CMPT-439
Numerical Computation

Fall 2020

Organizational Details

Class Meeting:
11:00am-12:15pm; Tuesday, Friday; RLC-105

Instructor: Dr. Miaomiao Zhang

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e-mail: mzhang01@manhattan.edu

Office hours:
Tuesday, Friday 12:20pm-1:20pm (online) and by appointment

*if you need to talk to me, e-mail to schedule a conversation

Textbooks (optional)

- **Gerald, C.F. and Wheatley, P.O., Applied Numerical Analysis, 7th Edition, Pearson, 2004, ISBN 0-321-13304-8**
- **Chapra S., Applied Numerical Methods with MATLAB for Engineers and Scientists, 4th Edition, McGraw Hill, 2018, ISBN 9780073397962**

Grading

Grading Method

Homework	50%
Midterm Exam	25%
Final Project	25%

Necessary conditions for “A”:

1) All homework projects turned in
2) Midterm test grade 90+
3) Course project grade 90+

Necessary conditions for A-:

1) All homework projects turned in
2) Midterm test grade 85+
3) Course project grade 85+

Grading Scale:

93+	→ A
90+	→ A-
85+	→ B+
80+	→ B
75+	→ B-
70+	→ C+
65+	→ C
60+	→ C-
50+	→ D
50-	→ F

Methods of Evaluation

- Homework assignments will be given throughout the semester (11-12 in total)
- Each project must be defended by presenting a written report with the results and demonstrating a working Matlab program
- Without a working program 40% points will be deducted
- Each assignment will be due. <u>10% of the points</u> will be deducted for every day an assignment is past due.

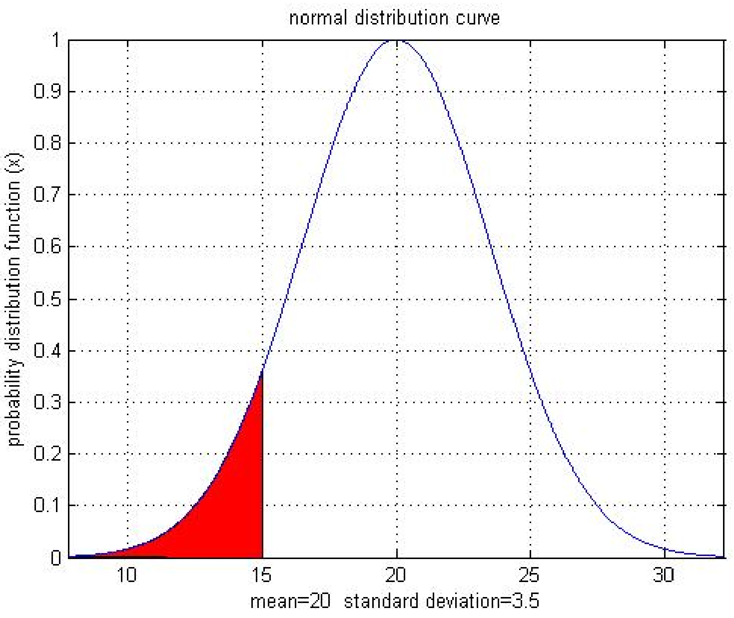
Final Project

- A **final project** is a major software design project, on which students will work in teams
- Each team will design a software system for solving problems covered in one of the sections of the course, integrating various methods in a software system with a user friendly graphical interface
- A course project shall be completed by its oral presentation and a detailed written report submitted

Why use Numerical Methods?

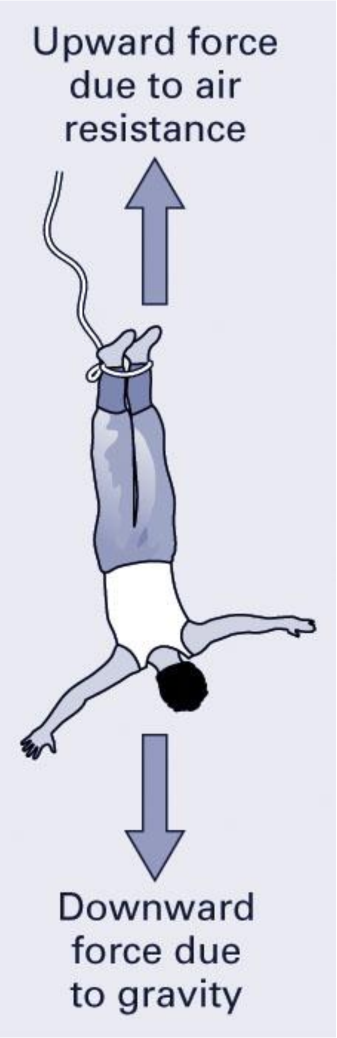
- To solve those problems that cannot be solved exactly

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}\,du$$



You've got a Problem

- Suppose that a bungee-jumping company hires you.
- You're given the task of predicting the *velocity* of a jumper as a function of time during the free-fall part of the jump.
- This information will be used as part of a larger analysis to determine the *length* and required *strength* of the bungee cord for jumpers of different *mass*.



