$\bar{X}$

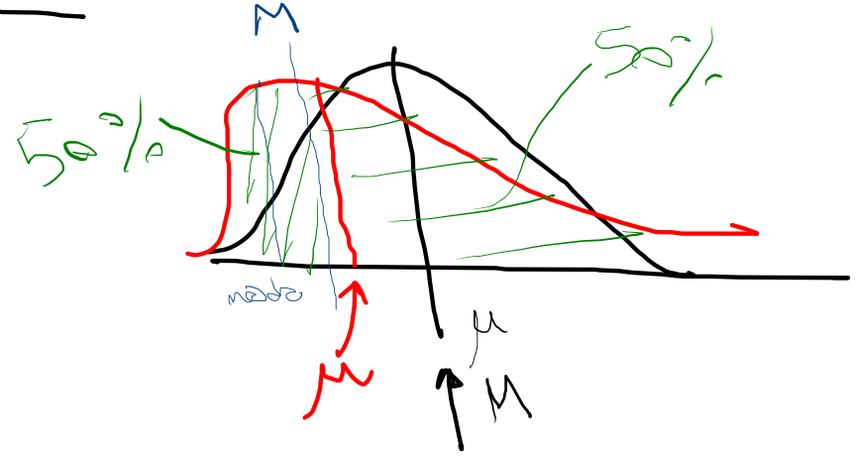
$$F(x) = P(\bar{X} \leq x) \rightarrow \text{cdf}$$

$$f(x) = \frac{\partial F}{\partial x} \rightarrow \text{pdf}$$

não-decrescente

não-negativa

Media

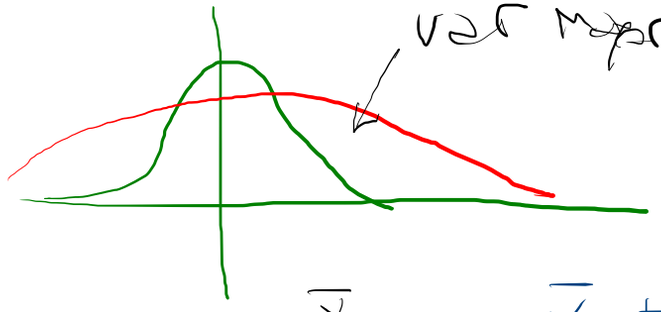


Media → centro de gravedad

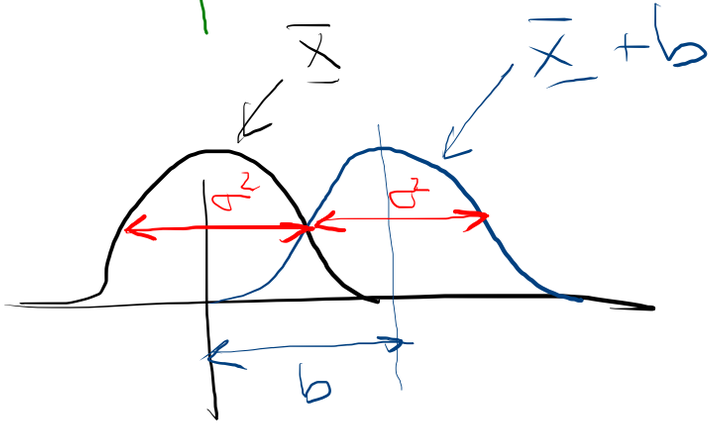
Mediana → percentil 50%

Varianza

Medida dispersión o variabilidad



σ^2 → ancha de la distrib.

