \sim N(\theta, 1/\sigma^2)$$

$$\theta = w^T x$$

ML Supervisado



$$\arg \max_{\theta} \prod_{i=1}^n f_{\theta}(y_i/x_i) \Rightarrow \arg \max_{\theta} \ln \left( \prod_{i=1}^n f_{\theta}(y_i/x_i) \right)$$

$$\arg \max_{\theta} \sum_{i=1}^n \ln f_{\theta}(y_i/x_i)$$

$$\arg \max_{\theta} \sum_{i=1}^n \ln \left( \frac{1}{\sqrt{2\pi}} e^{-\frac{(y_i - w^T x_i)^2}{2}} \right)$$

$$\arg \max_{\theta} \sum_{i=1}^n \left( \ln \left( \frac{1}{\sqrt{2\pi}} \right) + \ln \left( e^{-\frac{(y_i - w^T x_i)^2}{2}} \right) \right)$$

$$\arg \max_w \left( K - \frac{1}{2} \sum_{i=1}^n \underbrace{(y_i - w^T x_i)^2}_{\text{ECM} \rightarrow \text{RL}} \right)$$

Regresión lineal  $\rightarrow$  Linea en W

$$\phi(x) = [x_1, x_2, x_1^2, x_2^2, x_1^3, \dots]$$

$$\begin{bmatrix} x_{1,1} & x_{1,2} & y_1 \\ \vdots & \vdots & \vdots \\ x_{n,1} & x_{n,2} & y_n \end{bmatrix}$$

$$\hat{y} = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1^2 + w_4 x_2^2 + \dots$$

$$\phi(x) = [x_1, x_2, \cos(x_1), \sin(x_2)]$$

$$\underbrace{(\phi^T \phi)^{-1}}_{\text{matrix}} \phi^T y = w$$

$\downarrow$   
 $O(n^3)$   
Optimization

$\theta_{MV}$  Estimador Pontual

$$\hat{\theta} = g(x_1, \dots, x_n) \quad \hat{\theta} = \text{v.a.}$$

$$\begin{cases} \text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta \\ \text{Var}(\hat{\theta}) = E[(\hat{\theta} - E[\hat{\theta}])^2] \end{cases}$$

ML  $\rightarrow$  ECM

$$y = f(x) + \varepsilon \quad \varepsilon \sim N(0, \sigma^2)$$

$$E[(\hat{y} - y)^2] = \text{Bias}(\hat{y})^2 + \text{Var}(\hat{y}) + \underbrace{\sigma^2}_{\text{Error Irreducible}}$$

(3) Bayes

$\theta$  es un V.A.  $p(\theta)$   
 distribución a priori } Familia

Enfoque  
Bayesiano -  
Regresión  
Lineal

$$f_{x_1, x_2, \dots, x_n / \theta} = \prod_{i=1}^n f_{x_i / \theta}(x_i / \theta) \quad \text{Verosimilitud}$$

$f_{\theta}(\theta)$   
 distribución a  
 priori de  $\theta$

OBJETIVO:  $\frac{f_{\theta / x_1, x_2, \dots, x_n}(\theta / x_1, x_2, \dots, x_n)}{f_{x_{n+1} / x_1, x_2, \dots, x_n}}$

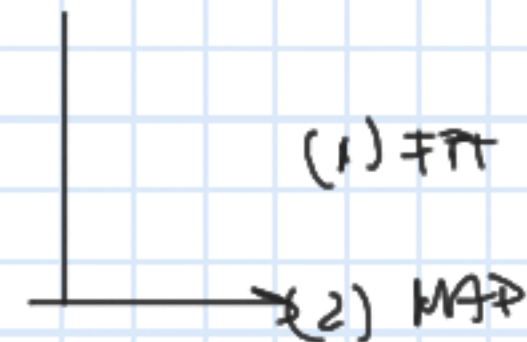
$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$f_{\theta / x_1, x_2, \dots, x_n}(\theta, x_1, x_2, \dots, x_n) = \frac{f_{\theta, x_1, x_2, \dots, x_n}(\theta, x_1, x_2, \dots, x_n)}{f_{x_1, x_2, \dots, x_n}}$$

$$f_{\theta / x_1, x_2, \dots, x_n}(\theta, x_1, x_2, \dots, x_n) = \frac{f_{x_1, x_2, \dots, x_n / \theta}(x_1, x_2, \dots, x_n / \theta) \cdot f_{\theta}(\theta)}{f_{x_1, x_2, \dots, x_n}}$$

