# Predicciones



## Procesos estocásticos con tendencia determinística

Supongamos el proceso Y formado por una componente estocástica X y una tendencia determinística mu, de la forma:

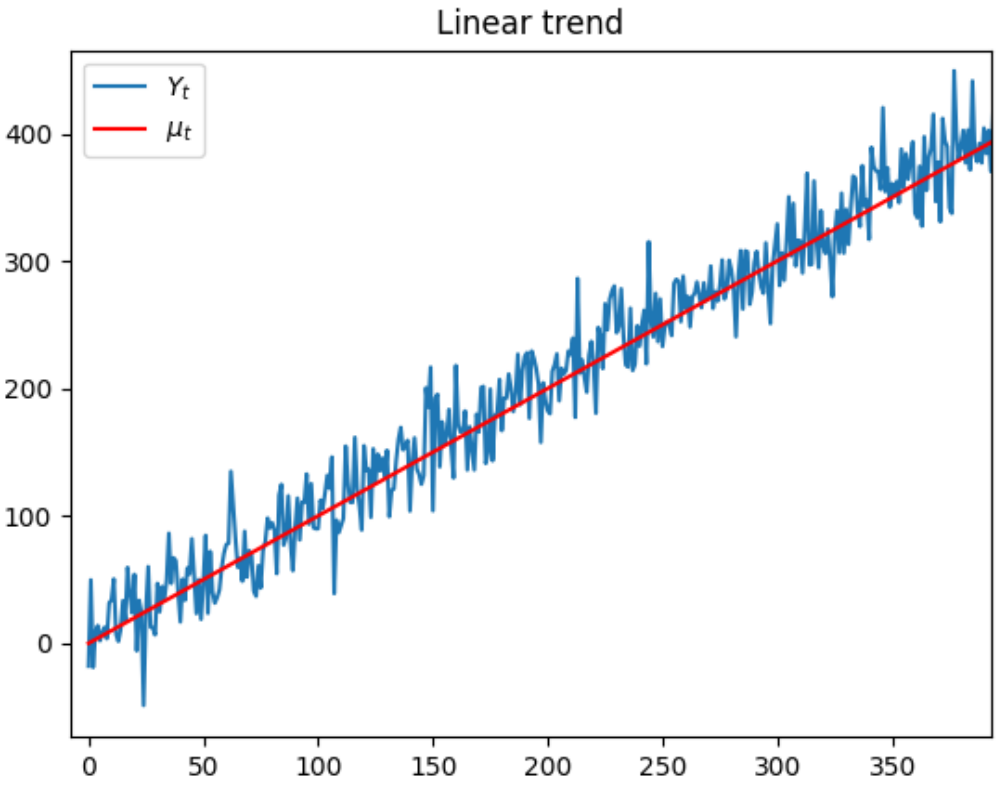
$$Y_t = X_t + \mu_t$$

Como hemos visto la tendencia puede tener distintas representaciones:

$$\text{Tendencia determinística } \mu_t := \begin{cases} \text{Constante} & \mu_t = \mu \\ \text{Lineal} & \mu_t = \beta_0 + \beta_1 t \\ \text{Cuadrática} & \mu_t = \beta_0 + \beta_1 t + \beta_2 t^2 \\ \text{Cíclica} & \mu_t = \mu_{t-T} \\ \text{Senoidal} & \mu_t = \beta \cos(\omega t + \phi) \end{cases}$$

### Tendencia Lineal

```
N=1000
mu=np.arange(0,N)
X=np.random.normal(5,25,N)
Y=X+mu
plt.plot(Y)
plt.plot(mu,color='red')
plt.title("Linear trend")
plt.legend(['$Y_t$','$\mu_t$'])
plt.show()
```



