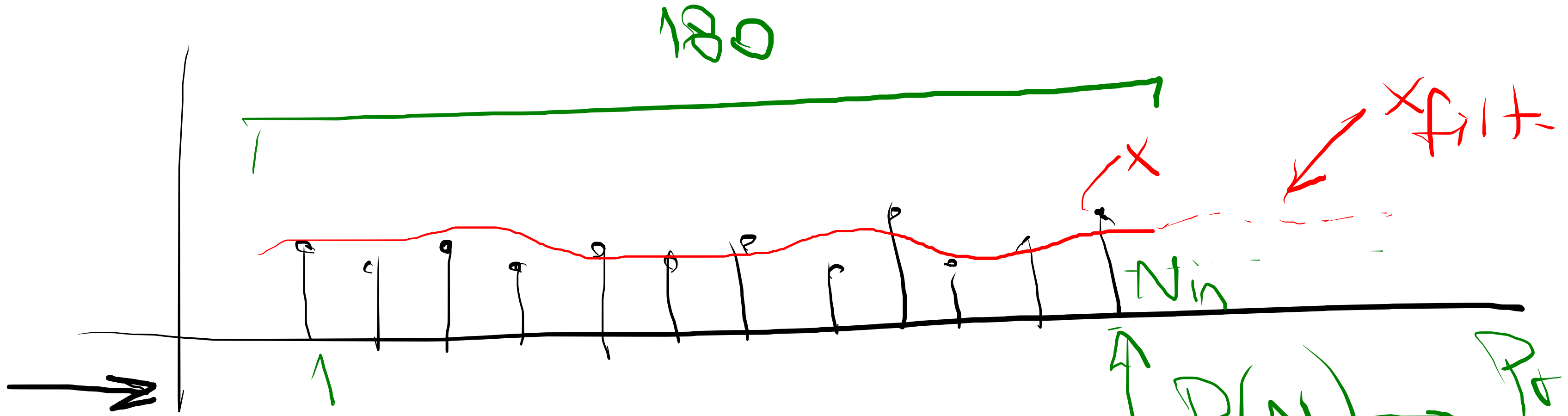
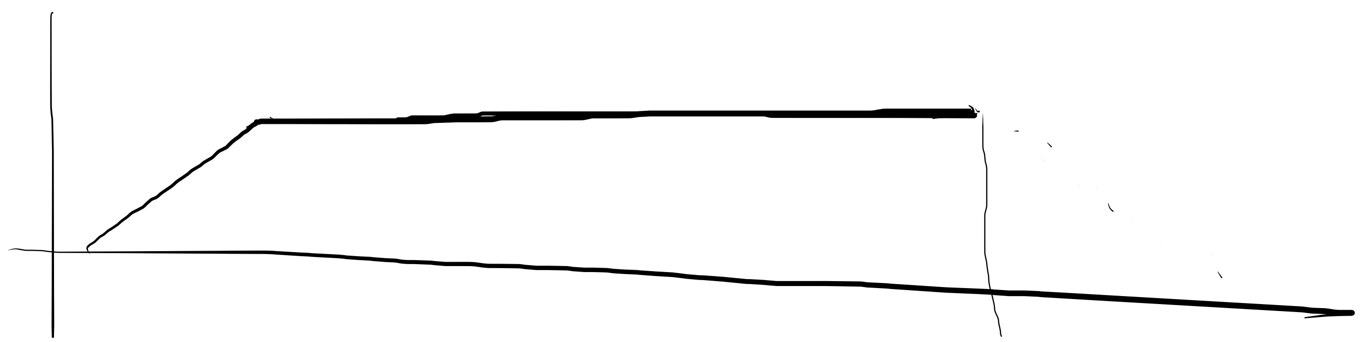
Ej. 3

