AuSRoS

Australian School of Robotic Systems

B1 - Motion Planning

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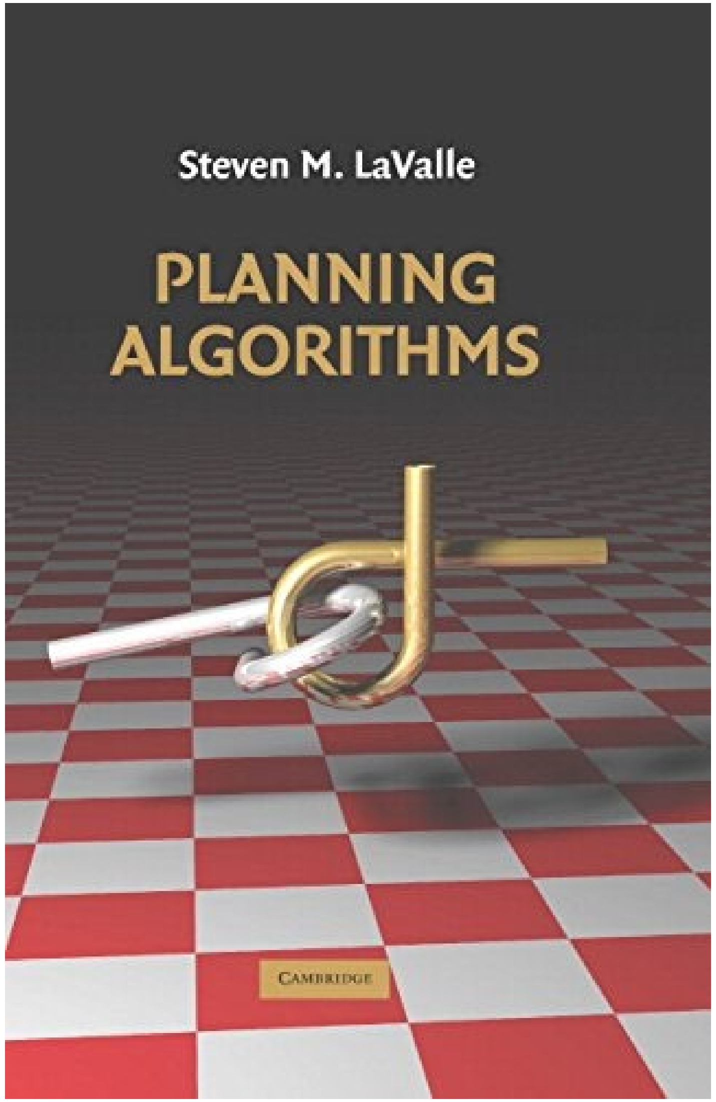
Some content adapted from Hanna Kurniawati's Adv. AI Course

Open Motion Planning Library
https://ompl.kavrakilab.org/

MoveIt

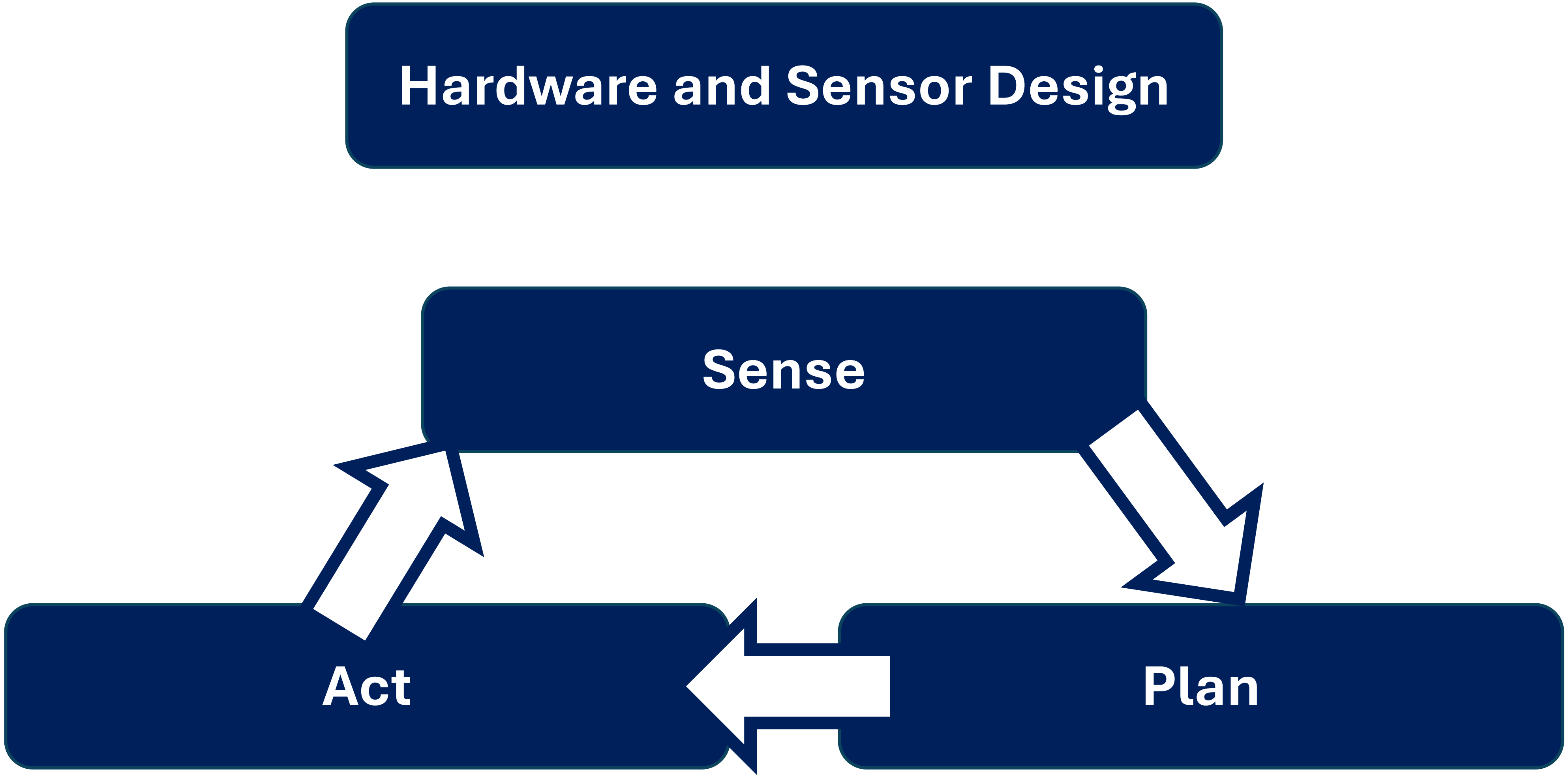
Moveit
https://moveit.ros.org/



https://lavalle.pl/planning/

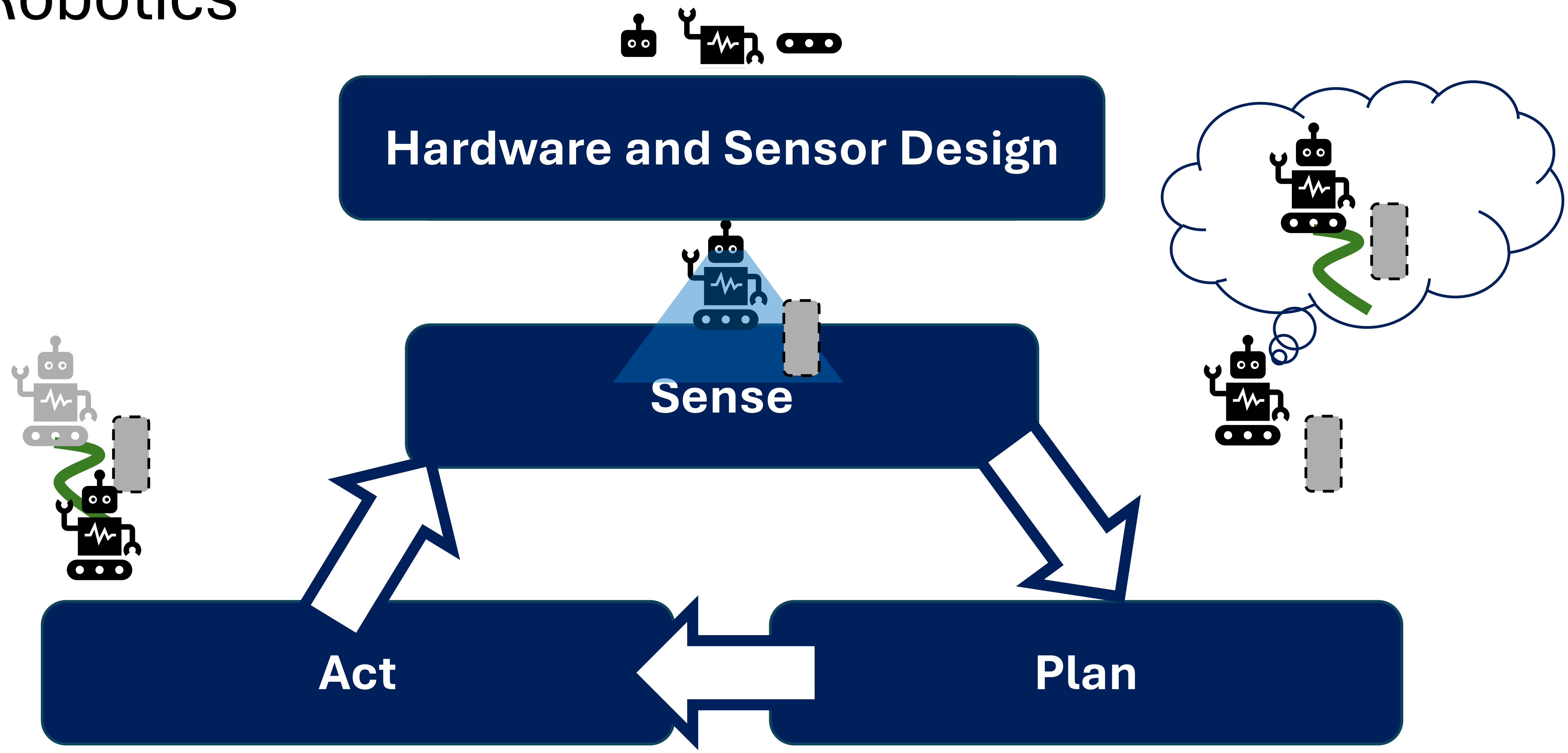
Planning

Robotics

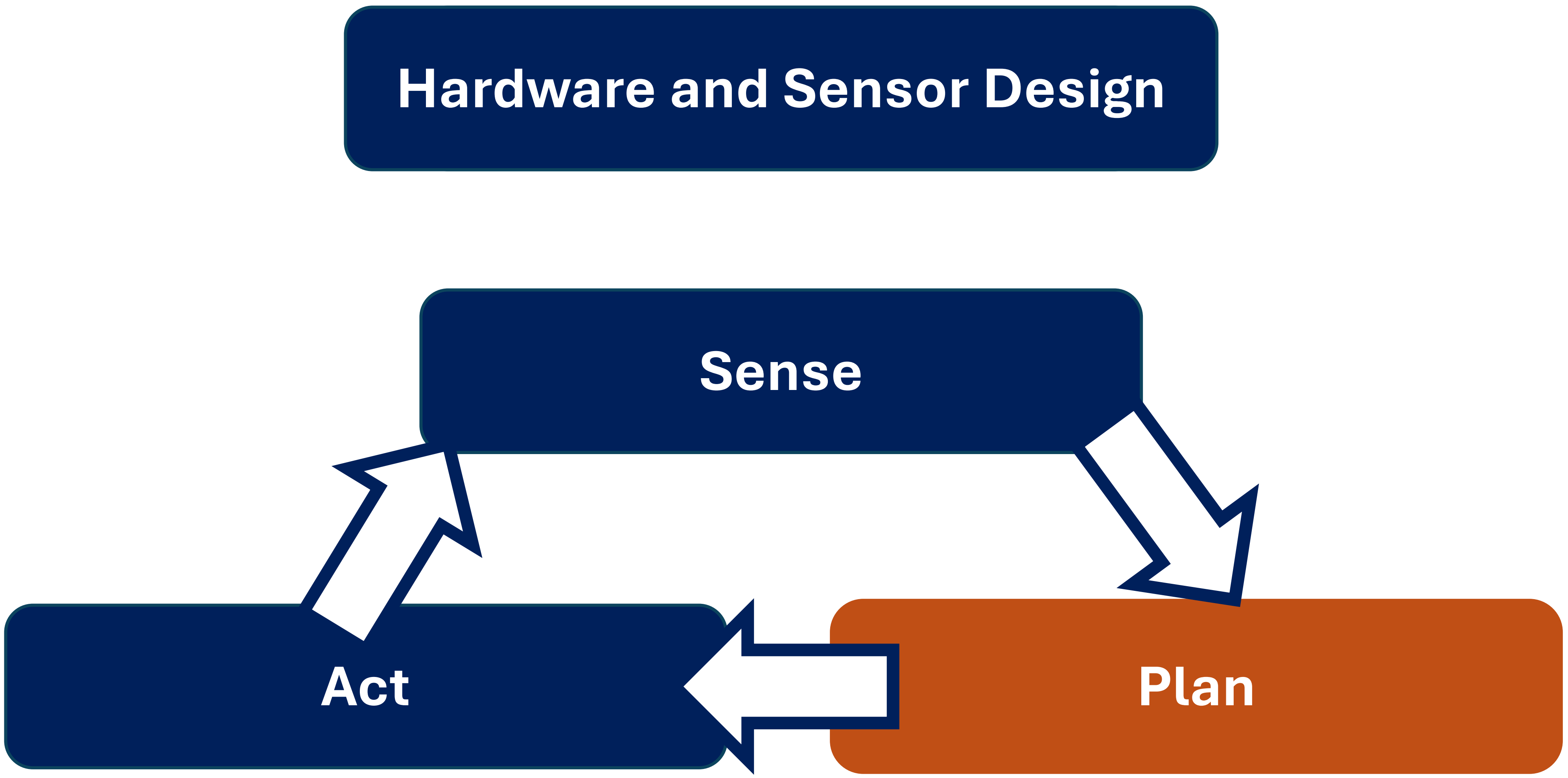


Keywords: Sense, Plan, Act

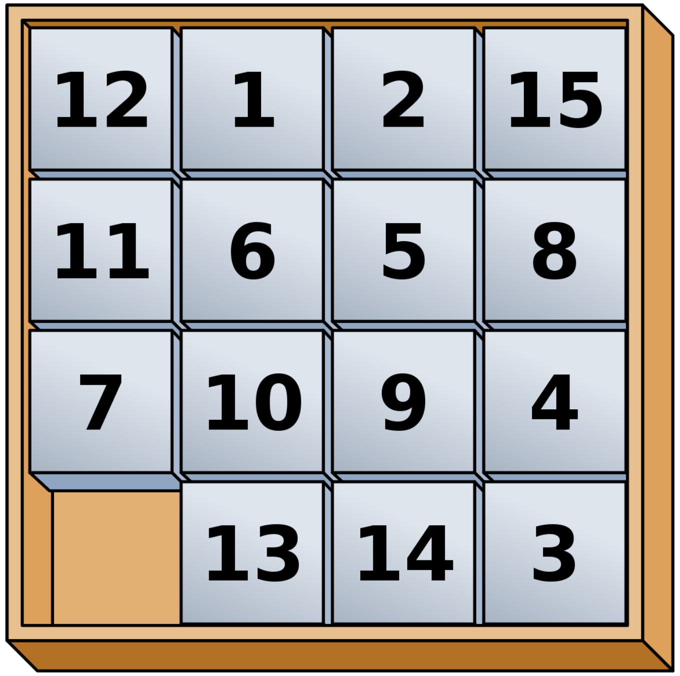
Robotics



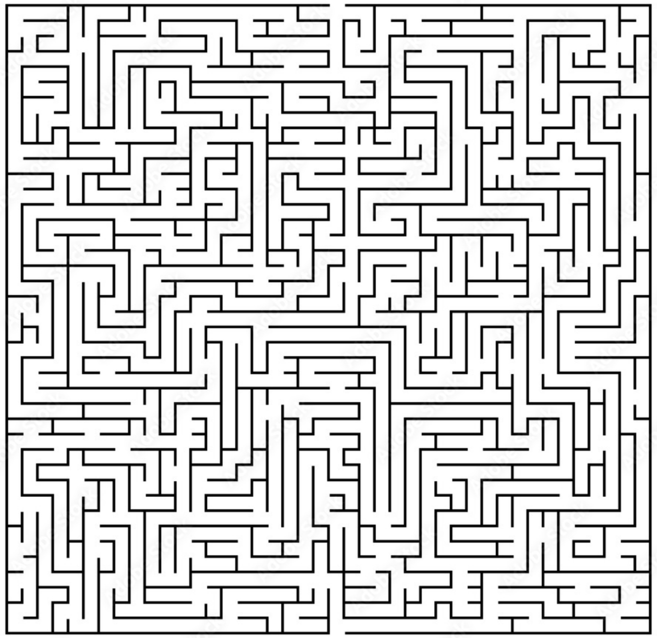
Robotics



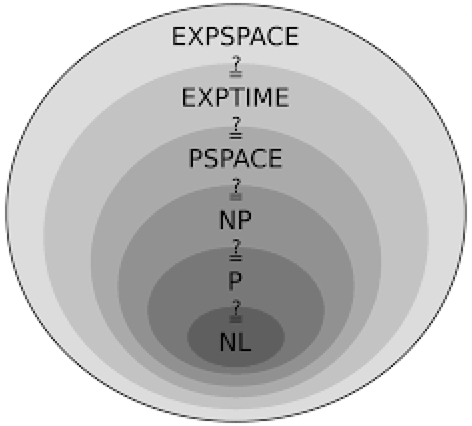
Computationally Hard Problems



wikipedia 15 puzzle

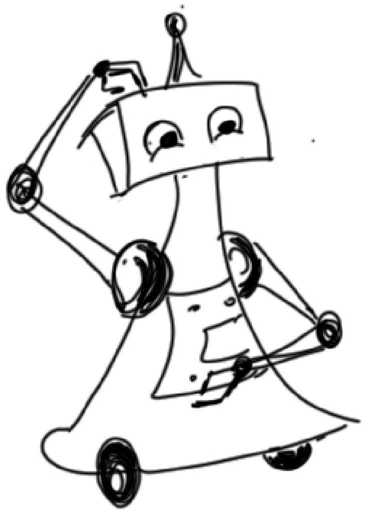


chess.com

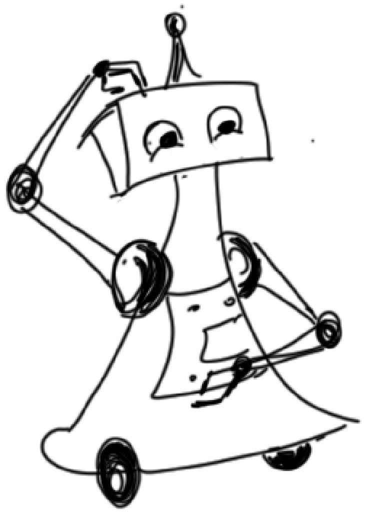


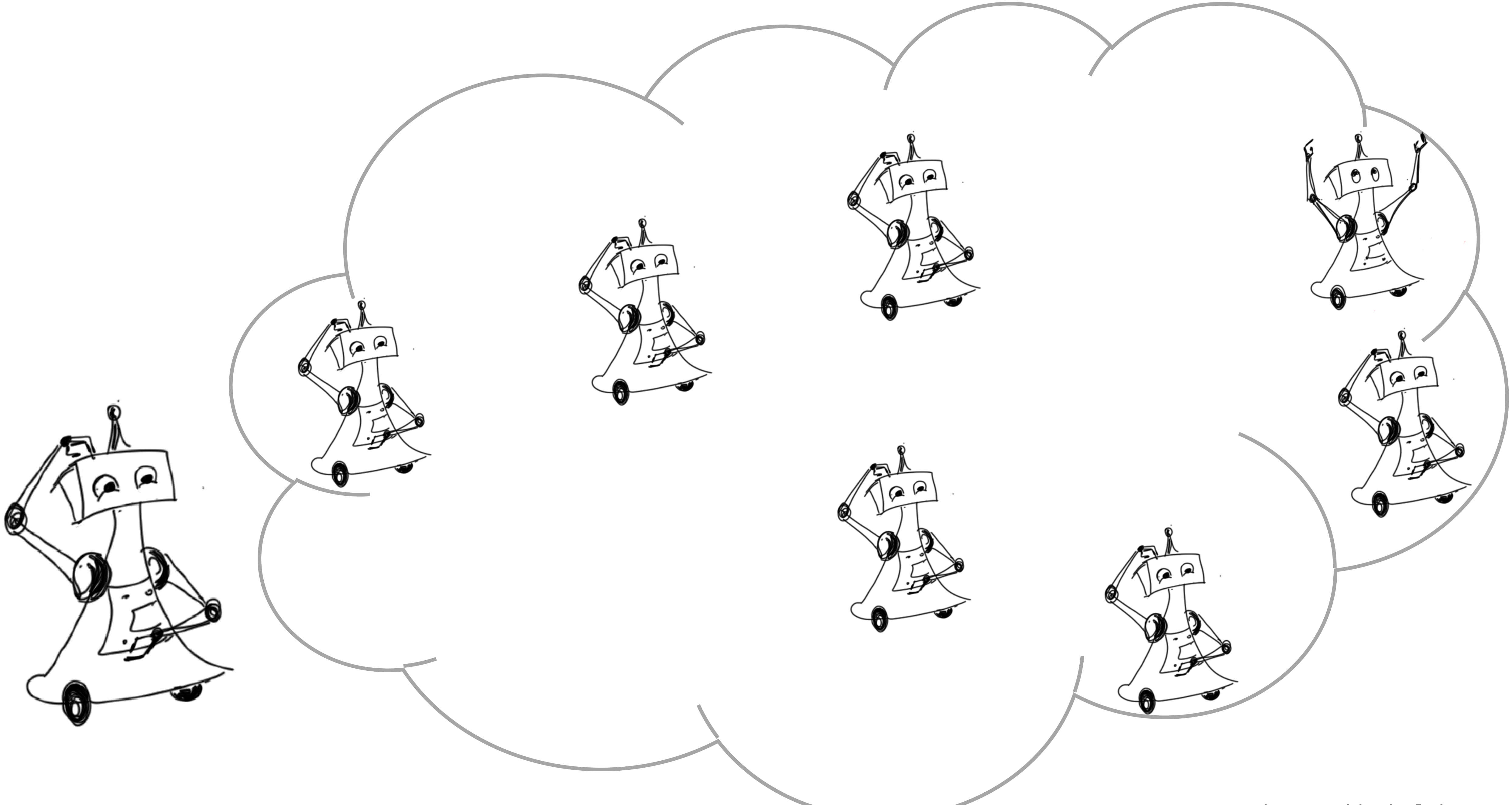
Keywords: Computational Complexity

Two roads diverged in a wood…

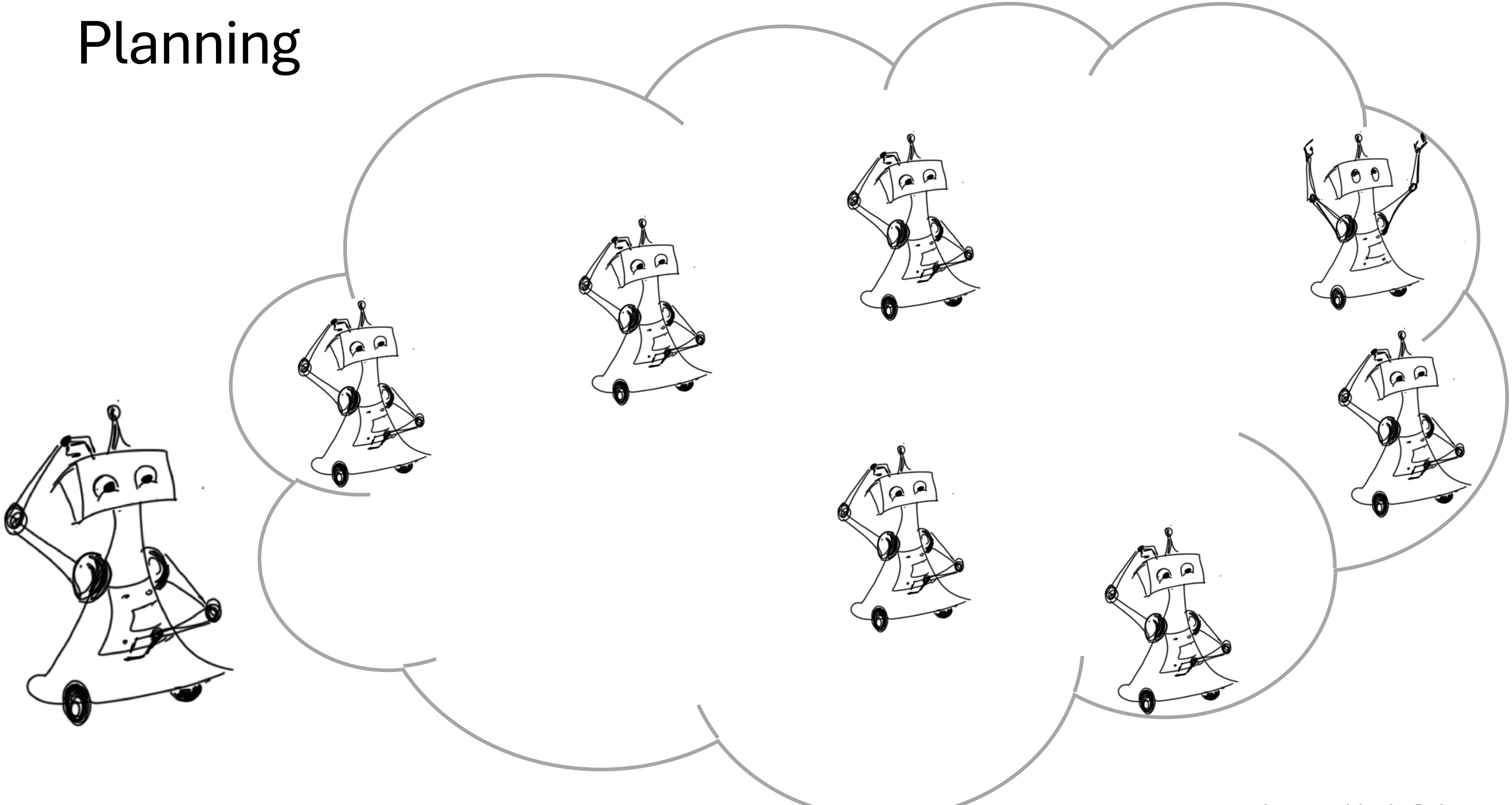


Many roads diverged in a wood

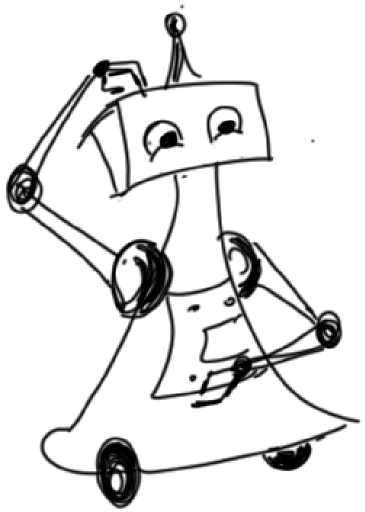




Planning



Two roads diverged in a wood, and the robot took the one to its goal.



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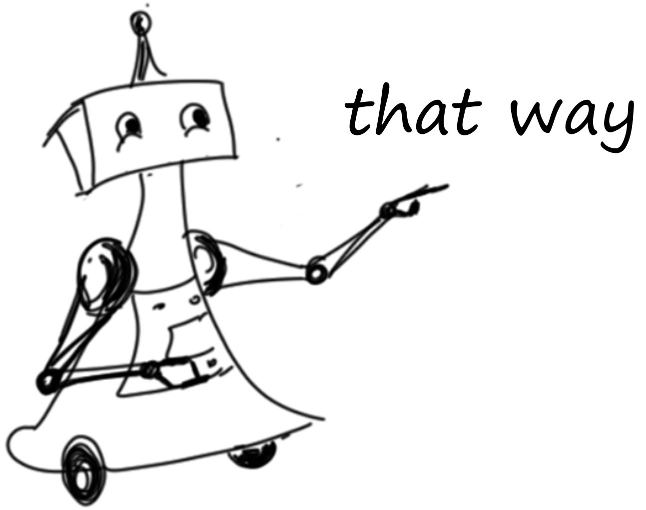


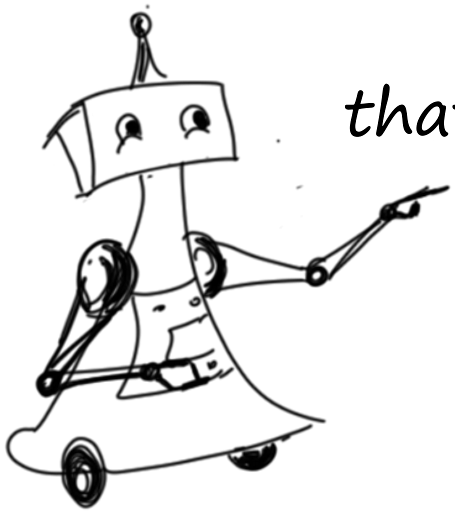
Keywords: Long horizon, Goal, Constraint

Planning

Long-horizon
goal-directed
reasoning
while
respecting constraints

Think many steps ahead.
Reach objective.

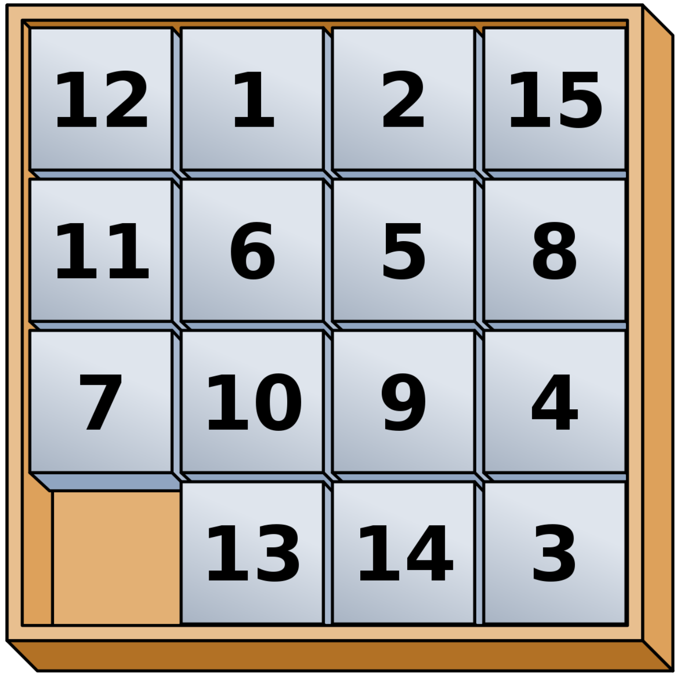
Not every move is valid.



