

## Midterm Notes

Decimal to Anything  $\Rightarrow$  Divide by the desired base. If decimal, multiply.

Anything to Decimal  $\Rightarrow a_i x b^i + \dots + a_2 x b^2 + a_1 x b^1 + a_0 x b^0 + a_{-1} x b^{-1} + \dots$

Hexa to Binary and Vice-Versa  $\Rightarrow$  4 bits

Octal to Binary and Vice-Versa  $\Rightarrow$  3 bits

Octal to Hexa OR Hexa to Octal  $\Rightarrow$  First convert to binary. Then, convert to what you are looking for.

Decimal to BCD  $\Rightarrow$  Consider each digit separately into 4 bits.

BCD to Decimal  $\Rightarrow$  Break up each section into 4 bits.

### Boolean Algebra

	(a)	(b)
Postulate 2	$x+0=x$	$x \cdot 1=x$
Postulate 5	$x+x'=1$	$xx'=0$
Theorem 1	$x+x=x$	$xx=x$
Theorem 2	$x+1=1$	$x \cdot 0=0$
Theorem 3 (involution)	$(x')'=x$	
Postulate 3 (commutative)	$x+y=y+x$	$xy=yx$
Theorem 4 (associative)	$x+(y+z)=(x+y)+z$	$x(yz)=(xy)z$
Postulate 4 (distributive)	$x(y+z)=xy+xz$	$x+yz=(x+y)(x+z)$
Theorem 5 (de Morgan's Law)	$(x+y)'=x'y'$	$(xy)'=x'+y'$
Theorem 6 (absorption)	$x+xy=x$	$x(x+y)=x$

minterm  $\Rightarrow$  1 (multiplication)      output =  $\sum m(\text{row \#s})$

maxterm  $\Rightarrow$  0 (addition)      output =  $\prod M(\text{row \#s})$

Note: The duration of this test is 1:15min.

1. (10 points total)

a) Write the truth table for an AND logic gate (3 pts)

X	Y	F
0	0	0
0	1	0
1	0	0
1	1	1

b) Write the truth table for an XOR logic gate (3pts)

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	0

c) Apply DeMorgan's theorem to the following (4pts) (Hint: Function Complement)

$$\begin{aligned} \text{i. } (xy' + z')' &= \overline{(x\bar{y} + \bar{z})} \\ &= \boxed{(\bar{x} + y)(z)} \end{aligned}$$

$$\begin{aligned} \text{ii. } (A'BC')' &= \overline{(\bar{A}B\bar{C})} \\ &= \boxed{A + \bar{B} + C} \end{aligned}$$



2. (10 points total)

a) Convert  $(30.625)_{10}$  to binary (base 2) (go out to maximum of 3 bits for the fractional part) (5pts)

	Result	Remainder
$2 \overline{) 30}$	15	0
$2 \overline{) 15}$	7.5	1
$2 \overline{) 7}$	3.5	1
$2 \overline{) 3}$	1.5	1
$2 \overline{) 1}$	0.5	1

$$(11110.101)_2$$

$$\begin{array}{r} 2.000 \\ \times 0.625 \\ \hline 10000 \\ 40000 \\ + 1200000 \\ \hline 1.250000 \end{array}$$

 $w_1 = 1$ 

$$\begin{array}{r} 2.00 \\ \times 0.25 \\ \hline 1000 \\ + 4000 \\ \hline 0.5000 \end{array}$$

 $w_2 = 0$ 

$$\begin{array}{r} 2.0 \\ \times 0.5 \\ \hline 1.0 \end{array}$$

 $w_3 = 1$ b) Now convert  $(30)_{10}$  to hexadecimal (base 16) (5pts)

	Result	Remainder
$16 \overline{) 30.000}$	1.875	16.000
$-16 \downarrow$		$\times 0.875$
$140 \downarrow$		1,80000
$-128 \downarrow$		1120000
$120 \downarrow$		$+ 12800000$
$-112 \downarrow$		14000000
$80 \downarrow$		
$-80 \downarrow$		
$0$		

$$(1E)_{16}$$

	Result	Remainder
$16 \overline{) 1.0000}$	0.0625	16.0000
$-16 \downarrow$		$\times 0.0625$
$40 \downarrow$		1,800000
$-32 \downarrow$		1,3200000
$80 \downarrow$		$+ 96000000$
$-80 \downarrow$		1.00000000
$0$		

