HW6 - Discrete environment / algorithm

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In [1]:
```
import gymnasium as gym
import numpy as np
import random
from collections import deque
import torch
import torch.nn as nn
import torch.optim as optim
import matplotlib.pyplot as plt
import time
import seaborn as sns
```

Custom MountainCar Environment (Discrete Actions):

In [2]:
```
class SimpleMountainCarEnv:
    def __init__(self):
        self.position_bounds = [-1.2, 0.6]
        self.velocity_bounds = [-0.07, 0.07]
        self.force = 0.001
        self.gravity = 0.0025
        self.max_steps = 200
        self.reset()

    def reset(self):
        self.position = np.random.uniform(-0.6, -0.4)
        self.velocity = 0.0
        self.steps = 0
        return np.array([self.position, self.velocity])

    def step(self, action):
        force_effect = (action - 1) * self.force
        gravity_effect = -np.cos(3 * self.position) * self.gravity

        self.velocity += force_effect + gravity_effect
        self.velocity = np.clip(self.velocity, *self.velocity_bounds)

        self.position += self.velocity
        self.position = np.clip(self.position, *self.position_bounds)

        if self.position in self.position_bounds:
            self.velocity = 0.0

        self.steps += 1
        done = False
        reward = -1

        if self.position >= 0.5:
            done = True
            reward = 1

        if self.steps >= self.max_steps:
            done = True

        return np.array([self.position, self.velocity]), reward, done
```

State Discretization and Utility Functions:

In [3]:
```
n_position_buckets = 30
n_velocity_buckets = 30

def discretize_state(state):
    position, velocity = state
    pos_scale = (position - (-1.2)) / (0.6 - (-1.2))
    vel_scale = (velocity - (-0.07)) / (0.07 - (-0.07))

    pos_bucket = int(pos_scale * (n_position_buckets - 1))
    vel_bucket = int(vel_scale * (n_velocity_buckets - 1))

    pos_bucket = min(n_position_buckets - 1, max(0, pos_bucket))
    vel_bucket = min(n_velocity_buckets - 1, max(0, vel_bucket))

    return (pos_bucket, vel_bucket)

def moving_average(data, window_size):
    return np.convolve(data, np.ones(window_size)/window_size, mode='valid')
```

Q-Learning Training Function:

In [4]:
```
def train_q_learning():
    env = SimpleMountainCarEnv()
    q_table = np.zeros((n_position_buckets, n_velocity_buckets, 3))
```

```python
    episode_rewards = []

    alpha = 0.1
    gamma = 0.99
    epsilon = 1.0
    epsilon_decay = 0.999
    epsilon_min = 0.05
    n_episodes = 10000

    start_time = time.time()

    for episode in range(n_episodes):
        state_continuous = env.reset()
        state_disc = discretize_state(state_continuous)
        total_reward = 0
        done = False

        while not done:
            if np.random.random() < epsilon:
                action = np.random.choice([0, 1, 2])
            else:
                action = np.argmax(q_table[state_disc])

            next_state_continuous, reward, done = env.step(action)
            next_state_disc = discretize_state(next_state_continuous)

            best_next_action = np.argmax(q_table[next_state_disc])
            td_target = reward + gamma * q_table[next_state_disc + (best_next_action,)]
            td_error = td_target - q_table[state_disc + (action,)]
            q_table[state_disc + (action,)] += alpha * td_error

            state_disc = next_state_disc
            total_reward += reward

        episode_rewards.append(total_reward)

        if epsilon > epsilon_min:
            epsilon *= epsilon_decay

        if (episode + 1) % 1000 == 0:
            print(f"Q-learning Episode {episode + 1}, Total Reward: {total_reward}, Epsilon: {epsilon:.3f}")

    end_time = time.time()
    print(f"Q-learning Training finished! Time elapsed: {end_time - start_time:.2f} seconds")
    return q_table, episode_rewards
```

Evaluating Q-Learning Performance (Visual & Statistical):