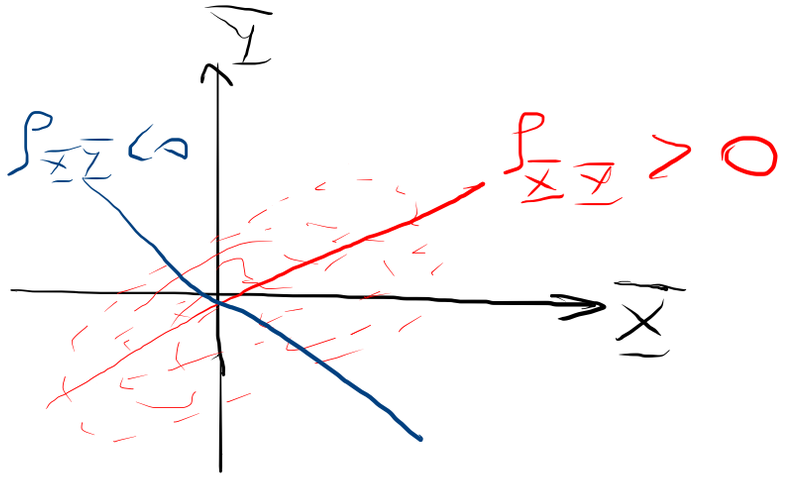


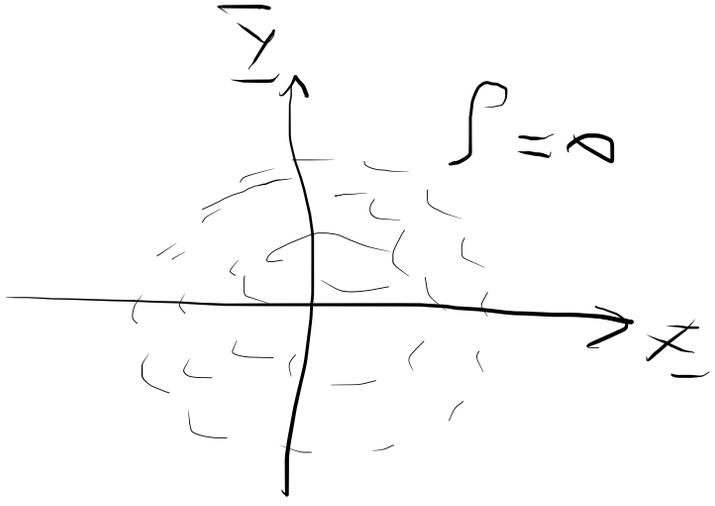
$E[h] = 1{,}7\ m$

$var[h] = 0{,}5\ m^2$

$\sigma[h] = \sqrt{0{,}5}\ m$

Índice de Correlación





Función generadora de Momentos

$$M_X(t) = E\left[e^{tX}\right]$$

$$E\left[X^n\right] = \left.\frac{d^n M_X(t)}{dt^n}\right|_{t=0} = \left.\frac{d^n}{dt^n} E\left[e^{tX}\right]\right|_{t=0} = M_X^{(n)}(0)$$

$$E\left[X^1\right] = E\left[X e^{tX}\right]\Big|_{t=0} = E[X]$$

$$f_X = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$M_X(t) = E[e^{tX}] = \int_0^\infty e^{tx} \lambda e^{-\lambda x} dx = \lambda \int_0^\infty e^{(t-\lambda)x} dx \qquad t < \lambda$$

$$= \frac{\lambda}{\lambda - t}$$

$$E[X] = \left(\frac{\lambda}{\lambda - t}\right)'\Big|_{t=0} = \frac{1}{\lambda}$$

$$E[X^2] = \frac{2}{\lambda^2}$$

$$\text{var}[X] = E[X^2] - \left(E[X]\right)^2 = \frac{1}{\lambda^2}$$

Estadística de Orden K

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \xrightarrow[\text{ascendente}]{\text{ordenar}} \begin{bmatrix} x_{(1)} \\ \vdots \\ x_{(N)} \end{bmatrix} = X_{ord}$$

x_i no es necesariamente $= x_{(i)}$

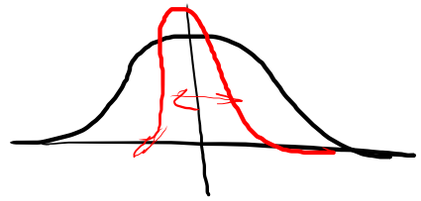
$x_{ord}(K) \rightarrow$ estadística de orden k.