3. (15 points total)

a) Convert 01100 to 2's complement (do not add signed bit) (5pts)

$$\begin{array}{r} 1111 \\ - 01100 \\ \hline 10011 \end{array}$$

$$\begin{array}{r} 10011 \\ + 1 \\ \hline 10100 \end{array}$$

$$(10100)_{2's}$$

b) 00111 - 01100 using 2's complement (hint: convert 01100 to 2's complement and add) (7pts)

$$\begin{array}{r} 11111 \\ - 01100 \\ \hline 10011 \end{array}$$

$$\begin{array}{r} 10011 \\ + 1 \\ \hline (10100)_{2's} \end{array}$$

$$\begin{array}{r} 00111 \\ + 10100 \\ \hline 11011 \end{array}$$

$$\begin{array}{r} 00100 \\ + 1 \\ \hline -(00101)_{2's} \end{array}$$

c) Write the answer to part b in decimal (base 10) with a negative or positive sign (3pts)

$$(00101)_2$$

$$(1 \times 2^2) + (1 \times 2^0) =$$

$$4 + 1 = 5$$

$$(-5)_{10}$$

4. a) Simplify

$$F = (x + y' + z')(x' + z')$$

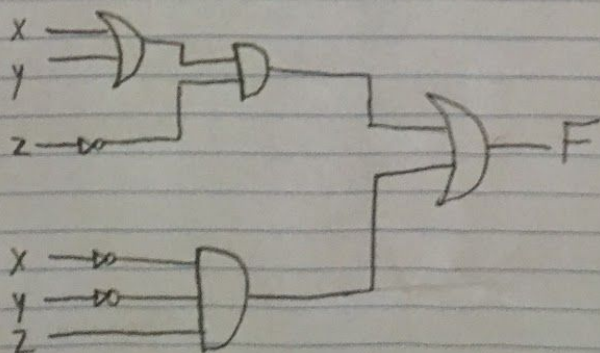
Postulate 4(a)

$$F = x'x + x'y' + x'z' + z'x + z'y' + z'z'$$

$$F = 0 + x'y' + x'z' + z'x + z'y' + 0 \quad \text{Postulate 5(b)}$$

$$F = x'y' + x'z' + z'x + z'y'$$

b)  $F = (x + y)z' + x'y'z$





5. (25 points total)

- a) Given the Minterm list, write the 1) truth table and 2) Sum of Products form (SOP) 3) Product of Sum (POS) (9 pts)

$$F = \sum m(0,1,4,7)$$

	X	Y	Z	F
min 0	0	0	0	1
min 1	0	0	1	1
max 2	0	1	0	0
max 3	0	1	1	0
min 4	1	0	0	1
max 5	1	0	1	0
max 6	1	1	0	0
min 7	1	1	1	1

	yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
X	00	01	11	10	
$\bar{x}$ 0	1	1	0	0	
x 1	1	0	1	0	

$$F = \bar{x}\bar{y} + \bar{y}\bar{z}$$

- b) Using any method that you choose to write the above truth table in simplified Sum of Products form (5pts)

$$F = (\bar{x} \cdot \bar{y} \cdot \bar{z}) + (\bar{x} \cdot \bar{y} \cdot z) + (x \cdot \bar{y} \cdot \bar{z}) + (x \cdot y \cdot z)$$

$$F = \sum m(0,1,4,7)$$

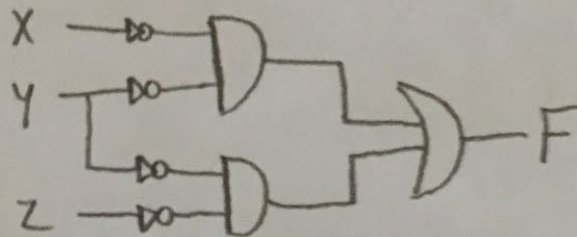
- c) Using any method that you choose to write the above truth table in simplified Product of Sums form (5pts)

$$F = (x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)$$

$$F = \pi M(2,3,5,6)$$

d) Draw the circuit described in the expression derived in part (a) of this problem (6pts)

$$F = \bar{x}\bar{y} + \bar{y}\bar{z}$$



6. (15 points)

Simplify the following expression using a 3 variable K-map

$$F = xyz' + x'yz + xyz + x'yz'$$

$$2^3 = 8$$

		$yz$			
		$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	$y\bar{z}$ 11	$yz$ 10
$\bar{x}$	0	0	0	1	1
$x$	1	0	0	1	1

$$F = y$$



7. (15 points)

Use a 4 variable (A, B, C, D) K-map to minimize the following minterms

$$F = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 10)$$

$$2^4 = 16$$

	$CD$	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$
$AB$	00	01	11	10
$\bar{A}\bar{B}$	0 1	1 1	3	2 1
$\bar{A}B$	4 1	5 1	7	6 1
$AB$	12	13	15	14
$A\bar{B}$	8 1	9 1	11	10 1

$$F = \bar{A}\bar{C} + \bar{A}C\bar{D} + \bar{B}\bar{D}$$