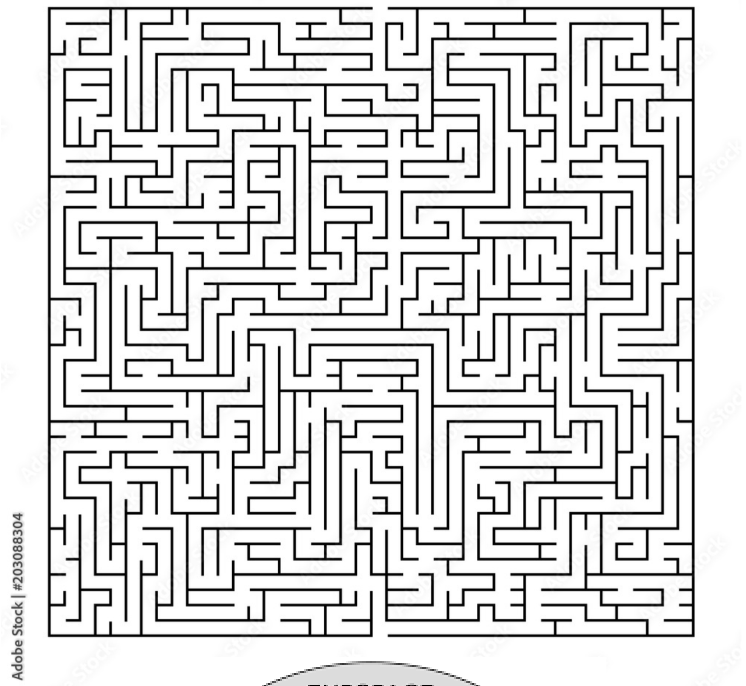
that way

Keywords: Long horizon, Goal, Constraint

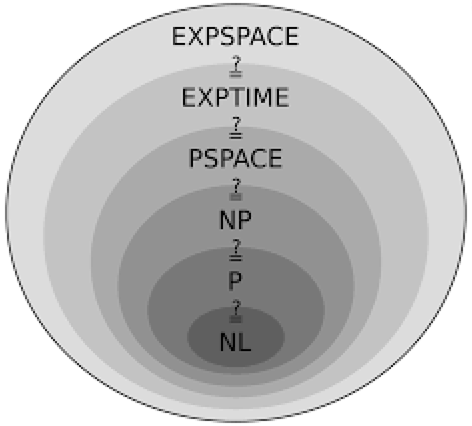
How do we represent a robot planning problem?



wikipedia 15 puzzle



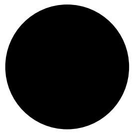
chess.com



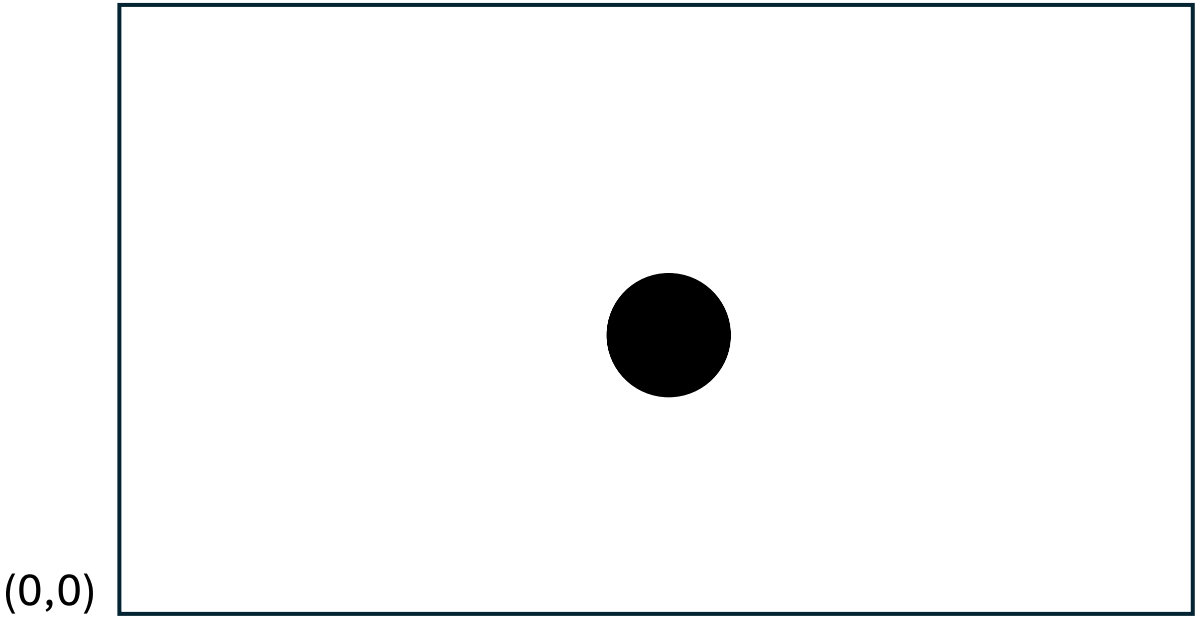
Representations and Abstractions

But first, what’s the point?

How do we represent or model a robot?



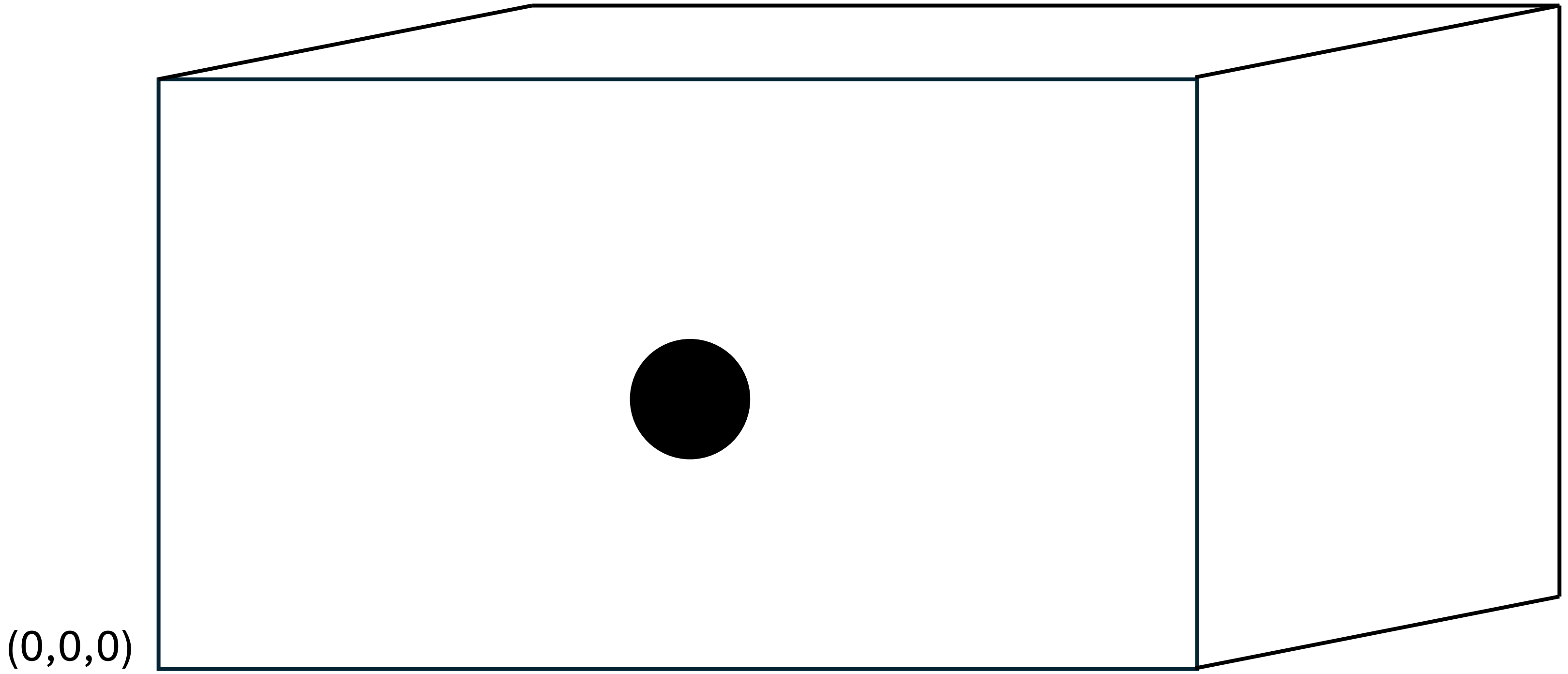
A point



A coordinate in 2D is (x,y) wrt some origin (0,0)

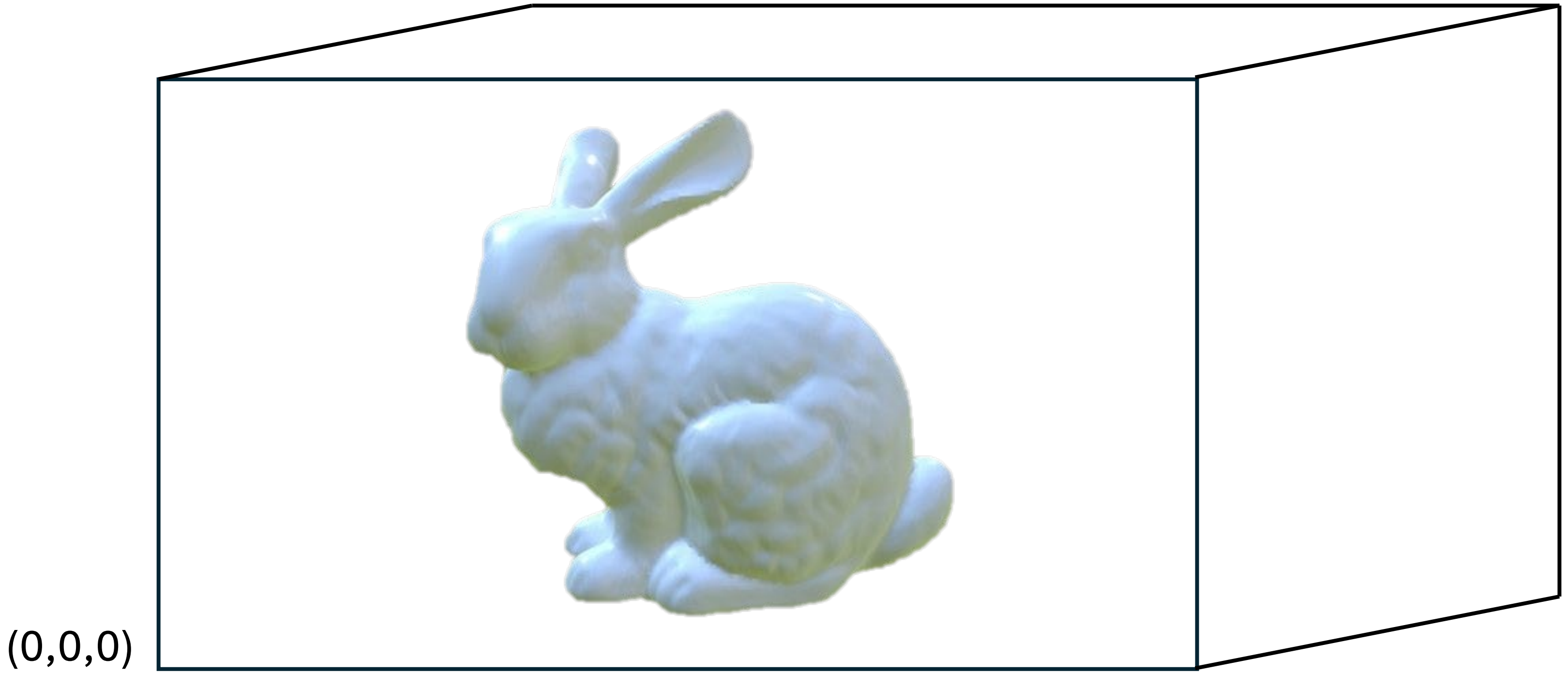
Keywords: Coordinate System

A point



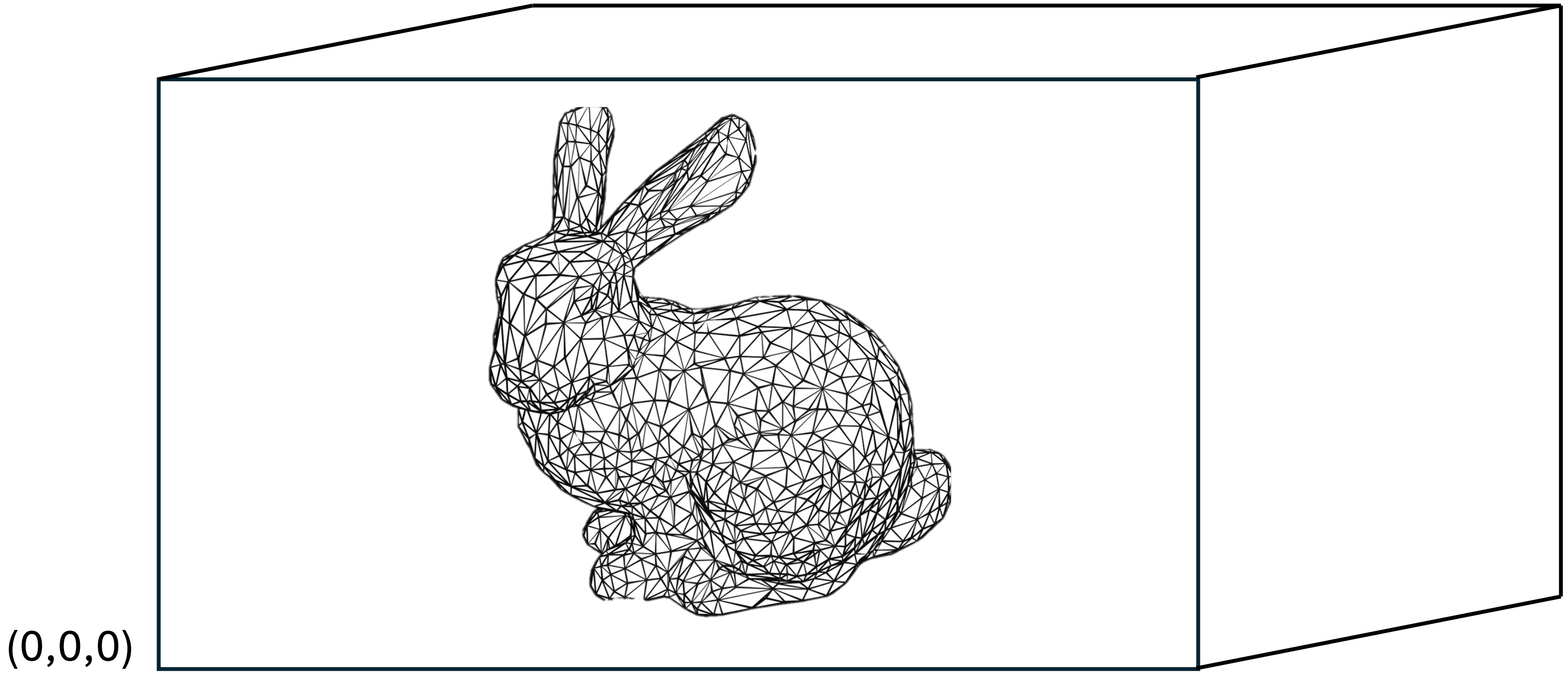
A coordinate in 3D is (x,y,z) wrt some origin (0,0,0)

A rigid body



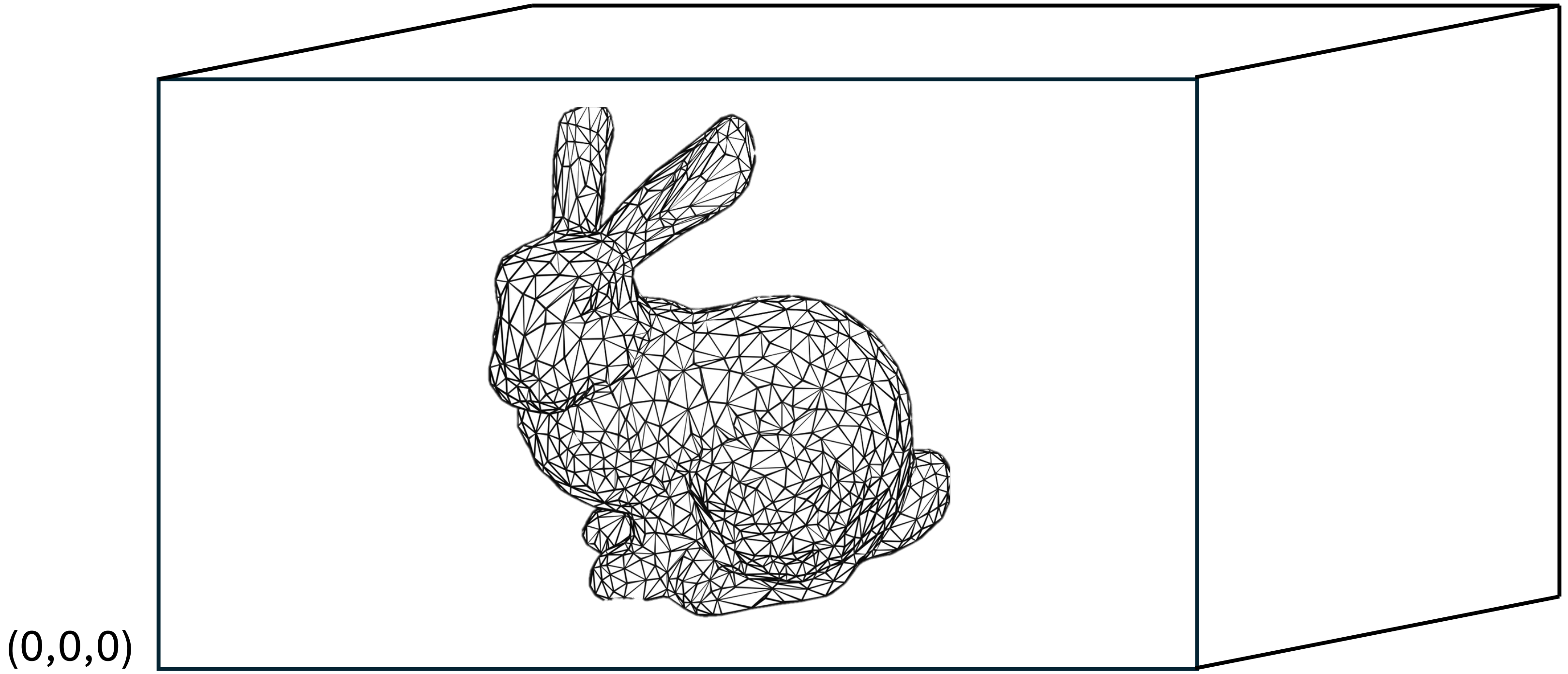
Keywords: Rigid Body

A rigid body



A set of points

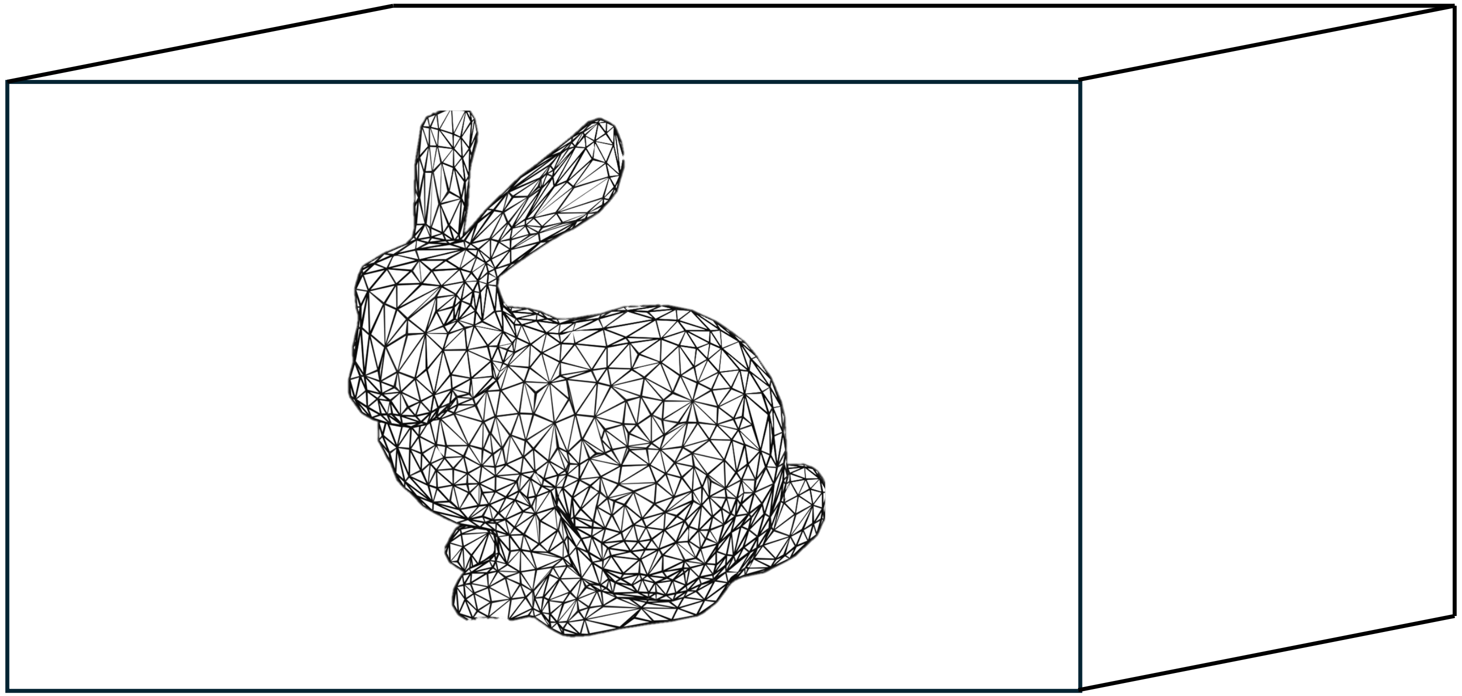
A rigid body



A set of points such that their relative distance $\underbrace{\text{does not change}}_{\text{rigid}}$

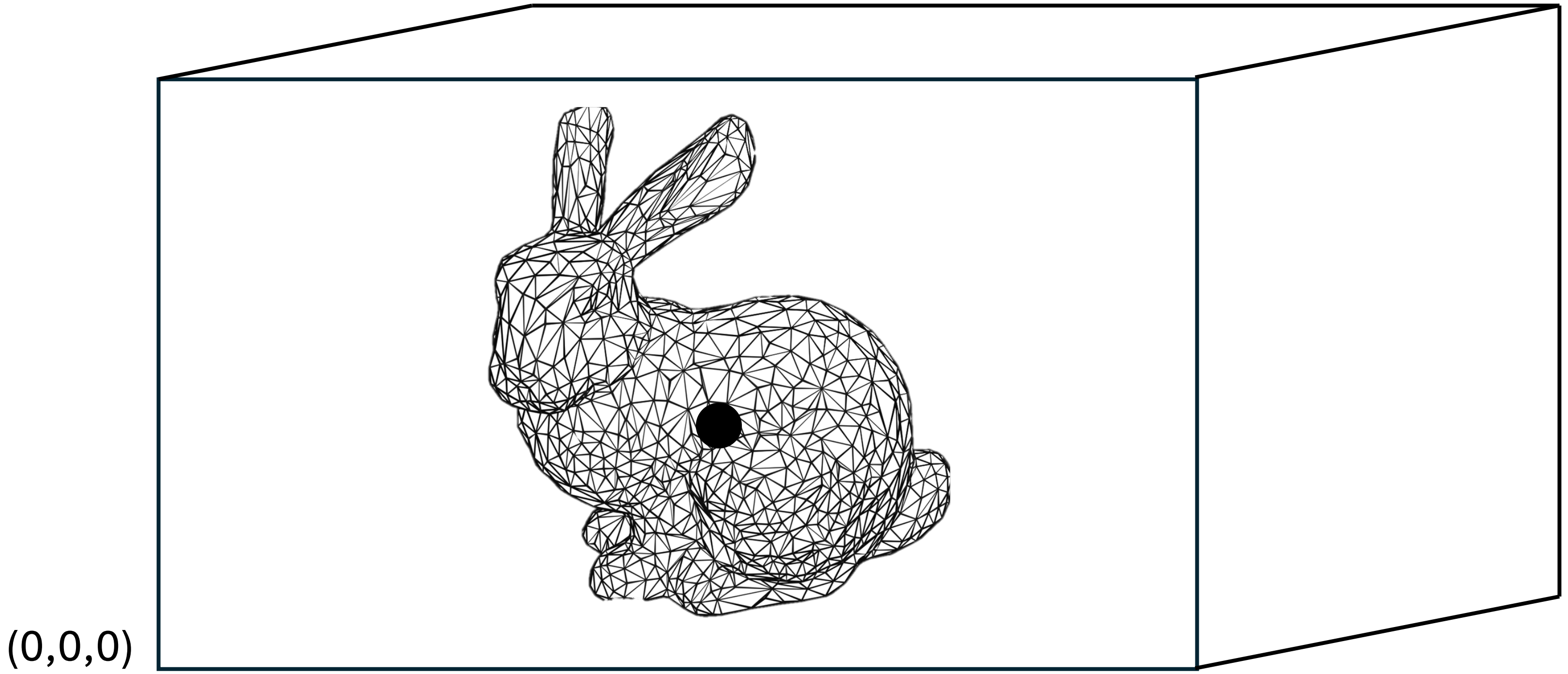
How do we represent a rigid body?

(0,0,0)



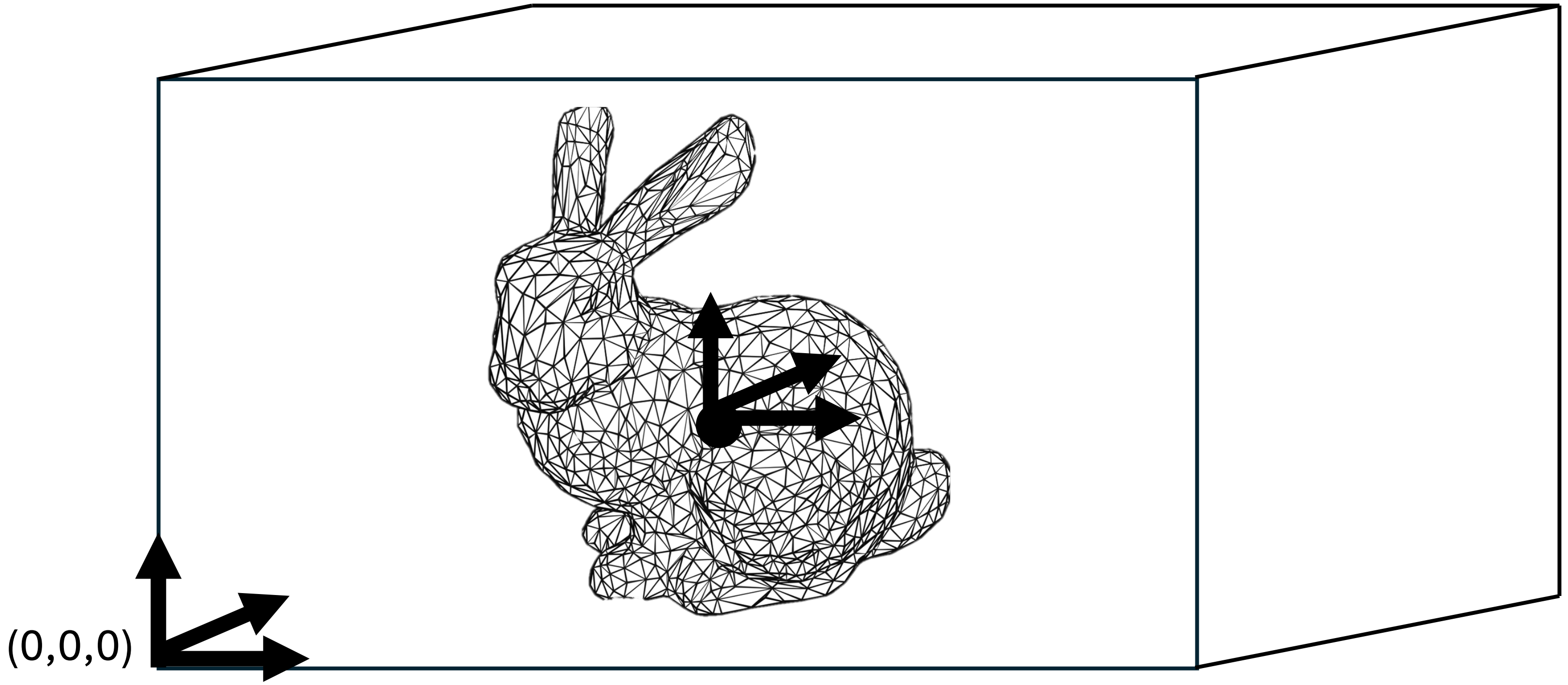
Option 1: A set of points.

How do we represent a rigid body?



Option 2: One predefined point **and … ?**

How do we represent a rigid body?