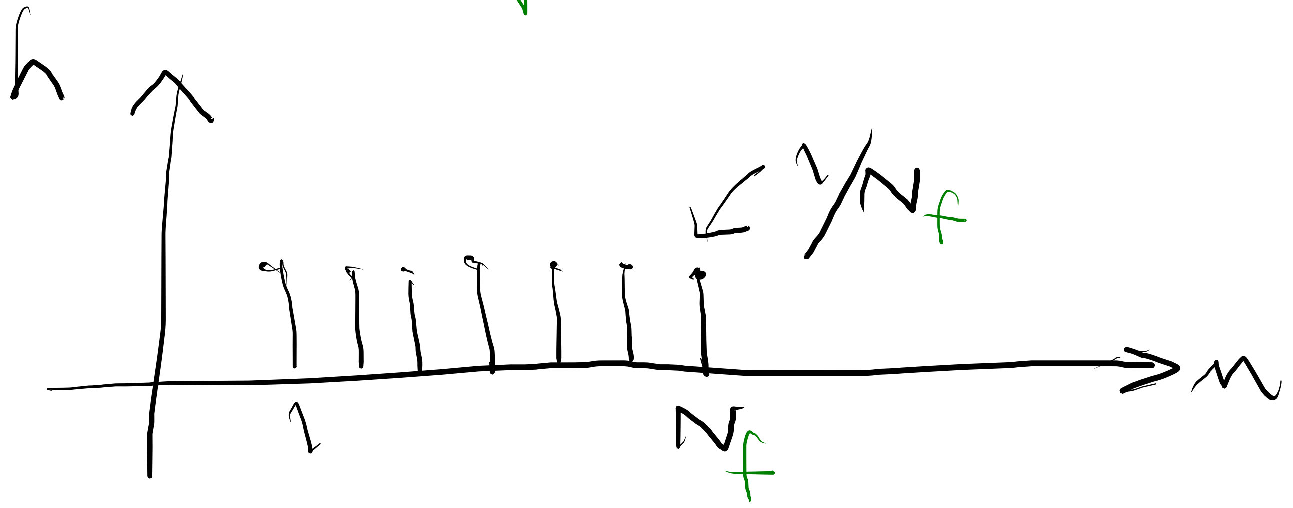


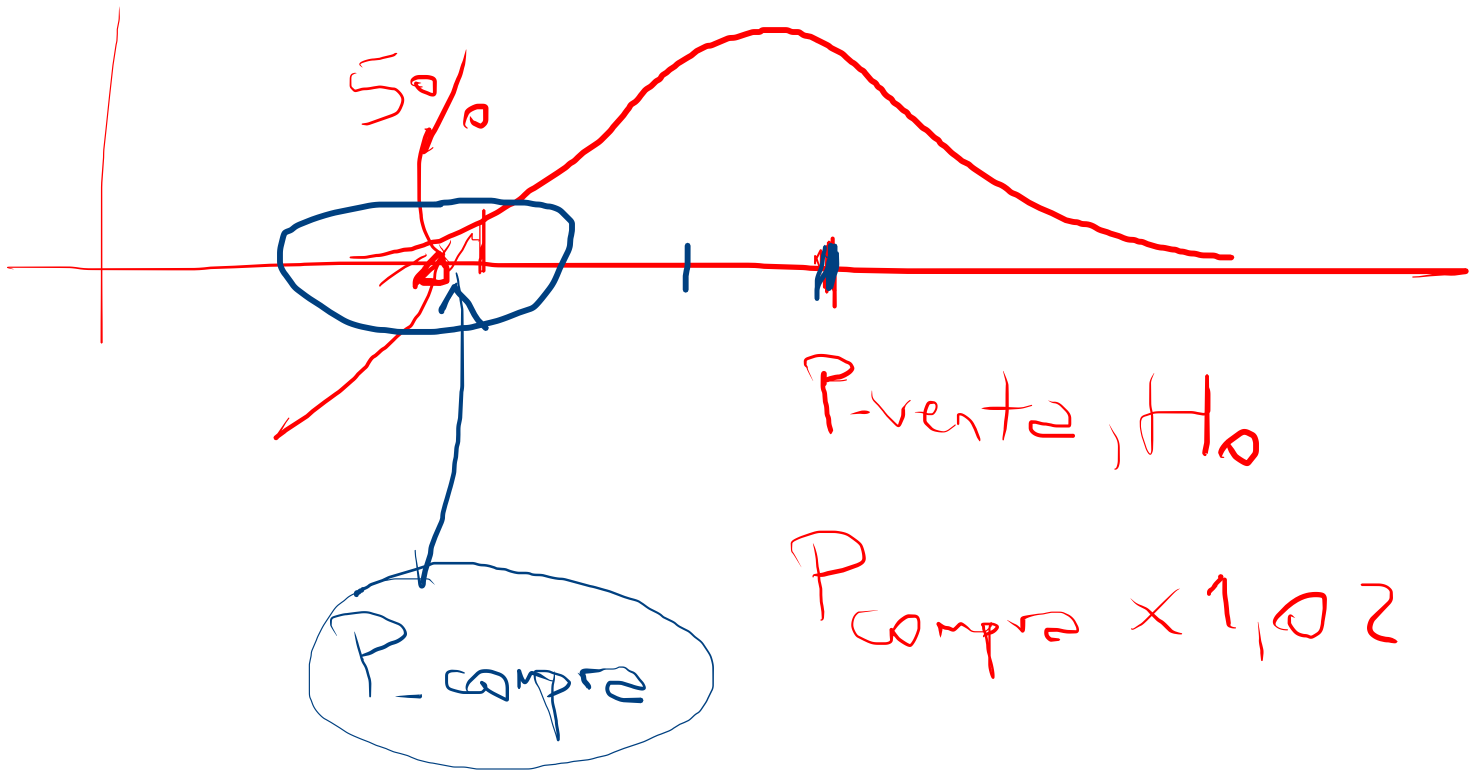
$P \geq 95\%$