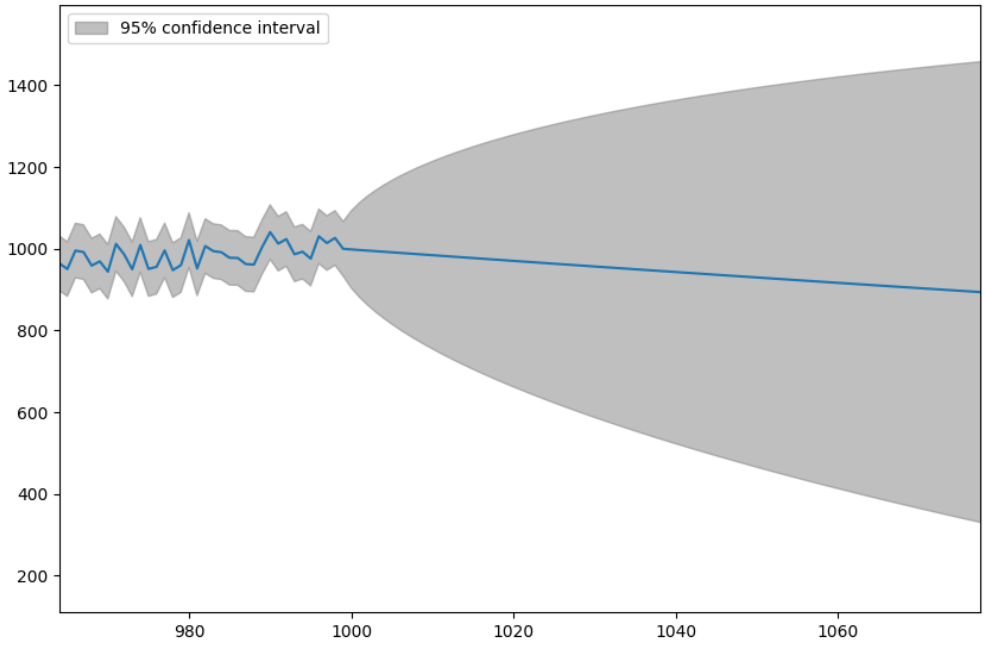

## Modelo AR(1)

```
from statsmodels.graphics.tsaplots import plot_predict
from statsmodels.tsa.arima_process import arma_generate_sample
from statsmodels.tsa.arima.model import ARIMA
from sklearn.metrics import mean_squared_error

ar1 = ARIMA(Y, order=(1, 0, 0), trend="n")
ar1_res = ar1.fit()
print(ar1_res.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 1000
Model:                 ARIMA(1, 0, 0)   Log Likelihood               -4950.416
Date:                mar, 16 nov 2021   AIC                           9904.832
Time:                        14:11:27   BIC                           9914.647
Sample:                             0   HQIC                          9908.562
                                - 1000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.9986      0.002    649.713      0.000       0.996       1.002
sigma2      1160.9614     50.960     22.782      0.000    1061.082    1260.841
===================================================================================
Ljung-Box (L1) (Q):                 276.92   Jarque-Bera (JB):                 0.49
Prob(Q):                              0.00   Prob(JB):                         0.78
Heteroskedasticity (H):               1.08   Skew:                             0.04
Prob(H) (two-sided):                  0.49   Kurtosis:                         3.08
===================================================================================
```

## Predicción

```
fig, ax = plt.subplots(figsize=(10, 8))
fig = plot_predict(ar1_res, start=1,end=2000, ax=ax)
legend = ax.legend(loc="upper left")
plt.show()
```




## Resultados

Lo que se observa es que  a medida que aumenta el largo de la predicción aumenta también el intervalo de confianza del 95%, haciendo que la predicción acumule error. Esto se puede ver con la forma general del proceso ARIMA. Si $\Psi_1, \Psi_2, \ldots, \Psi_{l-1}$ son los coeficientes del proceso, se puede observar que la varianza de la predicción del proceso AR(1) es:

$$\text{var}(e_t(\ell)) = \sigma_e^2(1 + \Psi_1^2 + \Psi_2^2 + \ldots + \Psi_{\ell-1}^2)$$

es decir que la varianza aumenta a medida que se agregan términos de predicción o bien aumenta $\ell$.  Para el caso AR1 además se puede usar el resultado de la serie geométrica y ver que:

