

Basis pursuit II

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Summary

1 Robustness

2 Referências

How do we handle noise in the measurements in the basis pursuit? I.e. what additional assumptions do we need to obtain similar results, if we consider the problem

$$\min \{ \|z\|_1 \mid z \in \mathbb{C}^N, \ \|Az - y\| \leq \eta \} ? \quad (P_{1,\eta})$$

It depends on the norm, in which we measure the noise, i.e. the norm of $\|Az - y\|$.

Robust null space property

At first, we consider the situation where we measure the noise in the ℓ^1 -norm, i.e. $\|Az - y\| = \|Az - y\|_1$.

Definition 1.1: Robust null space property

A matrix $A \in \mathbb{C}^{m \times N}$ is said to satisfy the **robust null space property** with respect to $\|\cdot\|$ with the constants $\rho \in (0, 1)$ and $\tau > 0$ relative to a set $S \subseteq [N]$ iff

$$\forall v \in \mathbb{C}^N : \|v_S\|_1 \leq \rho \|v_{\bar{S}}\|_1 + \tau \|Av\|. \quad (\text{RNSP}(\|\cdot\|, \rho, \tau, S))$$

A satisfies the robust null space property of **order** s with respect to $\|\cdot\|$ with the constants $\rho \in (0, 1)$ and $\tau > 0$ RNSP($\|\cdot\|, \rho, \tau, s$) iff A satisfies RNSP($\|\cdot\|, \rho, \tau, S$) for all sets $S \subseteq [N]$ with $|S| \leq s$.

Motivation dafür: TODO

Theorem 1.2: Characterization of RNSP

A matrix $A \in \mathbb{C}^{m \times N}$ satisfies $\text{RNSP}(\|\cdot\|, \rho, \tau, S)$ if and only if

$$\forall x, z \in \mathbb{C}^N : \quad \|z - x\|_1 \leq \frac{1 + \rho}{1 - \rho} (\|z\|_1 - \|x\|_1 + 2\|x_S\|) + \frac{2\tau}{1 - \rho} \|A(x - z)\|.$$

Corollary 1.3

Assume that $A \in \mathbb{C}^{m \times N}$ satisfies $\text{RNSP}(\|\cdot\|, \rho, \tau, s)$ with $0 < \rho < 1$ and $\tau > 0$ and let $x \in \mathbb{C}^N$. Then, if

$$\mathcal{L}_x := \left\{ x^\# \in \mathbb{C}^N \mid \|x^\#\|_1 = \min\{\|z_1\| \mid \|Ax - Az\| \leq \eta\} \right\}$$

is the solution set of the problem $(P_{1,\eta})$ with $y = Ax$, then

$$\sup_{x^\# \in \mathcal{L}_x} \|x - x^\#\|_1 \leq \frac{2(1+\rho)}{1-\rho} \sigma_s(x)_1 + \frac{4\tau}{1-\rho} \eta,$$

i.e. the solution set \mathcal{L}_x is contained in a ball of radius $\frac{2(1+\rho)}{1-\rho} \sigma_s(x)_1 + \frac{4\tau}{1-\rho} \eta$ around x in the ℓ^1 -norm.

A Tabela mostra um modelo de tabela.

Table 1: Exemplo de tabela

Pessoa	Idade	Peso	Altura
Marcos	26	68	178
Ivone	22	57	162
Sueli	40	65	153

Fonte: Produção do próprio autor.

A Figura mostra a logomarca promocional da Universidade Federal do Espírito Santo (UFES).

Figure 1: Logomarca Promocional UFES

Fonte: Produção do próprio autor.

Texto referente à conclusão.

- Estudar ...
- Explorar ...
- Analisar ...

Obrigado!
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