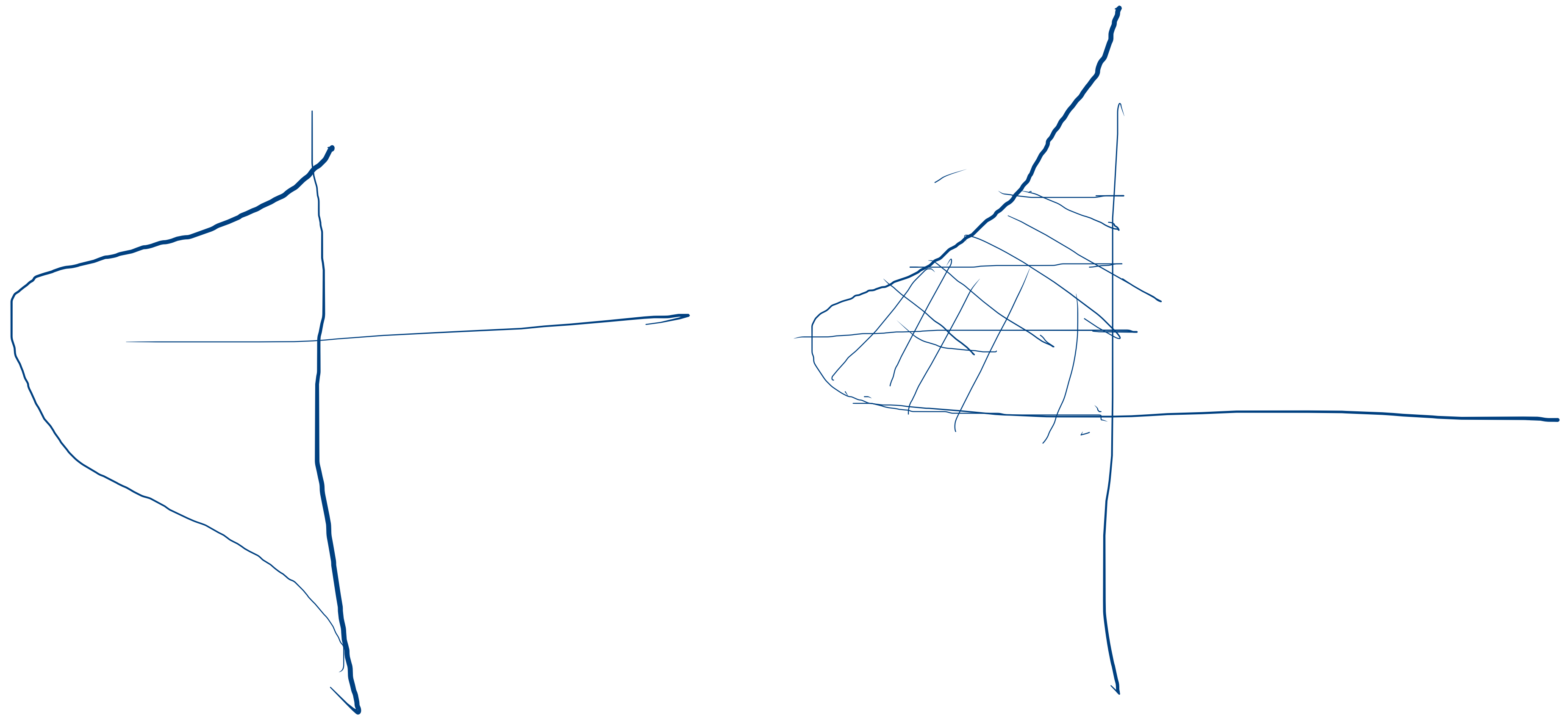
$P(N_{in}) \rightarrow$ Precio de compra $\rightarrow P_{venta} \geq 1{,}02\, P_{compra}$