How do we solve an engineering and scientific problem?

- Problem Description
- Mathematical Model
- Utilization of Mathematical Model
- Using the Solution

A Mathematical Model

- A mathematical model is usually represented as a functional relationship of the form

$$\text{Dependent Variable} = f\left(\text{independent variables},\ \text{parameters},\ \text{forcing functions}\right)$$

- *Dependent variable*: Characteristic that usually reflects the state of the system
- *Independent variables*: Dimensions such as time and space along which the systems behavior is being determined
- *Parameters*: reflect the system's properties or composition
- *Forcing functions*: external influences acting upon the system

Newton's 2nd law of Motion

- States that "*the time rate change of momentum of a body is equal to the resulting force acting on it*."
- The model is formulated as

$$\boldsymbol{F = m\,a}$$

$\boldsymbol{F}$=net force acting on the body (N)

$\boldsymbol{m}$=mass of the object (kg)

$\boldsymbol{a}$=its acceleration (m/s^2)

Characteristics of a Mathematical Model

- Formulation of Newton's 2nd law has several characteristics that are typical for mathematical models of the physical world:
  - It describes a natural process or system in mathematical terms
  - It represents an idealization and simplification of reality
  - Finally, it yields reproducible results, consequently, can be used for predictive purposes

Modeling of Physical Phenomena

- Some mathematical models of physical phenomena may be much more complex

- Complex models may not be solved exactly or require more sophisticated mathematical techniques than simple algebra for their solution

  - Example, modeling of a bungee-jumper

$$F = ma; a = \frac{dv}{dt}$$

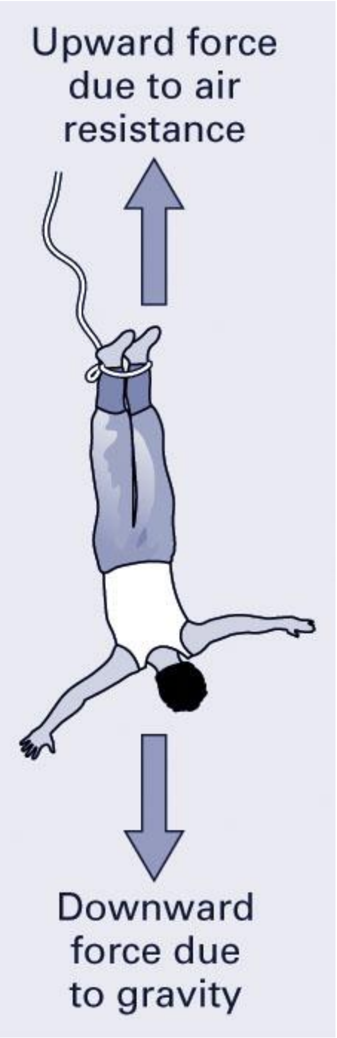
$$a = \frac{dv}{dt} = \frac{F}{m}$$

$$F = F_D + F_U$$

$$F_D = mg$$

$$F_U = -c_d v^2$$

$$\frac{dv}{dt} = \frac{mg - c_d v^2}{m}$$



- Dependent variable
  - velocity *v*
- Independent variables
  - time *t*
- Parameters
  - mass *m*
  - drag coefficient c_d
- Forcing function
  - gravitational acceleration g

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

- This is a differential equation and it is written in terms of the differential rate of change dv/dt of the variable that we are interested in predicting.
- If the bungee jumper is initially at rest (v=0 at t=0), then using Calculus we obtain