$x_{(1)} = \text{mín } x.$

$x_{(N)} = \text{máx } x.$

μ, σ^2 iid

$$\hat{X} = \frac{X_1 + \cdots + X_N}{N} \rightarrow E[\hat{X}] = \mu \qquad \sigma^2[\hat{X}] = \frac{N\sigma^2}{N^2} = \frac{\sigma^2}{N}$$

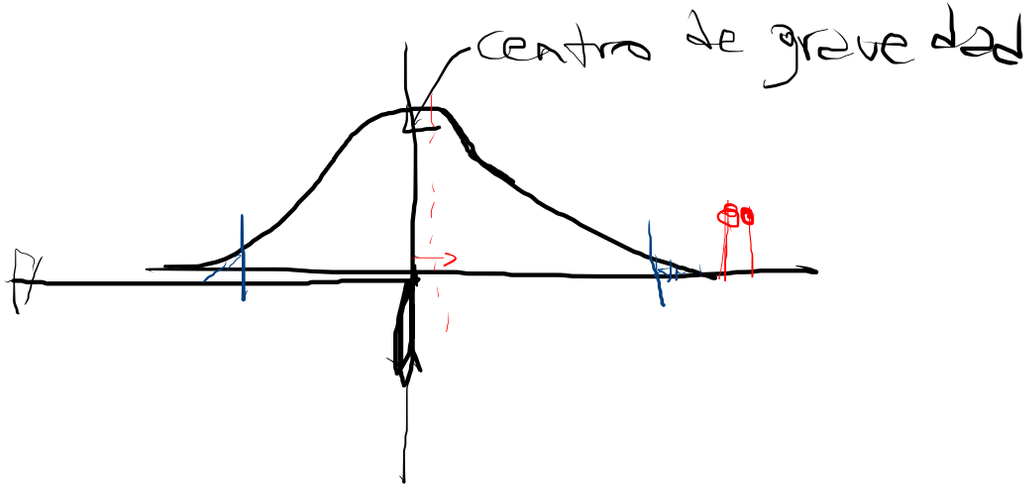


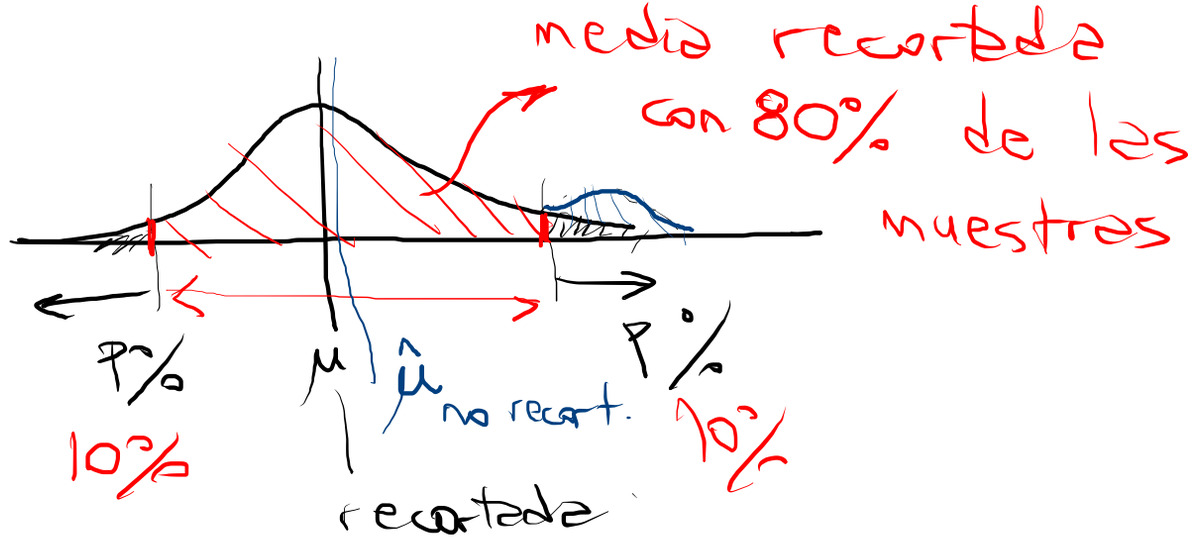
Media Muestral y Recortada

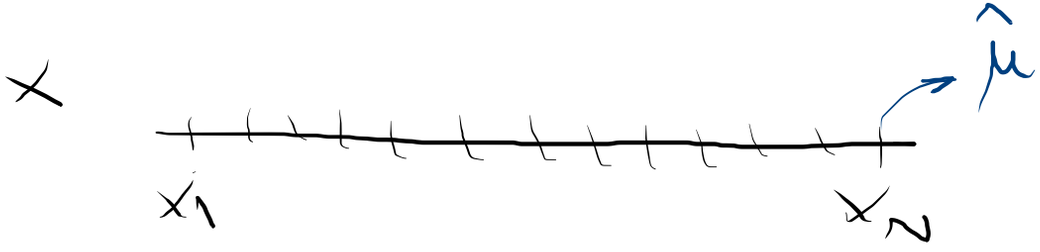
$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