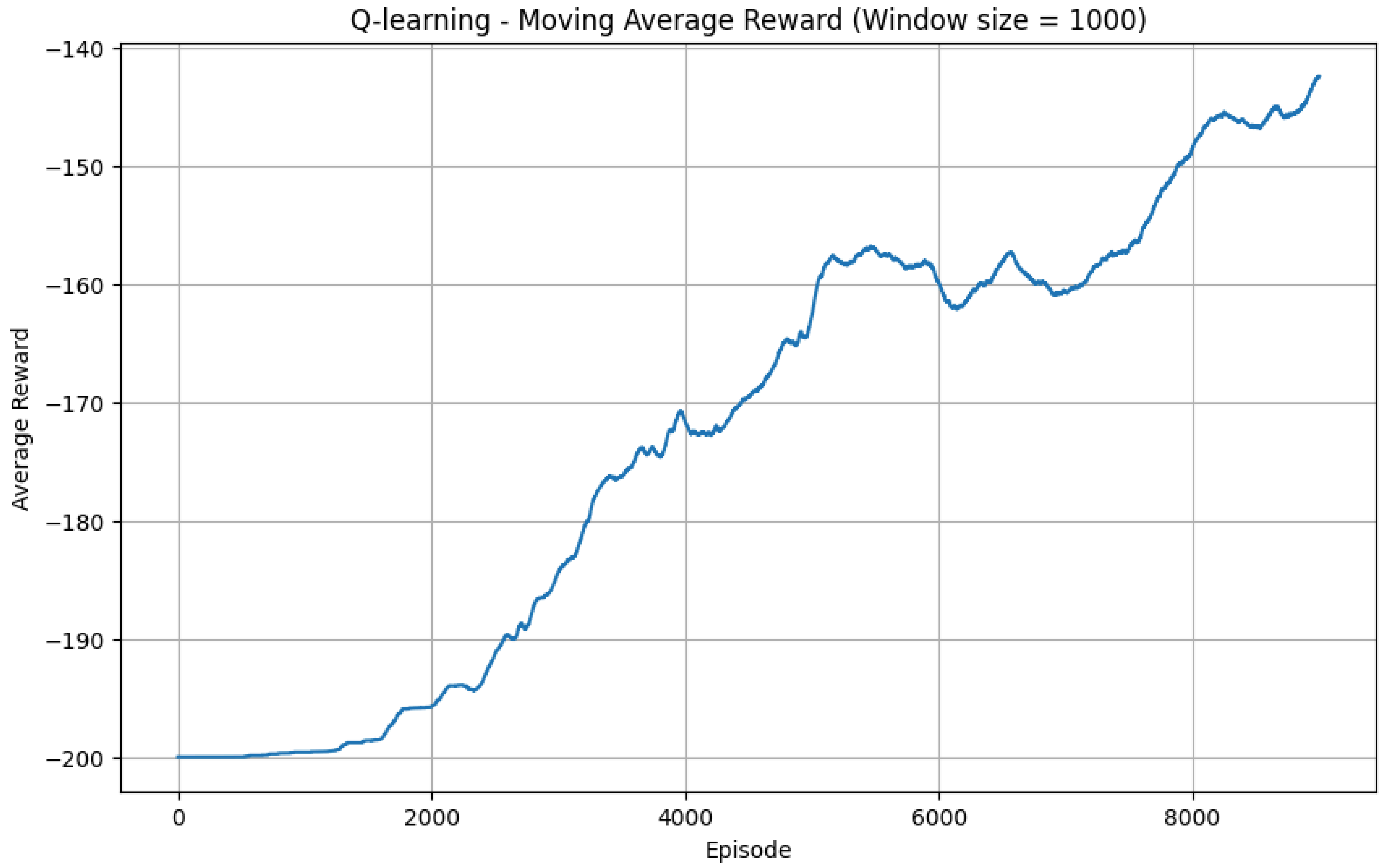
In [5]:
```python
def evaluate_q_learning_visual(q_table):
    env_eval = gym.make("MountainCar-v0", render_mode="human")
    state, _ = env_eval.reset()
    state_disc = discretize_state(state)
    done = False
    total_reward = 0

    while not done:
        action = np.argmax(q_table[state_disc])
        next_state, reward, terminated, truncated, _ = env_eval.step(action)
        done = terminated or truncated
        state_disc = discretize_state(next_state)
        total_reward += reward
        env_eval.render()
        time.sleep(0.01)

    print(f"\nFinal Evaluation Total Reward (Q-learning): {total_reward}")
    env_eval.close()

def evaluate_q_learning(q_table, n_runs=10):
    env = gym.make("MountainCar-v0")
    steps_list = []

    for _ in range(n_runs):
        state = env.reset()
        state = state[0] if isinstance(state, tuple) else state
        state_disc = discretize_state(state)
        done = False
        steps = 0

        while not done and steps < 200:
            action = np.argmax(q_table[state_disc])
            result = env.step(action)

            if len(result) == 5:
                next_state, _, terminated, truncated, _ = result
                done = terminated or truncated
            else:
                next_state, _, done, _ = result
```

```
            state_disc = discretize_state(next_state)
            steps += 1

        steps_list.append(steps)

    env.close()
    return np.mean(steps_list)
```

Run Q-Learning Training and Evaluation:

In [6]:
```
if __name__ == "__main__":
    q_table, reward_history = train_q_learning()
    evaluate_q_learning_visual(q_table)
    avg_steps = evaluate_q_learning(q_table)
    window = 1000
    moving_avg = np.convolve(reward_history, np.ones(window)/window, mode='valid')
    plt.figure(figsize=(10, 6))
    plt.plot(moving_avg)
    plt.title(f"Q-learning - Moving Average Reward (Window size = {window})")
    plt.xlabel("Episode")
    plt.ylabel("Average Reward")
    plt.grid(True)
    plt.show()
    print(f"\nAverage steps over evaluation runs: {avg_steps}")
```

```
Q-learning Episode 1000, Total Reward: -200, Epsilon: 0.368
Q-learning Episode 2000, Total Reward: -200, Epsilon: 0.135
Q-learning Episode 3000, Total Reward: -190, Epsilon: 0.050
Q-learning Episode 4000, Total Reward: -153, Epsilon: 0.050
Q-learning Episode 5000, Total Reward: -200, Epsilon: 0.050
Q-learning Episode 6000, Total Reward: -145, Epsilon: 0.050
Q-learning Episode 7000, Total Reward: -161, Epsilon: 0.050
Q-learning Episode 8000, Total Reward: -143, Epsilon: 0.050
Q-learning Episode 9000, Total Reward: -148, Epsilon: 0.050
Q-learning Episode 10000, Total Reward: -109, Epsilon: 0.050
Q-learning Training finished! Time elapsed: 25.77 seconds

Final Evaluation Total Reward (Q-learning): -143.0
```




```
Average steps over evaluation runs: 131.6
```

Double Q-Learning Training Function:

In [7]:
```
def train_double_q_learning():
    q_table_1 = np.zeros((n_position_buckets, n_velocity_buckets, 3))
    q_table_2 = np.zeros((n_position_buckets, n_velocity_buckets, 3))
    episode_rewards = []

    alpha = 0.1
    gamma = 0.99
    epsilon = 1.0
    epsilon_decay = 0.995
    epsilon_min = 0.1
    n_episodes = 10000

    env = SimpleMountainCarEnv()
    start_time = time.time()
```

```python
    for episode in range(n_episodes):
        state_continuous = env.reset()
        state_disc = discretize_state(state_continuous)
        total_reward = 0
        done = False

        while not done:
            if np.random.random() < epsilon:
                action = np.random.choice([0, 1, 2])
            else:
                q_sum = q_table_1[state_disc] + q_table_2[state_disc]
                action = np.argmax(q_sum)

            next_state_continuous, reward, done = env.step(action)
            next_state_disc = discretize_state(next_state_continuous)

            if np.random.random() < 0.5:
                best_next_action = np.argmax(q_table_1[next_state_disc])
                td_target = reward + gamma * q_table_2[next_state_disc + (best_next_action,)]
                td_error = td_target - q_table_1[state_disc + (action,)]
                q_table_1[state_disc + (action,)] += alpha * td_error
            else:
                best_next_action = np.argmax(q_table_2[next_state_disc])
                td_target = reward + gamma * q_table_1[next_state_disc + (best_next_action,)]
                td_error = td_target - q_table_2[state_disc + (action,)]
                q_table_2[state_disc + (action,)] += alpha * td_error

            state_disc = next_state_disc
            total_reward += reward

        episode_rewards.append(total_reward)

        if epsilon > epsilon_min:
            epsilon *= epsilon_decay

        if (episode + 1) % 1000 == 0:
            print(f"Double Q-learning Episode {episode + 1}, Total Reward: {total_reward}, Epsilon: {epsilon:.3f}")

    end_time = time.time()
    print(f"Double Q-learning Training finished! Time elapsed: {end_time - start_time:.2f} seconds")
    return q_table_1, q_table_2, episode_rewards
```

