1. Simplify the following expressions using postulates and theorems of Boolean algebra.

a) $ABC + A'B + ABC'$

$AB(C+C') + A'B$ Postulate 4a (distributive)

$AB(1) + A'B$ Postulate 5a

$AB + A'B$

$B(A+A')$ Postulate 4a (distributive)

$B(1)$ Postulate 5a

$\boxed{B}$

b) $(BC' + A'D)(AB' + CD')$

Postulate 4a (distributive)

$(BC'AB' + BC'CD' + A'DAB' + A'DCD')$

Postulate 5b

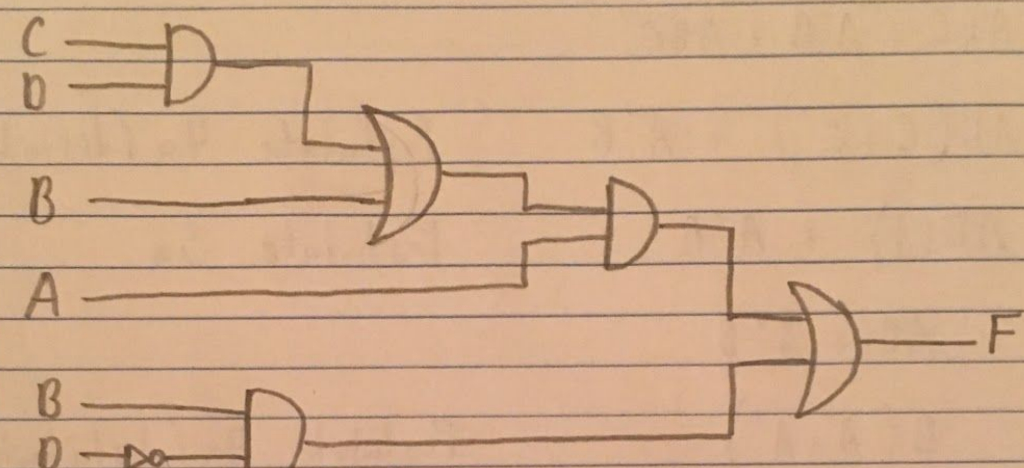
$C'A(0) + BD'(0) + DB'(0) + A'C(0)$

Theorem 2

$0 + 0 + 0 + 0$

$\boxed{0}$

2. Implement $F = A(B + CD) + BD'$ in basic gates.



3. Simplify the following Boolean functions using a Karnaugh map.

a) $F(X,Y,Z) = \Sigma(0,2,4,5,6)$

$X \backslash YZ$	$\bar{Y}\bar{Z}$ 00	$\bar{Y}Z$ 01	YZ 11	$Y\bar{Z}$ 10
$\bar{X}$ 0	(0) 1	(1) 0	(3) 0	(2) 1
X 1	(4) 1	(5) 1	(7) 0	(6) 1

$$\boxed{F(X,Y,Z) = \bar{Z} + X\bar{Y}}$$

b) $F(W,X,Y,Z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$

$WX \backslash YZ$	$\bar{Y}\bar{Z}$ 00	$\bar{Y}Z$ 01	YZ 11	$Y\bar{Z}$ 10
$\bar{W}\bar{X}$ 00	(0) 1	(1) 1	(3) 0	(2) 1
$\bar{W}X$ 01	(4) 1	(5) 1	(7) 0	(6) 1
WX 11	(12) 1	(13) 1	(15) 0	(14) 1
$W\bar{X}$ 10	(8) 1	(9) 1	(11) 0	(10) 0

$$\boxed{F(W,X,Y,Z) = \bar{Y} + \bar{W}\bar{Z} + X\bar{Z}}$$

c) $F = A'B'C' + B'CD' + A'BCD' + AB'C'$

$F = A'B'C'(1) + B'CD'(1) + A'BCD' + AB'C'(1)$

Postulate 5a

$F = A'B'C'(D+D') + B'CD'(A+A') + A'BCD' + AB'C'(D+D')$

Postulate 4a (distributive)

$F = A'B'C'D + A'B'C'D' + B'CD'A + B'CD'A' + A'BCD' + AB'C'D + AB'C'D'$

$F = \sum(1, 0, 10, 2, 6, 9, 8)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0: 1	1: 1	3: 0	2: 1
$\bar{A}B$ 01	4: 0	5: 0	7: 0	6: 1
AB 11	12: 0	13: 0	15: 0	14: 0
$A\bar{B}$ 10	8: 1	9: 1	11: 0	10: 1

$\boxed{F = \bar{B}\bar{C} + \bar{B}\bar{D} + \bar{A}C\bar{D}}$

d) $F = A'B + BC' + B'C'$

$F = A'B(1) + BC'(1) + B'C'(1)$

Postulate 5a

$F = A'B(C+C') + BC'(A+A') + B'C'(A+A')$

Postulate 4a (distributive)

$F = A'BC + A'BC' + BC'A + BC'A' + B'C'A + B'C'A'$

$F = \Sigma(3, 2, 6, 4, 0)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	^{0} 1	^{1} 0	^{3} 1	^{2} 1
A 1	^{4} 1	^{5} 0	^{7} 0	^{6} 1

$$\boxed{F = \bar{C} + \bar{A}B}$$

4) Giving the truth table, derive the canonical SOP and POS

	X	Y	Z	F	
Min 0	0	0	0	1	$\rightarrow \overline{X}\cdot\overline{Y}\cdot\overline{Z}$
Max 1	0	0	1	0	$\rightarrow X+Y+\overline{Z}$
Min 2	0	1	0	1	$\rightarrow \overline{X}\cdot Y\cdot\overline{Z}$
Max 3	0	1	1	0	$\rightarrow X+\overline{Y}+\overline{Z}$
Max 4	1	0	0	0	$\rightarrow \overline{X}+Y+Z$
Max 5	1	0	1	0	$\rightarrow \overline{X}+Y+\overline{Z}$
Max 6	1	1	0	0	$\rightarrow \overline{X}+\overline{Y}+Z$
Min 7	1	1	1	1	$\rightarrow X\cdot Y\cdot Z$

a) Canonical SOP

$$F=(\overline{X}\cdot\overline{Y}\cdot\overline{Z})+(\overline{X}\cdot Y\cdot\overline{Z})+(X\cdot Y\cdot Z)$$

$$F=\Sigma m(0,2,7)$$

b) Canonical POS

$$F=(X+Y+\overline{Z})(X+\overline{Y}+\overline{Z})(\overline{X}+Y+Z)(\overline{X}+Y+\overline{Z})(\overline{X}+\overline{Y}+Z)$$

$$F=\Sigma M(1,3,4,5,6)$$

5. Design a circuit that counts the number of 1's present in 3 inputs A, B, and C. Its output is a two-bit number X1X0, representing that count in binary. Assume active-HIGH logic.

a) Truth Table

Input			Number of 1's	Output	
A	B	C		X1	X0
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	2	1	0
1	0	0	1	0	1
1	0	1	2	1	0
1	1	0	2	1	0
1	1	1	3	1	1

b) $2^3 = 8 \rightarrow X1$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0			1	
A 1		1	1	1

$$\boxed{X1 = AC + BC + AB}$$

$2^3 = 8 \rightarrow X0$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0		1		1
A 1	1		1	

$$\boxed{X0 = A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + \bar{A}B\bar{C}}$$

c)