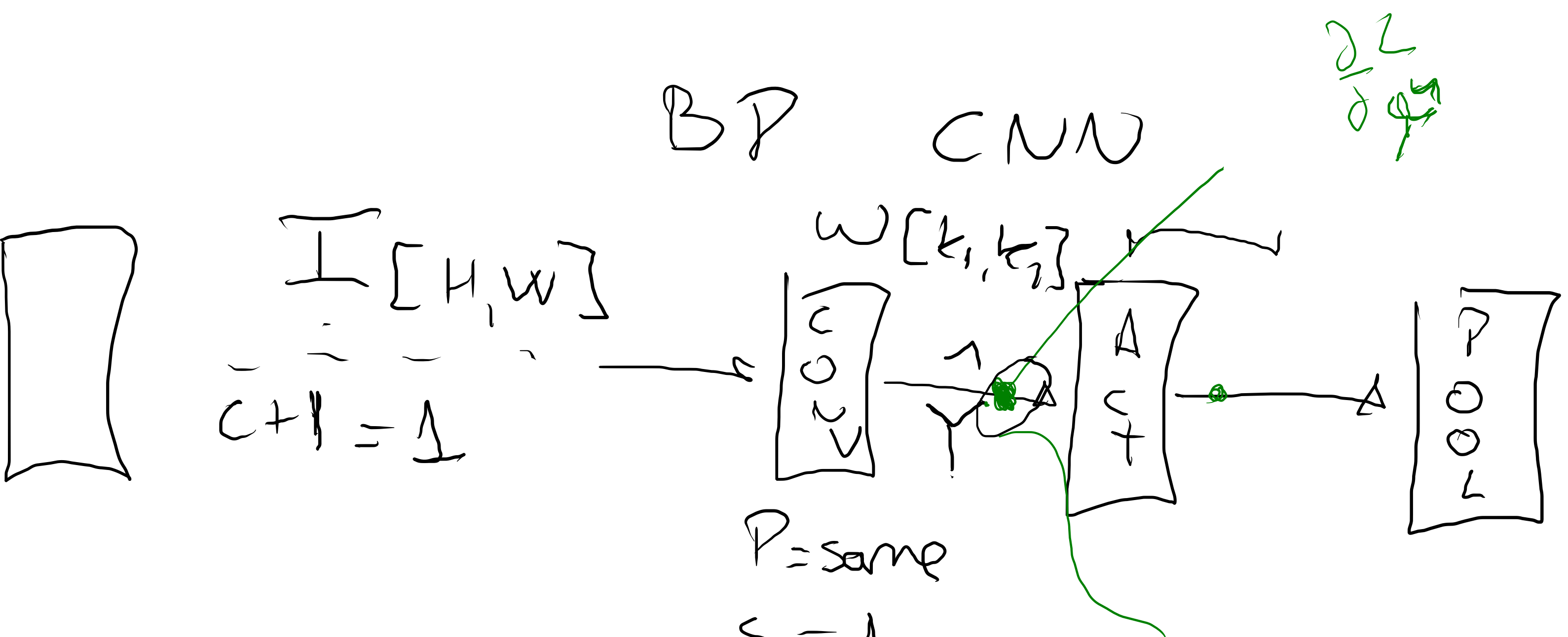
$$\hat{Y}(\hat{A}, X)$$

BP CNN



P = same

s = 1

Chout = 1

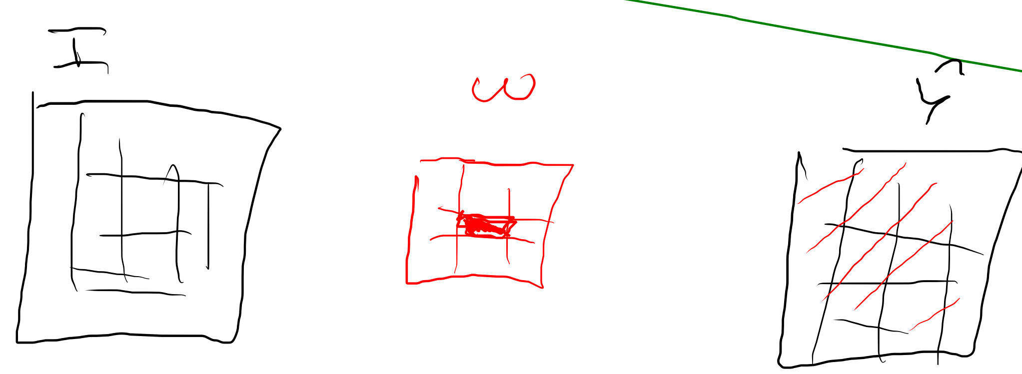
$$\hat{Y}_{(i,j)} = \sum_{m=0}^{k_1-1} \sum_{n=0}^{k_2-1} I(i+m, j+n)\, w(m,n) + b$$

Loss ; $L(I, w)$

$\frac{\partial L}{\partial I}$ → para pasar el gradiente

$\frac{\partial L}{\partial w}$ → para optimizar los pesos

1º hallar $\frac{\partial L}{\partial w}$



$L(Y - \hat{Y})$

$\frac{\partial L}{\partial w(m',n')}$ → Toda $\hat{Y}$ varía con $w(m',n')$

$$\frac{\partial L}{\partial w(m',n')} = \underbrace{\sum_{i=0}^{H-k_1} \sum_{j=0}^{W-k_2}}\ \frac{\partial L}{\partial \hat{Y}_{(i,j)}} \cdot \frac{\partial \hat{Y}_{(i,j)}}{\partial w(m',n')}$$

es conocido... de la capa siguiente

$$\frac{\partial \hat{Y}(i,j)}{\partial w(m',n')} = \frac{\partial}{\partial w(m',n')} \underbrace{\sum_{m=0}^{k_1-1} \sum_{n=0}^{k_2-1} I(i+m, j+n)\, w(m,n)} + b$$