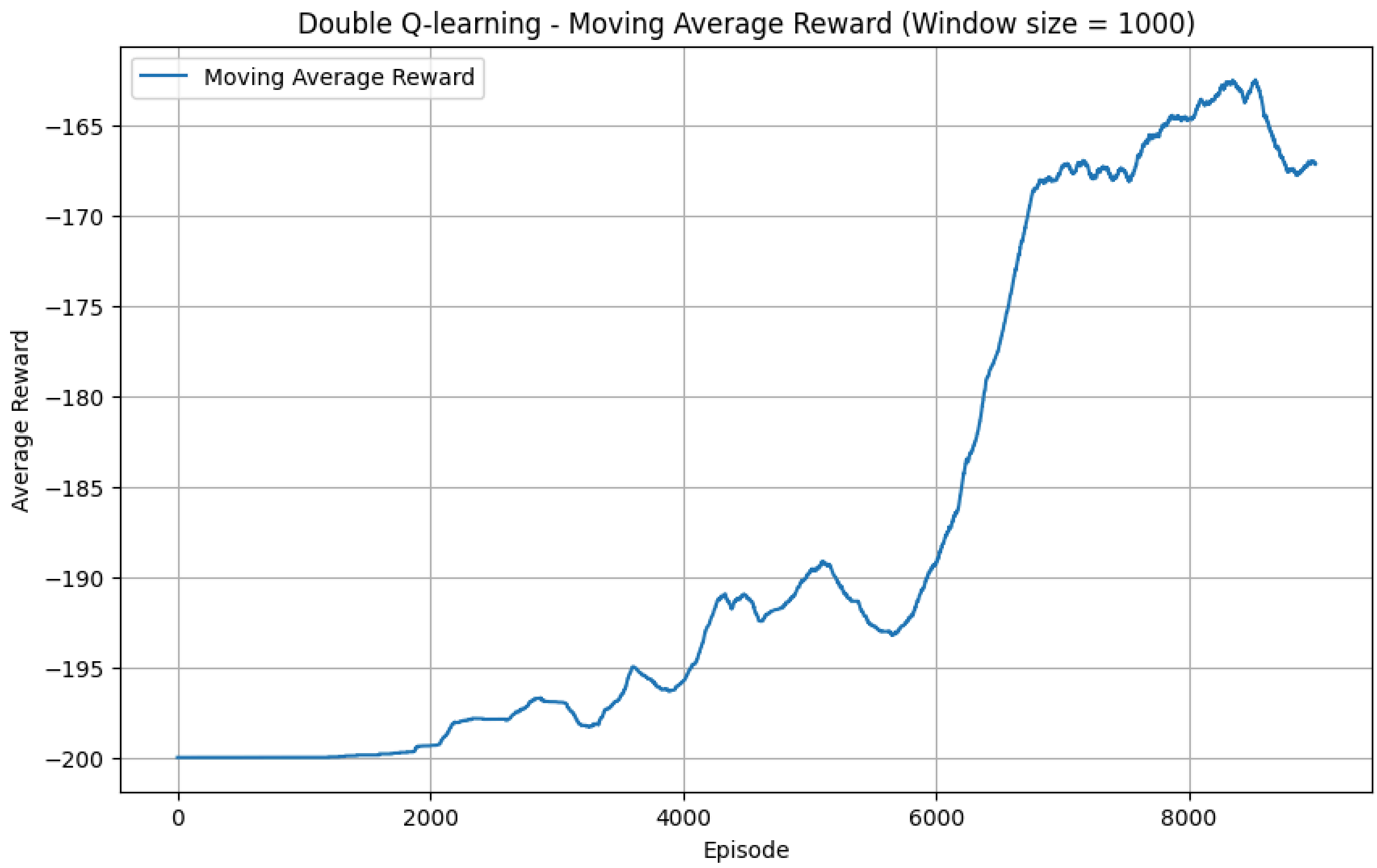
Q-Learning Evaluation Functions:

In [8]:
```python
def evaluate_double_q_learning(q_table_1, q_table_2):
    env_eval = gym.make("MountainCar-v0", render_mode="human")
    state, _ = env_eval.reset()
    state_disc = discretize_state(state)
    done = False
    total_reward = 0

    while not done:
        q_sum = q_table_1[state_disc] + q_table_2[state_disc]
        action = np.argmax(q_sum)

        next_state, reward, terminated, truncated, _ = env_eval.step(action)
        done = terminated or truncated
        state_disc = discretize_state(next_state)
        total_reward += reward
        env_eval.render()
        time.sleep(0.01)

    print(f"\nFinal Evaluation Total Reward (Double Q-learning): {total_reward}")
    env_eval.close()
    return total_reward
```

Run Double Q-Learning Training and Evaluation:

In [9]:
```python
if __name__ == "__main__":
    q1, q2, double_q_rewards = train_double_q_learning()
    avg_reward_dq = evaluate_double_q_learning(q1, q2)
    window = 1000
    moving_avg_dq = np.convolve(double_q_rewards, np.ones(window)/window, mode='valid')
    plt.figure(figsize=(10, 6))
    plt.plot(moving_avg_dq, label="Moving Average Reward")
    plt.title(f"Double Q-learning - Moving Average Reward (Window size = {window})")
    plt.xlabel("Episode")
    plt.ylabel("Average Reward")
    plt.grid(True)
    plt.legend()
    plt.show()
    print(f"Average reward in final evaluation: {avg_reward_dq}")
```

```
Double Q-learning Episode 1000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 2000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 3000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 4000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 5000, Total Reward: -194, Epsilon: 0.100
Double Q-learning Episode 6000, Total Reward: -169, Epsilon: 0.100
Double Q-learning Episode 7000, Total Reward: -148, Epsilon: 0.100
Double Q-learning Episode 8000, Total Reward: -190, Epsilon: 0.100
Double Q-learning Episode 9000, Total Reward: -171, Epsilon: 0.100
Double Q-learning Episode 10000, Total Reward: -153, Epsilon: 0.100
Double Q-learning Training finished! Time elapsed: 29.42 seconds

Final Evaluation Total Reward (Double Q-learning): -186.0
```




```
Average reward in final evaluation: -186.0
```