$$\frac{f_{\theta / x_1, x_2, \dots, x_n}(\theta / x_1, x_2, \dots, x_n)}{f_{x_{n+1} / x_1, x_2, \dots, x_n}} \propto \underbrace{f_{x_1, x_2, \dots, x_n / \theta}}_{\text{verosimilitud}} \cdot \underbrace{f_{\theta}(\theta)}_{\text{priori}}$$

función de verosimilitud  
vs  
estimador





## Maximum a Posteriori

$$\arg \max_{\theta} \prod_{i=1}^n f_{y_i, x_i, \theta} f_{\theta}(\theta)$$

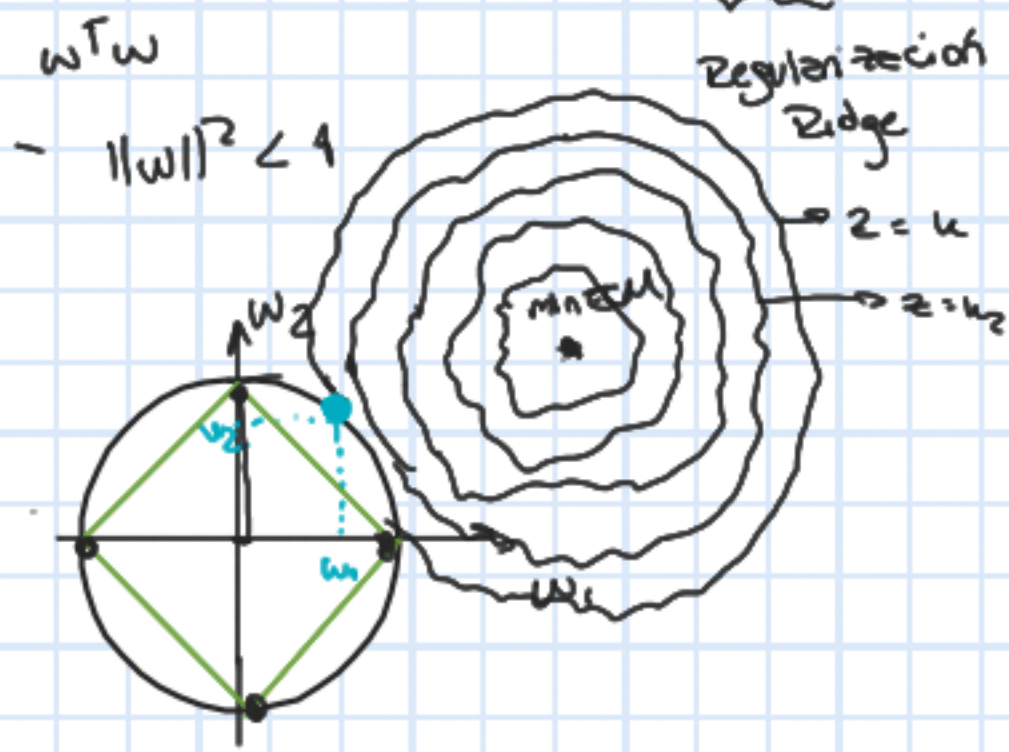
$$\arg \min_{\theta} -\ln \left( \prod_{i=1}^n f_{y_i, x_i, \theta} \right) - \ln f_{\theta}(\theta) \quad \textcircled{A}$$

## Bayes Regression Linear

$$H \begin{cases} y = w^T x \\ f_{w_1, w_2, \dots, w_m} \sim N(0, \frac{1}{\lambda} I) \end{cases}$$

$$f_{w_1, w_2, \dots, w_m} \propto \exp \left( -\frac{1}{2} \bar{w}^T \left( \frac{1}{\lambda} I \right)^{-1} \bar{w} \right) \\ \propto \exp \left( -\frac{1}{2} \lambda w^T w \right)$$

$$\textcircled{A} \arg \min_w -\ln \sum_{i=1}^n f_{y_i, x_i, w} + \underbrace{\lambda w^T w}_{\lambda \|w\|_2^2} \Rightarrow \theta_{MAP} = (X^T X + \lambda I)^{-1} X^T y$$



Elastic Net

$$k_1 \|w\|^2 + k_2 \|w\|$$

Laplaceans

$\lambda \|w\|$   
Regularization  
Lasso

Selection  
features