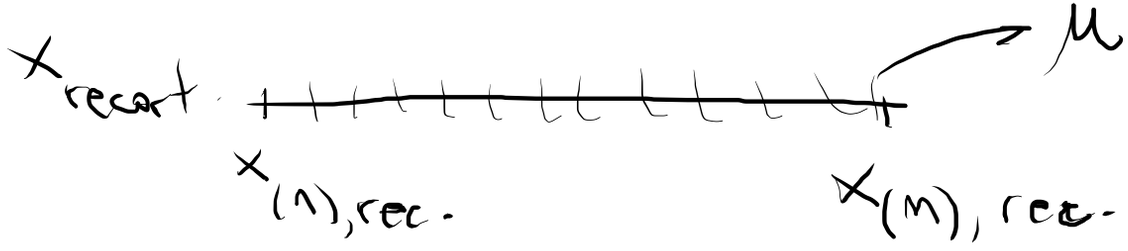
$$s^2 = \underbrace{\frac{1}{N-1}}_{\text{estimado insesgado}} \sum_{i=1}^{N} (x_i - \hat{\mu})^2$$

Usando datos $\longrightarrow$ perdiendo grados de libertad.

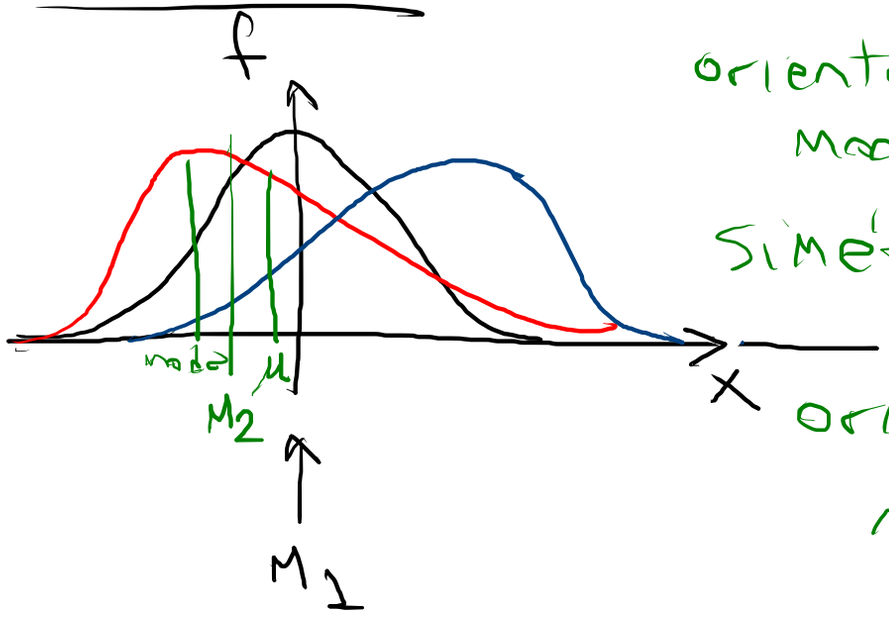

centro de gravedad


media recortada
con 80% de los
nuestres
P%
μ
μ̂ no recort.
P%
10%
10%
recortada


x
μ̂
x1
xN


xrecort
μ
x(1),rec.
x(n),rec.