DQN Implementation: Replay Buffer, Network Architecture, and Training Loop:

In [10]:
```
class ReplayBuffer:
    def __init__(self, capacity=100000):
        self.buffer = deque(maxlen=capacity)

    def push(self, transition):
        self.buffer.append(transition)

    def sample(self, batch_size):
        batch = random.sample(self.buffer, batch_size)
        state, action, reward, next_state, done = zip(*batch)
        return (
            torch.tensor(np.array(state), dtype=torch.float32),
            torch.tensor(action, dtype=torch.int64),
            torch.tensor(reward, dtype=torch.float32),
            torch.tensor(np.array(next_state), dtype=torch.float32),
            torch.tensor(done, dtype=torch.float32)
        )

    def __len__(self):
        return len(self.buffer)

class DQN(nn.Module):
    def __init__(self, input_dim, output_dim):
        super(DQN, self).__init__()
        self.fc1 = nn.Linear(input_dim, 128)
        self.fc2 = nn.Linear(128, 64)
        self.fc3 = nn.Linear(64, output_dim)

    def forward(self, x):
        x = torch.relu(self.fc1(x))
        x = torch.relu(self.fc2(x))
        return self.fc3(x)

def train_dqn():
    env = SimpleMountainCarEnv()
    alpha = 0.0005
    gamma = 0.98
    epsilon = 1.0
    epsilon_min = 0.01
```

```
    epsilon_decay = 0.995
    batch_size = 128
    n_episodes = 1000
    max_steps = 200
    target_update_freq = 20

    input_dim = 2
    output_dim = 3

    policy_net = DQN(input_dim, output_dim)
    target_net = DQN(input_dim, output_dim)
    target_net.load_state_dict(policy_net.state_dict())
    target_net.eval()

    optimizer = optim.Adam(policy_net.parameters(), lr=alpha)
    criterion = nn.MSELoss()

    replay_buffer = ReplayBuffer()

    def select_action(state, epsilon):
        if random.random() < epsilon:
            return random.choice([0, 1, 2])
        else:
            state_t = torch.tensor(state, dtype=torch.float32).unsqueeze(0)
            with torch.no_grad():
                q_values = policy_net(state_t)
            return q_values.argmax().item()

    episode_rewards = []
    start_time = time.time()

    for episode in range(n_episodes):
        state = env.reset()
        total_reward = 0
        done = False

        for step in range(max_steps):
            action = select_action(state, epsilon)
            next_state, reward, done = env.step(action)

            replay_buffer.push((state, action, reward, next_state, done))
            state = next_state
            total_reward += reward

            if len(replay_buffer) >= batch_size:
                states, actions, rewards, next_states, dones = replay_buffer.sample(batch_size)

                q_values = policy_net(states).gather(1, actions.unsqueeze(1)).squeeze(1)

                with torch.no_grad():
                    max_next_q_values = target_net(next_states).max(1)[0]
                    targets = rewards + gamma * max_next_q_values * (1 - dones)

                loss = criterion(q_values, targets)
                optimizer.zero_grad()
                loss.backward()
                optimizer.step()

            if done:
                break

        episode_rewards.append(total_reward)
        epsilon = max(epsilon_min, epsilon * epsilon_decay)

        if (episode + 1) % target_update_freq == 0:
            target_net.load_state_dict(policy_net.state_dict())

        if (episode + 1) % 100 == 0:
            print(f"DQN Episode {episode + 1}, Total Reward: {total_reward}, Epsilon: {epsilon:.3f}")

    end_time = time.time()
    print(f"DQN Training finished! Time elapsed: {end_time - start_time:.2f} seconds")
    return policy_net, episode_rewards
```

DQN Evaluation: Visualizing Agent Performance:

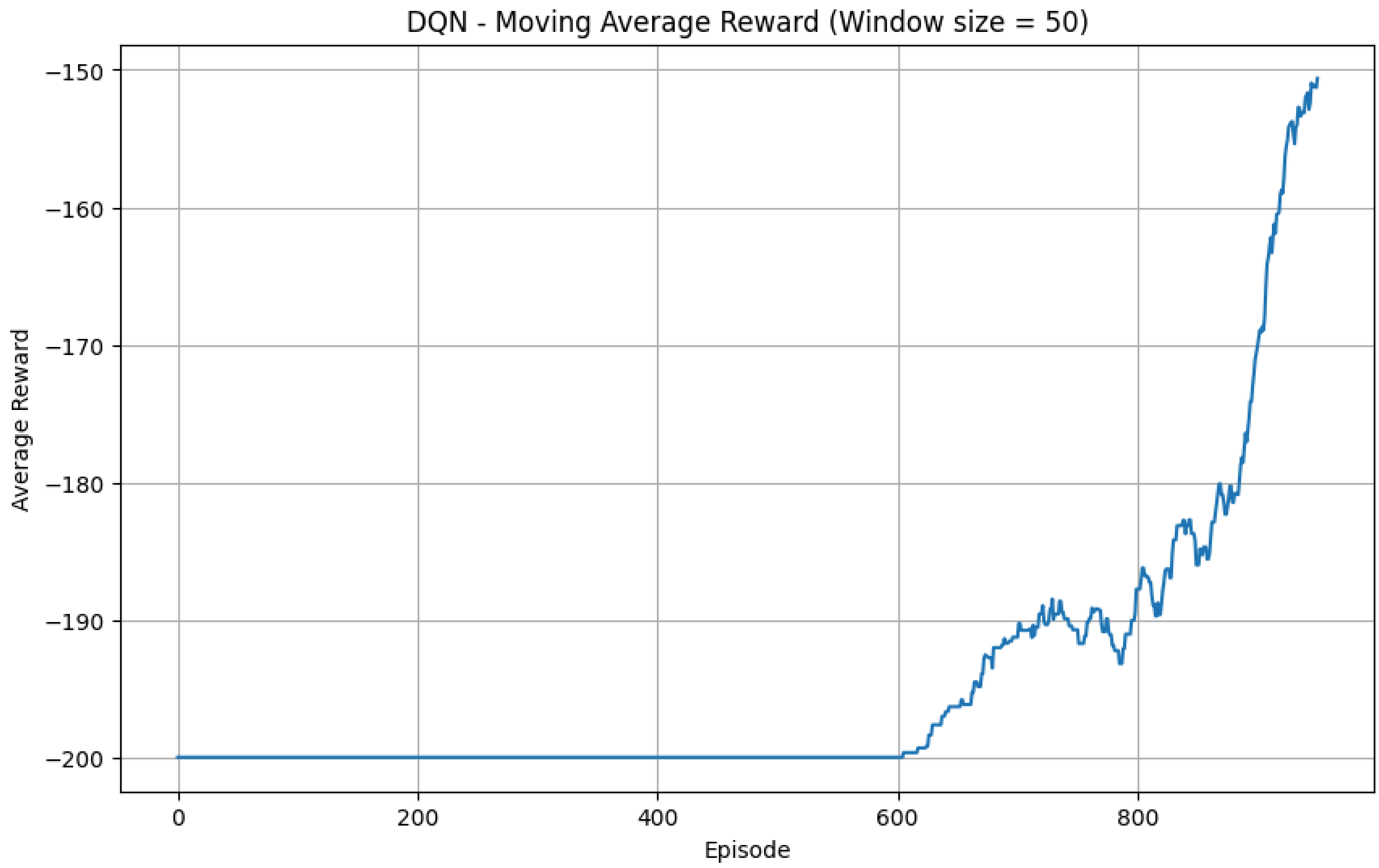
In [11]:
```
def evaluate_dqn(policy_net):
    env_eval = gym.make("MountainCar-v0", render_mode="human")
    state_continuous, _ = env_eval.reset()
    total_reward = 0
    done = False

    while not done:
        state_t = torch.tensor(state_continuous, dtype=torch.float32).unsqueeze(0)
        with torch.no_grad():
            q_values = policy_net(state_t)
        action = q_values.argmax().item()
```

```
        next_state, reward, terminated, truncated, _ = env_eval.step(action)
        done = terminated or truncated
        state_continuous = next_state
        total_reward += reward
        env_eval.render()
        time.sleep(0.01)

    print(f"\nFinal Evaluation Total Reward (DQN): {total_reward}")
    env_eval.close()
    return total_reward
```