Independent variable

$$v(t) = \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}}\, t\right)$$

Dependent variable

Forcing function

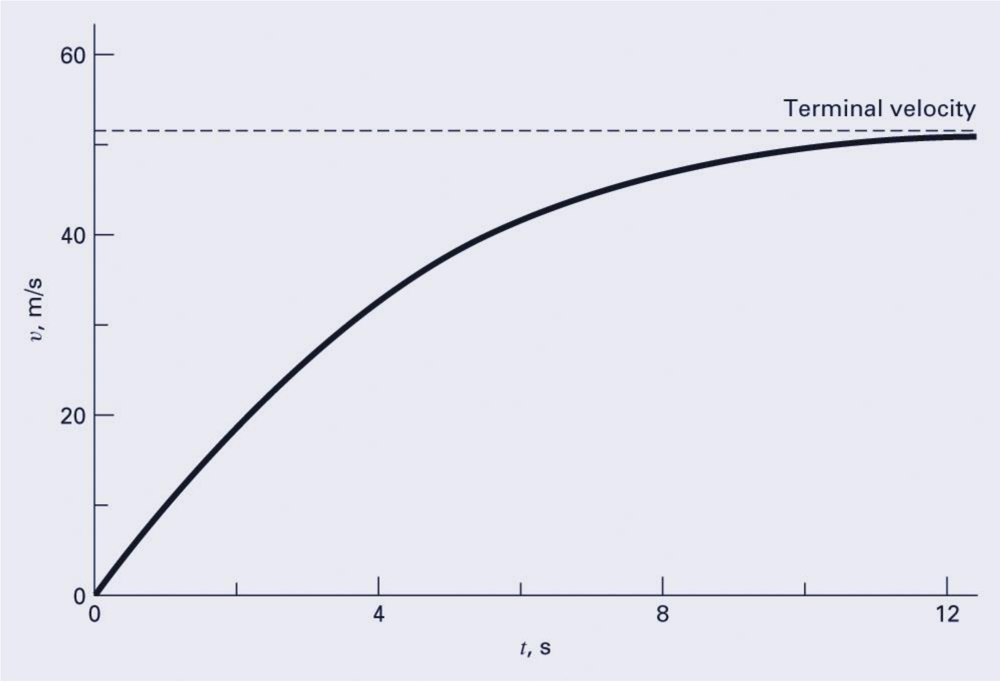
Parameters

* tanh is the hyperbolic tangent that can be either computed directly or via the exponential func

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Model Results

Using a computer (or a calculator), the model can be used to generate a graphical representation of the system. For example, the graph below represents the velocity of a 68.1 kilogram jumper, assuming a drag coefficient of 0.25 kilograms per mile



Numerical Modeling

Some system models will be given as implicit functions or as differential equations - these can be solved either using analytical methods or numerical methods.

Example - the bungee jumper velocity equation from before is the analytical solution to the differential equation

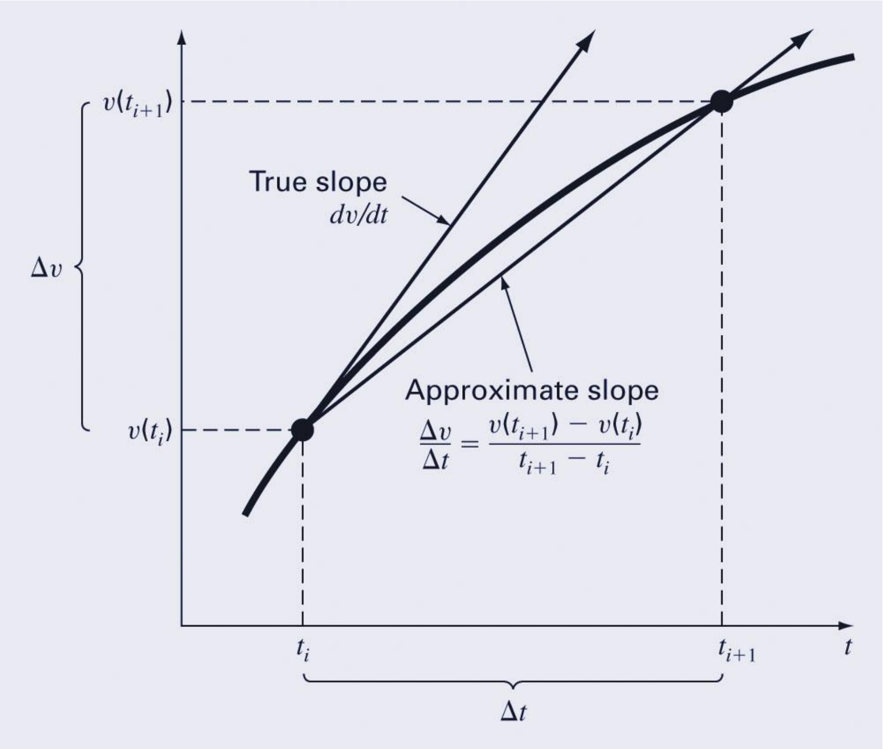
$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

where the change in velocity is determined by the gravitational forces acting on the jumper versus the drag force.