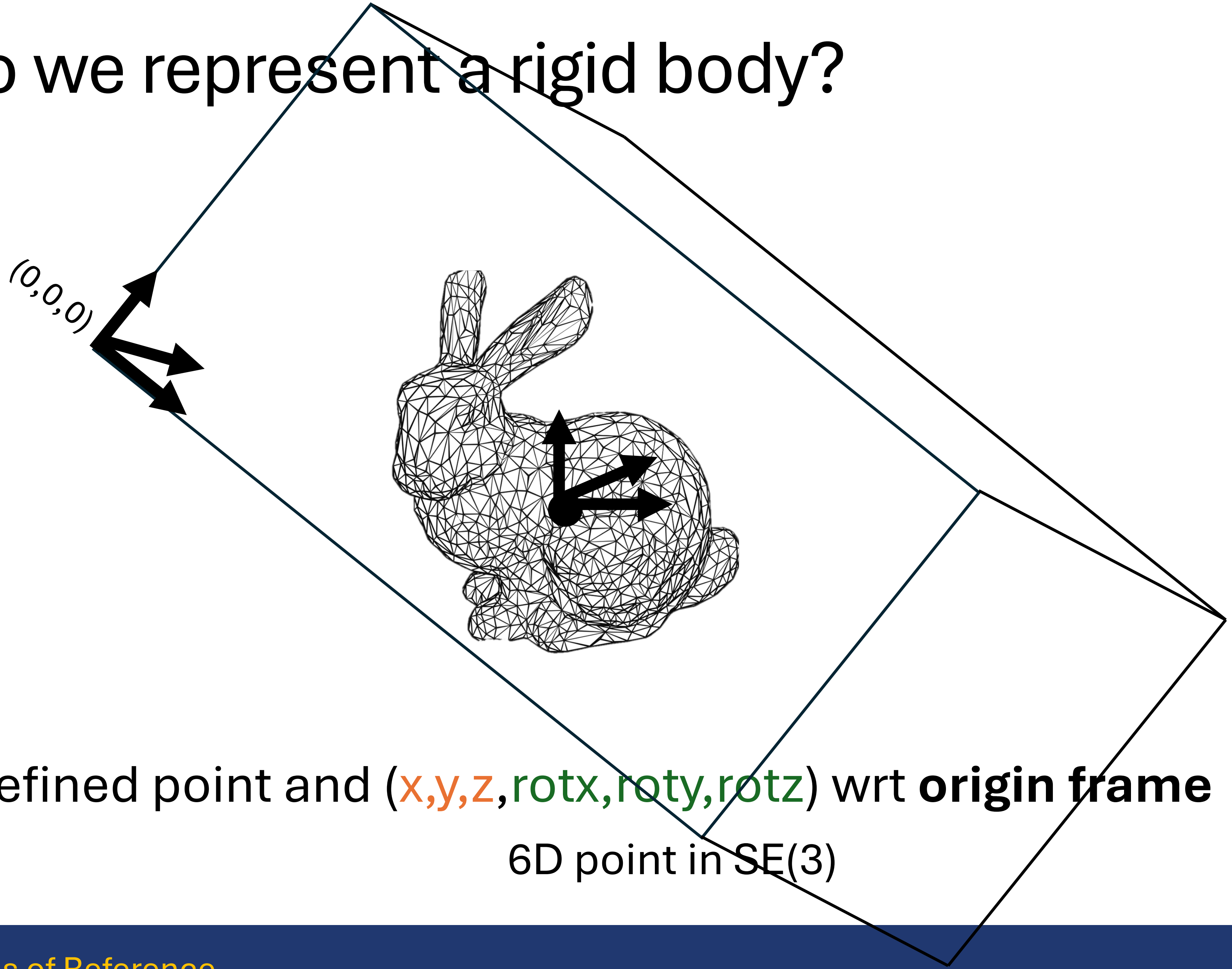


One predefined point and (x,y,z,rotx,roty,rotz) wrt origin frame

6D point in SE(3)

Keywords: SE(3), Frames of Reference

How do we represent a rigid body?



One predefined point and (x,y,z,rotx,roty,rotz) wrt **origin frame**

6D point in SE(3)

Keywords: SE(3), Frames of Reference

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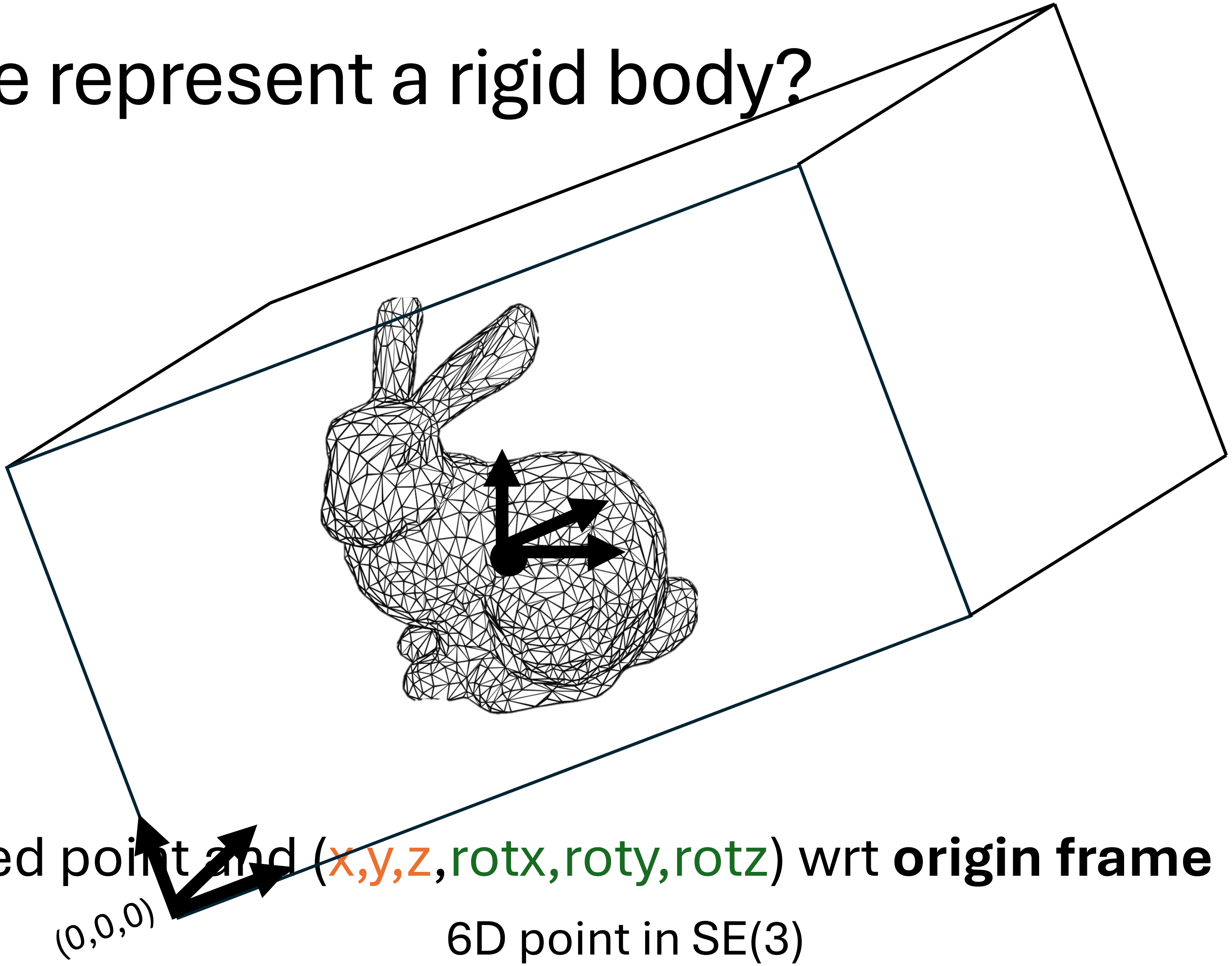

One predefined point and (x,y,z,rotx,roty,rotz) wrt **origin frame**

6D point in SE(3)

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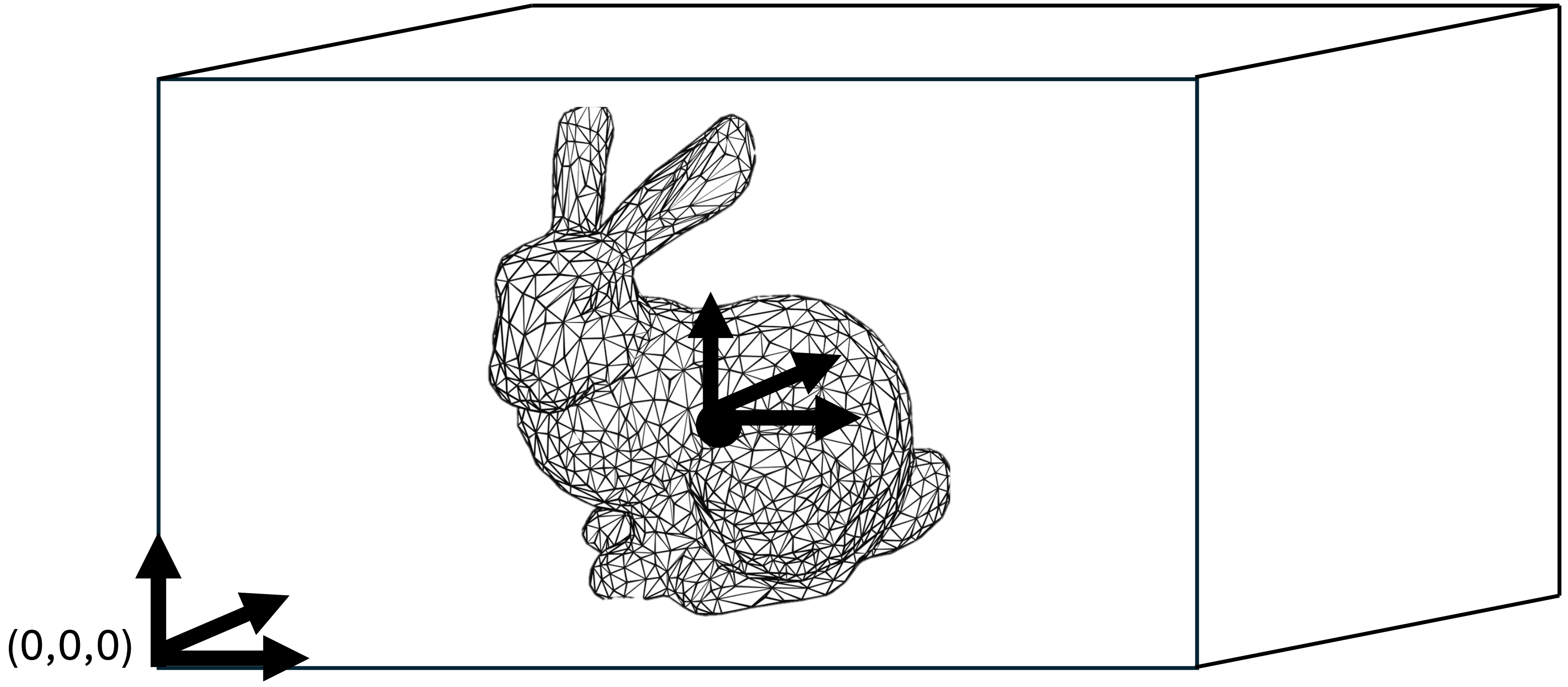

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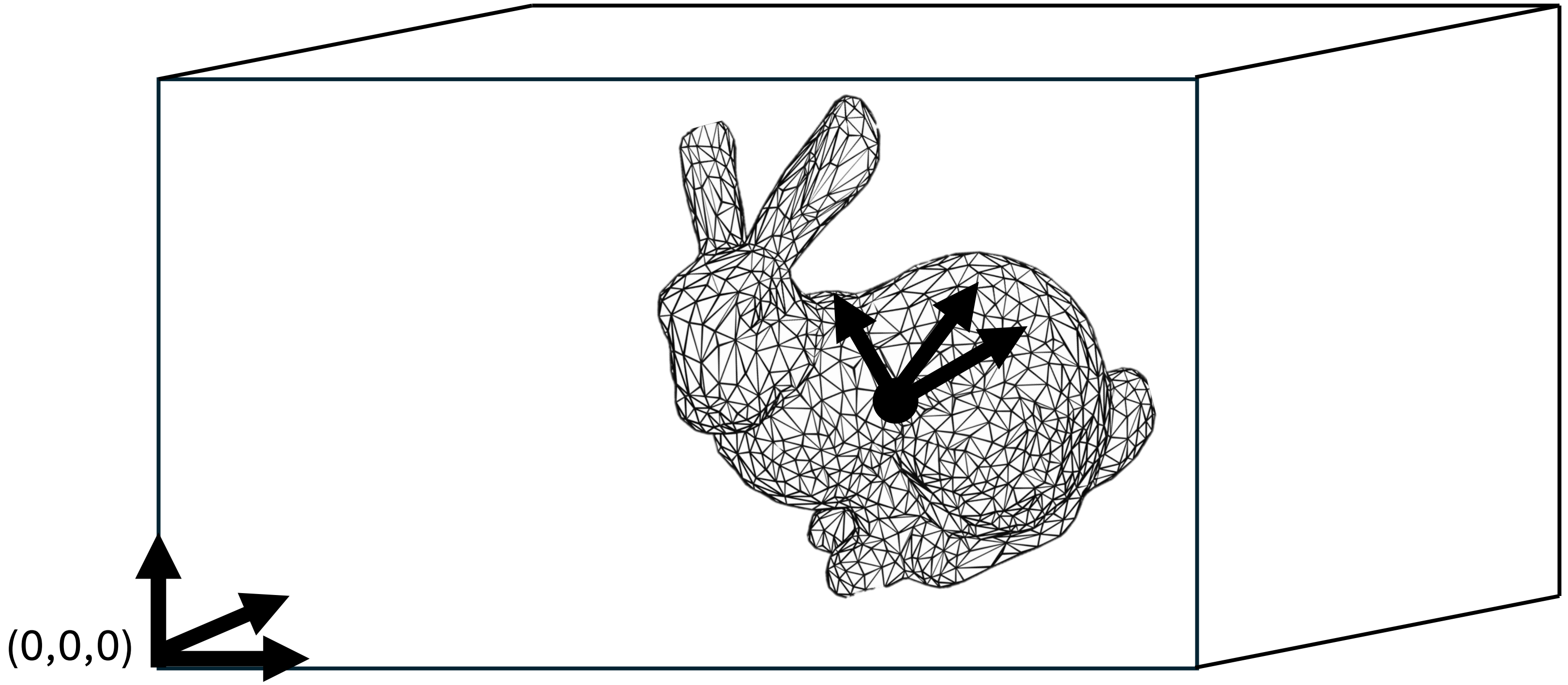
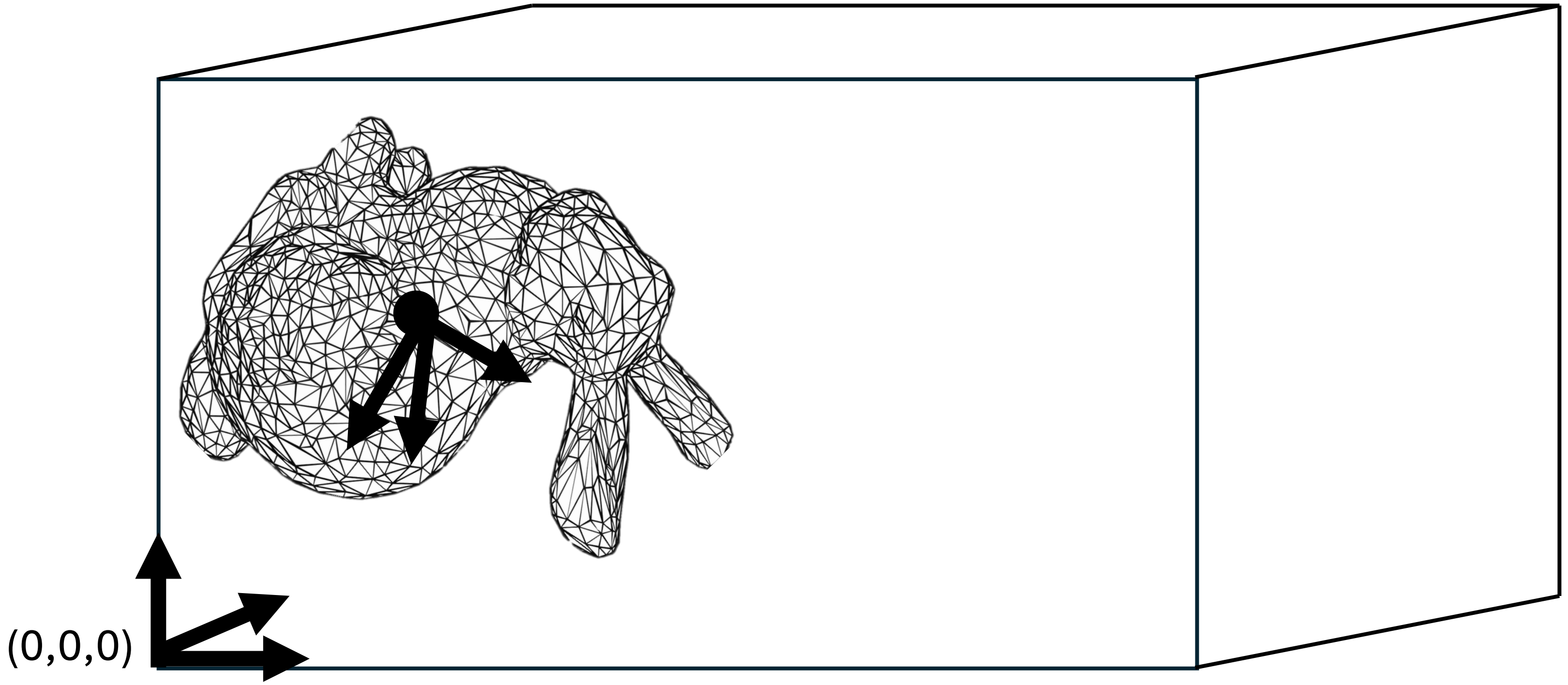


One predefined point and (x,y,z,rotx,roty,rotz) wrt origin frame

6D point in SE(3)

Keywords: SE(3), Frames of Reference

Rigid Body Poses

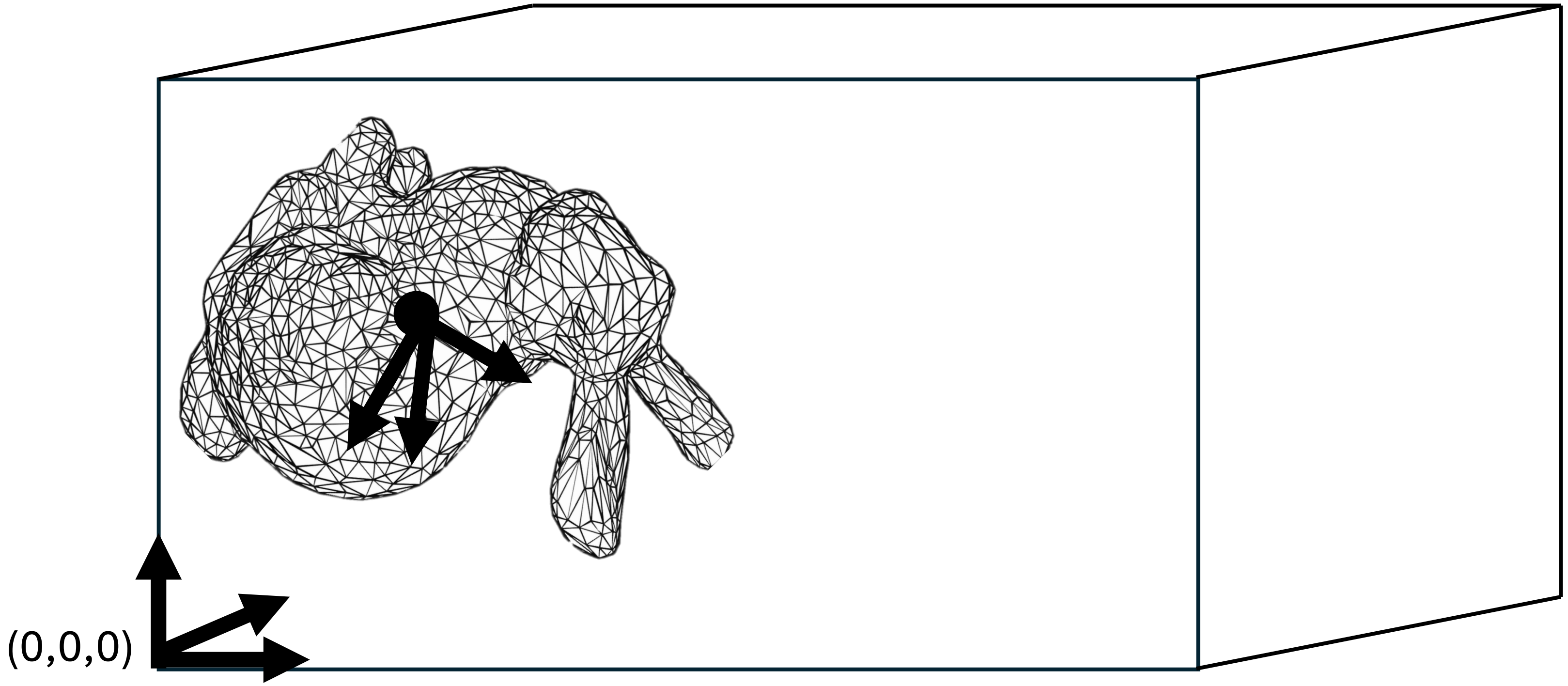


One predefined point and (x,y,z,rotx,roty,rotz) wrt origin frame

Pose in SE(3)

Keywords: SE(3), Frames of Reference

Rigid Body Poses



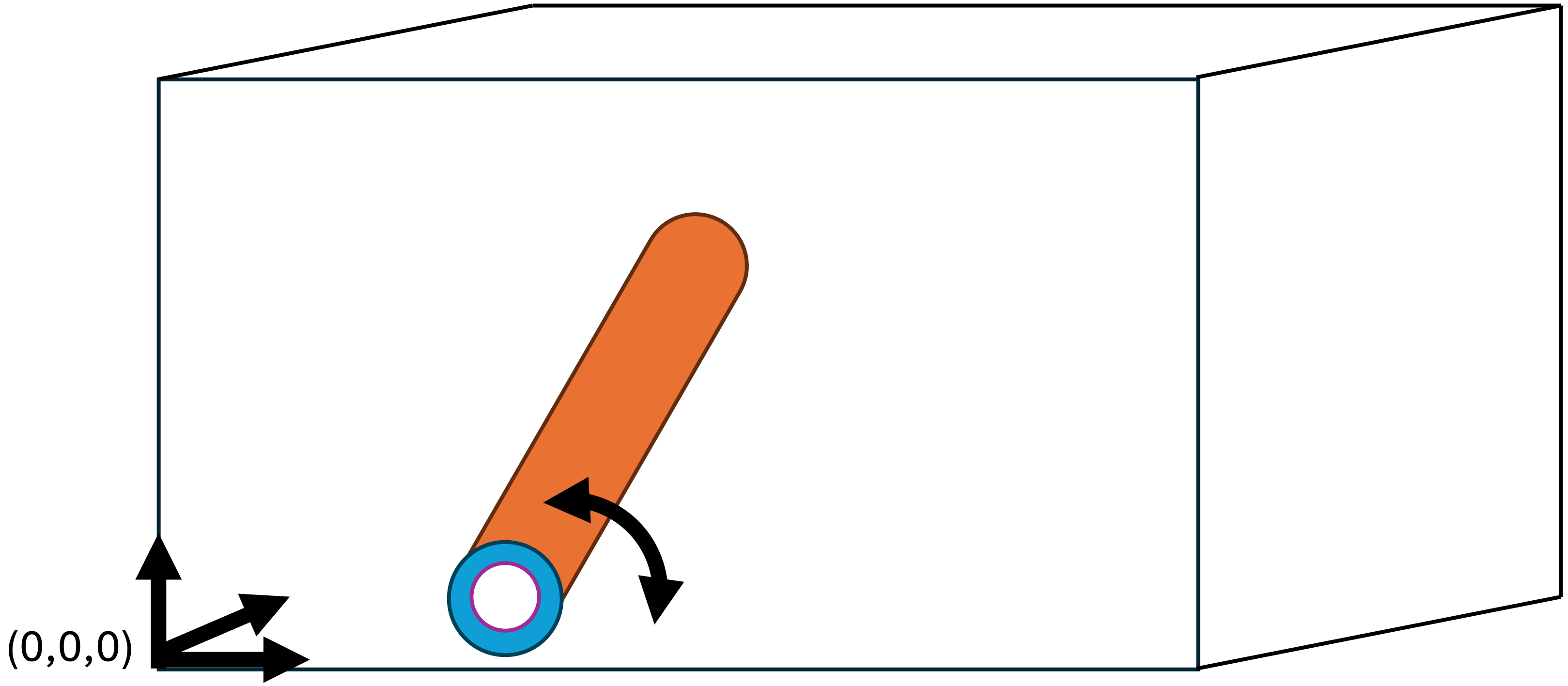
One predefined point and (x,y,z,rotx,roty,rotz) wr

Pose in SE(3)

3D rotations can have several representations – Euler angles, axis-angle, rotation matrix, **quaternion**

Keywords: 3D Transformations, Quaternions

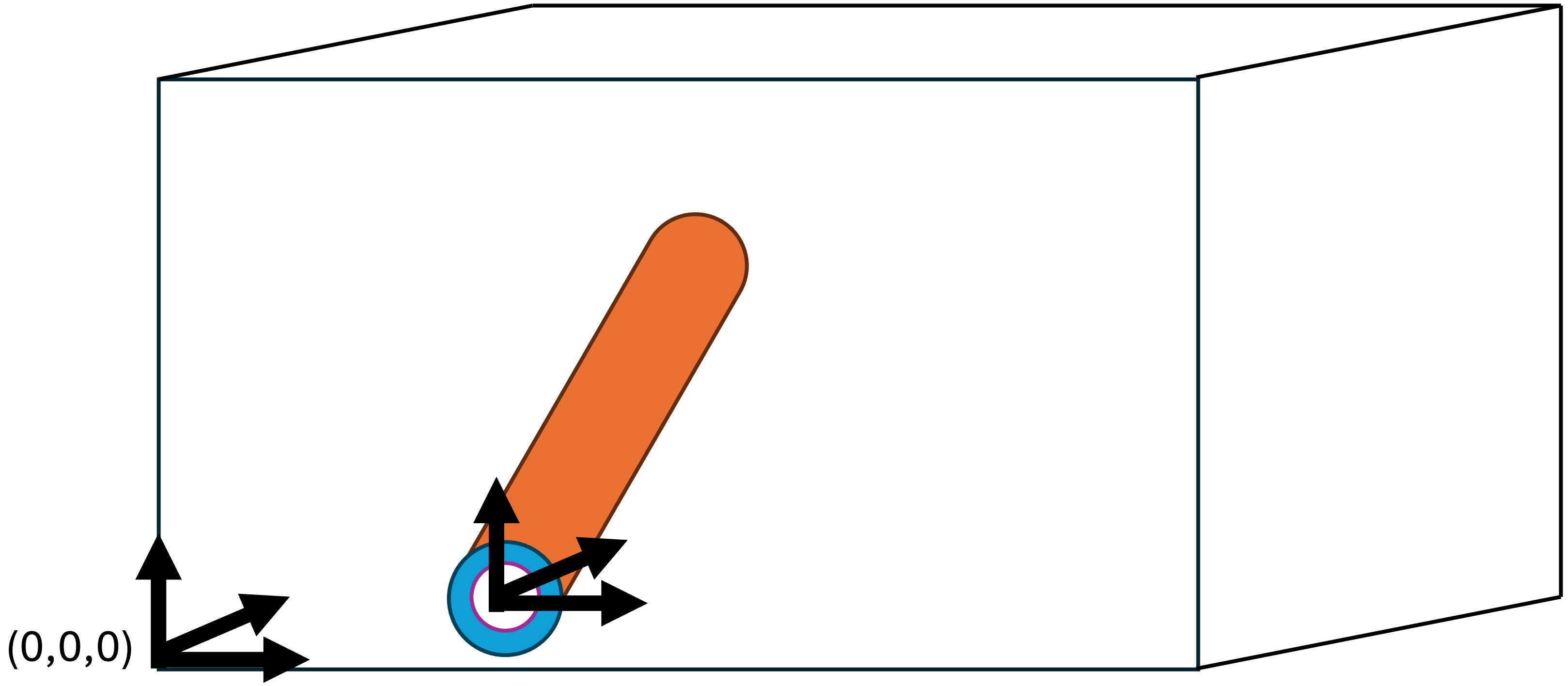
How do we represent a single-jointed robot?



One predefined point and (x,y,z,rotx,roty,rotz) wrt origin frame

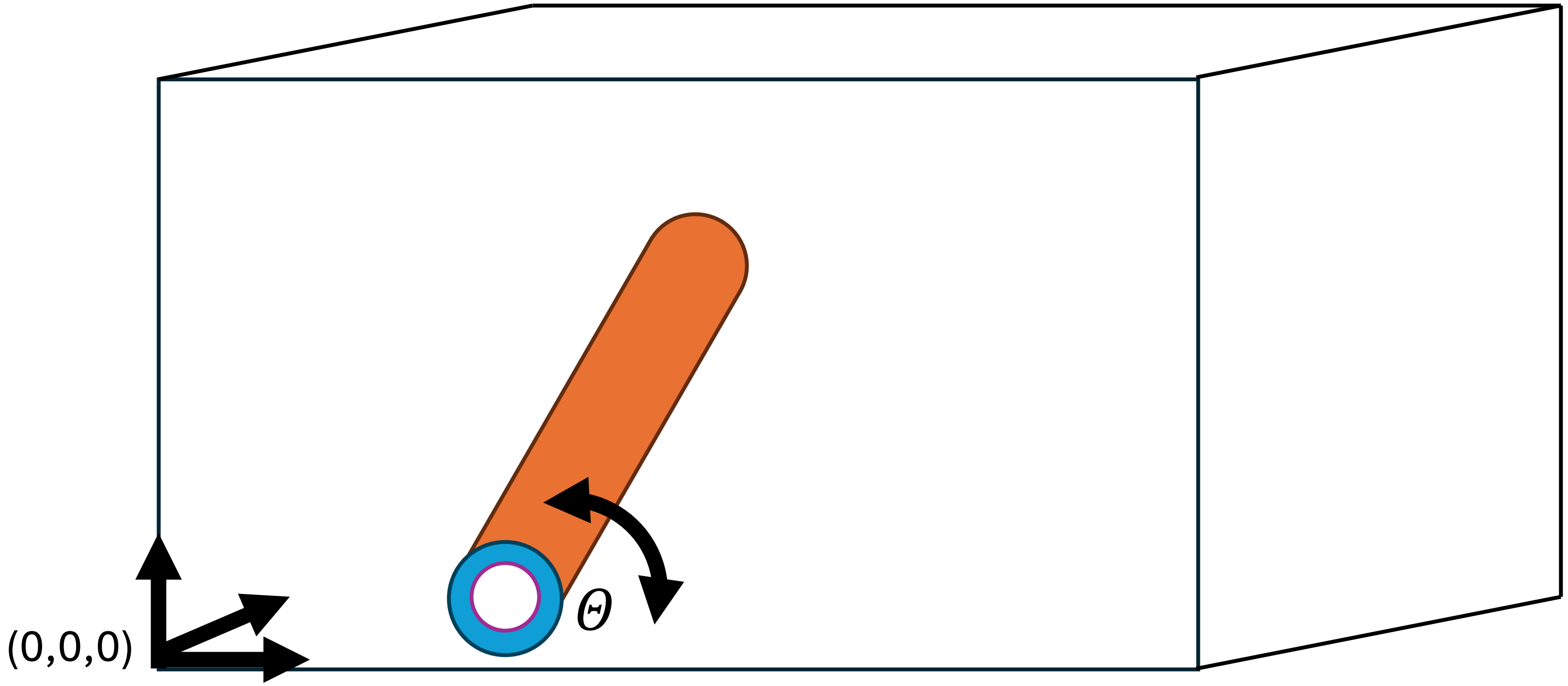
Keywords: SE(3), Frames of Reference

How do we represent a single-jointed robot?