DQN Training and Performance Visualization

In [12]:
```
if __name__ == "__main__":
    policy_net, dqn_rewards = train_dqn()

    window_size = 50
    moving_avg = np.convolve(dqn_rewards, np.ones(window_size) / window_size, mode='valid')

    plt.figure(figsize=(10, 6))
    plt.plot(moving_avg)
    plt.title("DQN - Moving Average Reward (Window size = 50)")
    plt.xlabel("Episode")
    plt.ylabel("Average Reward")
    plt.grid(True)
    plt.show()

    avg_reward = evaluate_dqn(policy_net)
    print(f"Average reward in final evaluation: {avg_reward}")
```

```
DQN Episode 100, Total Reward: -200, Epsilon: 0.606
DQN Episode 200, Total Reward: -200, Epsilon: 0.367
DQN Episode 300, Total Reward: -200, Epsilon: 0.222
DQN Episode 400, Total Reward: -200, Epsilon: 0.135
DQN Episode 500, Total Reward: -200, Epsilon: 0.082
DQN Episode 600, Total Reward: -200, Epsilon: 0.049
DQN Episode 700, Total Reward: -200, Epsilon: 0.030
DQN Episode 800, Total Reward: -200, Epsilon: 0.018
DQN Episode 900, Total Reward: -200, Epsilon: 0.011
DQN Episode 1000, Total Reward: -140, Epsilon: 0.010
DQN Training finished! Time elapsed: 285.13 seconds
```




```
Final Evaluation Total Reward (DQN): -166.0
Average reward in final evaluation: -166.0
```

General Evaluation Functions for RL Models:

In [13]:
```
def evaluate_model(model, model_type, n_runs=10):
    env = gym.make("MountainCar-v0")
    steps_list = []
    successes = 0

    for _ in range(n_runs):
        state = env.reset()
        state = state[0] if isinstance(state, tuple) else state
        done = False
        steps = 0
```

```
        while not done and steps < 200:
            if model_type == "Q-learning":
                state_disc = discretize_state(state)
                action = np.argmax(model[state_disc])
            elif model_type == "Double Q-learning":
                state_disc = discretize_state(state)
                action = np.argmax(model[0][state_disc] + model[1][state_disc])
            else:  # DQN
                state_t = torch.tensor(state, dtype=torch.float32).unsqueeze(0)
                with torch.no_grad():
                    action = model(state_t).argmax().item()

            next_state, _, terminated, truncated, _ = env.step(action)
            done = terminated or truncated
            state = next_state
            steps += 1

        steps_list.append(steps)
        if steps < 200:
            successes += 1

    env.close()
    return np.mean(steps_list), successes/n_runs

def evaluate_steps_to_goal(model, model_type, n_episodes=100):
    env = gym.make("MountainCar-v0")
    steps_history = []

    for _ in range(n_episodes):
        state = env.reset()
        state = state[0] if isinstance(state, tuple) else state
        done = False
        steps = 0

        while not done and steps < 200:
            if model_type == "Q-learning":
                state_disc = discretize_state(state)
                action = np.argmax(model[state_disc])
            elif model_type == "Double Q-learning":
                state_disc = discretize_state(state)
                action = np.argmax(model[0][state_disc] + model[1][state_disc])
            else:  # DQN
                state_t = torch.tensor(state, dtype=torch.float32).unsqueeze(0)
                with torch.no_grad():
                    action = model(state_t).argmax().item()

            next_state, _, terminated, truncated, _ = env.step(action)
            done = terminated or truncated
            state = next_state
            steps += 1

        steps_history.append(steps)

    env.close()
    return steps_history
```