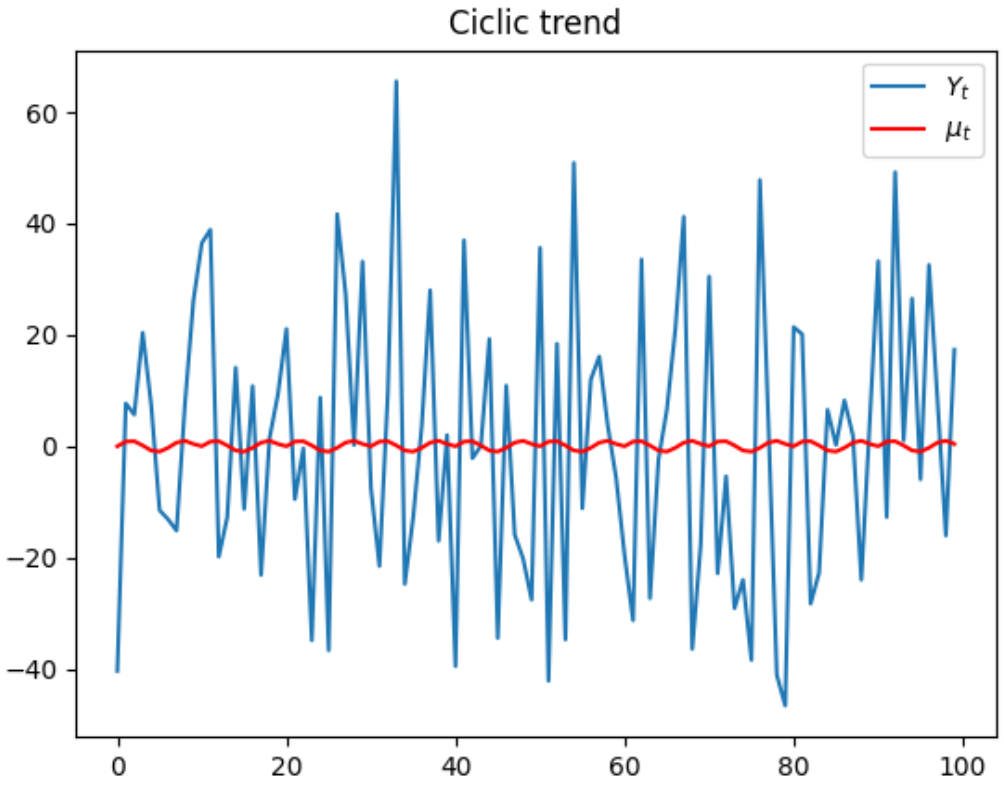
$$\text{var}(e_t(\ell)) = \sigma_e^2 \left( \frac{1 - \phi^{2\ell}}{1 - \phi^2} \right)$$

cuando el retardo $\ell$ es muy largo, se puede aproximar y ver que el resultado es la autocovarianza de Y.

$$\text{var}(e_t(\ell)) \approx \left( \frac{\sigma_e^2}{1 - \phi^2} \right) \approx \text{var}(Y_t) = \gamma_0$$