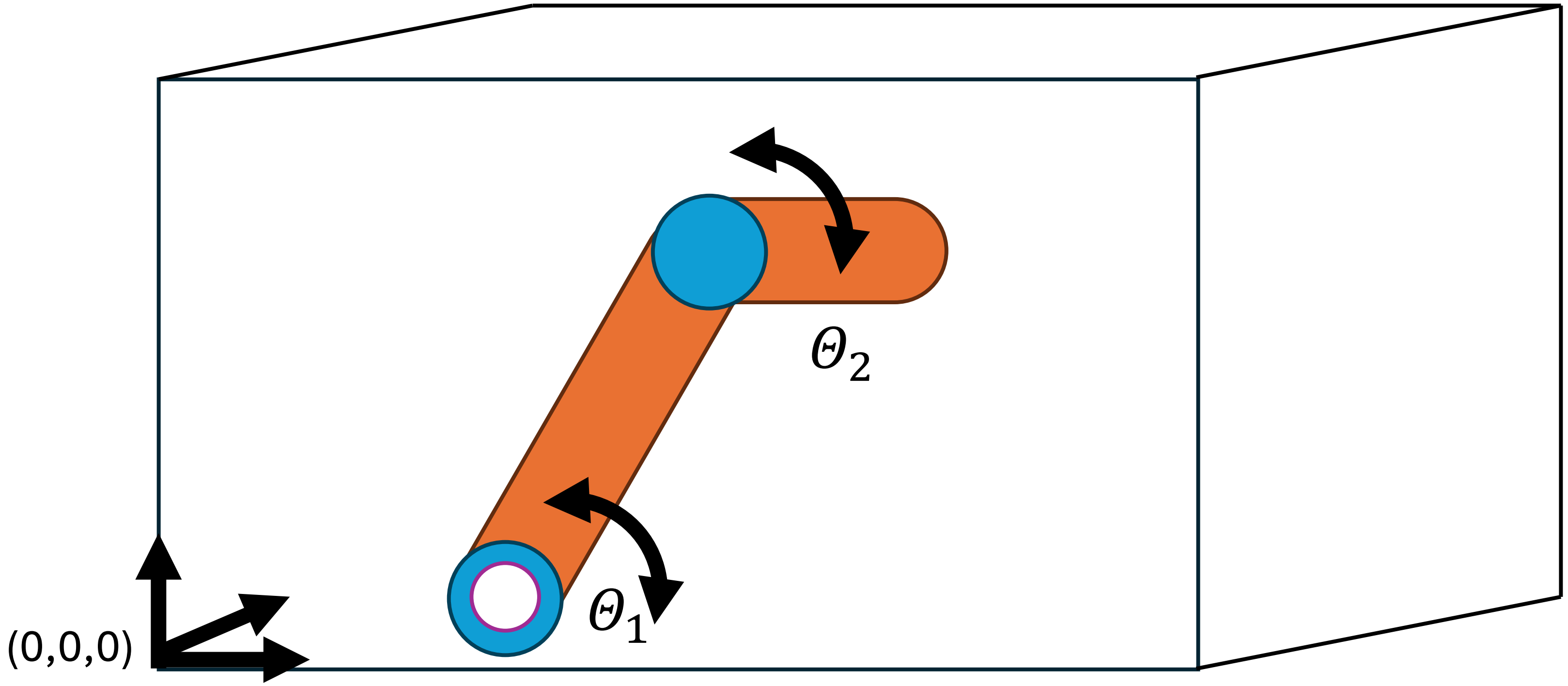
One predefined point and (x,y,z,rotx,roty,rotz) **and ...?**

How do we represent a single-jointed robot?



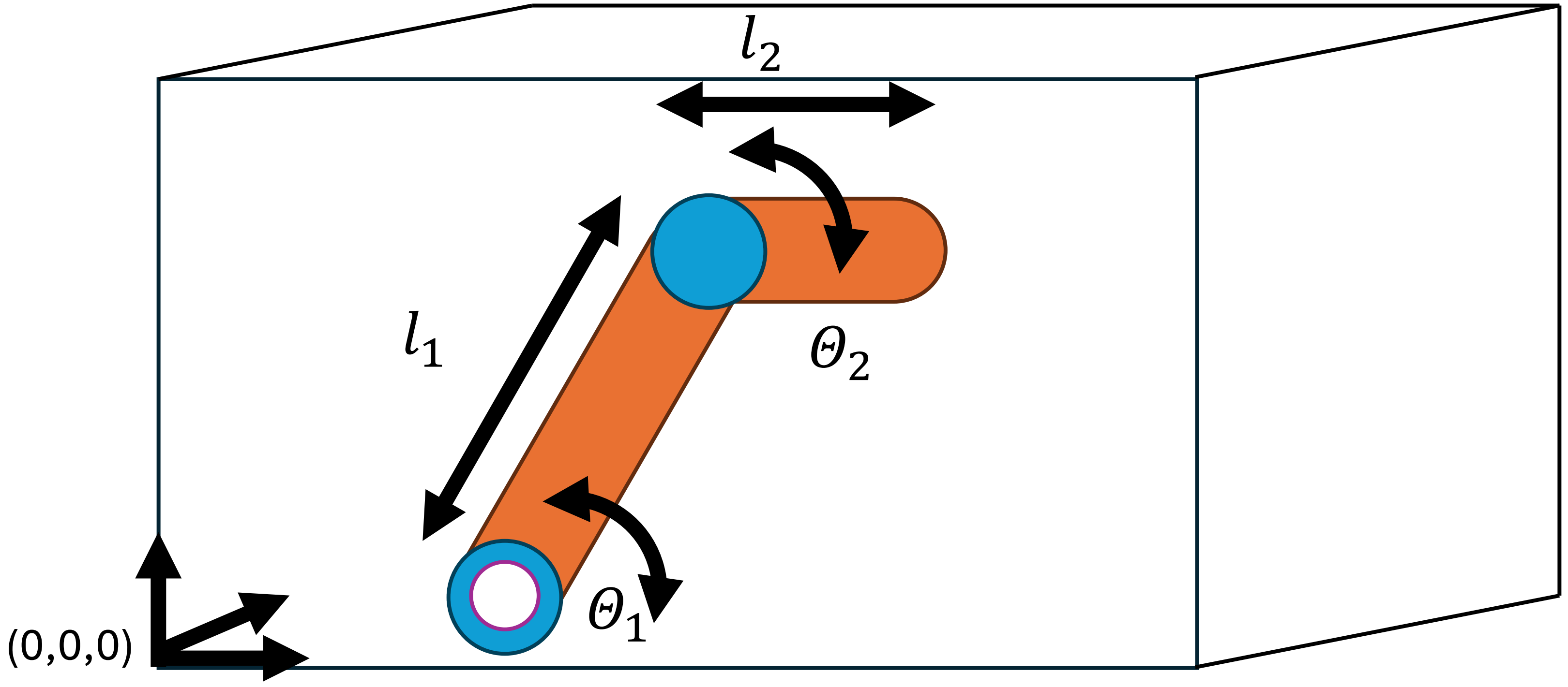
Base frame pose and angle Θ

More joints



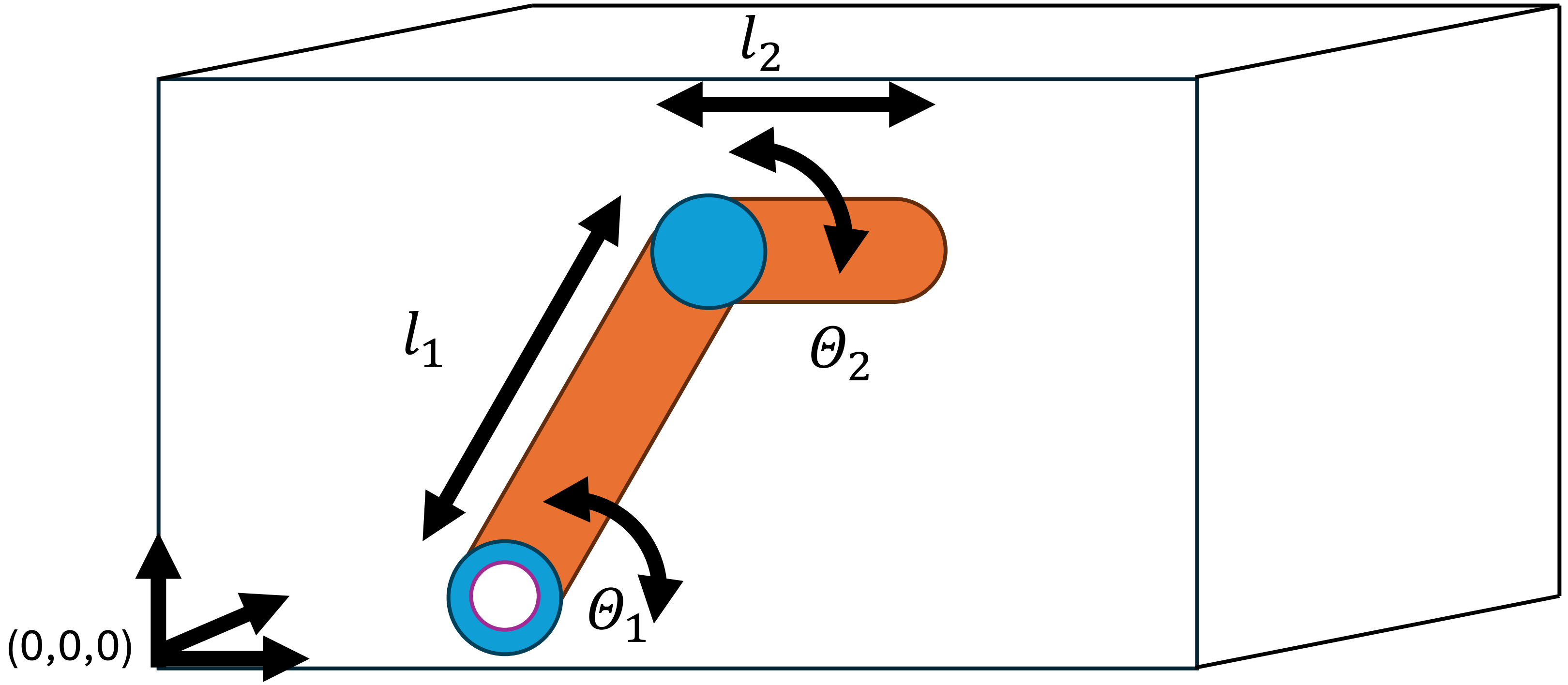
Base frame pose, angle Θ_1 **and …?**

More joints



Base frame pose, l_1, l_2, angles Θ_1, Θ_2

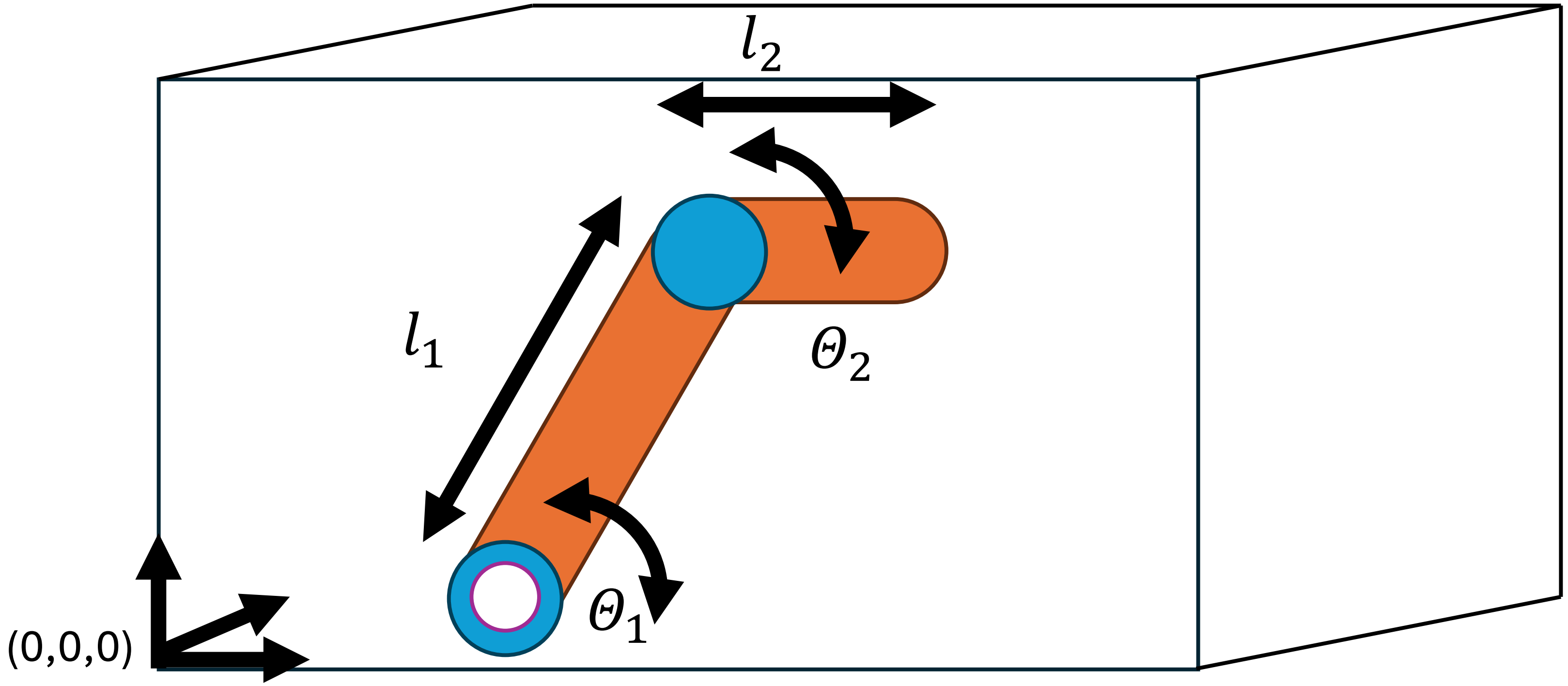
More joints



$\underbrace{\text{Base frame pose, } l_1, l_2}$ angles Θ_1, Θ_2

Model of robot, DH Parameters,
URDF, Simulation

More joints



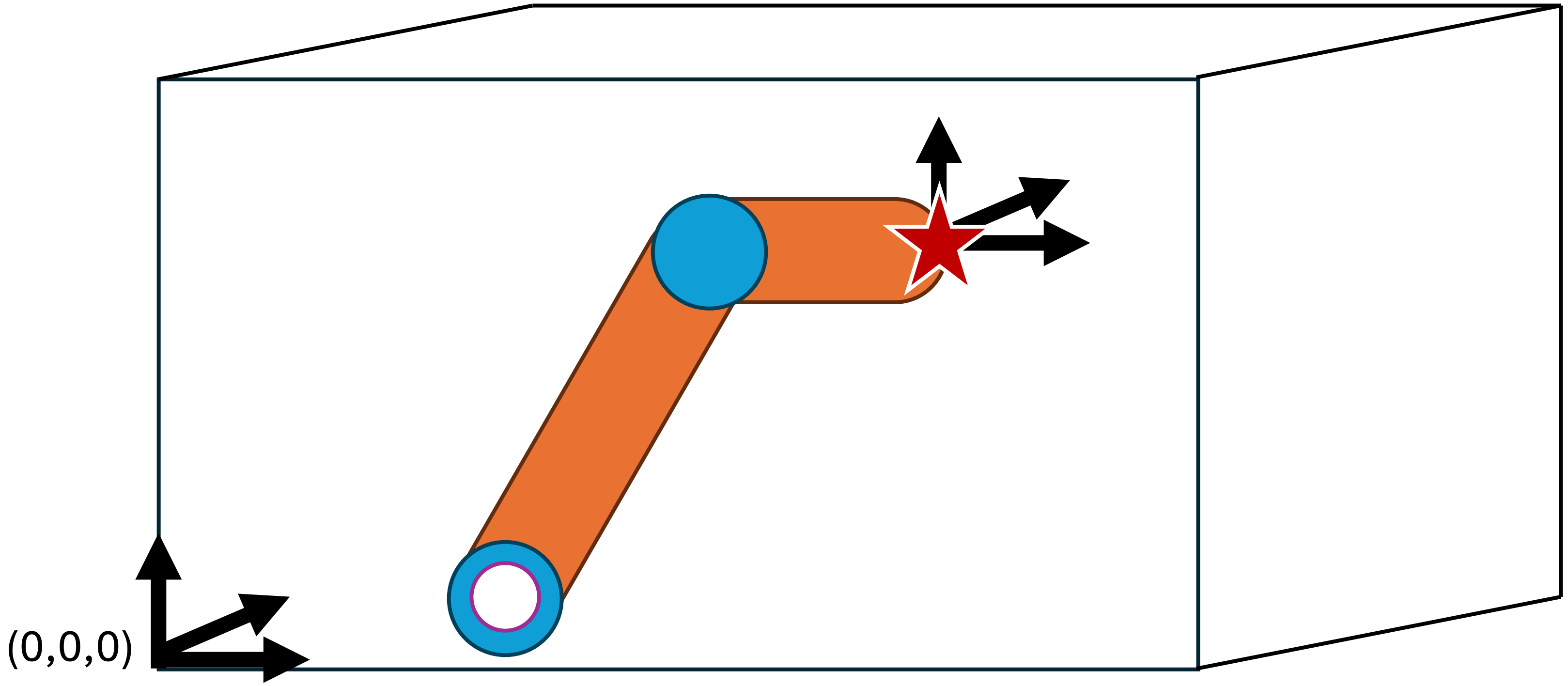
$\underbrace{\text{Base frame pose, } l_1, l_2}$

Model of robot, DH Parameters, URDF, Simulation

$\underbrace{\text{angles } \Theta_1, \Theta_2}$

Degrees of freedom (DoF)

Forward Kinematics

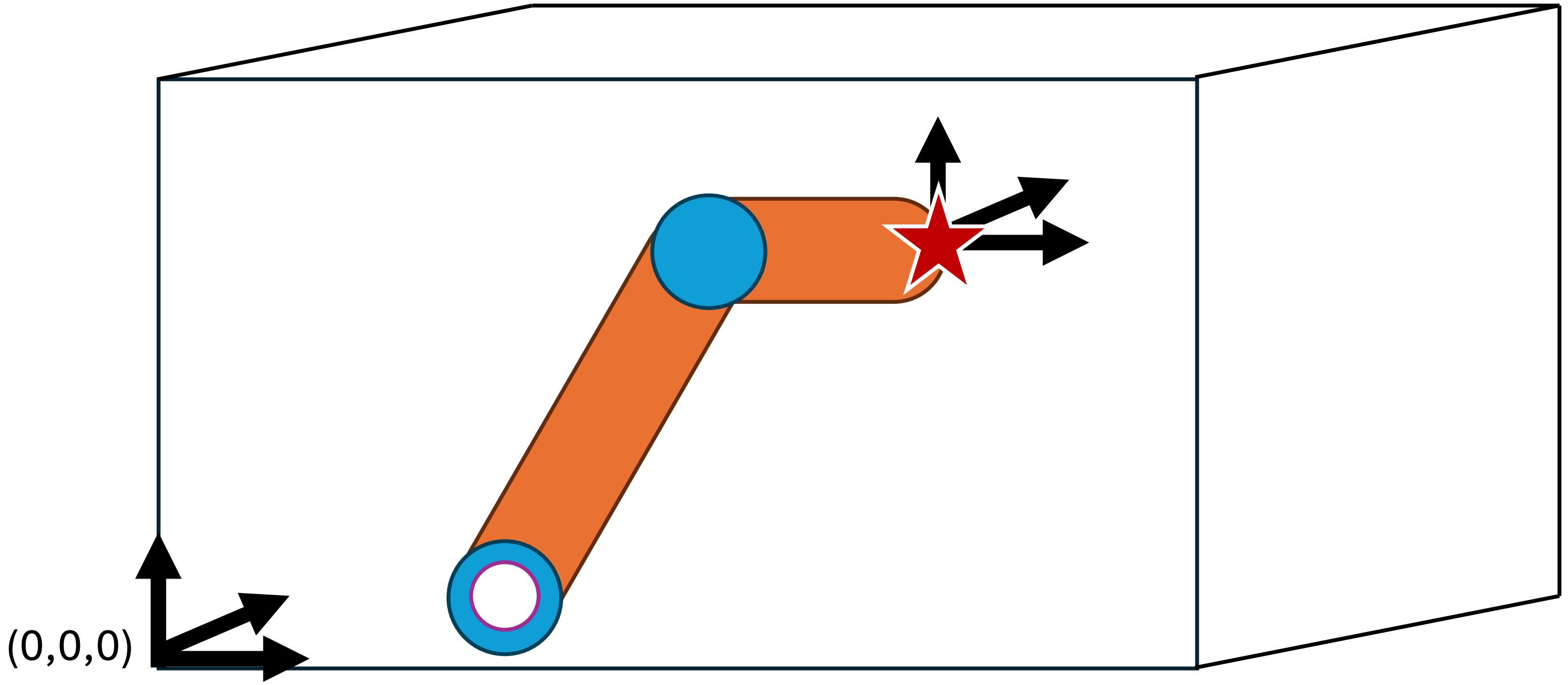


Forward Kinematics – given joint angles what is the pose of a part of the robot?

Θ_1, Θ_2

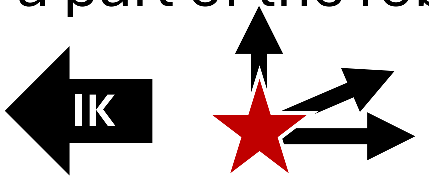
Keywords: FK

Inverse Kinematics



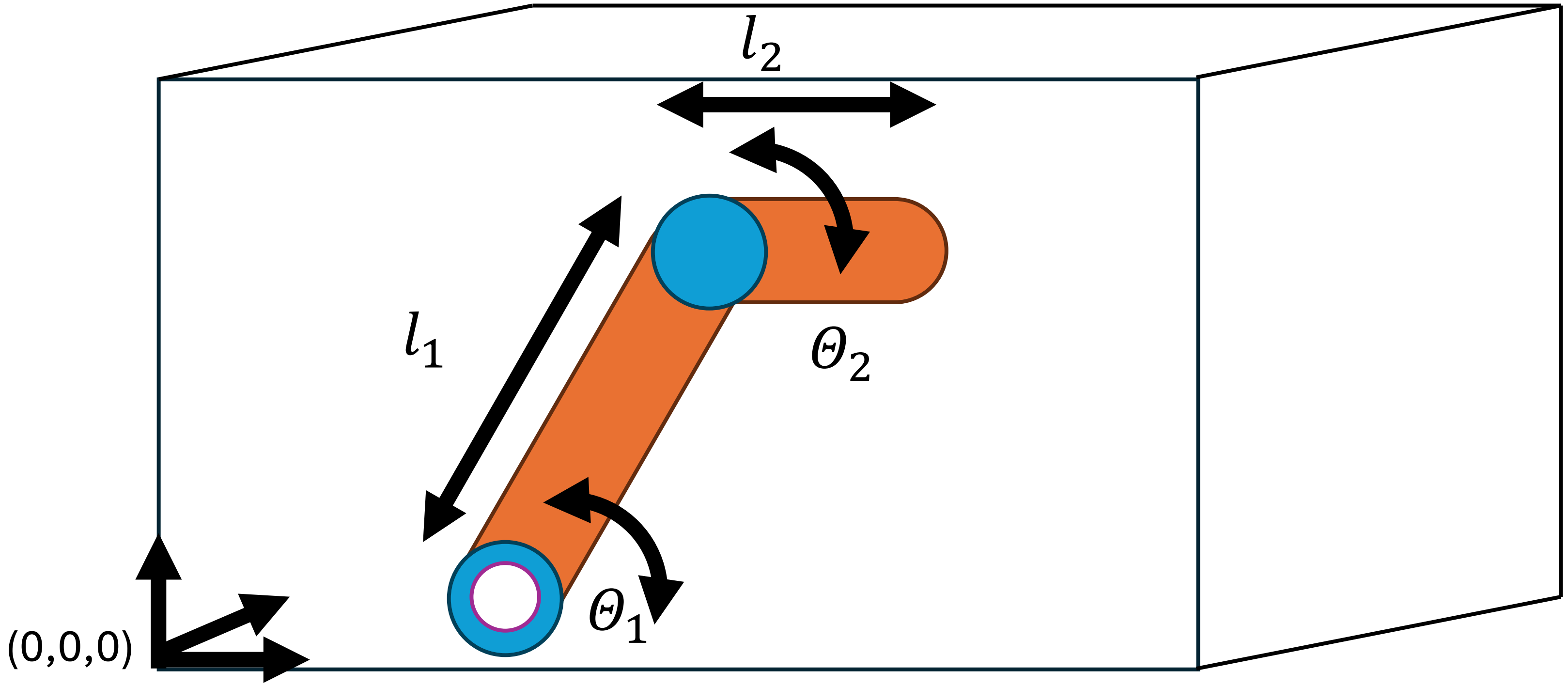
Inverse Kinematics – given pose of a part of the robot what are joint angles?

Θ_1, Θ_2



Keywords: IK

What changes in the robot?



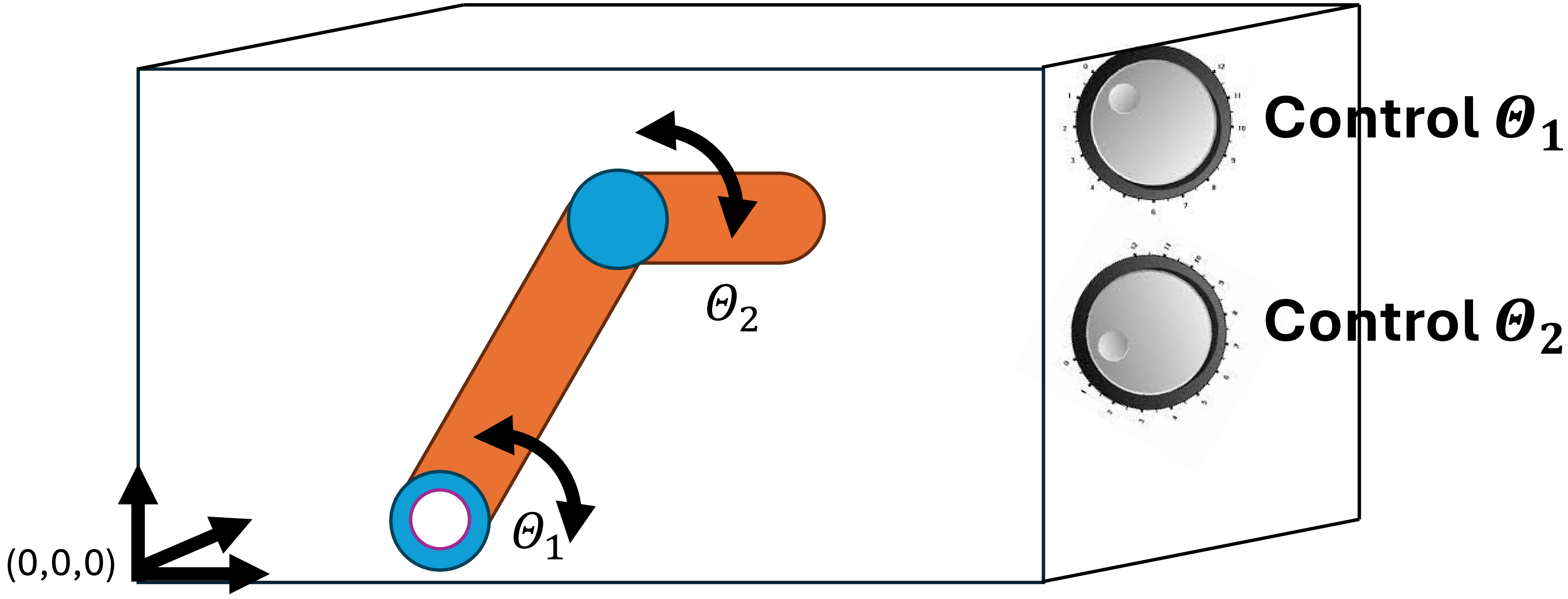
$\underbrace{\text{Base frame pose, } l_1, l_2}$

Model of robot, DH Parameters, URDF, Simulation

$\underbrace{\textbf{angles } \boldsymbol{\Theta_1}, \boldsymbol{\Theta_2}}$

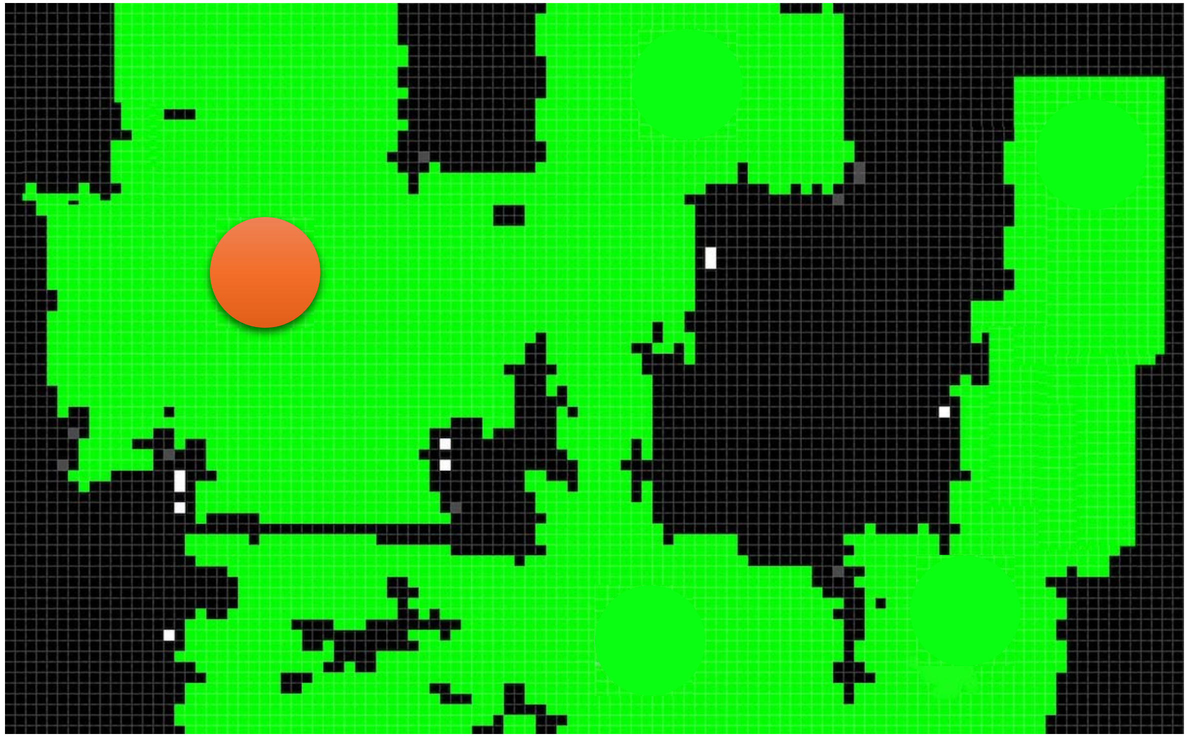
Degrees of freedom (DoF)