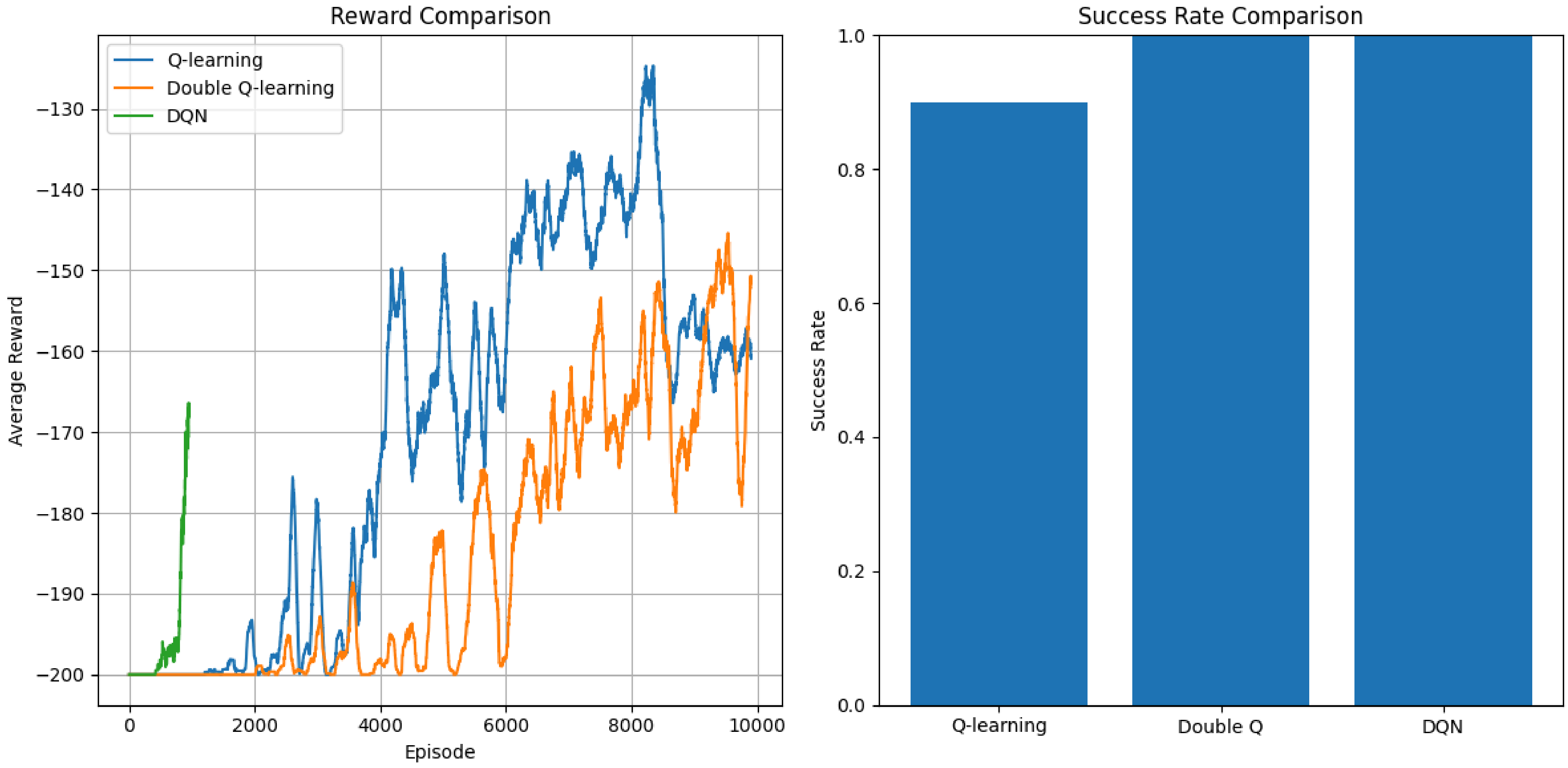
In this section, the program **trains all the algorithms, evaluates their performance on the MountainCar environment, and compares the results using tables and plots**. It also uses a **boxplot and cumulative success plot to show the distribution of steps to reach the goal and the success rate over episodes.

In [15]:
```
if __name__ == "__main__":
    # Train and evaluate all models
    print("Training Q-learning...")
    start_time = time.time()
    q_table, q_rewards = train_q_learning()
    time_q = time.time() - start_time

    print("\nTraining Double Q-learning...")
    start_time = time.time()
    q1_table, q2_table, dq_rewards = train_double_q_learning()
    time_dq = time.time() - start_time

    print("\nTraining DQN...")
    start_time = time.time()
    dqn_model, dqn_rewards = train_dqn()
    time_dqn = time.time() - start_time

    # Evaluate models
    q_steps, q_success = evaluate_model(q_table, "Q-learning")
    dq_steps, dq_success = evaluate_model((q1_table, q2_table), "Double Q-learning")
    dqn_steps, dqn_success = evaluate_model(dqn_model, "DQN")

    # Generate results table
    results = [
        ["Q-learning", q_steps, q_success, len(q_rewards), time_q],
        ["Double Q-learning", dq_steps, dq_success, len(dq_rewards), time_dq],
        ["DQN", dqn_steps, dqn_success, len(dqn_rewards), time_dqn]
    ]
```

