1)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \left(\frac{\partial L}{\partial \theta}\right) = T_m - B_{eq}\dot\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \left(\frac{\partial L}{\partial \theta}\right) + B_{eq}\dot\theta = T_m$$

$$\frac{d}{dt}\left[J_{eq}\dot\theta + \frac{1}{2}J_{link}(2\dot\theta + 2\dot\alpha + 0) - 0\right] - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\alpha}\right) + B_{eq}\dot\theta = T_m$$

$$J_{eq}\ddot\theta + J_{link}\ddot\theta - \frac{d}{dt}(2\dot\theta + 2\dot\alpha)\frac{1}{2}J_{link} + B_{eq}\dot\theta = T_m$$

$$(J_{eq} + J_{link})\ddot\theta + J_{link}\ddot\alpha + B_{eq}\dot\theta = T_m$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\alpha}\right) - \frac{\partial L}{\partial \alpha} = 0$$

$$J_{link}(\ddot\theta + \ddot\alpha) - (-k_{stiff}\alpha) = 0$$

$$J_{link}\ddot\theta + J_{link}\ddot\alpha + k_{stiff}\,\alpha = 0$$

2)

$\mathcal{L}_0$ (equation 20)

$\mathcal{L}_0$ (equation 21)

Sub equation ② into equation ②⓪

$$G_{link}(S) = \frac{\theta_r(S)}{\theta(S)} = \frac{\frac{k_{stiff}}{J_{link}}}{s^2 + \frac{k_{stiff}}{J_{link}}}$$

G_{hub} can be obtained from algebraic manipulation of equation 20

3)

$$\lim_{s \to 0} G_{link}(S) = \frac{\frac{k_{stiff}}{J_{link}}}{s^2 + \frac{k_{stiff}}{J_{link}}}$$

$$\frac{\theta_r(S)}{\theta(S)} = 1$$

$$\theta_r(S) = \theta(S)$$

4)

Stable   if $k_{pd} < 0$ } A

Stable   if $k_{pd} < 7.6$ } B

5)

<u>Controller A</u>

Values of k_{pd} (root locations that intersect the imaginary axis (dotted line) satisfy design req.

<u>Controller B</u>

There are no Values of k_{pd} (root locations that intersect with the imaginary axis (dotted line), so no satisfy design req.

6)

increasing the demand q_{in}