2D configuration of a robot



θ_1, θ_2

2D configuration of a robot

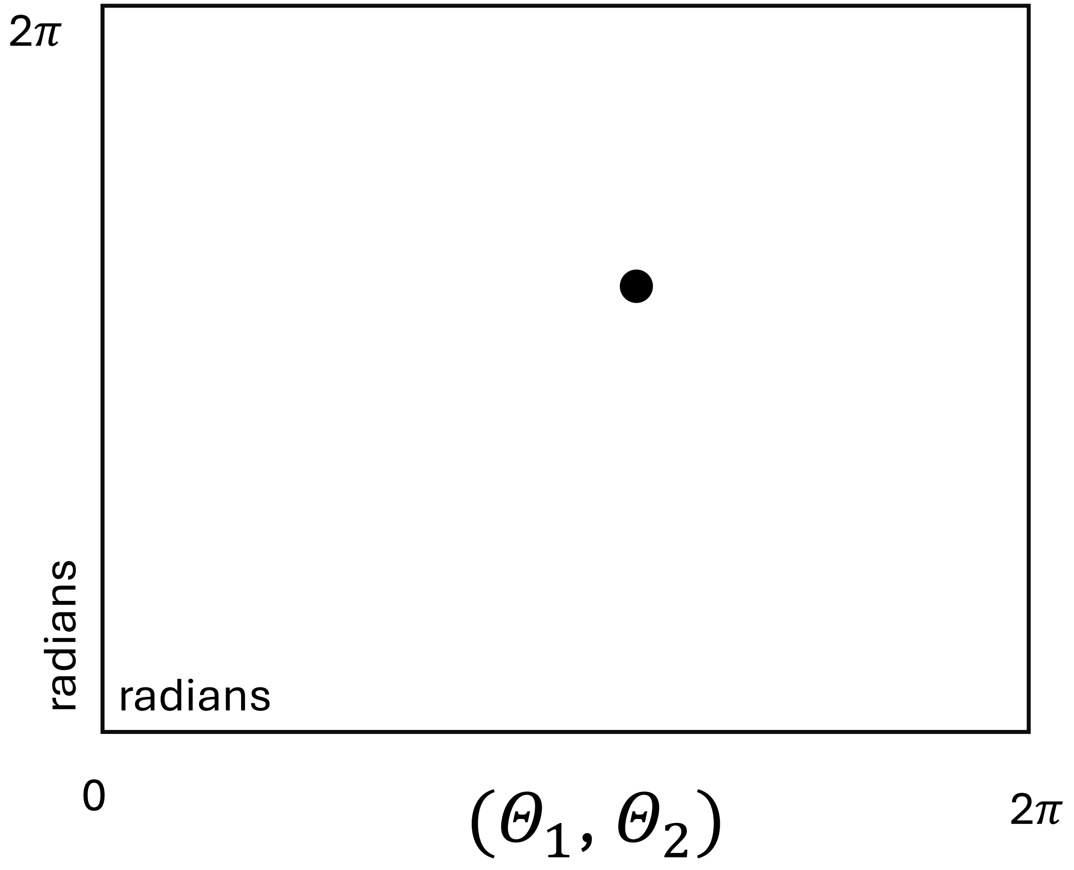


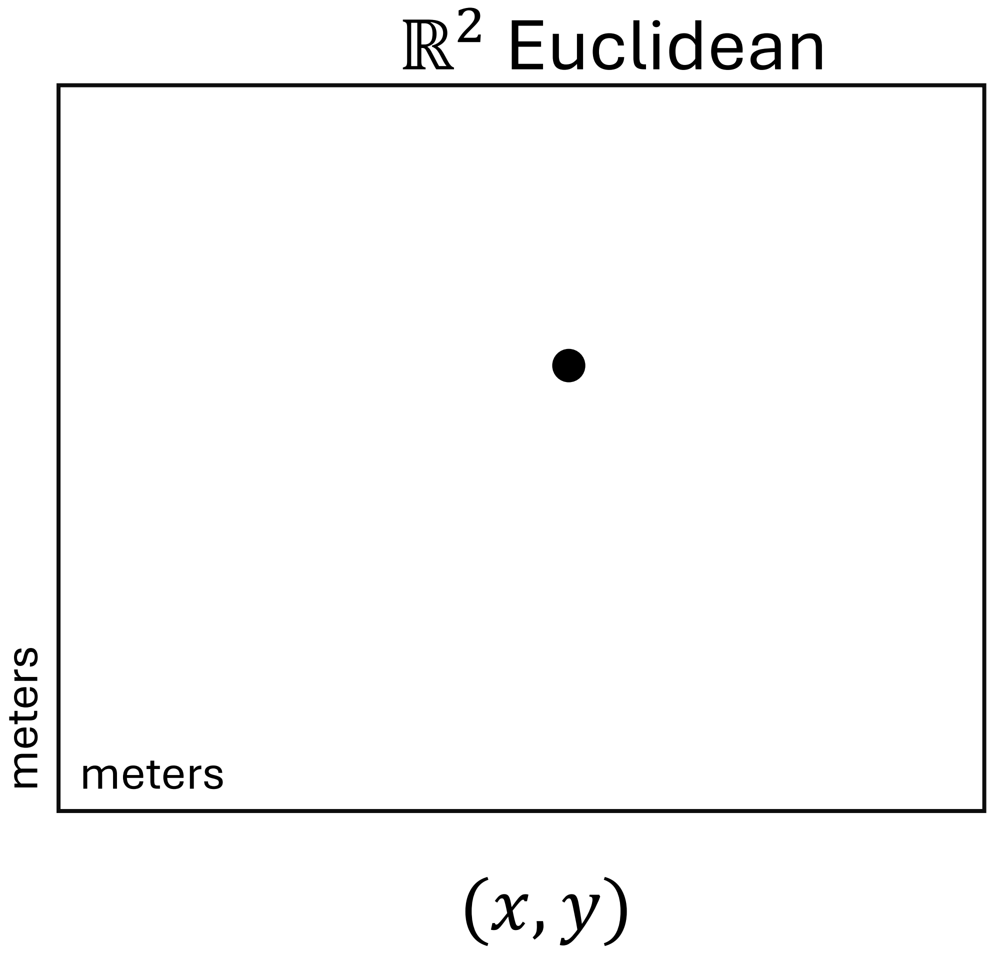
A circular robot on a floorplan

Control X

Control Y

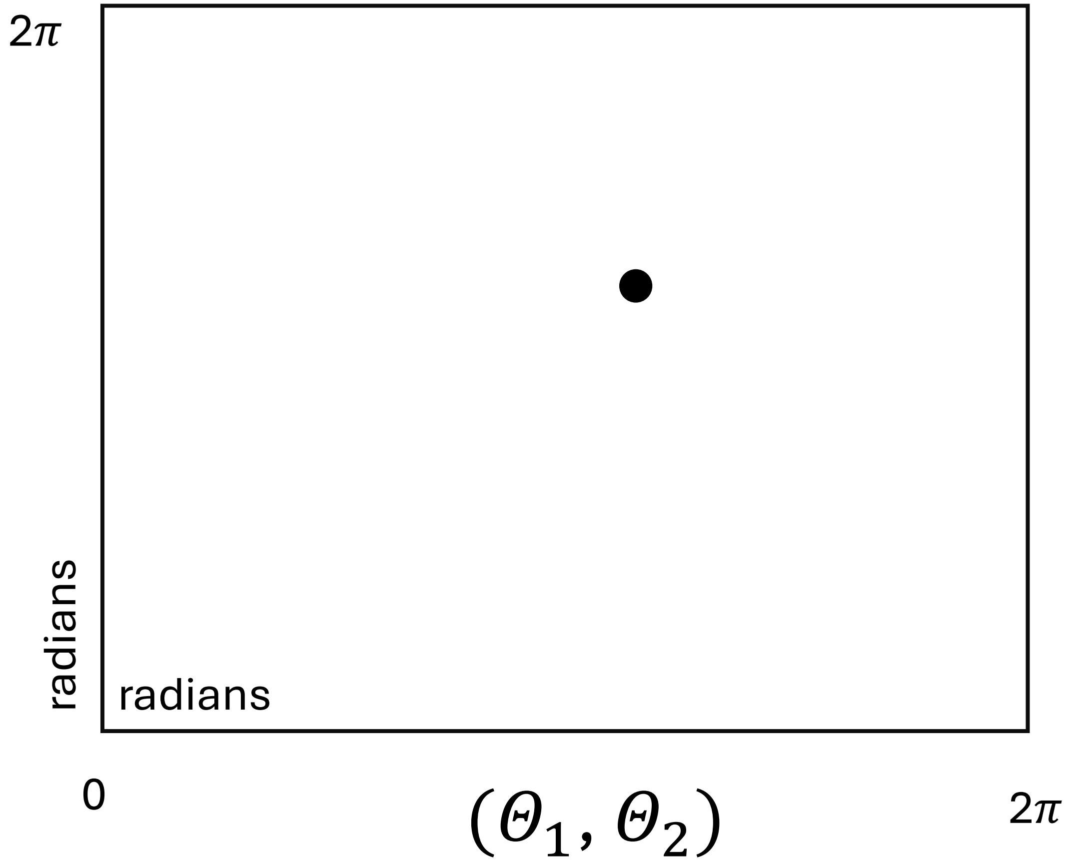
Topology of the Space



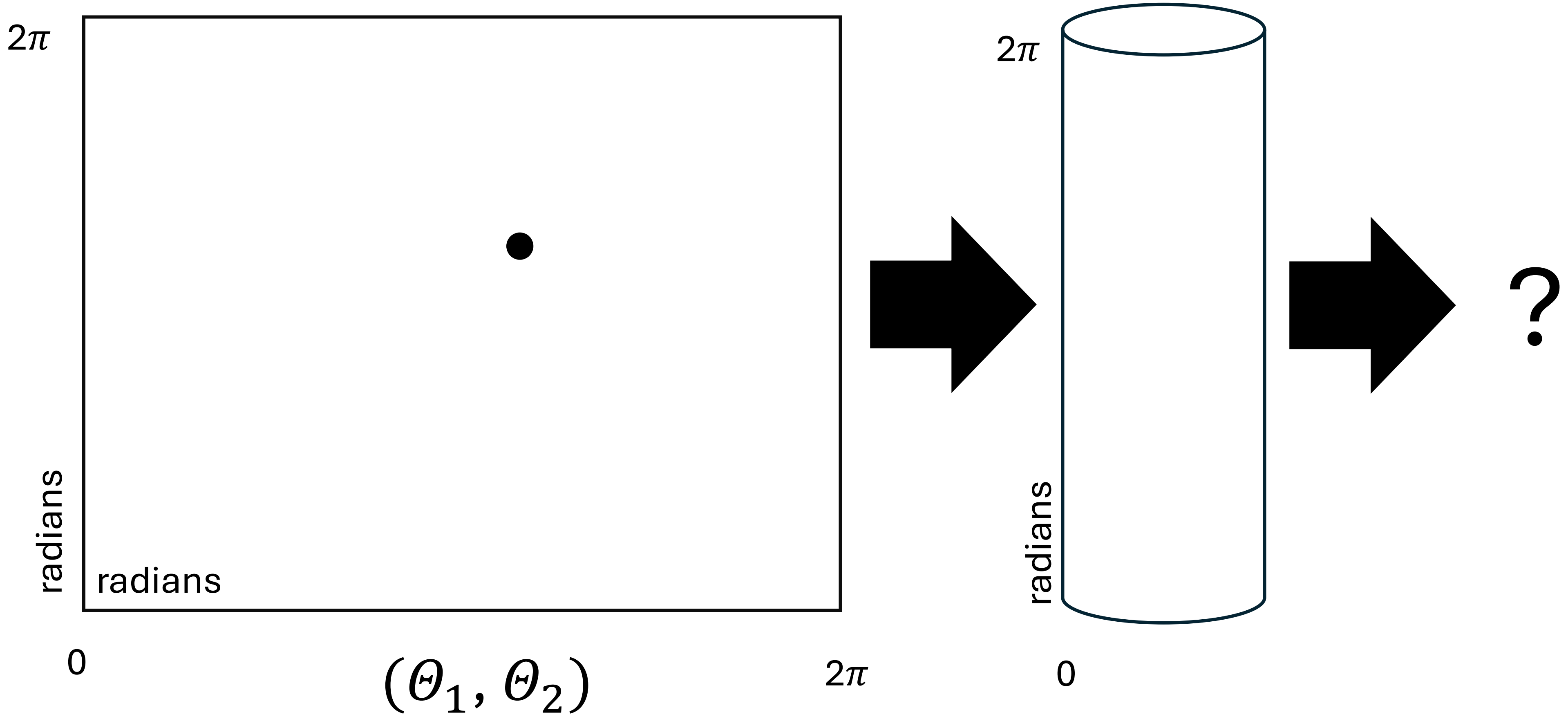


Keywords: Topology

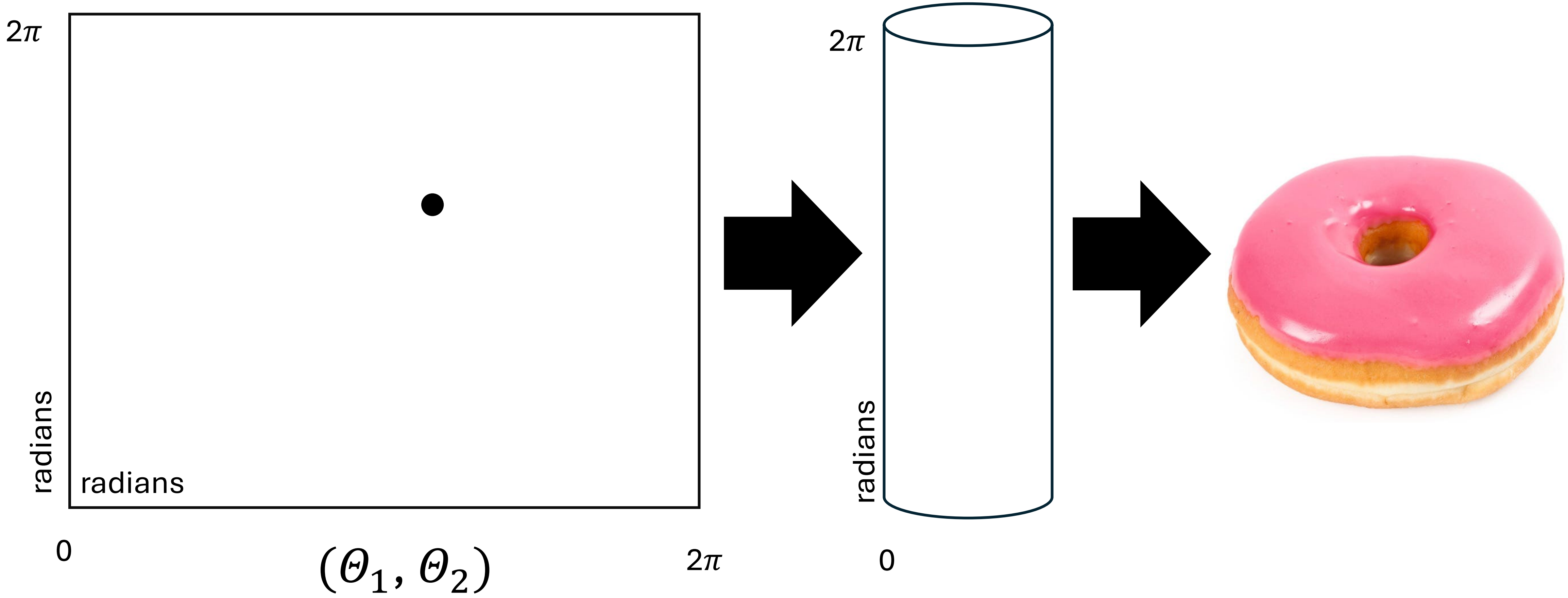
Topology of the Space



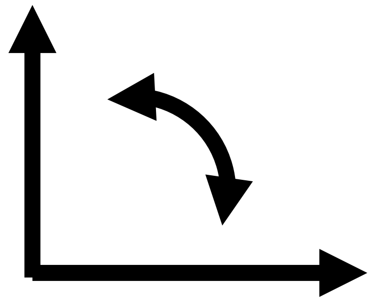
Topology of the Space



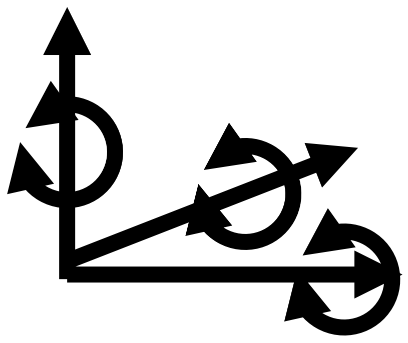
Topology of the Space



More Spaces

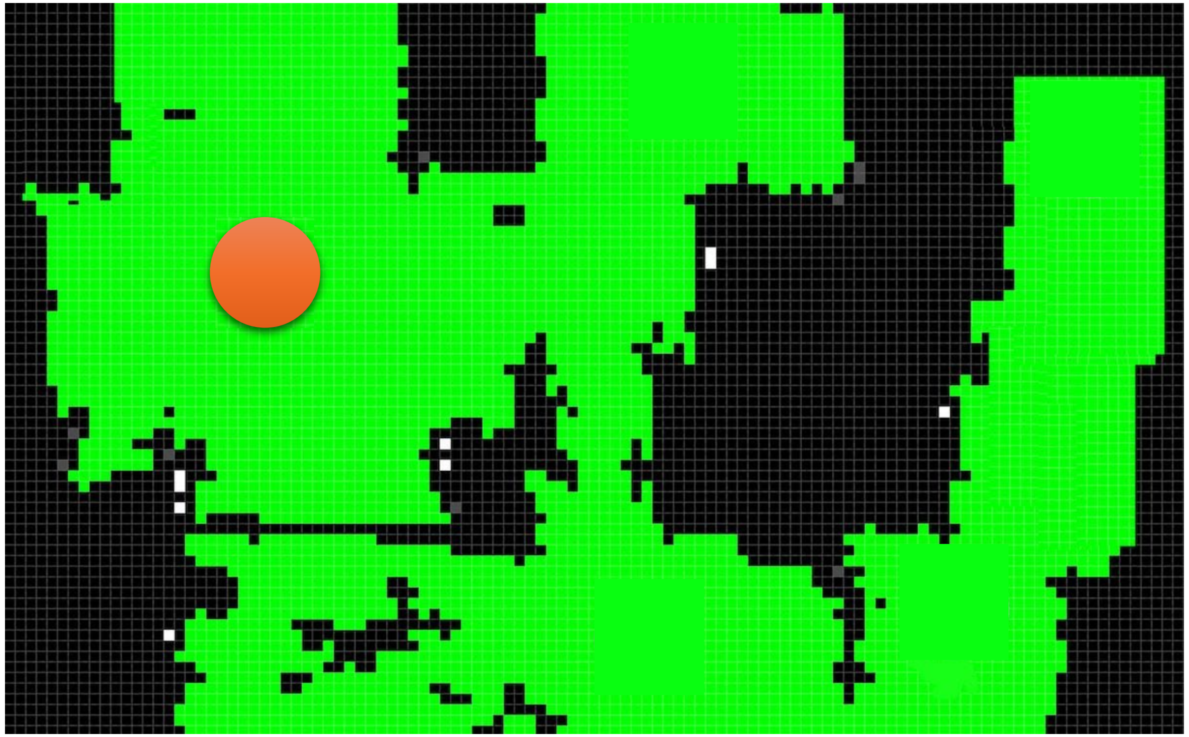


(x, y, θ) SE(2)



$(x, y, z, \theta_x, \theta_y, \theta_z)$ SE(3) poses

Configuration Spaces

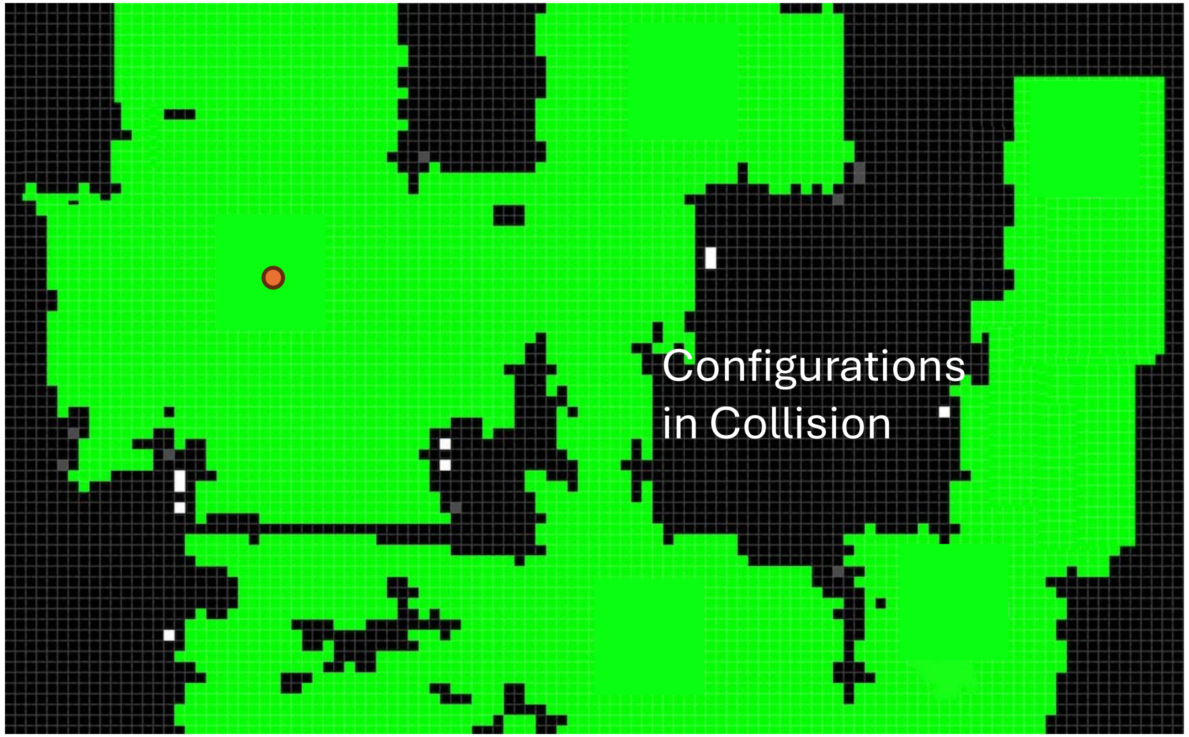


Is this a configuration space?
What are the dark regions?

A robot configuration is a **point** in the configuration space.

Lozano-Perez, Tomas. *Spatial planning: A configuration space approach*. Springer New York, 1990.

Configuration Spaces

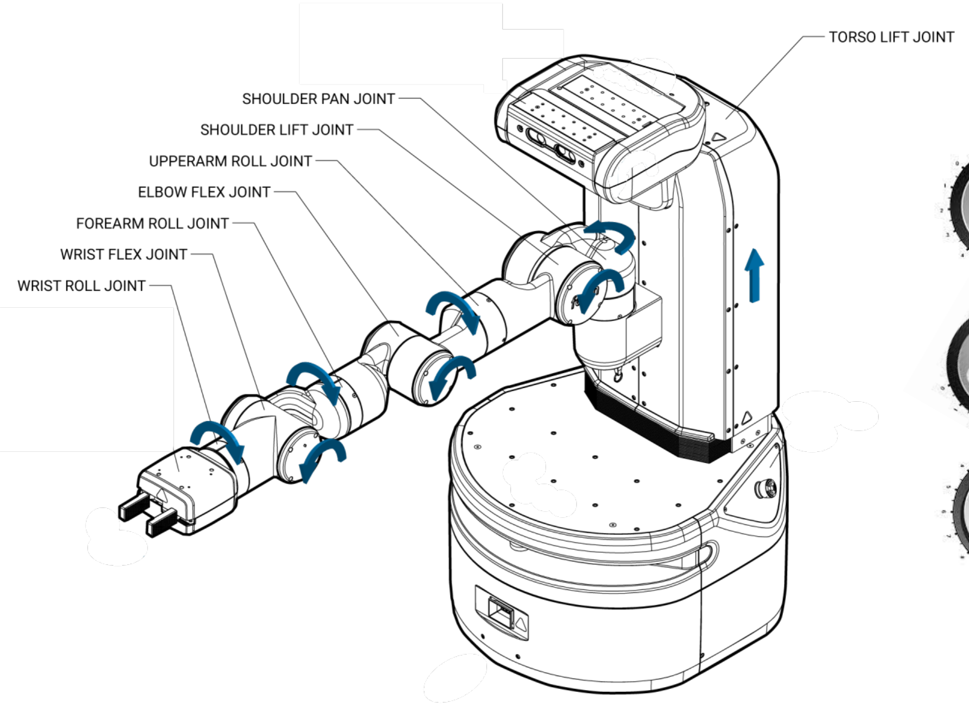


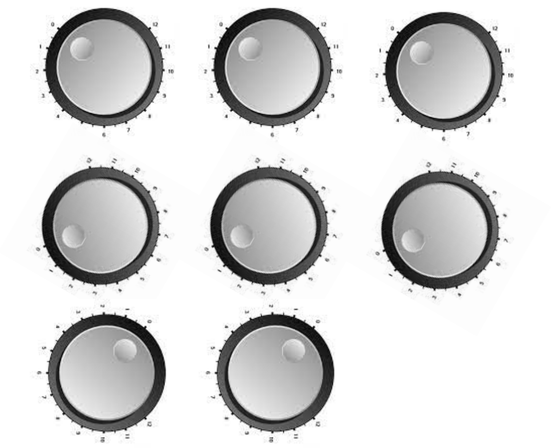
Feasible subset of configuration space.

A robot configuration is a point in the configuration space.

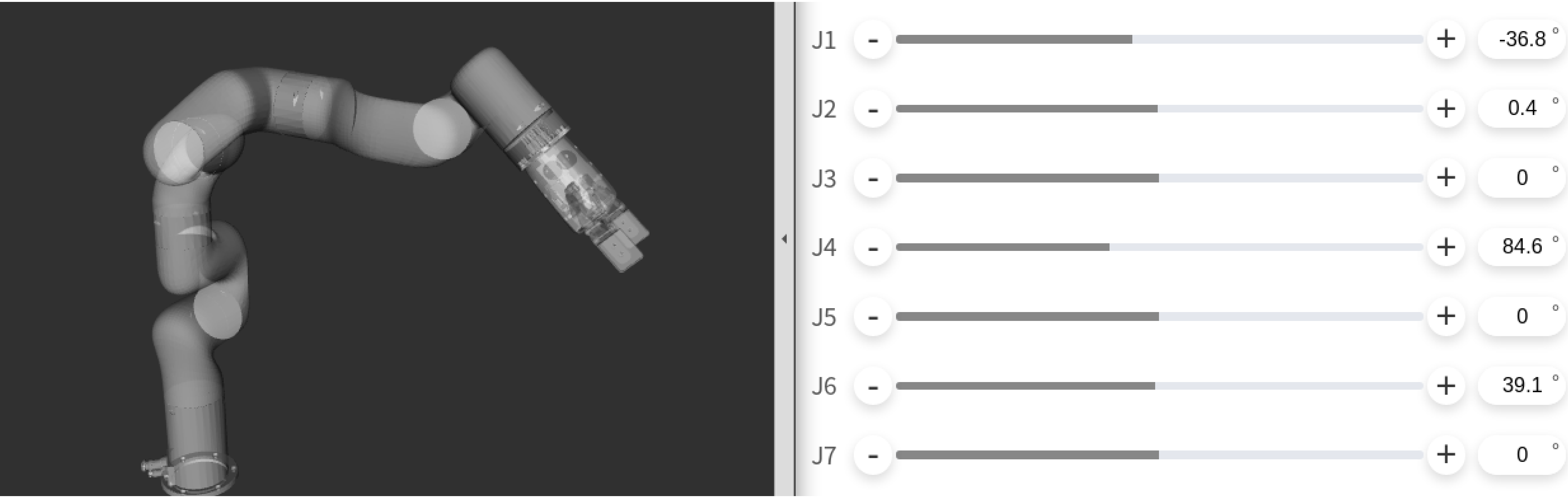
Keywords: Minkowski Sum, Collisions, Feasibility

Dimensionality of Configuration Spaces

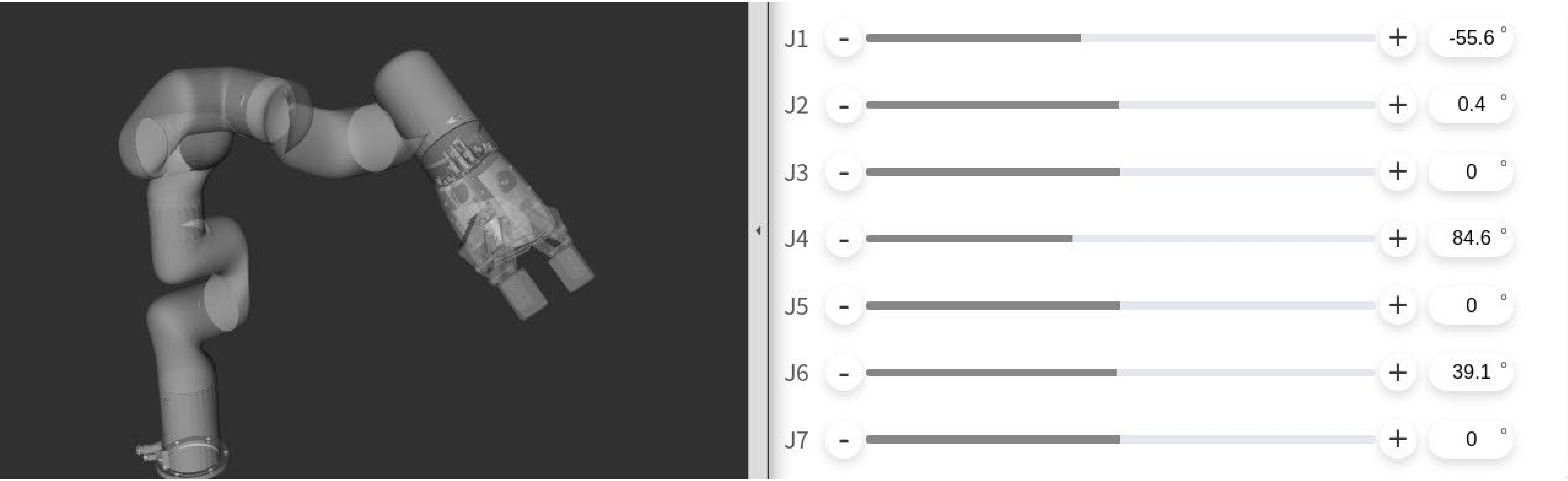




More the joints, more the dimensions.