## Tendencia cíclica

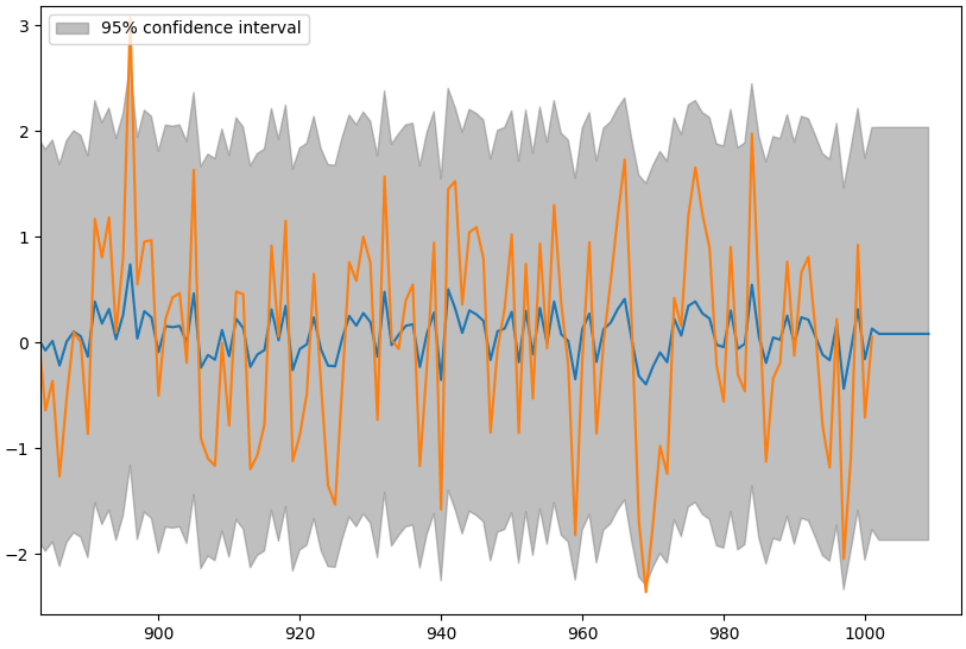
```
n=100
N=10000
mu=np.repeat(np.sin(np.arange(n)),int(N/n))
X=np.random.normal(5,25,int(N))
Y=X+mu
plt.plot(Y)
plt.plot(mu,color='red')
plt.title("Ciclic trend")
plt.legend(['$Y_t$','$\mu_t$'])
plt.show()
```




## Modelo MA(1)

```
# MA(1)
N =1000
b1 =0.25
e_t0 = np.random.normal(0,1)
e_t1 = np.random.normal(0,1)
y = np.append(e_t0,b1*e_t0+e_t1)  # y1 = b1*e_t0+e_t1

for i in range(N):
   e_t0 = e_t1
   e_t1 = np.random.normal(0,1)
   y = np.append(y,b1*e_t0+e_t1)


y.mean()
```

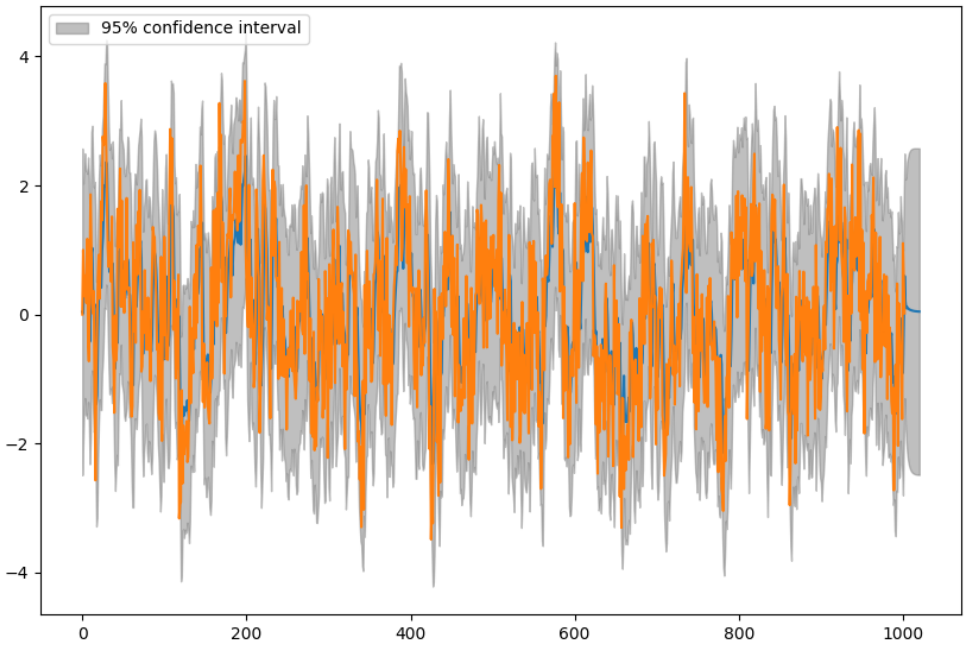
```
y.std()
plt.plot(y);plt.show()

# Predict
ma1 = ARIMA(y, order=(0, 0, 1))
ma1_res = ma1.fit()
print(ma1_res.summary())

fig, ax = plt.subplots(figsize=(10, 8))
fig = plot_predict(ma1_res, start=1,end=1010, ax=ax)
plt.plot(y)
legend = ax.legend(loc="upper left")
plt.show()

y_ma1=y
```