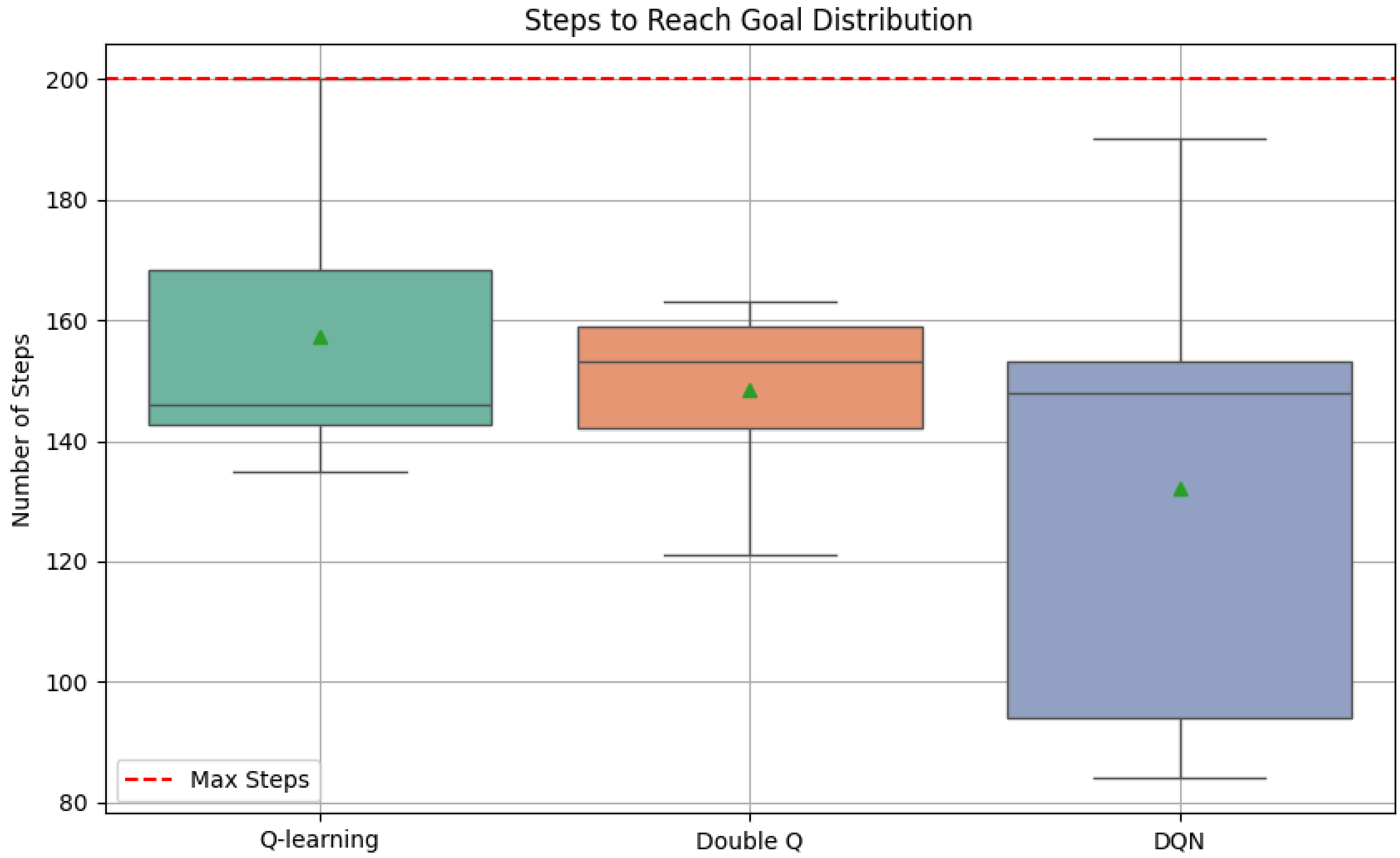
```python
print("\nModel Comparison Table:")
print("{:<20} {:<15} {:<15} {:<15} {:<15}".format(
    "Model", "Avg Steps", "Success Rate", "Episodes", "Train Time (s)"))
for row in results:
    print("{:<20} {:<15.1f} {:<15.2f} {:<15} {:<15.1f}".format(*row))

# Plot comparison charts
plt.figure(figsize=(12, 6))

# Reward plot
plt.subplot(1, 2, 1)
plt.plot(moving_average(q_rewards, 100), label="Q-learning")
plt.plot(moving_average(dq_rewards, 100), label="Double Q-learning")
plt.plot(moving_average(dqn_rewards, 50), label="DQN")
plt.title("Reward Comparison")
plt.xlabel("Episode")
plt.ylabel("Average Reward")
plt.legend()
plt.grid()

# Success rate bar chart
plt.subplot(1, 2, 2)
models = ["Q-learning", "Double Q", "DQN"]
success_rates = [q_success, dq_success, dqn_success]
plt.bar(models, success_rates)
plt.title("Success Rate Comparison")
plt.ylabel("Success Rate")
plt.ylim(0, 1)

plt.tight_layout()
plt.show()

# Time-to-goal evaluation
q_steps_history = evaluate_steps_to_goal(q_table, "Q-learning")
dq_steps_history = evaluate_steps_to_goal((q1_table, q2_table), "Double Q-learning")
dqn_steps_history = evaluate_steps_to_goal(dqn_model, "DQN")

# Calculate statistics
stats = {
    "Q-learning": {
        "mean": np.mean(q_steps_history),
        "std": np.std(q_steps_history),
        "success_rate": np.mean(np.array(q_steps_history) < 200)
    },
    "Double Q-learning": {
        "mean": np.mean(dq_steps_history),
        "std": np.std(dq_steps_history),
        "success_rate": np.mean(np.array(dq_steps_history) < 200)
    },
    "DQN": {
        "mean": np.mean(dqn_steps_history),
        "std": np.std(dqn_steps_history),
        "success_rate": np.mean(np.array(dqn_steps_history) < 200)
    }
}

# Print results
print("\nTime-to-Goal Performance:")
print("{:<20} {:<15} {:<15} {:<15}".format(
    "Model", "Mean Steps", "Std Dev", "Success Rate"))
for model, data in stats.items():
    print("{:<20} {:<15.1f} {:<15.1f} {:<15.2f}".format(
        model, data["mean"], data["std"], data["success_rate"]))

# Visualization 1: Boxplot comparison
plt.figure(figsize=(10, 6))
sns.boxplot(data=[q_steps_history, dq_steps_history, dqn_steps_history],
            palette="Set2",
            showmeans=True)
plt.xticks([0, 1, 2], ["Q-learning", "Double Q", "DQN"])
plt.title("Steps to Reach Goal Distribution")
plt.ylabel("Number of Steps")
plt.axhline(200, color='r', linestyle='--', label="Max Steps")
plt.legend()
plt.grid(True)

# Visualization 2: Cumulative success
plt.figure(figsize=(10, 6))
for name, steps in zip(["Q-learning", "Double Q", "DQN"], [q_steps_history, dq_steps_history, dqn_steps_history]):
    success = np.cumsum(np.array(steps) < 200)
    plt.plot(success, label=name)
plt.title("Cumulative Successful Episodes")
plt.xlabel("Episode")
plt.ylabel("Success Count")
plt.legend()
plt.grid(True)
```