7-dim configuration: (J1,J2,J3,J4,J5,J6,J7)



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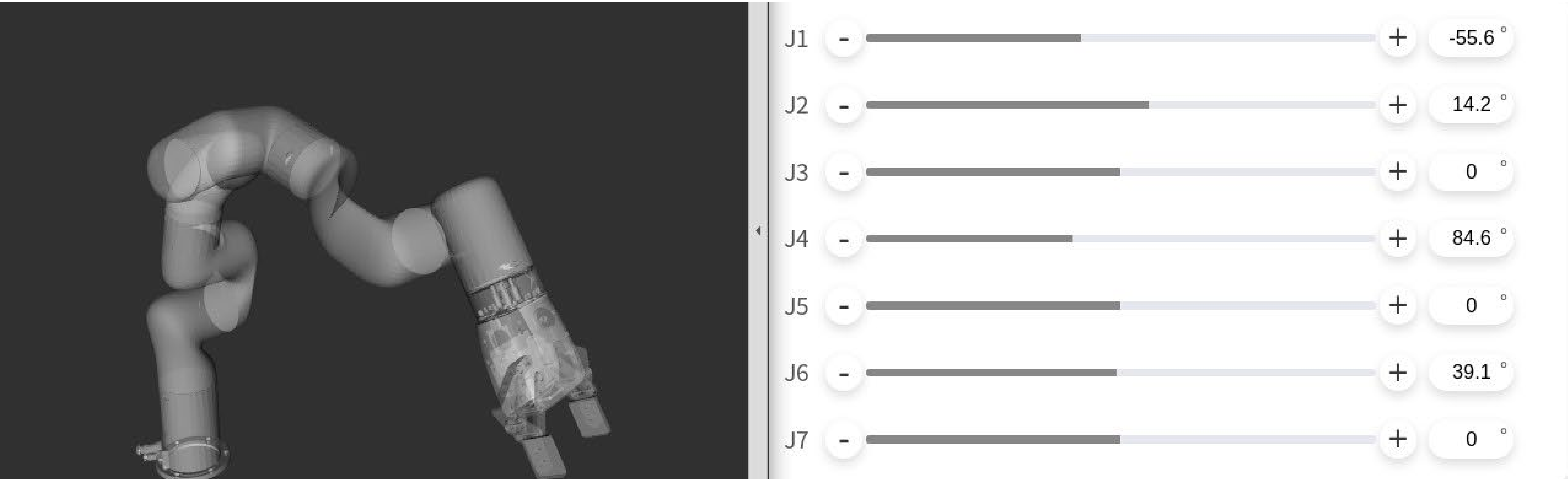

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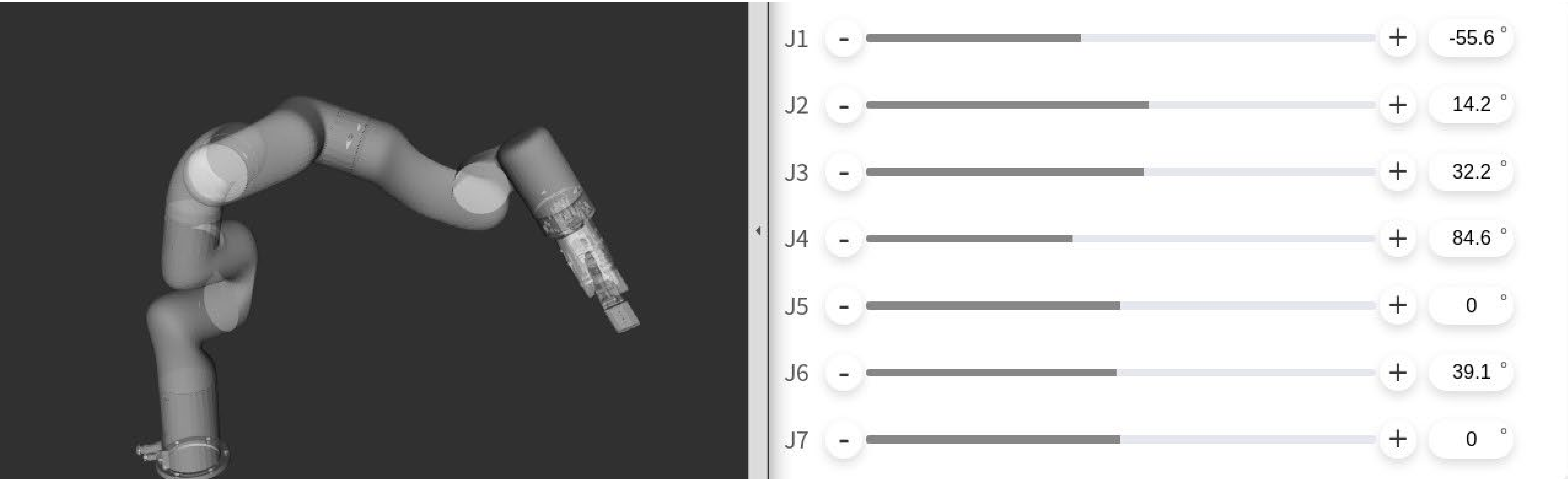

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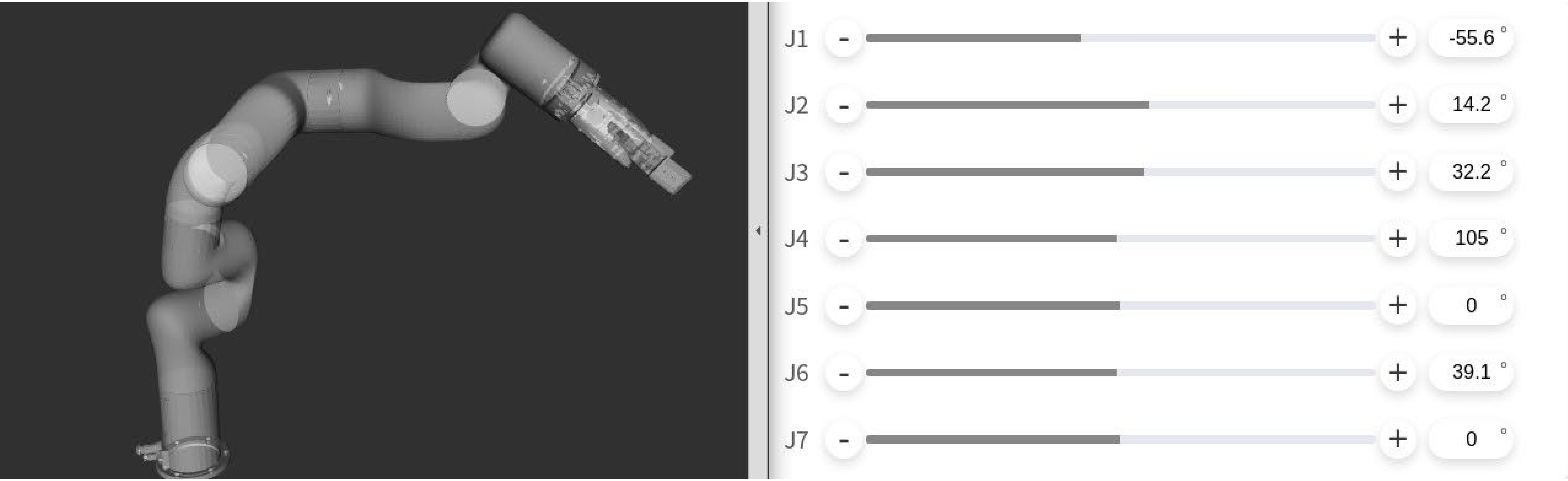
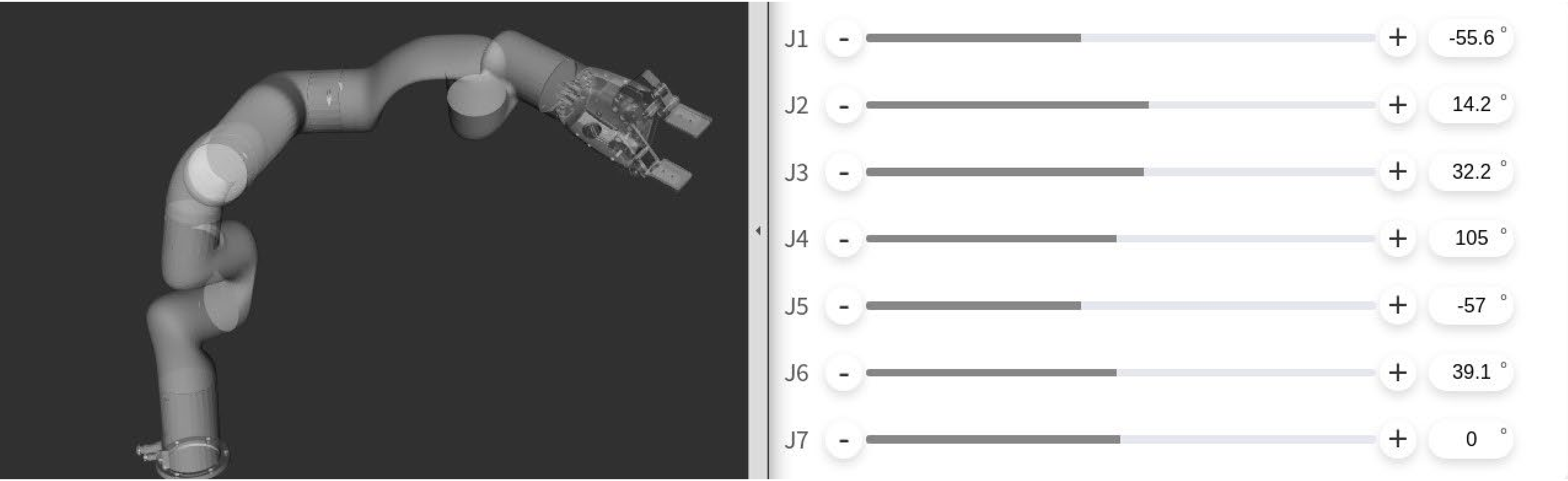
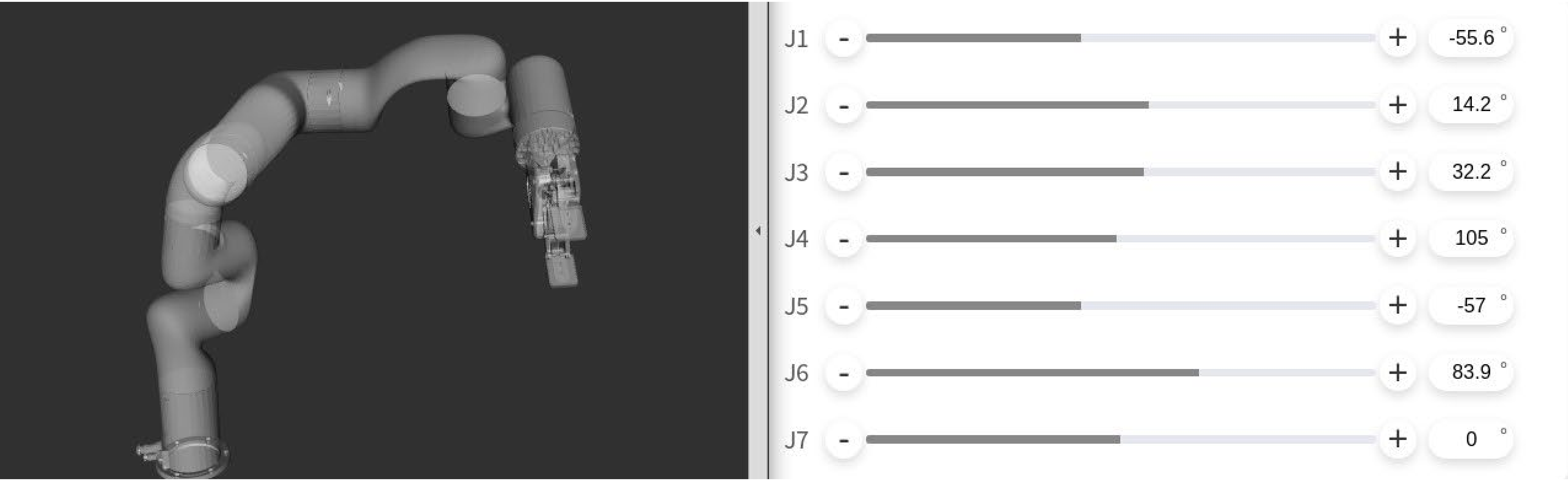


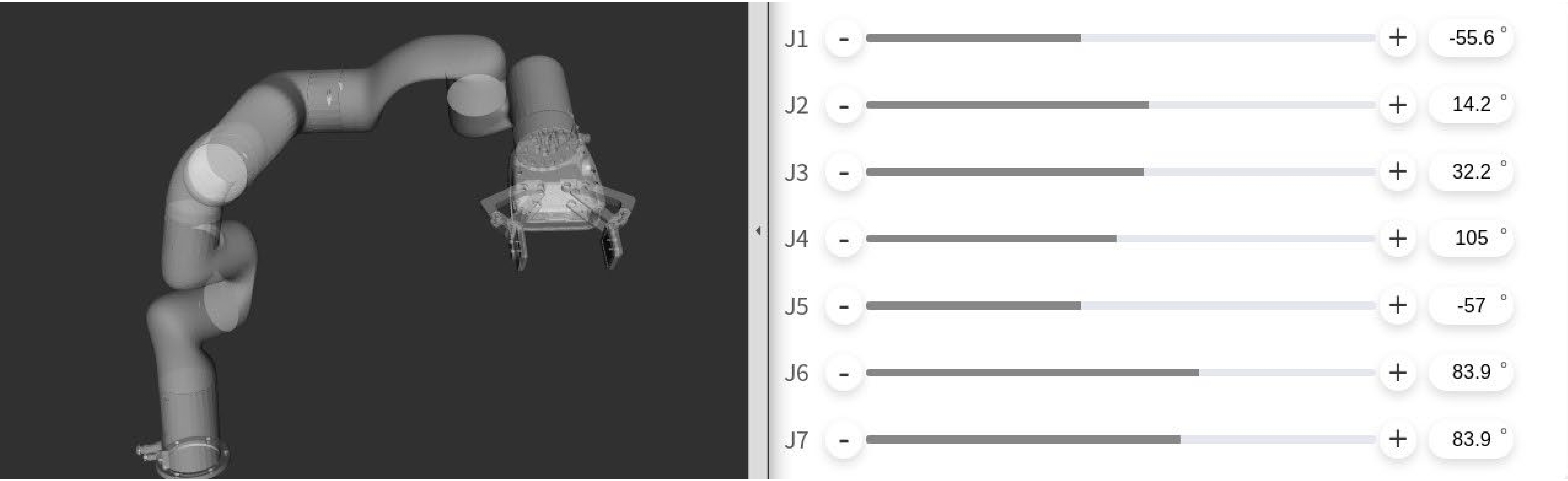
7-dim configuration: (J1,J2,J3,J4,J5,J6,J7)



7-dim configuration: (J1,J2,J3,J4,J5,J6,J7)

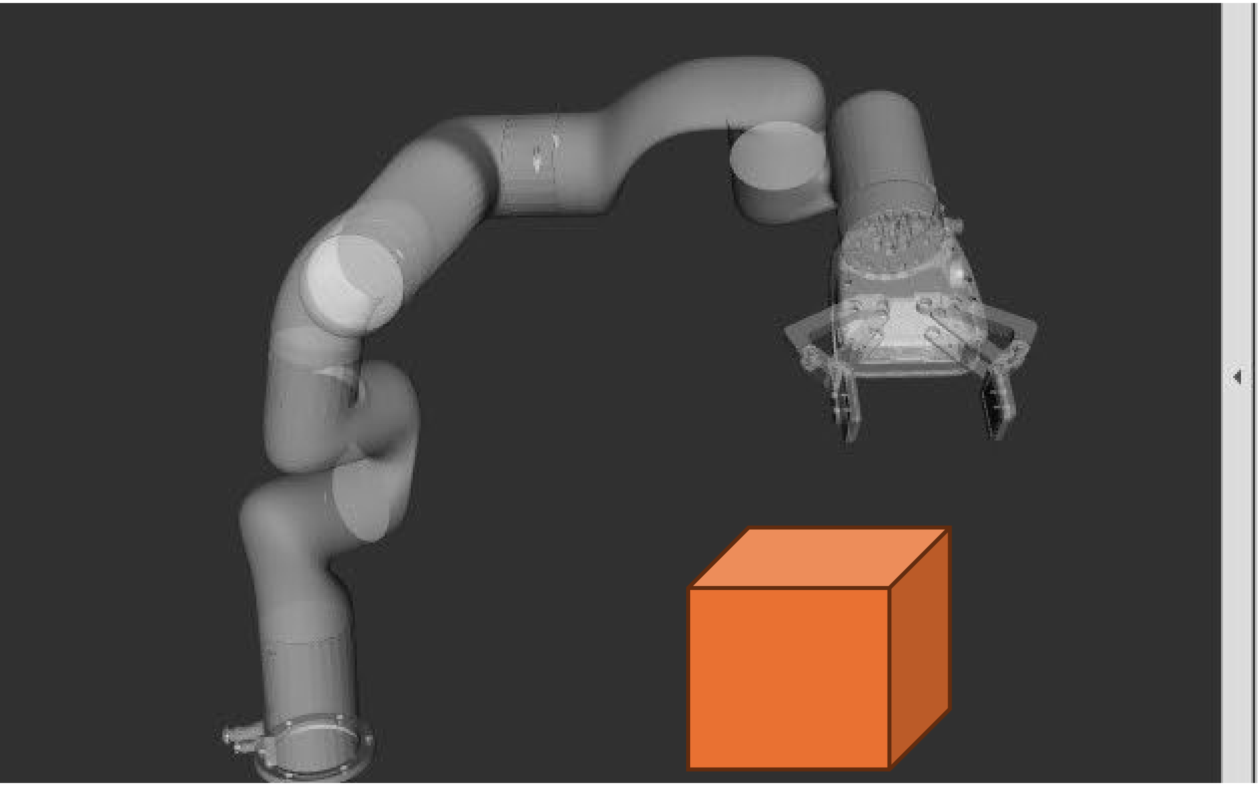


7-dim configuration: (J1,J2,J3,J4,J5,J6,J7)

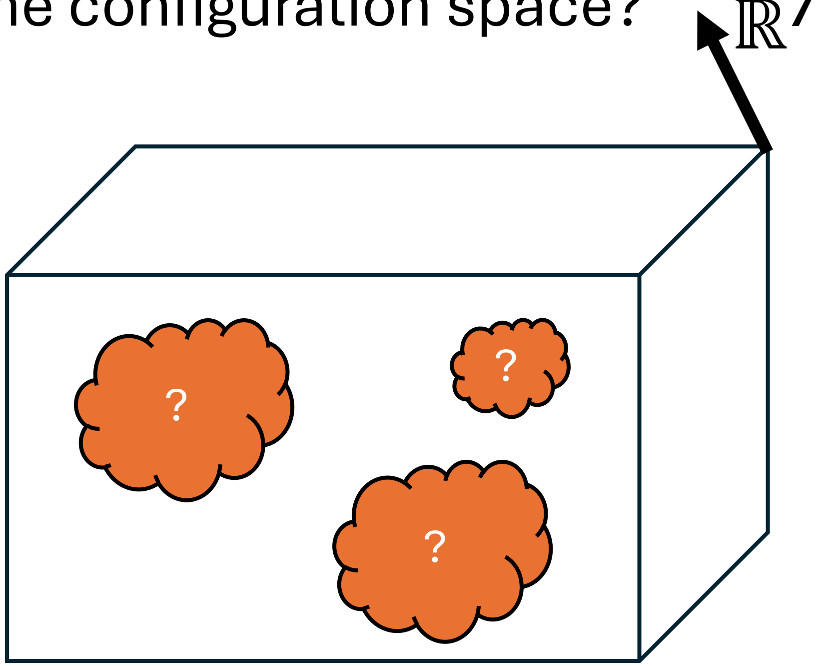


High dimensions cause combinatorial explosion of search space

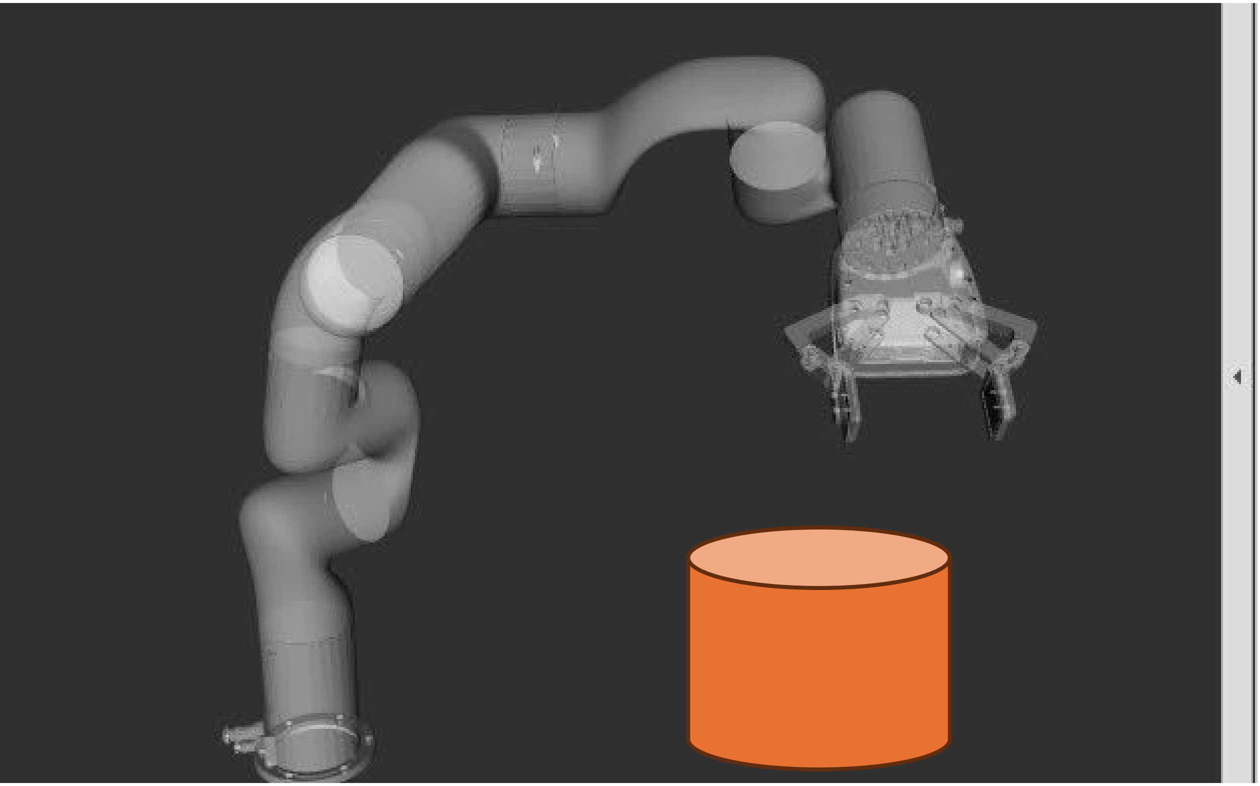
Keywords: Search Space, Combinatorial Explosion



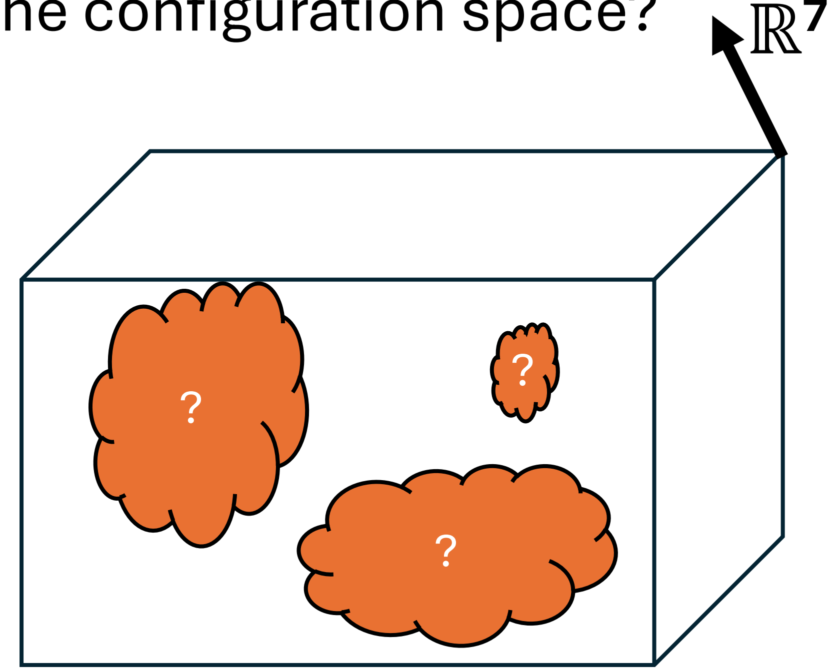
What does the cube look like in the configuration space?



Set of colliding configurations in high-dimensional configuration spaces.



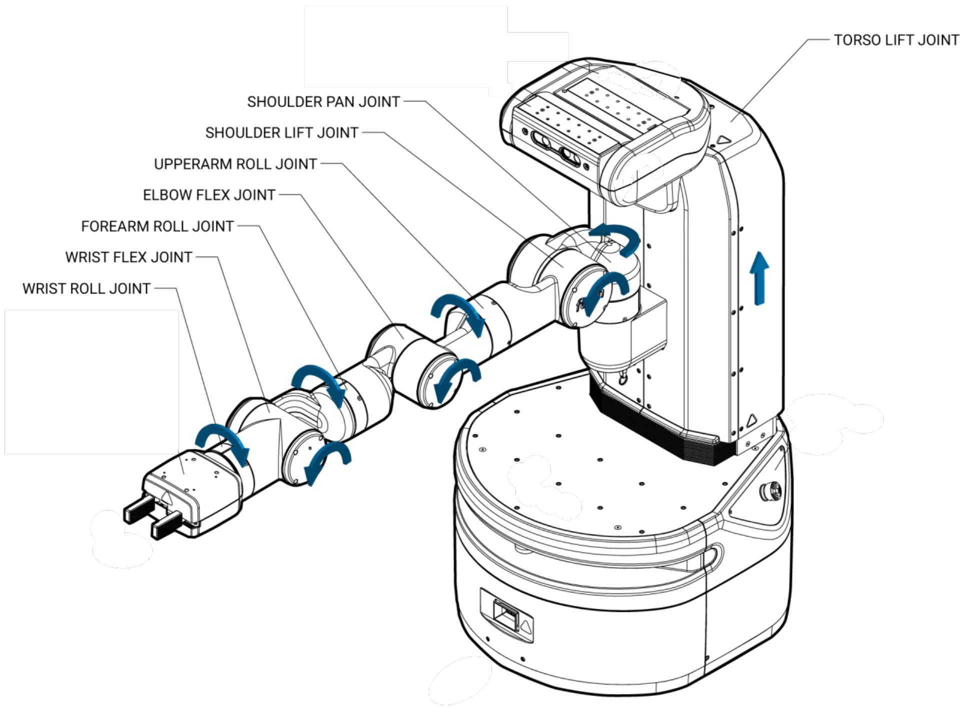
What does the cylinder look like in the configuration space?

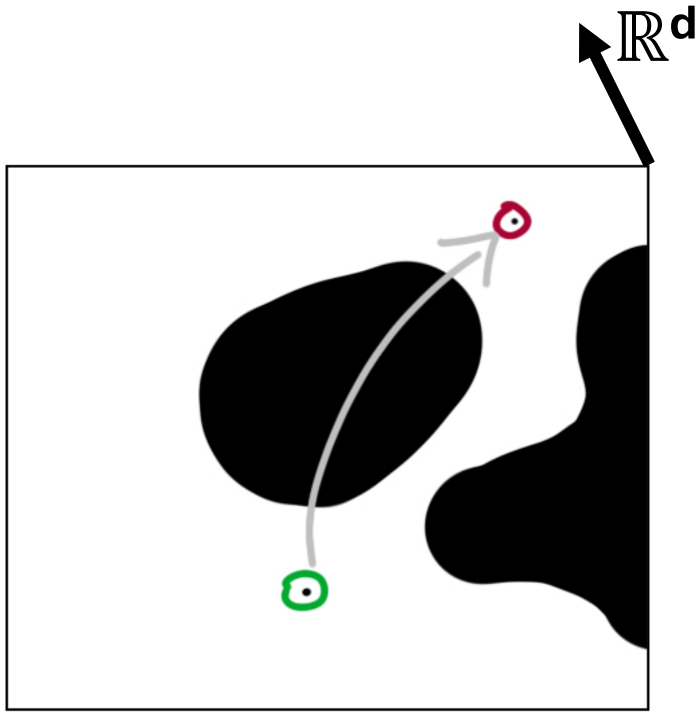


Set of colliding configurations in high-dimensional configuration spaces.