M.A.F.






5%
P-venta, H_0
P_compra
P compra x 1,02



Entropía

H T Fuente de información.

Información: $I_{\underline{X}=x} = \log_2 (1/p)$

$\hookrightarrow p(\underline{X}=x)$

$P_{ceca} = 1 \rightarrow I_{\underline{X}=ceca} = \log_2(1/1) = 0 \rightarrow$ no aporta información

$P_{cara} = 0 \rightarrow I_{\underline{X}=cara} = \log_2(1/0) = \infty \rightarrow$ sorpresa.

Entropía

$\log(1/a) = -\log a$

$H(\underline{X}) = E\left[\log_2\left(1/p(\underline{X})\right)\right] \rightarrow$ información promedio

$= \text{Esperanza} \rightarrow -\sum_x P(\underline{X}=x) \cdot \log_2\left(P(\underline{X}=x)\right)$

$$\sum_x \underbrace{P(\underline{X}=x)}_{\geq 0} \cdot \overbrace{\log_2\left(\frac{1}{\underbrace{P(\underline{X}=x)}_{\geq 1}}\right)}^{\geq 0} \geq 0$$

$H(\underline{X}) \geq 0$

$P \in [0,1] \Rightarrow \frac{1}{P} \in [1, +\infty)$

$$\underline{X} = \{x_1, x_2\}$$

$$P_1 = P(\underline{X} = x_1) = 1 = P$$

$$P_2 = P(\underline{X} = x_2) = 0 = 1 - P$$

$$H(\underline{X}) = E[I(\underline{X})] = \sum_x P(\underline{X} = x) \log_2 \left(1/P(\underline{X} = x) \right)$$

$$H(\underline{X}) = \underbrace{\overbrace{P_1 \log_2 (1/P_1)}^{0}}_{0} + \underbrace{\overbrace{P_2 \log_2 (1/P_2)}^{\infty}}_{0 \cdot \infty}$$