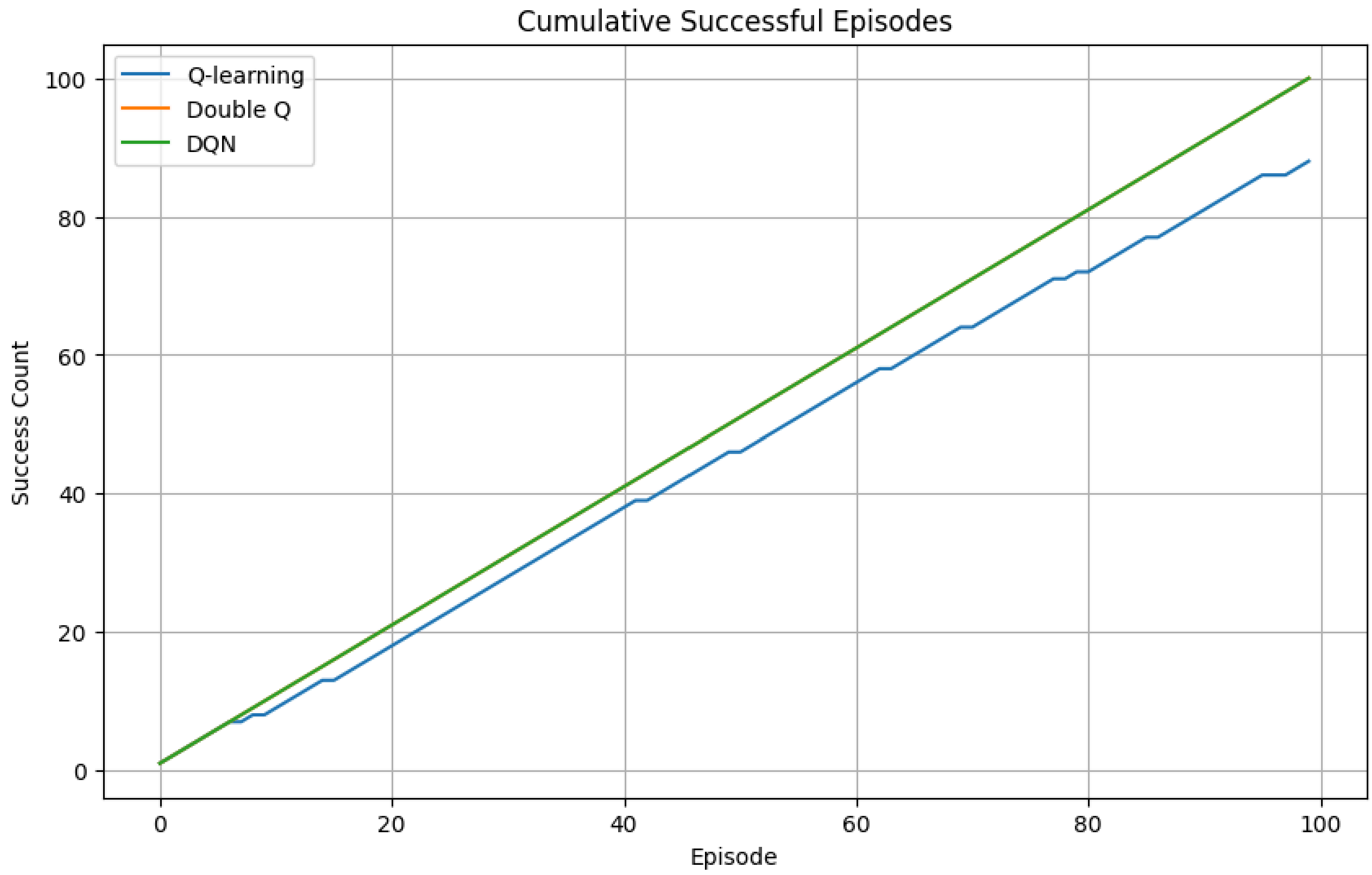
```
plt.show()
```

```
Training Q-learning...
Q-learning Episode 1000, Total Reward: -200, Epsilon: 0.368
Q-learning Episode 2000, Total Reward: -200, Epsilon: 0.135
Q-learning Episode 3000, Total Reward: -161, Epsilon: 0.050
Q-learning Episode 4000, Total Reward: -200, Epsilon: 0.050
Q-learning Episode 5000, Total Reward: -200, Epsilon: 0.050
Q-learning Episode 6000, Total Reward: -150, Epsilon: 0.050
Q-learning Episode 7000, Total Reward: -154, Epsilon: 0.050
Q-learning Episode 8000, Total Reward: -148, Epsilon: 0.050
Q-learning Episode 9000, Total Reward: -148, Epsilon: 0.050
Q-learning Episode 10000, Total Reward: -147, Epsilon: 0.050
Q-learning Training finished! Time elapsed: 25.67 seconds

Training Double Q-learning...
Double Q-learning Episode 1000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 2000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 3000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 4000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 5000, Total Reward: -160, Epsilon: 0.100
Double Q-learning Episode 6000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 7000, Total Reward: -200, Epsilon: 0.100
Double Q-learning Episode 8000, Total Reward: -153, Epsilon: 0.100
Double Q-learning Episode 9000, Total Reward: -163, Epsilon: 0.100
Double Q-learning Episode 10000, Total Reward: -159, Epsilon: 0.100
Double Q-learning Training finished! Time elapsed: 29.44 seconds

Training DQN...
DQN Episode 100, Total Reward: -200, Epsilon: 0.606
DQN Episode 200, Total Reward: -200, Epsilon: 0.367
DQN Episode 300, Total Reward: -200, Epsilon: 0.222
DQN Episode 400, Total Reward: -200, Epsilon: 0.135
DQN Episode 500, Total Reward: -200, Epsilon: 0.082
DQN Episode 600, Total Reward: -200, Epsilon: 0.049
DQN Episode 700, Total Reward: -200, Epsilon: 0.030
DQN Episode 800, Total Reward: -166, Epsilon: 0.018
DQN Episode 900, Total Reward: -200, Epsilon: 0.011
DQN Episode 1000, Total Reward: -155, Epsilon: 0.010
DQN Training finished! Time elapsed: 284.04 seconds

Model Comparison Table:
Model              Avg Steps       Success Rate    Episodes        Train Time (s)
Q-learning         155.2           0.90            10000           25.7
Double Q-learning  143.6           1.00            10000           29.4
DQN                139.2           1.00            1000            284.0
```




```
Time-to-Goal Performance:
Model              Mean Steps      Std Dev         Success Rate
Q-learning         157.2           21.4            0.88
Double Q-learning  148.4           12.6            1.00
DQN                132.1           30.3            1.00
```


Steps to Reach Goal Distribution
Number of Steps
200
180
160
140
120
100
80
Max Steps
Q-learning
Double Q
DQN


Cumulative Successful Episodes
Success Count
Q-learning
Double Q
DQN
100
80
60
40
20
0
Episode