Motion Planning
in
High-Dimensional Configuration Spaces

Motion Planning
in High-dimensional Configuration Spaces



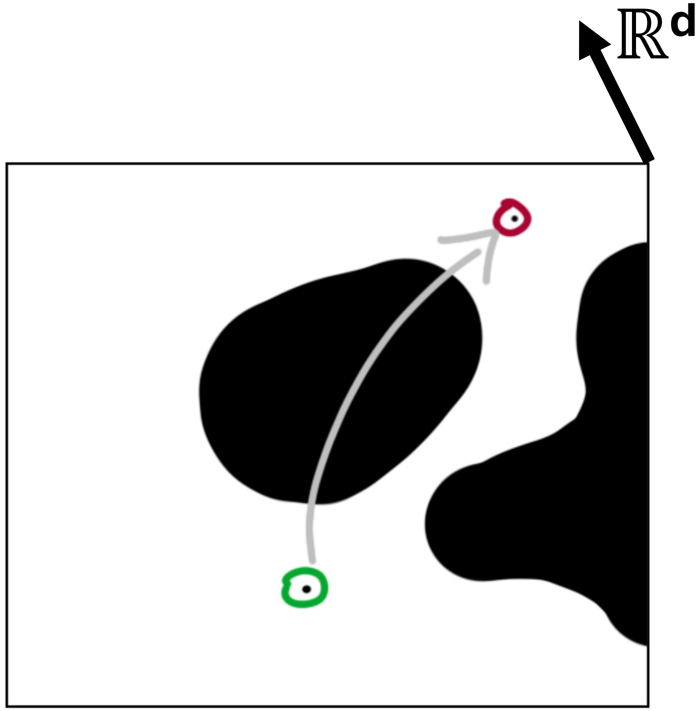


Start configuration,
goal,
configuration space → **Motion Planning** → Continuous curve
in feasible configuration space
connecting start and goal

Properties and Notes: Theoretical Guarantees

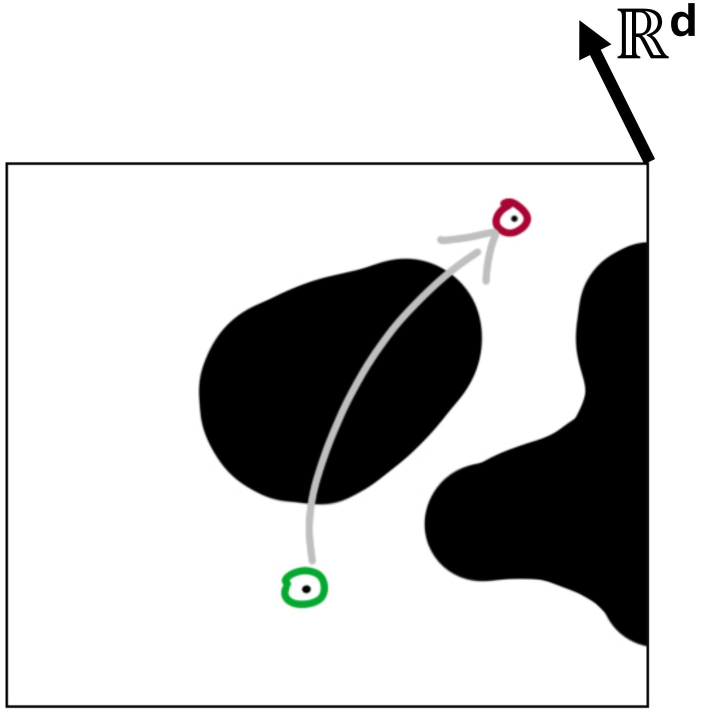
Completeness:
Guarantee a feasible solution
if one exists

Optimality:
Guarantee a cost-optimal solution
if one exists

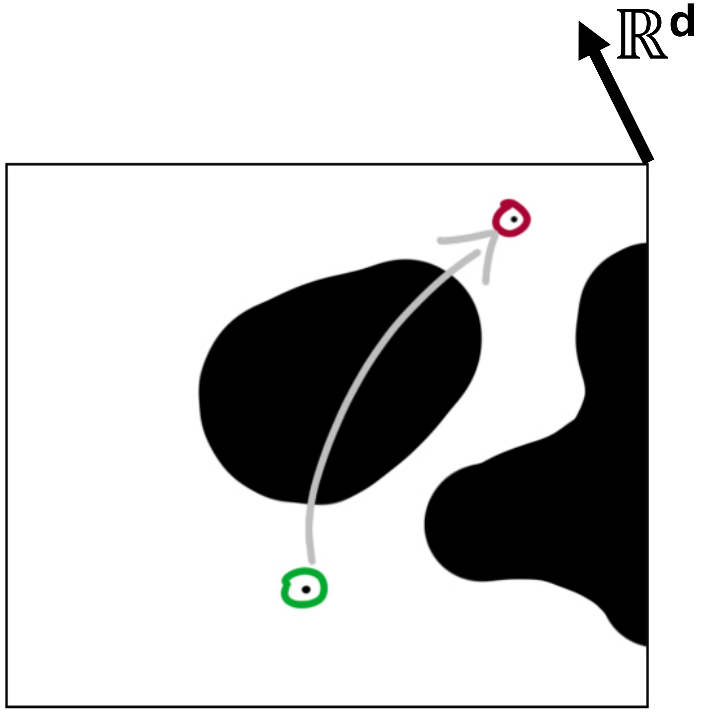


Keywords: Completeness, Optimality

Properties and Notes: Plans or Policies?



Plan: start to goal

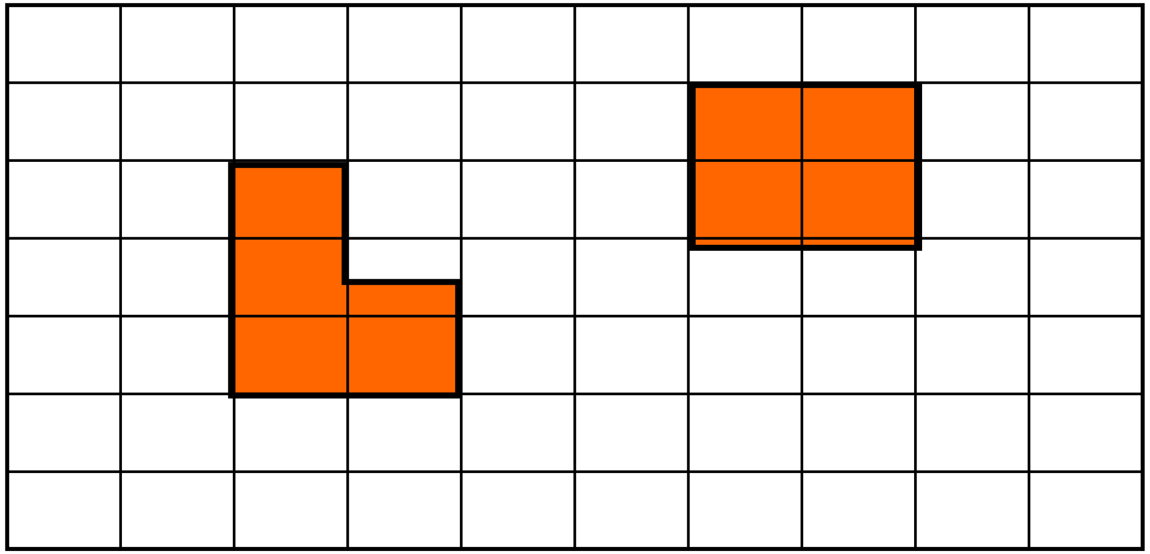


Policy: next action from any state

Keywords: Plans, Policies

Solution Strategies: Discretizing the Space

Connect a sequence of adjacent discretization cells from start to goal



Solution Strategies: Discretizing the Space

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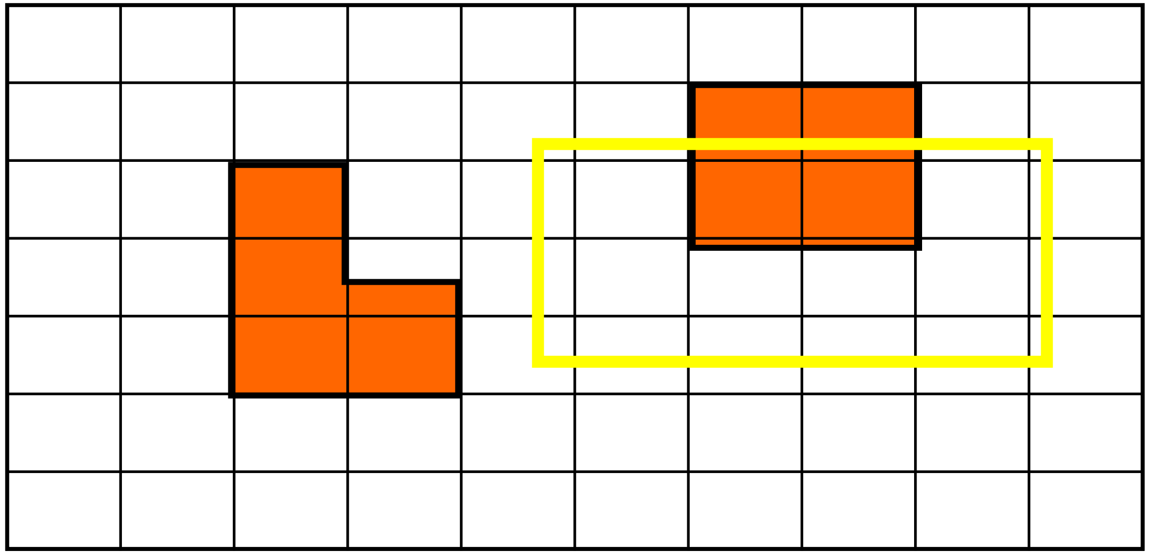
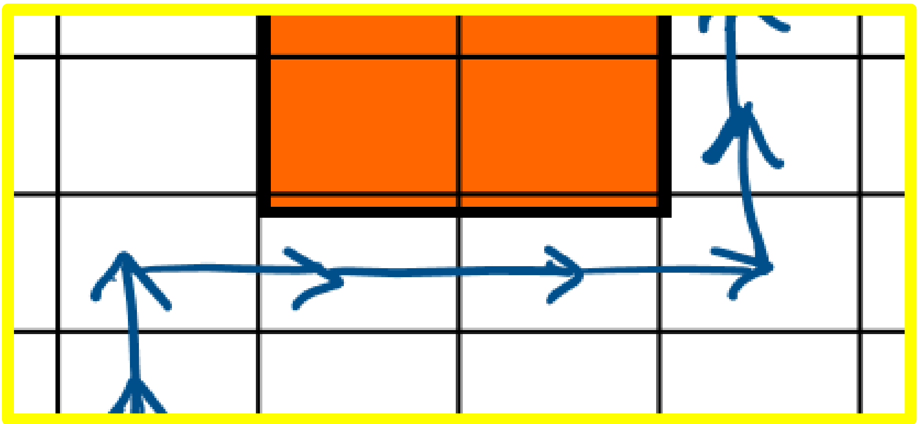
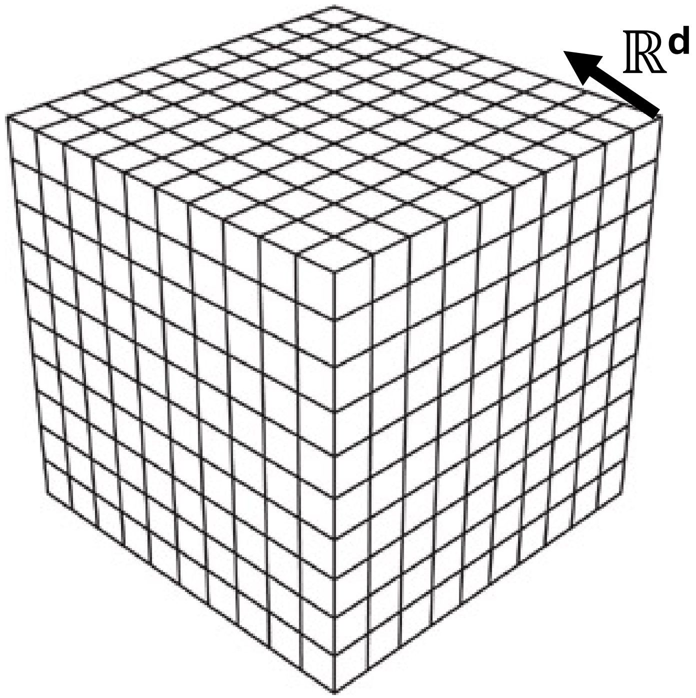
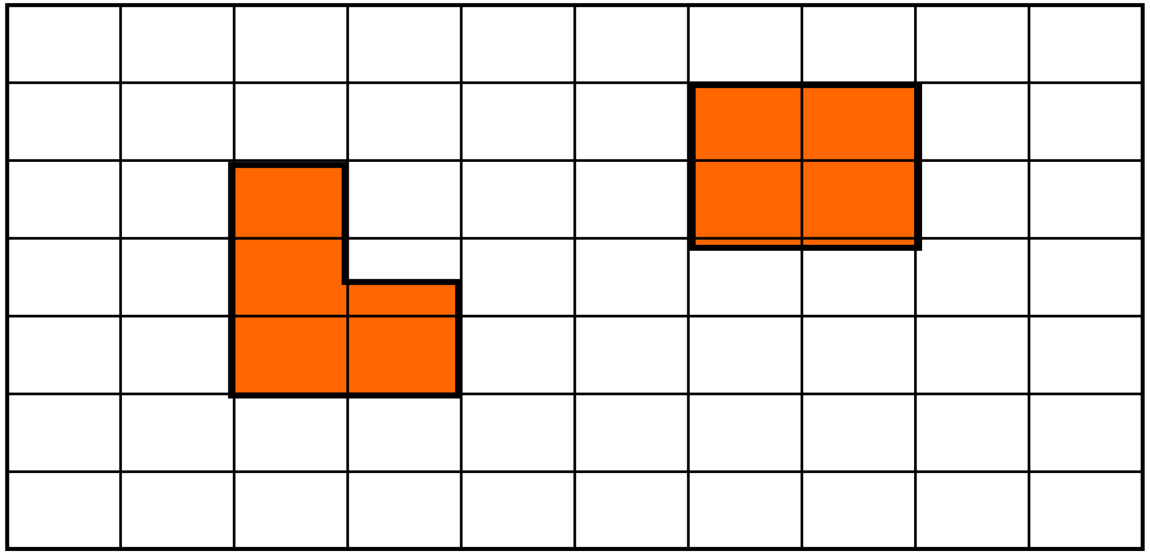




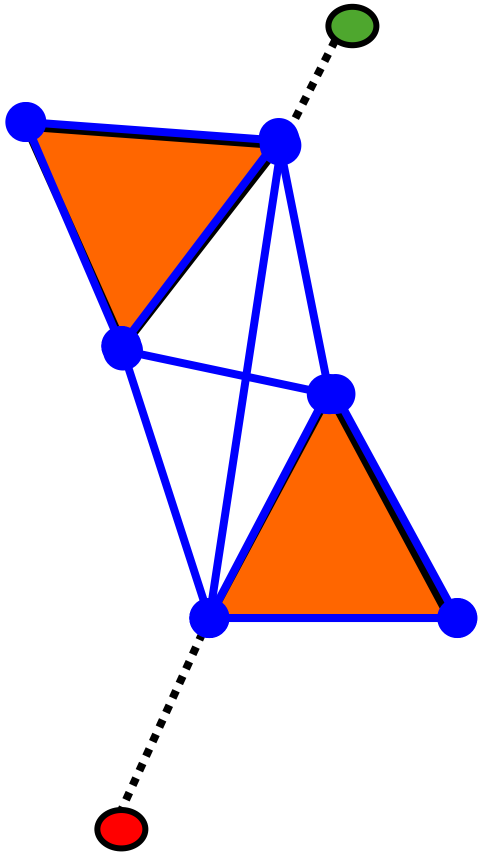
Solution Strategies: Discretizing the Space

Connect a sequence of adjacent discretization cells from start to goal

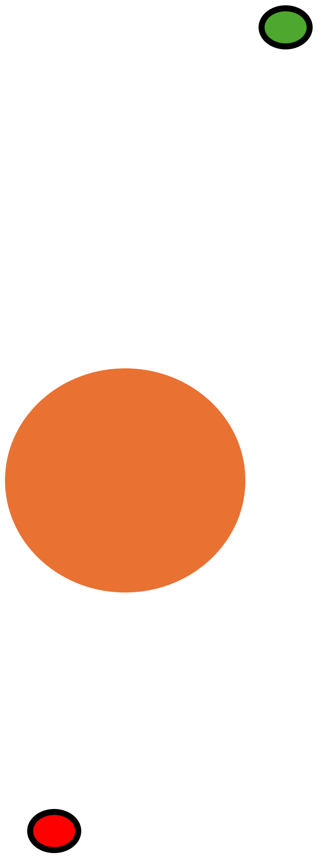




Other Solution Strategies



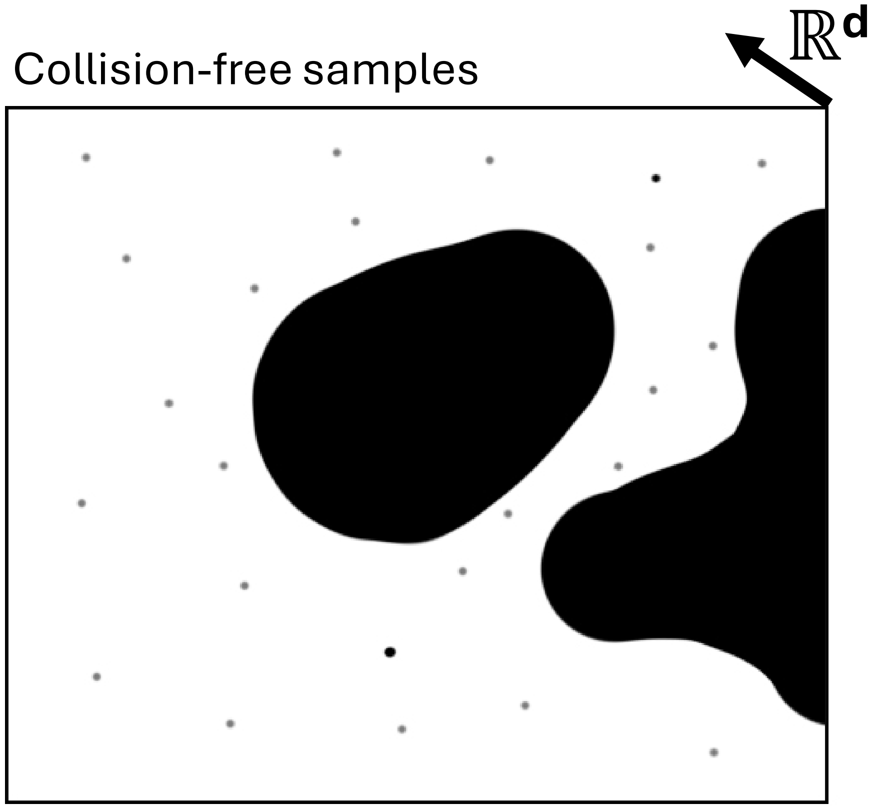
Visibility Graphs
Reasoning about geometry of obstacles does not scale to high dimensions.



Navigation Functions
Potential fields can have local minima.

Keywords: Visibility Graph, Navigation Function, Potential Function

Sampling-based Motion Planning



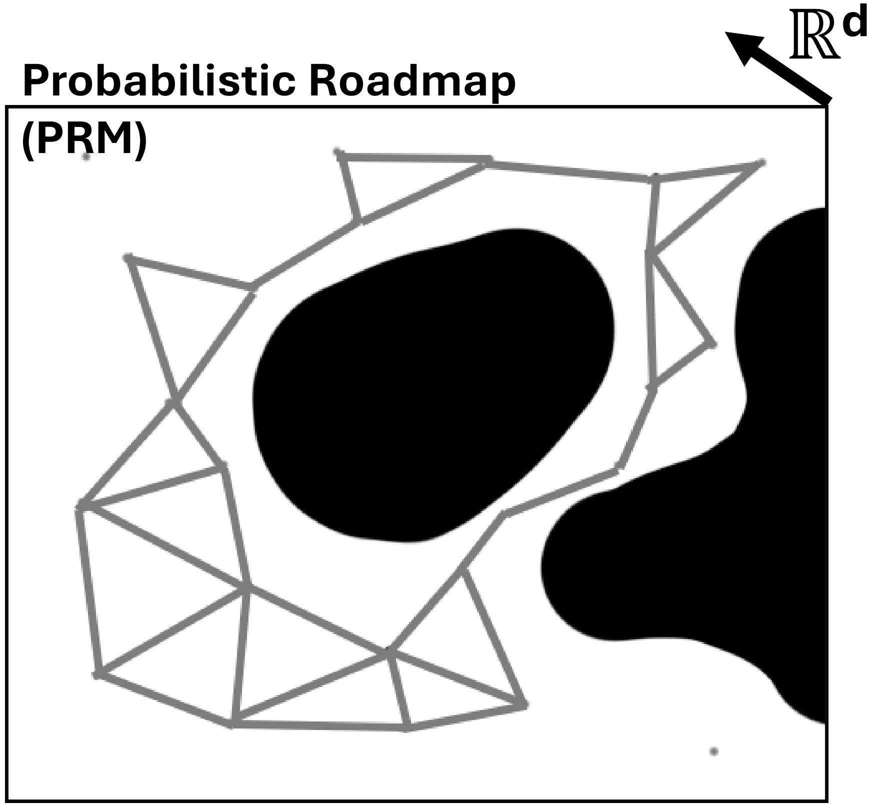
A sampling-based motion planner samples configurations

Kavraki et al 1996. Probabilistic roadmaps for path planning in high-dimensional configuration spaces."

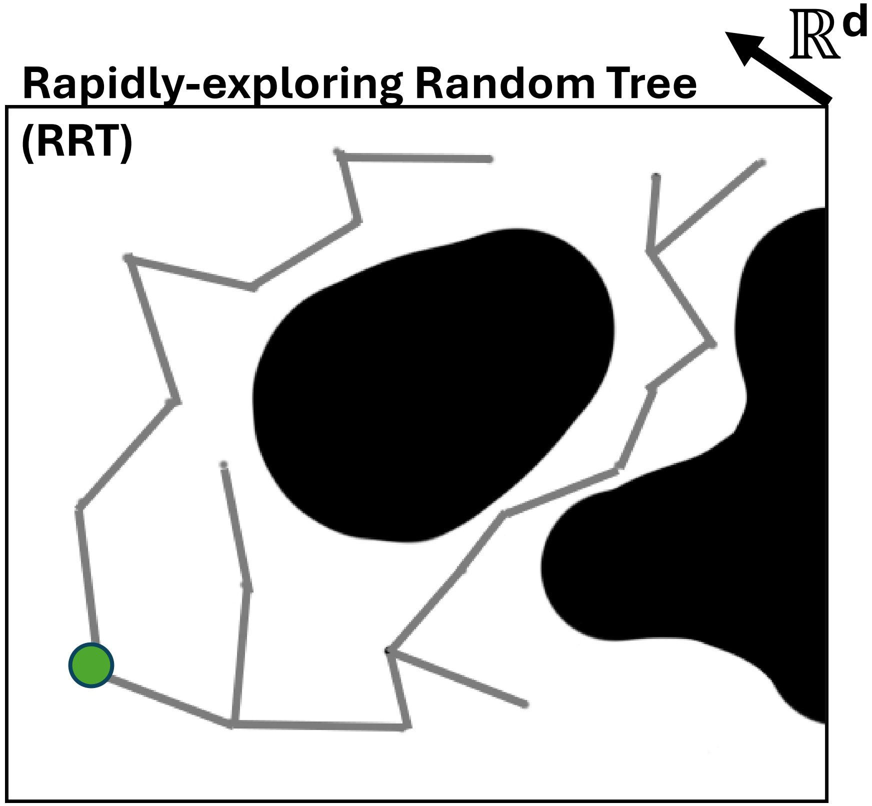
Keywords: Sampling-based Motion Planning

Sampling-based Motion Planning

Probabilistic Roadmap (PRM)



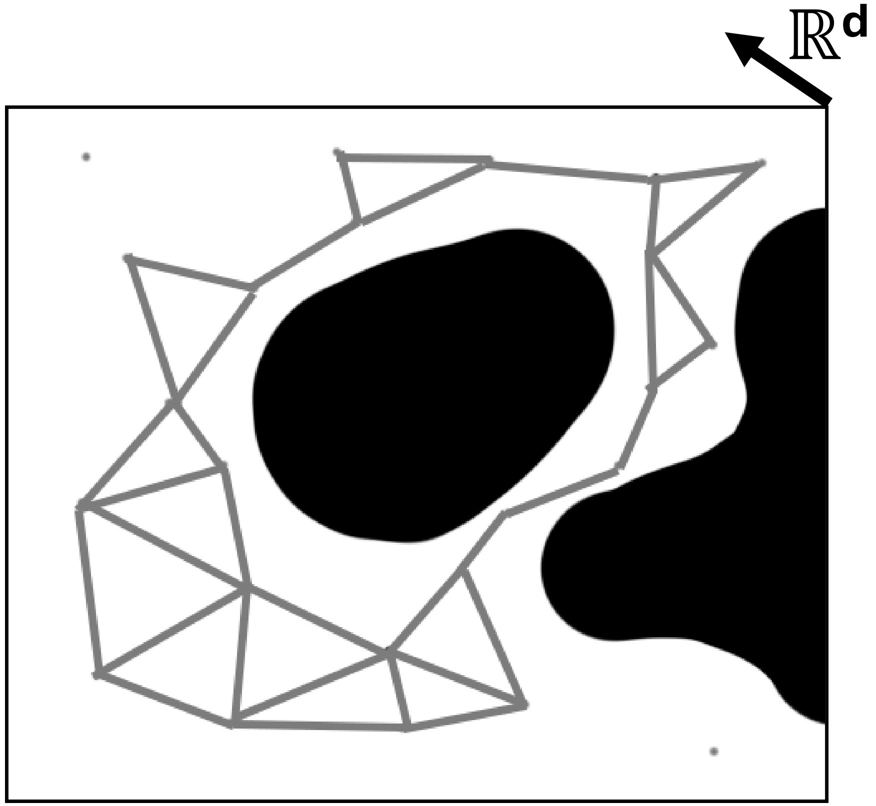
Rapidly-exploring Random Tree (RRT)



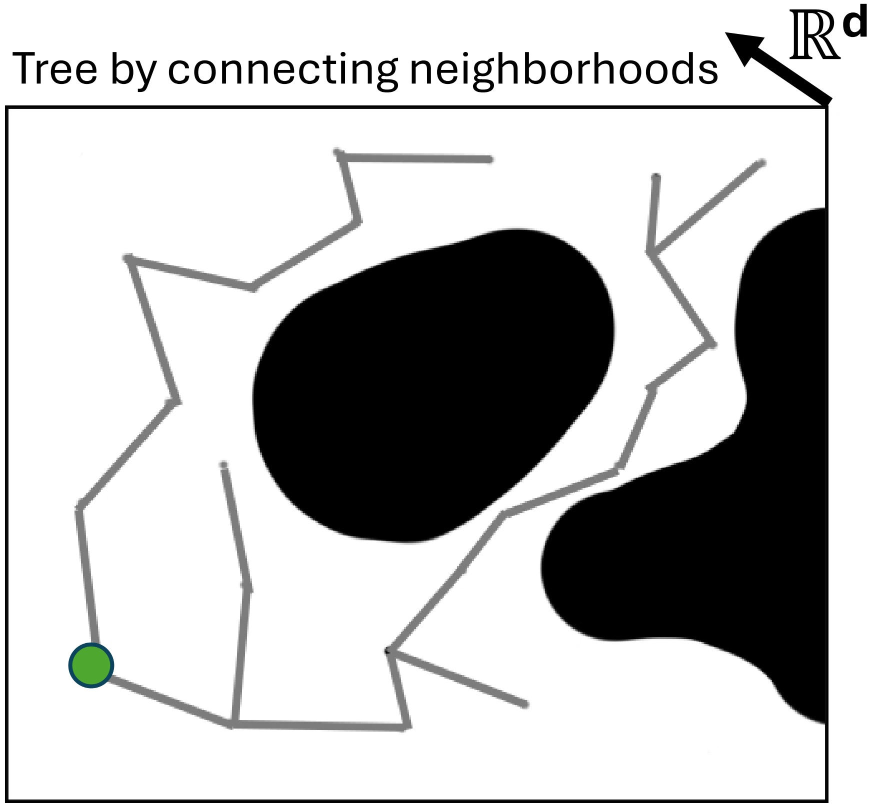
A roadmap or a tree is created by connecting neighborhoods in the collision-free or feasible configuration space.

Kavraki et al 1996. Probabilistic roadmaps for path planning in high-dimensional configuration spaces.“
LaValle et al 2001, "Randomized kinodynamic planning."

Sampling-based Motion Planning



Tree by connecting neighborhoods



A planning data structure is created by connecting neighborhoods in the collision-free or feasible configuration space.

Kavraki et al 1996. Probabilistic roadmaps for path planning in high-dimensional configuration spaces."
LaValle et al 2001, "Randomized kinodynamic planning."

Sampling-based Motion Planning

```
while problem not solved grow planning data structure
      sample collision-free configurations
      connect neighborhoods with local collision-free edges
      if start and goal are connected
            report path from start to goal
```

Sampling-based Motion Planning

```
while problem not solved grow planning data structure
      sample collision-free configurations
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            report path from start to goal
```
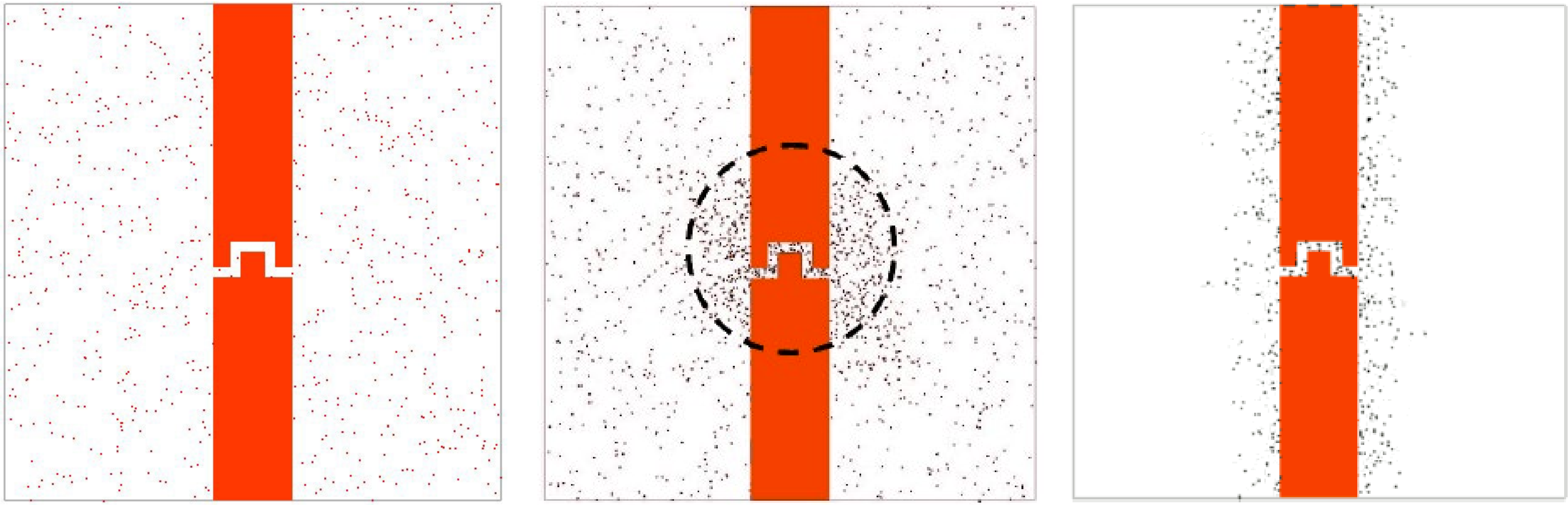

Key modules:

Sampler: *Sampling strategies – uniform, goal-biased, etc.*
Collision-checker: *Evaluate configurations and motions (hint: FK is involved)*
Nearest Neighbor: *NN data structures – KD Trees, Approximate NN (ANN), etc*
Local Planner: *Connecting nearby configurations – steering, forward propagation*
Search: *Heuristic search - A*, Dijkstra, Tree traversal*

Samplers



Sample from a distribution in the configuration space.
Tree-based methods can goal-bias by growing tree towards goal.

Keywords: Goal bias

Local Planning: Kinematic versus Kinodynamic

Select nearby configuration and connect to it
Kinematic
PRM, k-PRM, RRT-Connect
PRM*, RRT*, FMT*, RRG

Select a control input and connect to forward propagated state
Kinodynamic
EST, RRT, KPIECE,
SST, AO-RRT, BoE

A steering function can connect to a nearby configuration by solving a two-point boundary value problem.

Keywords: Kinematic, Kinodynamic, Steering Function, Two-point Boundary Value Problem

Local Planning: Graphs versus Trees

Kinematic

Properties of configuration space and geodesics connecting configurations.

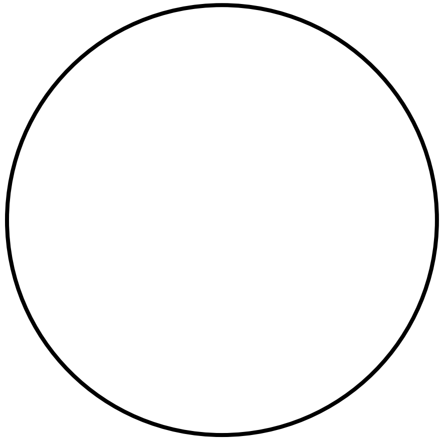
Kinodynamic

The space is typically referred to as state space.