Mediana



orientación positiva

moda → mediana → media

simétrica → coinciden, moda, M, μ

orientación negativa

media, mediana, moda

Cálculo de mediana

$$\begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \longrightarrow \text{ordena ascendente} \quad \begin{bmatrix} x_{(1)} \\ \vdots \\ x_{(N)} \end{bmatrix} \longrightarrow x_{(N/2)}$$

$$\begin{bmatrix} 2 \\ 5 \\ 7 \\ 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 \\ 2 \\ 5 \\ 7 \end{bmatrix} \longrightarrow \tilde{M}$$

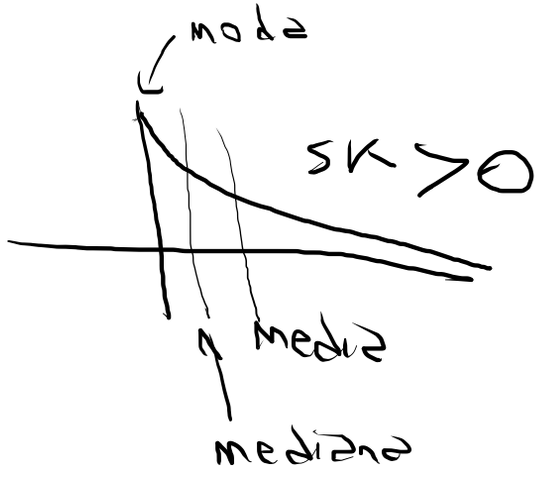
Skewness (sk) $\equiv$ → Momento de tercer orden

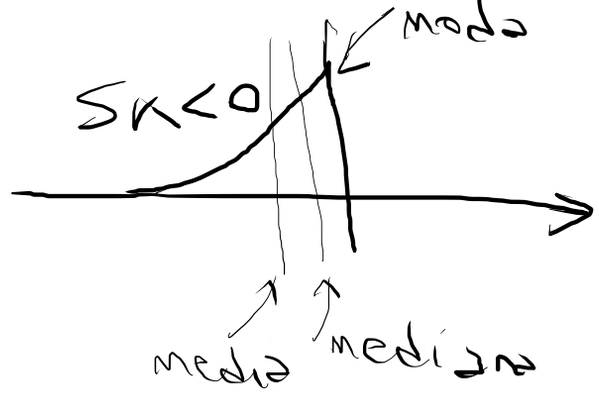
$$E[\bar{x}] = \int \overbrace{x^1 f(x)}^{\geq 0, < 0} \, dx$$

Simétrica → sk = 0

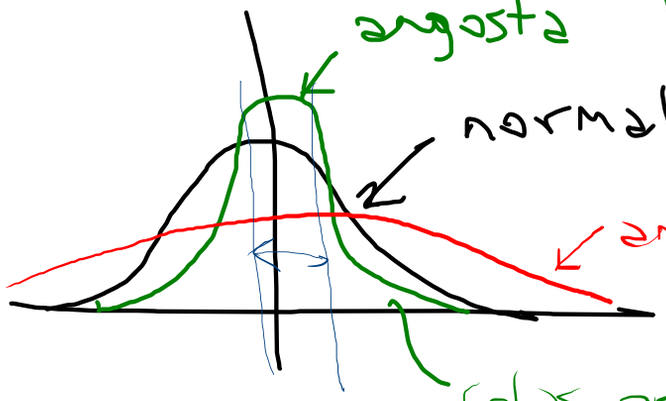
$$\sigma_{\bar{x}} = \int \overbrace{(x - E[\bar{x}])^2 f(x)}^{\geq 0} \, dx$$

≥ 0





Curtosis (Kurt.)