## Modelo AR(2)

```
N=1000
a1=0.4
a2=0.35
x=np.arange(2)
e_t=np.random.normal(0,1)
y=np.append(x,a1*x[1]+a2*x[0]+e_t)

for i in range(N):
   e_t=np.random.normal(0,1)
   y=np.append(y,a1*y[-1]+a2*y[-2]+e_t)


y.mean()
y.std()
#plt.plot(y);plt.show()
```

```
# Predict
l=20
ar2 = ARIMA(y, order=(2, 0, 0))
ar2_res = ar2.fit()
print(ar2_res.summary())

fig, ax = plt.subplots(figsize=(10, 8))
fig = plot_predict(ar2_res, start=0,end=N+l, ax=ax)
plt.plot(y)
legend = ax.legend(loc="upper left")
plt.show()
```




## Modelo ARMA(2,1)

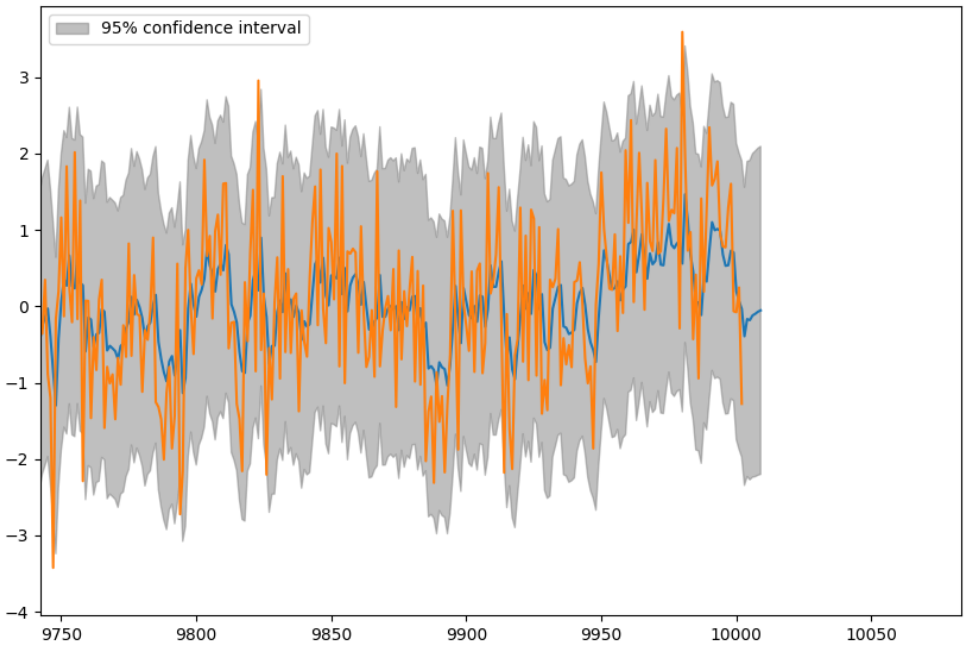
```
#ARMA

N=10000
a1=0.4
a2=0.3
b1 =-0.3

x=np.arange(2)
e_t=np.random.normal(0,1)
y=np.append(x,a1*x[1]+a2*x[0]+e_t)

for i in range(N):
   e_0 = e_t
   e_t=np.random.normal(0,1)
   y=np.append(y,a1*y[-1]+a2*y[-2]+ b1*e_0 + e_t)
```

```
y.mean()
y.std()
plt.plot(y);plt.show()
```