Numerical Methods

To solve the problem using a numerical method, note that the time rate of change of velocity can be approximated as:

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(ti)}{t_{i+1} - ti}$$



Euler's Method

Substituting the finite difference into the differential equation gives

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c_d}{m} v^2$$

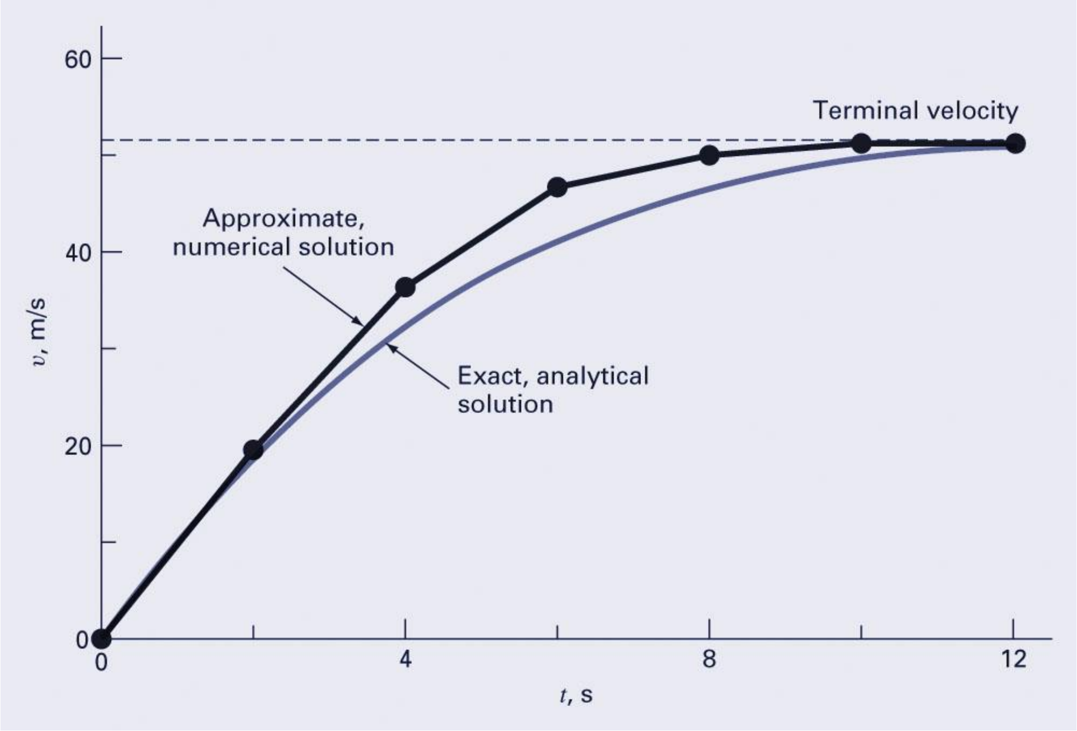
Solve for

$$v(t_{i+1}) = v(t_i) + \left(g - \frac{c_d}{m} v(t_i)^2\right)(t_{i+1} - t_i)$$

$$\text{new} = \text{old} + \text{slope} \times \text{step}$$

Numerical Results

Applying Euler's method in 2 s intervals yields:



How do we improve the solution?

- Smaller steps

Numerical Analysis: What it is?

- So, as we already mentioned:
- For many engineering problems, we can't obtain analytical solutions

- Numerical methods make it possible to approximate a solution, that is obtain a solution, which is close to the exact analytical solution

Areas of Numerical Analysis

- Estimation of errors in numerical procedures
- Solving nonlinear equations
- Solving sets of equations
- Interpolation and curve fitting
- Approximation of functions
- Numerical differentiation and integration
- Numerical solution of differential equations (ordinary and partial)
- Methods of optimization (finding min/max of a function)
- Fast numerical algorithms in signal processing (Fast Fourier Transform, Fast Walsh Transform, Fast Cosine Transform, etc.)

Five Categories of Numerical Methods