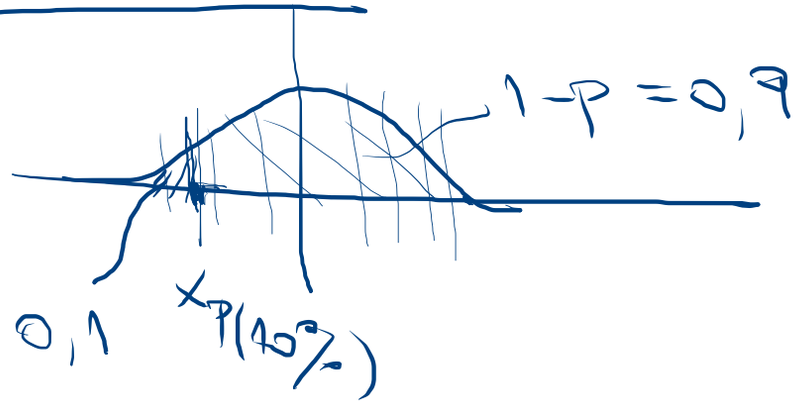
angosta Kurt > 3

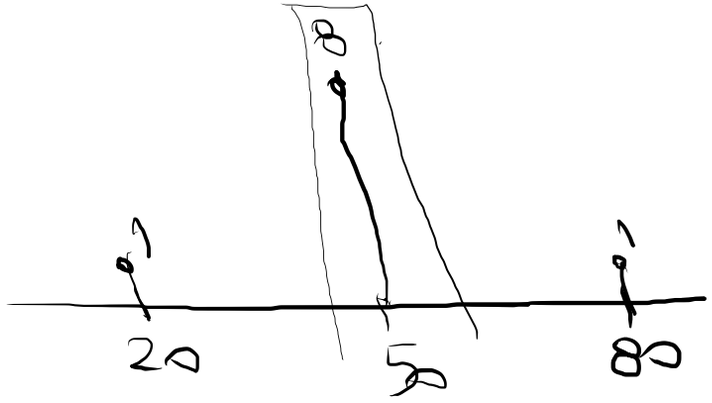
normal (Kurt = 3)

ancha, colas menos pronunciadas. (Kurt < 3)