$P_1 \to 1$, $P_1 \to 1$, $P_2 \to 0$, $P_2 \to 0$

$$\underbrace{\frac{\overbrace{\log_2(1/P_2)}^{\infty}}{1/P_2}}_{\infty}$$

L'Hopital $\longrightarrow 0$

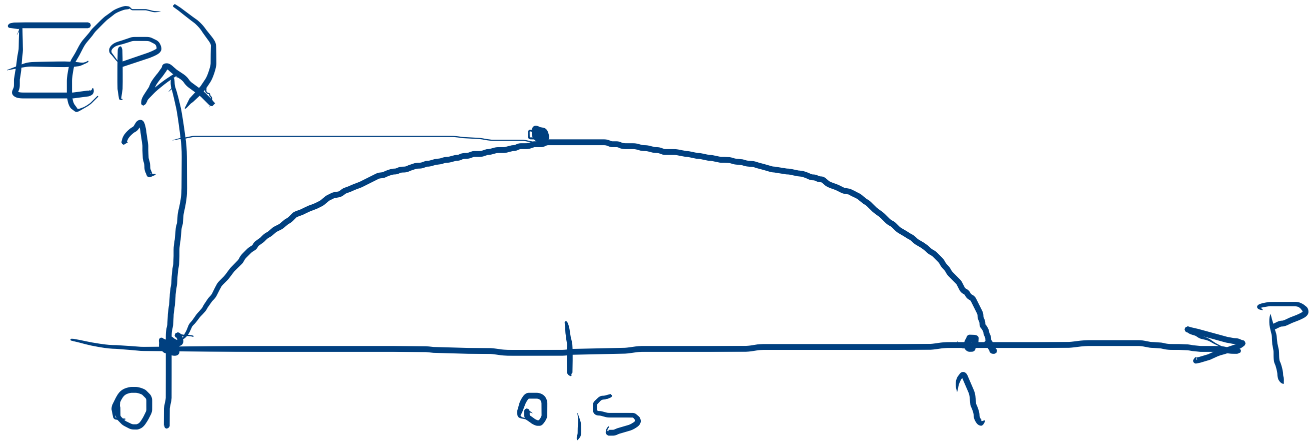
$$\lim_{P_2 \to 0} (\;\cdot\;)$$

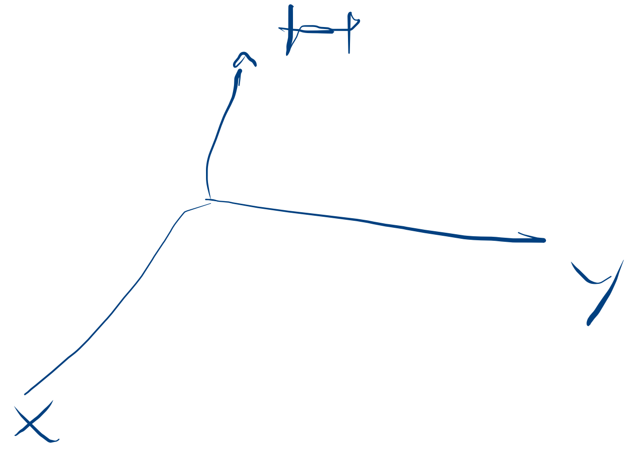
$$H(\underline{X}) = 0.$$

$$\lim_{P_2 \to 0} \frac{\log_2(1/P_2)}{1/P_2} \overset{L'H}{=} \lim_{P_2 \to 0} \frac{k_x \frac{1}{1/P_2} \cdot \cancel{(1/P_2)'}}{\cancel{(1/P_2)'}} = \lim_{P_2 \to 0} k_x P_2 = 0.$$

$P = P_1 = P_2 = 0{,}5$

$$E[I(\underline{x})] = -\left(0{,}5 \overbrace{\log_2(2^{-1})}^{-1} + 0{,}5 \overbrace{\log_2(2^{-1})}^{-1}\right) = 1.$$





Divergencia KL

En general, 'p' $\to P(X = x)$

$$\log \frac{a}{b} = \log a - \log b$$

Estimar $p \to q$.

$$D(p\|q) = \sum_x p(x) \log_2 \frac{p(x)}{q(x)}$$

Si estima perfectamente $p(x) \to q(x) = p(x)$

$$\Rightarrow D(p\|q) = 0$$

$$D(p\|q) \geq 0.$$

$$D_{p\|q} = -H_p + H_{p,q}$$

$$H_{p,q} = H_p + \underbrace{D_{p\|q}}_{\geq 0} \geq H_p$$

Información Mutua

$$D(P_{\underline{X}\underline{Y}}, P_{\underline{X}}P_{\underline{Y}}) = \sum P_{xy} \log \frac{P_{xy}}{P_x P_y} \triangleq I(x,y)$$

Si x, y son indeptes. $\Rightarrow I(x,y) = 0 \rightarrow$ no gano info. de x observando y, y vice versa.

$\hookrightarrow P_{xy} = P_x P_y$

$$\overbrace{I(x,y)}^{\geq 0} = H(x) - H(x|y).$$

$$H(x) - H(x|y) \geq 0 \Rightarrow H(x) \geq H(x|y) \leftarrow$$ condicionar disminuye H.

$\downarrow$ entropía $\quad\quad \uparrow$ determinismo