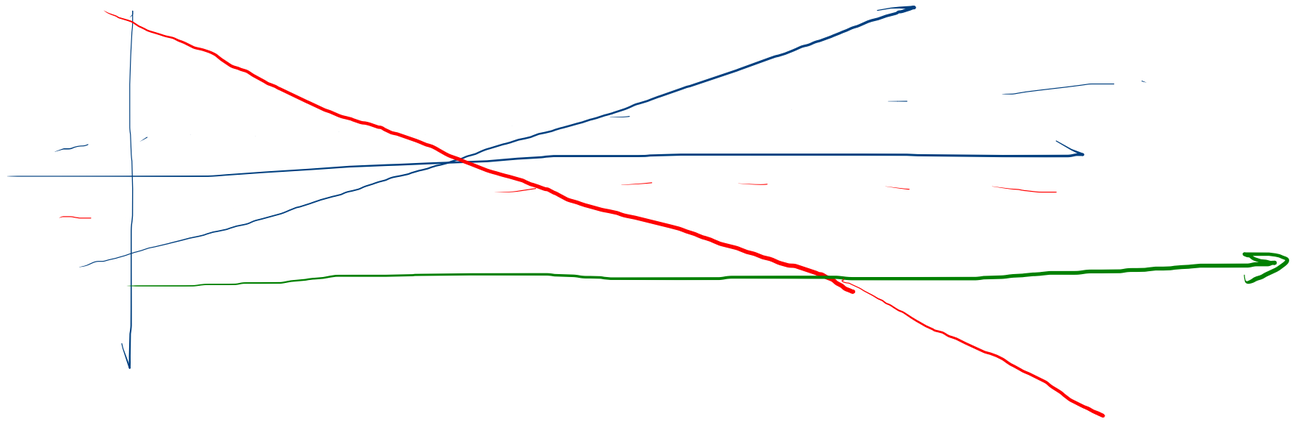


Entropía

Fuente de información

Información

$$I_{\underline{X}=x} = \log_2(1/p)$$

$$p(\underline{X}=x)$$

$$p_{cara} = 1 \Rightarrow I_{\underline{X}=cara} = \log_2(1/1) = 0$$

$$p_{cara} = 0 \Rightarrow I_{\underline{X}=cara} = \log_2(1/0) = \infty$$

} Sorpresa nula

$$\log\left(\frac{1}{p}\right) = \log_2 p^{-1} = -\log_2 p$$

$$E\left[\log\left(\frac{1}{p}\right)\right] = \sum_{x \in S} p_x \log_2\left(\frac{1}{p_x}\right) = -\sum p \log_2 p$$

≥ 0

$1 \geq p \geq 0$.

$$\frac{1}{p} \in [1, +\infty)$$

$X = \{x_1, x_2\}$

$P_1 = 1 \quad , P_2 = 0$

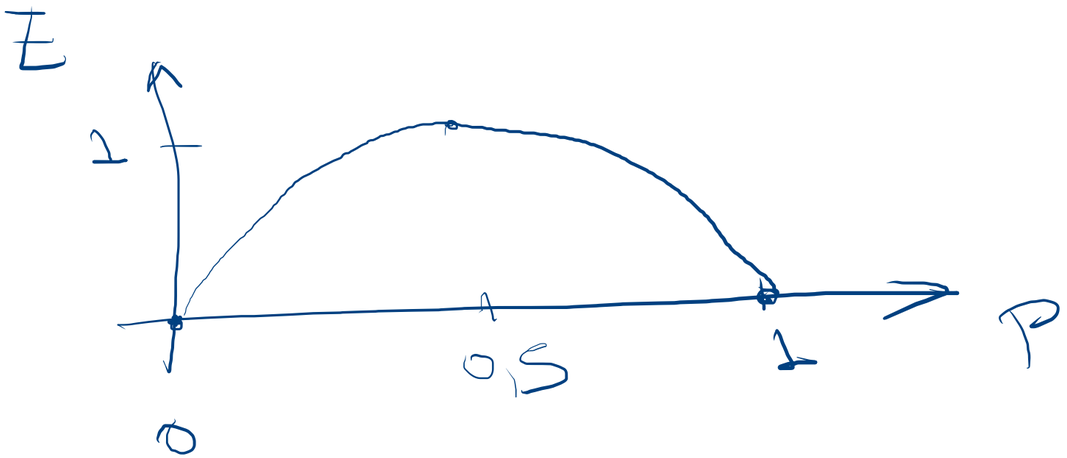
$$E[I(X)] = \underbrace{P_1}_{0} \underbrace{\log_2 \left(\underbrace{1/P_1}_{1} \right)}_{0} + \underbrace{P_2}_{0} \underbrace{\log_2 \left(\underbrace{1/P_2}_{\to \infty} \right)}_{\text{0}} = 0$$

$$0 \cdot \infty \overset{\text{L'Hopital}}{=} \frac{\infty}{\infty} = \frac{\log_2(1/P_2)}{1/P_2} \overset{\text{L'H}}{=} 0$$

$\lim_{P_2 \to 0}$

$p_1 = p_2 = 0{,}5 \qquad p_1 = p \qquad p_2 = 1-p$

$$E[I(x)] = -\left(0{,}5 \underbrace{\log_2\left(2^{-1}\right)}_{-1} + 0{,}5 \log_2\left(2^{-1}\right)\right) = 1$$



Divergencia KL

Conocemos p $\rightarrow$ $P(X = x)$

Estimar $p \rightarrow q$

$$D(p||q) = \sum p \log \frac{p}{q}$$

(la suma es la integral)

$p = q \Rightarrow D(p||q) = 0$

$D(p||q) \geq 0$

$$D = -H_p + H_{p,\mathfrak{F}} \geq 0$$

$$D > 0 \Rightarrow H_{p,\mathfrak{F}} > H_p$$

Información Mutua

$$D(P_{XY}, P_X P_Y) = \sum P_{XY} \log \frac{P_{XY}}{P_X P_Y} = I(X,Y)$$

Si X, Y son Indep. $\Rightarrow$ $I(X,Y) = 0$.

$H(X|Y) \leq H(X)$ $\rightarrow$ condicionar reduce la información

si X, Y son indep. $\Rightarrow H(X|Y) = H(X) \leq H(X)$

no son indep. $\Rightarrow$


entropia
determinismo