colas pronunciadas

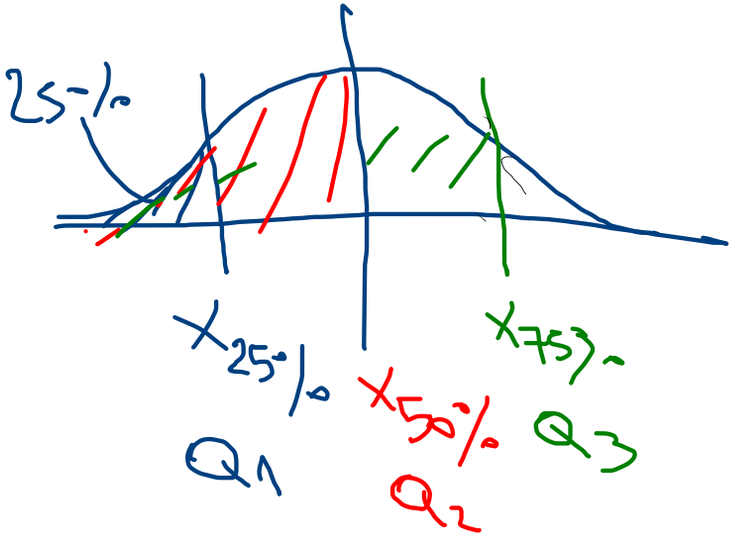
Percentil p





IQR = 0

pdf Muy concentrada

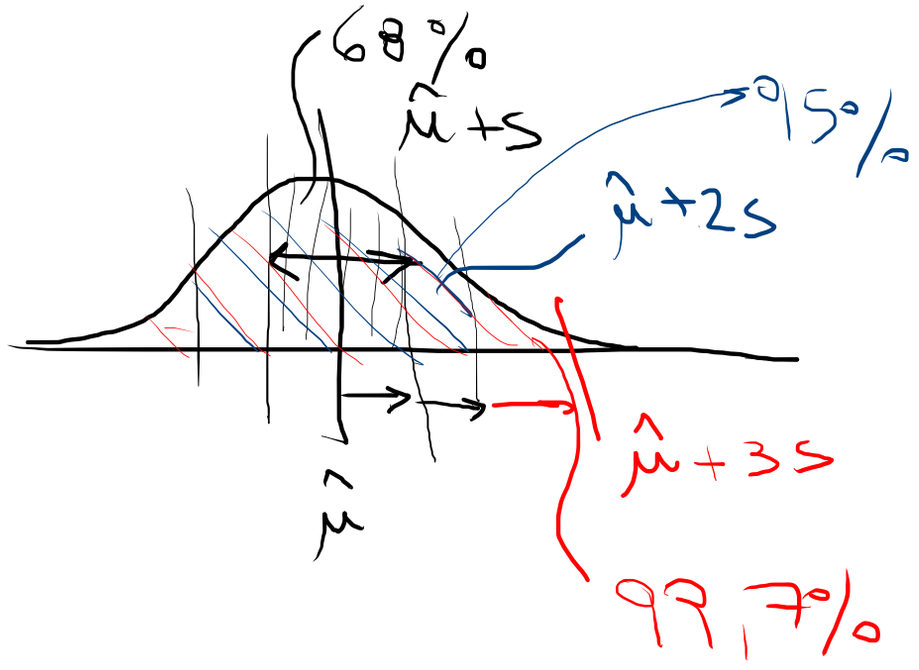


$$IQR = Q_3 - Q_1$$

Mínimos valores para describir conjunto de datos:

- Valor mínima, $X_{(1)}$
- Q_1
- Q_2
- Q_3
- Valor máximo, $X_{(N)}$

Regla Empírica



Recortemos la distrib. al 95% :

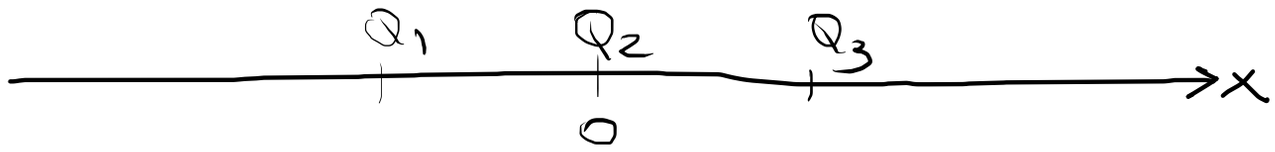
$rg = \text{Max} - \text{Min}$] recortado 95%

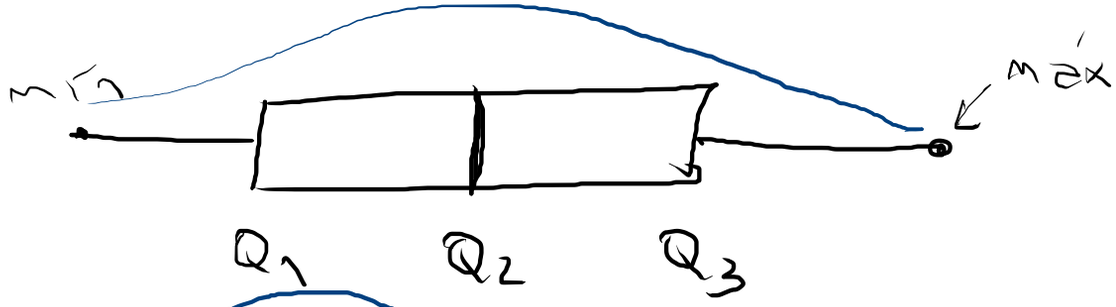
$\mu+2s - (\mu-2s) = 4s$

$\hat{s} = \frac{rg}{4}$.

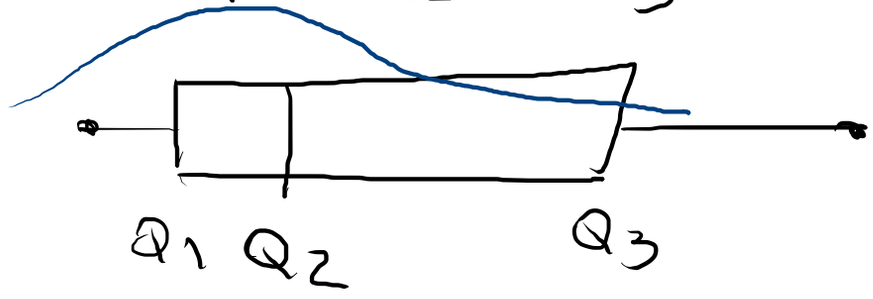
Diagrama de Box & Whiskers (Simple)

Q_1, Q_2, Q_3





ejemplo 1



ejemplo 2

Box & Whiskers Complete