The dynamics of the system can be simulated in a forward search tree.

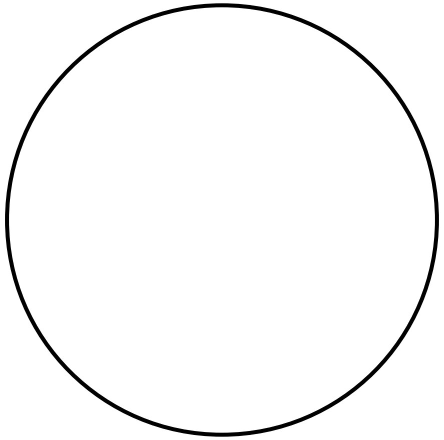
Keywords: State Space

Connection Strategies



Neighborhood radius

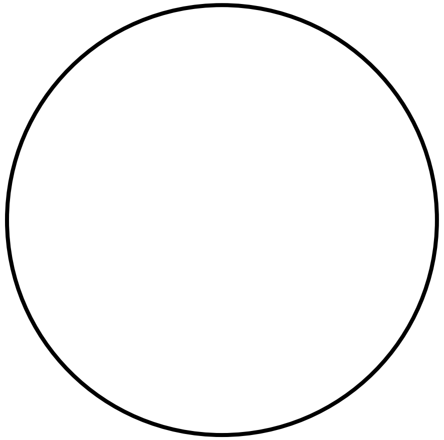
Nearest k



Grow trees
up to maximum extension

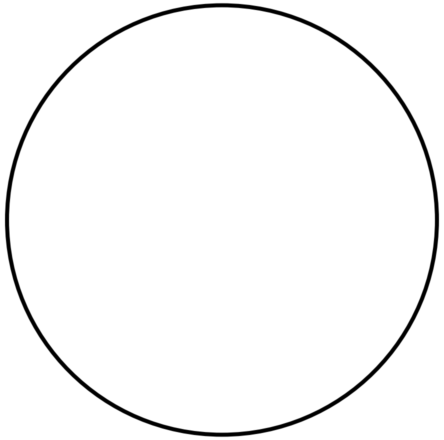
Keywords: Connection radius, Tree Extension

Can we choose any radius or k?



Neighborhood radius

Nearest k

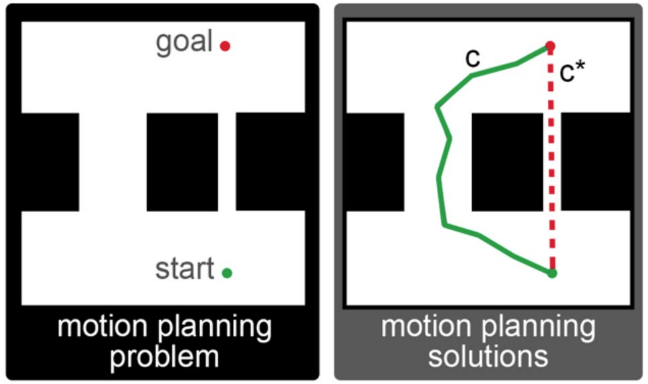


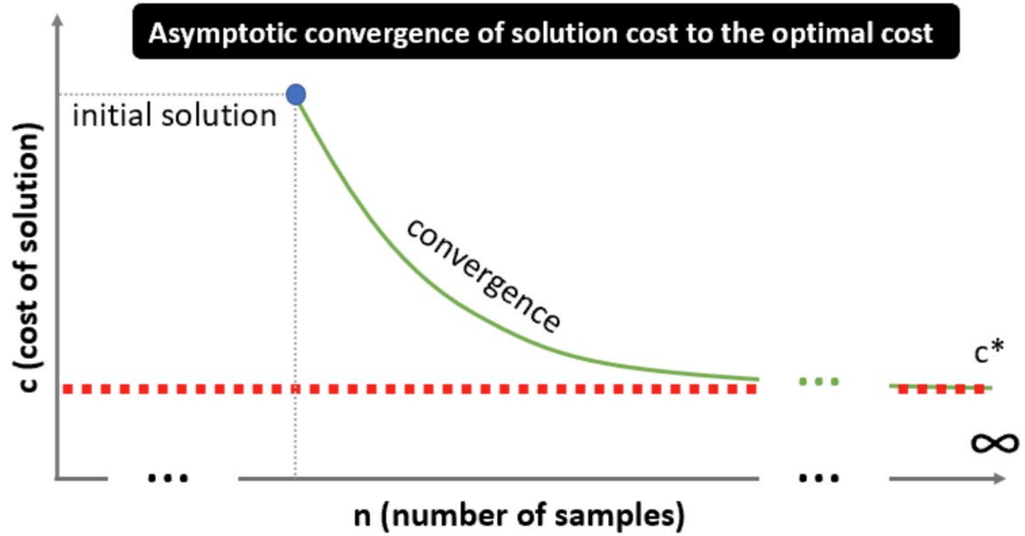
Grow trees
up to maximum extension

Probabilistic Guarantees

Probabilistic Completeness:
Asymptotically, in probability *guarantee* a feasible solution if one exists

Asymptotic Optimality:
Asymptotically, in probability *guarantee* a cost-optimal solution if one exists
Kinematic: e.g., PRM*, FMT*, RRT*
Kinodynamic: e.g., SST, AO-RRT



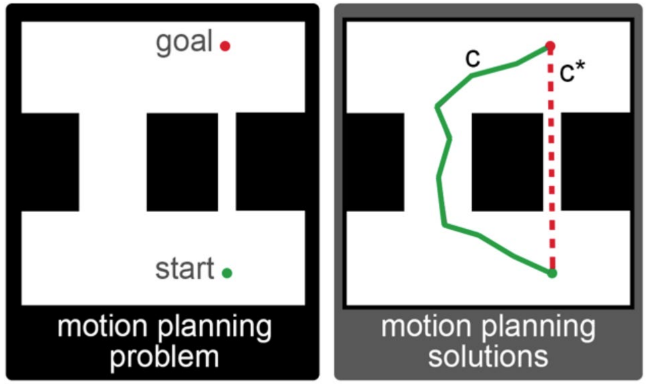


Bekris, K. E., & Shome, R. (2021). Asymptotically optimal sampling-based planners. Encyclopedia of Robotics.

Karaman, Sertac, and Emilio Frazzoli. "Sampling-based algorithms for optimal motion planning." *IJRR* 30, (2011).

Keywords: Probabilistic Completeness, Asymptotic Optimality

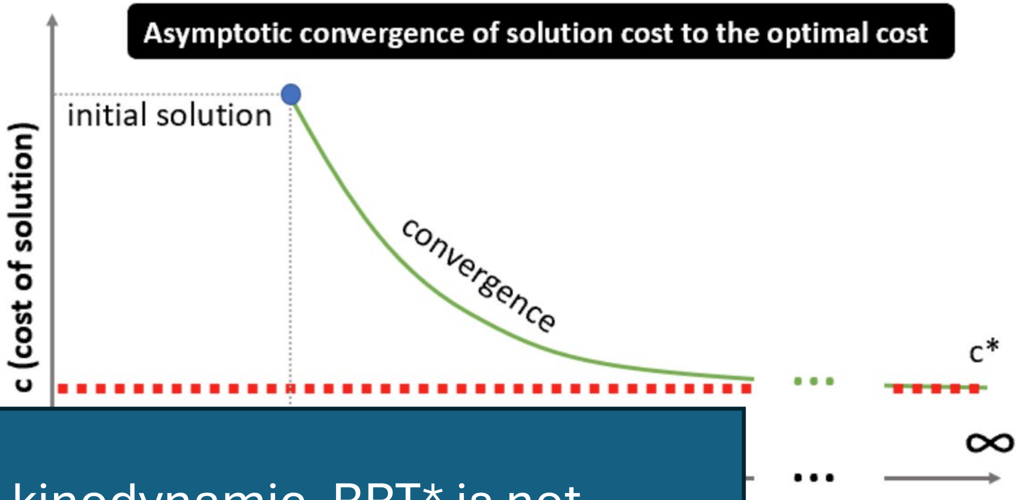
Probabilistic Guarantees



Probabilistic Completeness:
Asymptotically, in probability *guarantee* a feasible solution if one exists

Asymptotic Optimality:
Asymptotically, in probability *guarantee* a cost-optimal solution if one exists
Kinematic: e.g., PRM*, FMT*, **RRT***
Kinodynamic: e.g., SST, AO-RRT

RRT is kinodynamic. RRT* is not.



...ally optimal sampling-based planners. Encyclopedia of Robotics.

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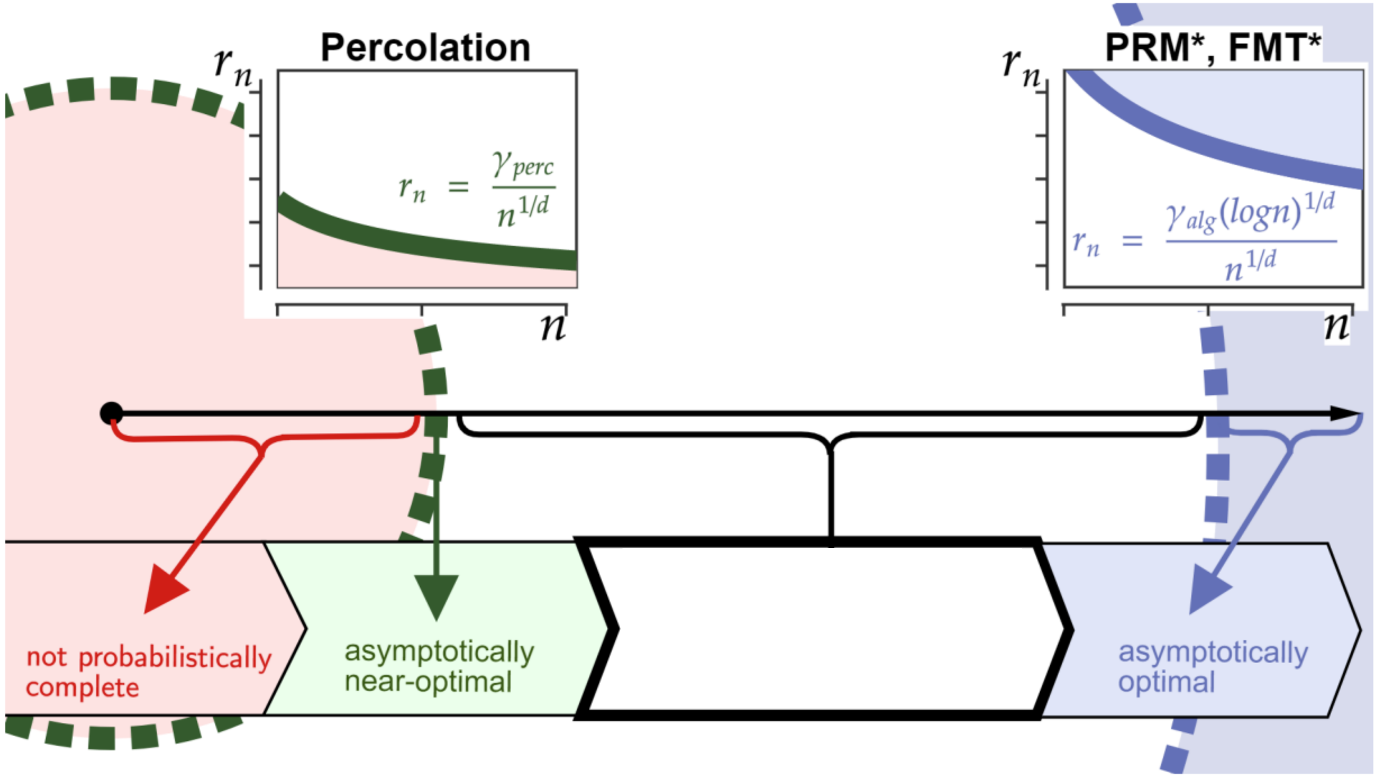
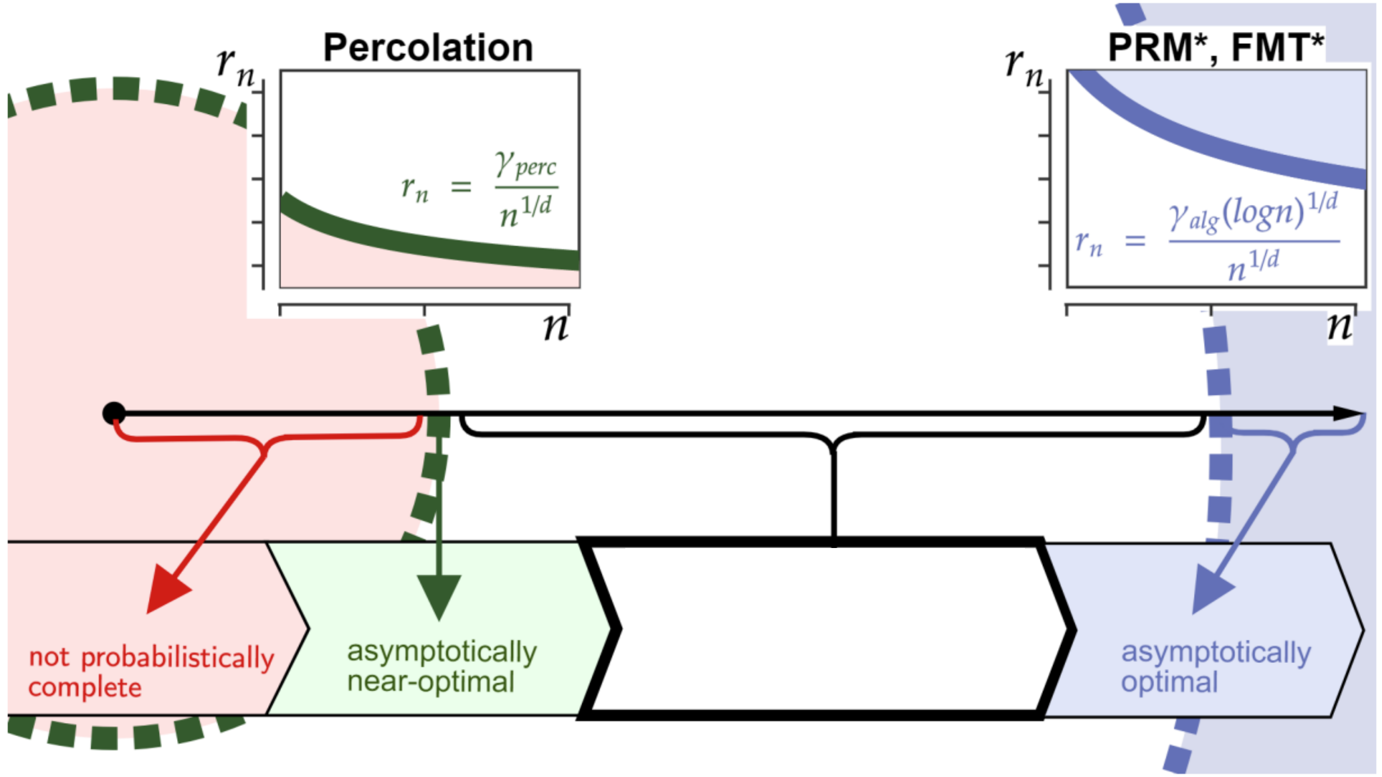
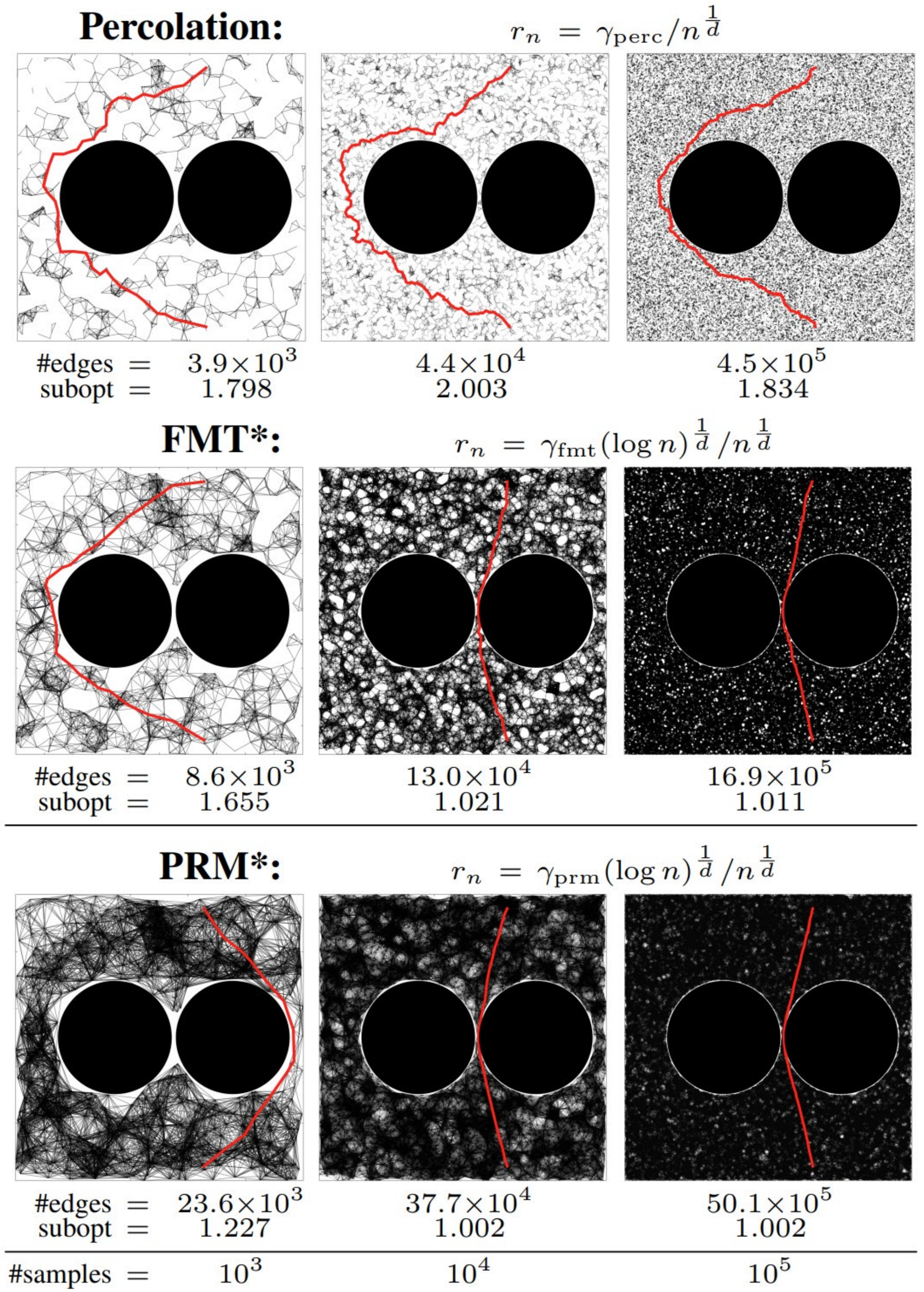
Choice of Connection Radius



Choice of Connection Radius



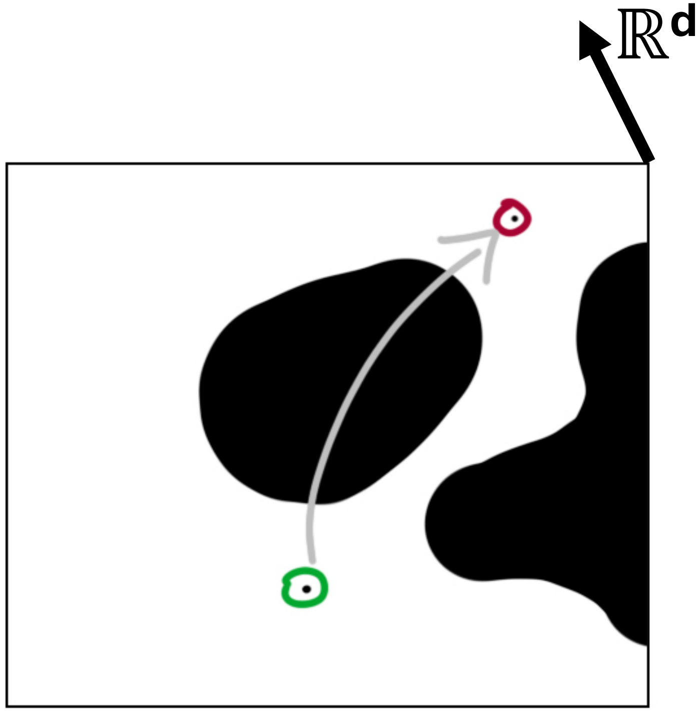


Sampling-based Motion Planning

Scales to high dimensions.

Kinematic and kinodynamic.

Probabilistic completeness and asymptotic optimality.



Some categories of high-dimensional planning problems can now be solved in microseconds.

Thomason, Wil, Zachary Kingston, and Lydia E. Kavraki. "Motions in microseconds via vectorized sampling-based planning." *ICRA* (2024).

Start configuration,
goal,
configuration space

Motion Planning

Continuous curve
in feasible configuration space
connecting start and goal

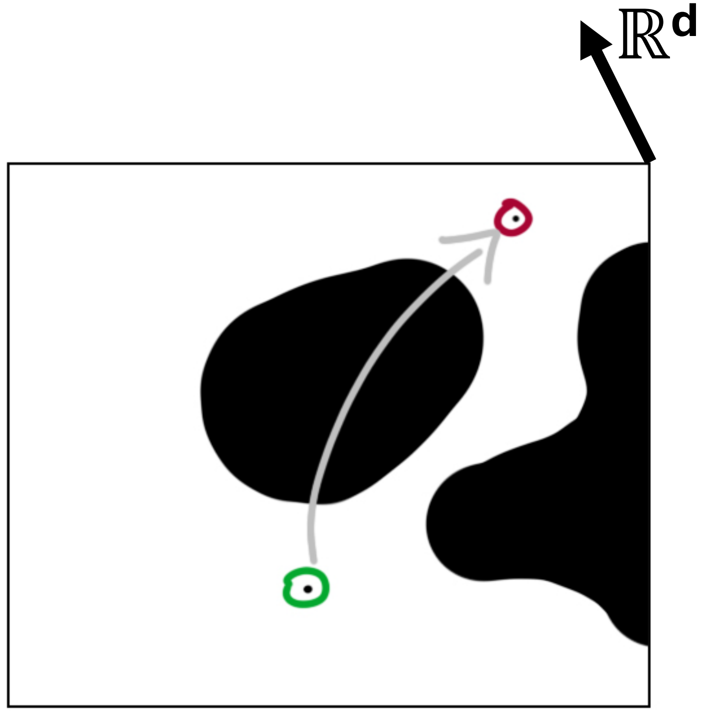
The general problem is still hard

The space is still high-dimensional.

Easier spaces (end-effector pose control) do not generalize.

The infeasible subset of space is not known beforehand.

Kinodynamic state spaces are typically more complex.



Start configuration,
goal,
configuration space

→ Motion Planning →

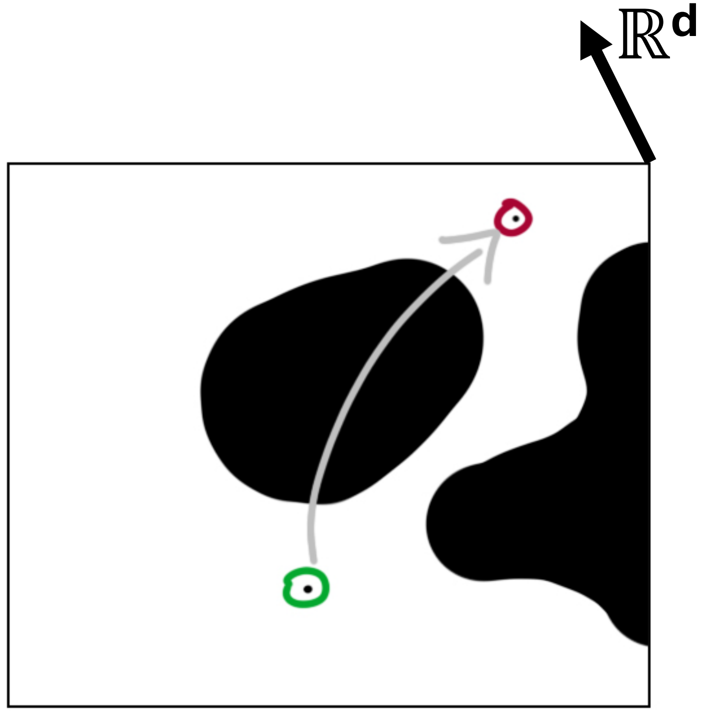
Continuous curve
in feasible configuration space
connecting start and goal

Wrapping Up

Other Strategies: Trajectory Optimization

Trajectory optimization
samples a trajectory representation
and optimizes it
subject to constraints and costs.

(CHOMP, TrajOPT, cuRobo, etc.)



Other Variants

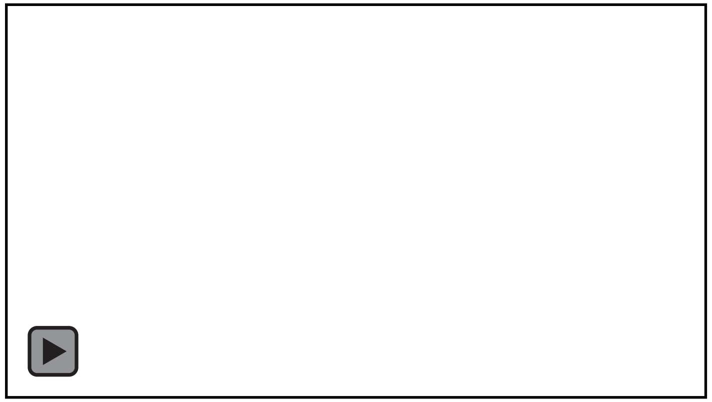
Motions over constraint manifolds.

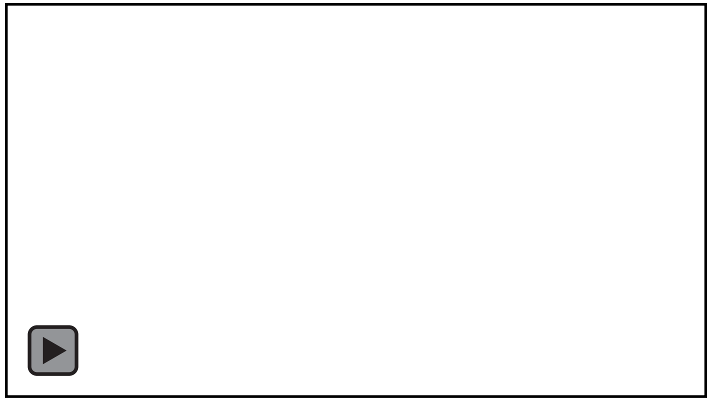
Multi-robot motions.

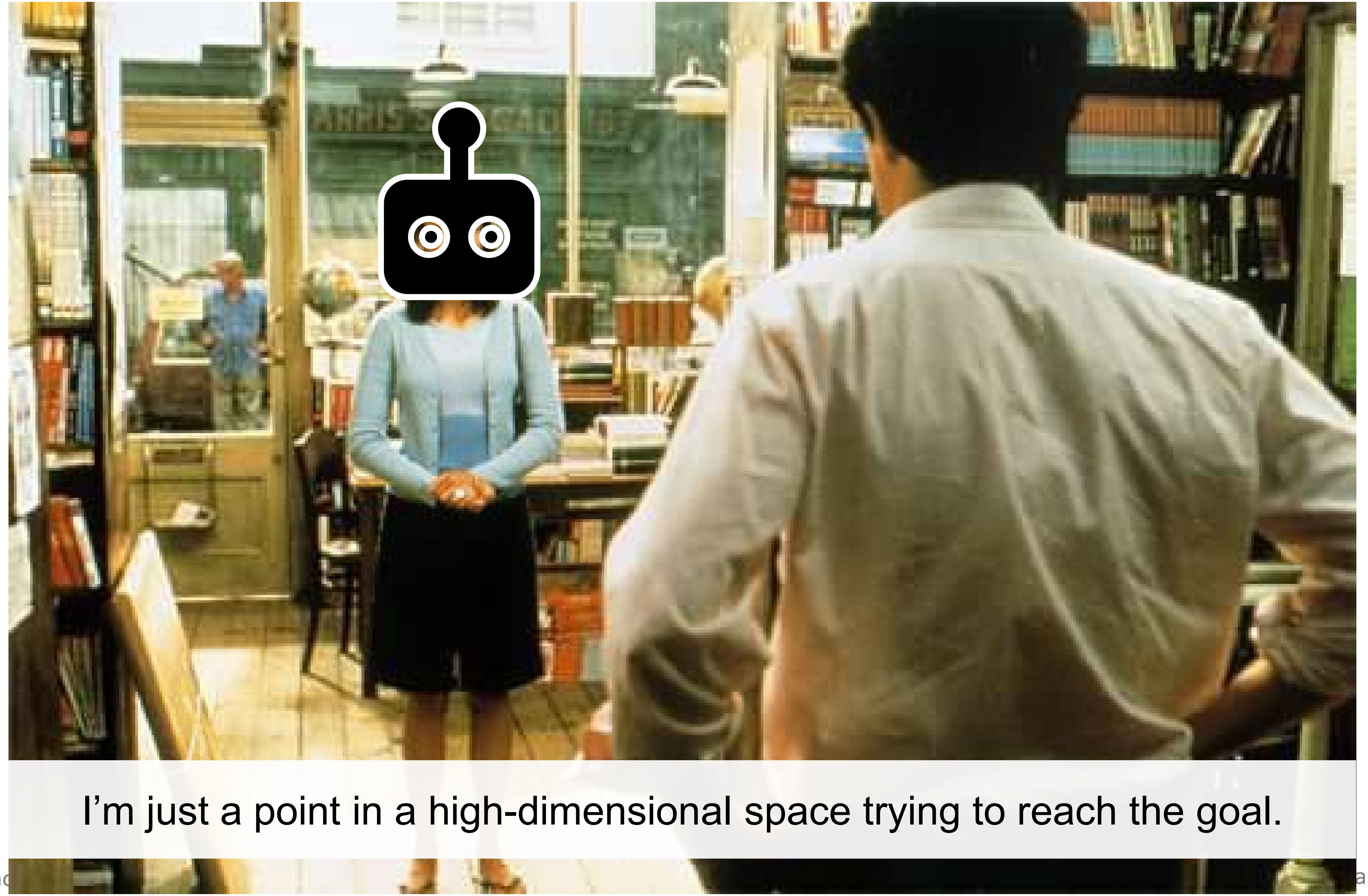
Motions under uncertainty.

Motions learned from demonstration.

…






I’m just a point in a high-dimensional space trying to reach the goal.