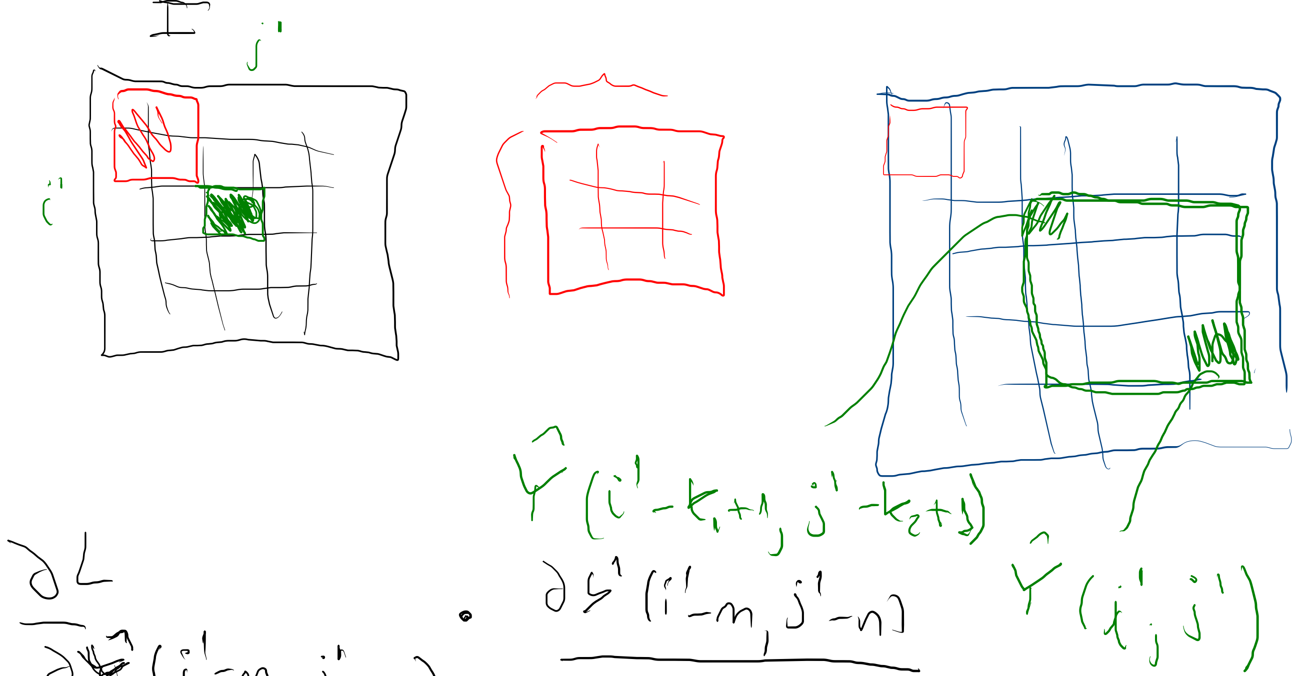
$$\left(\cdots + \underbrace{I(i+m', j+n')\, w(m',n')} + \cdots + b \right)$$

$$\frac{\partial \hat{Y}(i,j)}{\partial w(m',n')} = I(i+m', j+n')$$

$$\frac{\partial L}{\partial w(m',n')} = \sum_{i=0}^{H-k_1} \sum_{j=0}^{W-k_2} \frac{\partial L}{\partial \hat{Y}_{(i,j)}} \cdot I(i+m', j+n')$$

$$\boxed{\frac{\partial L}{\partial w(m',n')} = \frac{\partial L}{\partial Y(m',n')} \otimes I(m',n') = \left\{ \frac{\partial L}{\partial Y_{(m',n')}} \right\}_{180^\circ} * I_{(m',n')}}$$

$$\frac{\partial L}{\partial I(i',j')}$$



$$\frac{\partial L}{\partial I(i',j')} = \sum_{m=0}^{k_1-1} \sum_{n=0}^{k_2-1} \underbrace{\frac{\partial L}{\partial \hat{Y}(i'-m,j'-n)}}_{\text{conocido}} \cdot \frac{\partial \hat{Y}(i'-m,j'-n)}{\partial I(i',j')}$$

$$\frac{\partial \hat{Y}(i'-m,j'-n)}{\partial I(i',j')} = \frac{\partial}{\partial I(i',j')} \left[\sum_{p=0}^{k_1-1} \sum_{q=0}^{k_2-1} I(i'-m+p,\, j'-n+q) \cdot w(p,q) + b \right]$$

$p=m$ y $q=n$

entonces

$$I(i'\cancel{-m}\cancel{+m},\, j'\cancel{-n}\cancel{+n}) = I(i',j')$$

$$\frac{\partial \hat{Y}}{\partial I} = \boxed{\frac{\partial}{\partial I(i',j')}[\ ] = w(m,n)}$$

$$\frac{\partial L}{\partial I(i',j')} = \sum_{m=0}^{k_1-1} \sum_{n=0}^{k_2-1} \frac{\partial L}{\partial \hat{Y}(i'-m,j'-n)} \cdot w(m,n)$$

$$\boxed{\frac{\partial L}{\partial I(i',j')} = w(i',j') * \frac{\partial L}{\partial \hat{Y}}(i',j')}$$