## Predicción

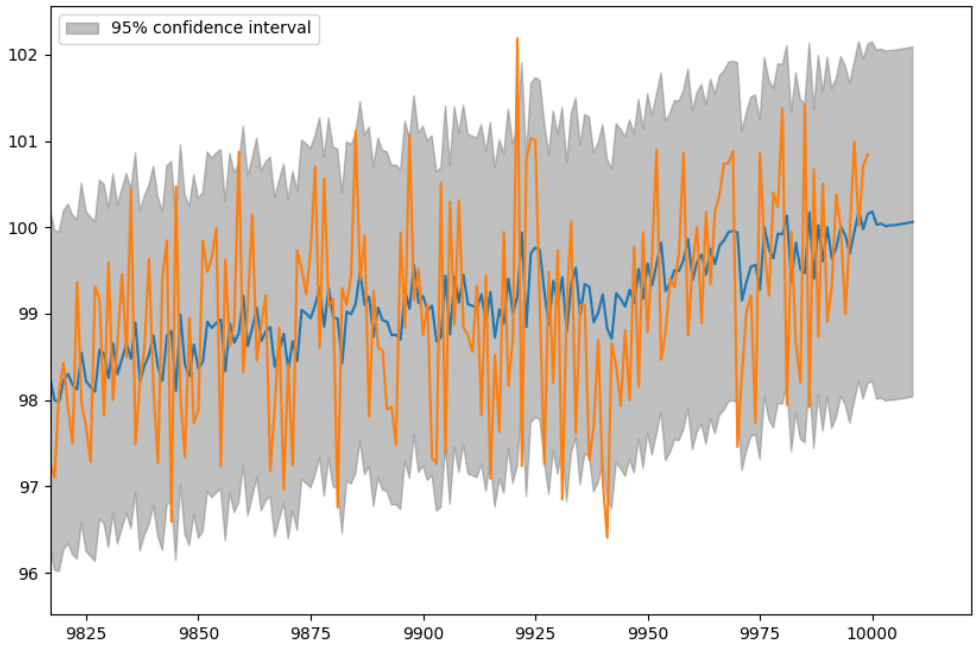
```
# Predict
y_arma = ARIMA(y, order=(2, 0, 1))
y_arma_res = y_arma.fit()
print(y_arma_res.summary())

fig, ax = plt.subplots(figsize=(10, 8))
fig = plot_predict(y_arma_res, start=1,end=N+10, ax=ax)
plt.plot(y)
legend = ax.legend(loc="upper left")
plt.show()
```

## Resultados



## Modelo ARIMA(2,1,1)

```
#ARIMA

N=10000
a1=0.35
a2=0.25
b1 =-0.35

x=np.arange(2)
e_t=np.random.normal(0,1)
y=np.append(x,a1*x[1]+a2*x[0]+e_t)

for i in range(N):
   e_0 = e_t
   e_t=np.random.normal(0,1)
   y=np.append(y,a1*y[-1]+a2*y[-2]+ b1*e_0 + e_t)

t = np.arange(int(N*0.01), step=0.01)
y=y[2:N+2]

y = y+t
y.mean()
y.std()
#plt.plot(y);plt.show()


# Predict
y_arma = ARIMA(y, order=(2, 1, 1), trend='t')
y_arma_res = y_arma.fit()
print(y_arma_res.summary())

fig, ax = plt.subplots(figsize=(10, 8))
fig = plot_predict(y_arma_res, start=1,end=N+10, ax=ax)
plt.plot(y)
legend = ax.legend(loc="upper left")
plt.show()
```


95% confidence interval
102
101
100
99
98
97
96
9825
9850
9875
9900
9925
9950
9975
10000